# Stock market crashes in 2007-2009: were we able to predict them? 

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#### Abstract

We investigate the stock market crashes in China, Iceland, and the US in the 2007-2009 period. The bond stock earnings yield difference model is used as a prediction tool. Historically, when the measure is too high, meaning that long bond interest rates are too high relative to the trailing earnings over price ratio, then there usually is a crash of $10 \%$ or more within four to twelve months. The model did in fact predict all three crashes. Iceland had a drop of fully $95 \%$, China fell by two thirds and the US by $57 \%$.


JEL: G14, G15, G12, G10

Keywords: stock market crash, bond-stock earnings yield model, FED model, China, Iceland and US stock markets

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## 1. Background

The second author started using the bond stock earnings yield model while in Tokyo consulting at the Yamaichi Research Institute in 1988. The model relates the yield on stocks (measured by the ratio of earnings to stock prices) to the yield on nominal Treasury bonds. The theory behind the model is that an optimal asset allocation between stocks and bonds is related to their relative yields and when the bond yield is too high, a market adjustment is needed and there is a shift out of stocks into bonds. If the adjustment is large, it causes an equity market correction (a decline of $10 \%$ within one year). Hence, there is a short term negative equity risk premium (ERP). A study of the October 1987 stock market crash illustrated the model and it was from this episode that the model was discovered in Ziemba's crash study group. Table 1 and Figure 1 show that the model went into the danger zone, that is the measure was above an upper confidence limit, in April 1987 with a spread of 3.39. The S\&P500 was then at 289.32. By the end of September the S\&P500 was at 318.66 with the measure higher at 4.14. After the crash at the end of October, the S\&P500 had fallen to 245.01 with the spread falling out of the danger zone to 1.64 . Table 1 has beginning of months values. Continuous daily values are in Figure 1 and one sees a sharp move up in the measure to 4.42 , in September 1987 way above the upper limit.

There are various ways that one can compute the upper and lower limits but experience has shown that with the various approaches, all of which use out of sample prior data, one usually has the same conclusion. In Figure 1, the limits are simply the trailing mean plus or minus a standard deviation

Figure 1: Bond and stock price earnings yield differential model for the S\&P500, 19801990, Source: Ziemba and Schwartz, 1991

measure so the one sided limits hold $95 \%$ of the probability. ${ }^{1}$

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## 2. Moving Average and Signal Chart

In the following sections we use a moving average and a rolling horizon standard deviation to establish the confidence levels. The $h$-day moving average at time $t$, denoted by $\mu_{t}^{h}$, and the corresponding rolling horizon standard deviation $\sigma_{t}^{h}$ are

$$
\begin{array}{r}
\mu_{t}^{h}=\frac{1}{h} \sum_{i=1}^{h} x_{t-i} \\
\sigma_{t}^{h}=\sqrt{\frac{1}{h} \sum_{i=1}^{h}\left(x_{t-i}-\mu_{t}^{h}\right)^{2}}
\end{array}
$$

Using rolling horizon means and standard deviations provide data consistency. This is used to compute confidence levels for the BSEYD measure. In particular, rolling horizon mean and standard deviation are not overly sensitive to the starting date of the bond yield or stock market data, or to the overall number of data points. However, the choice of the horizon parameter $h$ is subjective.

For our analysis of the US market, we use a five year horizon, so $h=1260$ as longer time horizons tend to generate a robust signal and eliminate false positives. Five years of historical bond and stocks data may be a difficult requirement outside of major markets. For Iceland and China, we use a one year rolling horizon, but tighten the confidence level in order to eliminate false positives.

October.

The figures show the signal calculated, respectively, on a standard onetail $95 \%$ normal distribution based confidence level and on an application of Cantelli's inequality. The conclusions are similar: over the period January 1, 1995 to April 6, 2011, two crash signals occurred: one in June 1999 and the other in June 2007 (as discussed in section 5).

The idea behind the BSEYD model is that a crash signal should occur whenever

$$
B S E Y D(t)>C L(t)
$$

where $C L(t)$ represents a one-tail confidence level. The level $C L$ acts as a time-varying threshold for the crash signal.

Equivalently, we define the signal directly as

$$
S I G N A L(t)=B S E Y D(t)-C L(t),
$$

So, a crash signal should occur whenever

$$
S I G N A L(t)=B S E Y D(t)-C L(t)>0
$$

Graphically, the threshold for the crash signal is now an horizontal line with value 0, as shown below in Figures 5(b), 6(b), 4 and 13. These graphs show a calculation of $S I G N A L(t)$ based respectively on a standard one-tail $95 \%$ confidence level and on an application of Cantelli's inequality.

An examination of the BSEYD spread distributions reveals their nonNormal nature (see Figures 3, 7, 11 and 19. As a result, standard confidence intervals which are based on a Gaussian assumption may prove inaccurate. In this case, we use Cantelli's inequality, a one-tailed version of Chebyshev's inequality, to derive a 'worst case' confidence level (see, for example, Problem 7.11.9 in Grimmett and Stirzaker, 2001).

Cantelli's inequality relates the probability that the distance between a random variable $X$ and its mean $\mu$ exceeds a number $k>0$ of standard deviations $\sigma$ to this distance

$$
P[X-\mu \geq k \sigma] \leq \frac{1}{1+k^{2}}
$$

or alternatively

$$
P\left[X-\mu \geq \sigma \sqrt{\frac{1}{\alpha}-1}\right] \leq \alpha
$$

where $\alpha=\frac{1}{1+k^{2}}$. The parameter $\alpha$ provides an upper bound for a one-tailed confidence level on any distribution, regardless of how different it is from a Normal distribution.

We use Cantelli's inequality with a one-tail confidence level to assess the relative strength of a signal. Using the one-tail confidence level, we obtain the crash signal. We then find the Cantelli probability $\alpha$ giving us a similar signal date. To generate the same signal as a standard $95 \%$ confidence level, we need to select $\alpha=27 \%$ in Cantelli's inequality. Similarly, to generate the same signal as a standard $99 \%$ confidence level, we need to select $\alpha=15.60 \%$

In the case of China, as discussed in section 3, we used a standard $99 \%$ confidence level to determine the signal. Based on Cantelli's inequality, we
expect in the worst case to have a crash signal $15.60 \%$ of the time. Retrospectively, Cantelli's inequality is rather severe since it places the threshold for a signal at a BSEYD spread of $3 \%$ or above, on 520 consecutive trading days from September 9, 2007 to February 2, 2009 out of the 2099 days (i.e. $24.77 \%$ of days) where this measure was computed. By contrast, if we consider the distribution over the entire period, we observe that the spread only exceeded $2.52 \%$ on $0.03 \%$ of all instances. To conclude, a standard $99 \%$ confidence level would have been sufficient to determine a clear crash signal.

In the case of Iceland, as discussed in section 4, we use a $95 \%$ confidence level to determine the signal, Cantelli's inequality suggests a worst case probability that $27 \%$ of observations could result in a signal. However, lowering the Cantelli probability $\alpha$ from $27 \%$ to $20 \%$ (corresponding roughly to a standard $97.7 \%$ confidence level) does not result in significant loss of responsiveness of the signal, at least for Glitnir and Kaupthing (see Figures 13b,d).

Ziemba (2011) gives an historical account of his use of this model from 1988 to 2011 in the US, Japan and other countries. Twelve times the signal was in the danger zone from 1948 to 1988 and each time the Nikkei Stock Average index fell at least $10 \%$ from the level when the signal was reached. During this forty year period, the index had twenty such falls of which eight occurred for reasons other than high interest rates relative to earnings. The BSEYD model also predicted the $-56 \%$ crash that started in January 1990 with a signal further in the danger zone than at any time since 1948.

The Fed model, which is the ratio not the difference like the original model, is mathematically equivalent to this model, as now shown.

The Fed model in its original 1996 form states the dependence of a fair stock price level $\tilde{p}(t)$ at time $t$ to the expected earnings, $E(t)$, and the most liquid (10- or 30-year) Treasury bond rate $r(t)$. Earnings expectations are incorporated in prices and discounted via

$$
\begin{equation*}
\tilde{p}(t)=\frac{E(t)}{r(t)}, t=1,2, \ldots \tag{1}
\end{equation*}
$$

Equity earnings per share $\gamma(t)$ is the expected earnings for a unit investment in the stock market with equity shares, $S(t)$, namely

$$
\gamma(t)=\frac{E(t)}{S(t)}
$$

Then

$$
S(t)=\frac{E(t)}{\gamma(t)}
$$

There is a direct relationship between the equity yield in Equation (2) and the long bond rate in Equation (1). The ratio of the current market value to the theoretical value is the Fed model bond stock yield ratio BSYR(t)

$$
\begin{equation*}
\frac{S(t)}{\tilde{p}(t)}=\frac{E(t) / \gamma(t)}{E(t) / r(t)}=\frac{r(t)}{\gamma(t)}=B S Y R(t) \tag{2}
\end{equation*}
$$

The bond stock earnings yield differential that we focus on in this paper is related to the valuation measure and the equity yield via

$$
\begin{gathered}
\frac{S(t)}{\tilde{p}(t)}-1=\frac{B S E Y D(t)}{\gamma(t)} \\
B S E Y D(t)=[B S Y R(t)-1] \gamma(t) .
\end{gathered}
$$

The differential reflects the difference between the current market value and its theoretical value. A more theoretically sound motivation for the predictive ability of the BSEYD is using the basic Gordon formula, where EP is the forward earnings yield (which Schwartz and Ziemba (2000) show is the best predictor of at least individual Japanese stock prices),
$\mathrm{E} / \mathrm{P}-$ nominal yield $=$ equity risk premium - real growth - inflation.
So the BSEYD can be used as a proxy for the unobservable right hand side economic variables.

For given equity yield the BSEYD and the BSYR can be used to identify zones of under and over valuation and forecast possible forthcoming market adjustments.

Koivu, Pennanen and Ziemba (2005) study the Fed model using a dynamic vector equilibrium correction model with data from 1980 to 2003 in the US, UK and Germany and show that the Fed model had predictive power in forecasting equity prices, earnings and bond yields. The model has been successful in predicting market turns, but in spite of its empirical success and simplicity, the model has been justifiabily criticized. First it does not consider the role played by time varying risk premiums in the portfolio selection process while it does consider a risk free government interest rate as the discount factor of future earnings. More seriously, the inflation illusion (the possible impact of inflation expectations on the stock market) as suggested by Modigliani and Cohn (1979) is not taken into consideration. Secondly, the model assumes the comparability of earning price ratios, a real quantity, with a nominal, bond induced, interest rate [Campbell and Vuolteenaho (2004), Asness (2000, 2003), and Ritter and Warr (2002) discuss these issues.]

Consigli, MacLean, Zhao and Ziemba (2009) propose a stochastic model of equity returns based on an extension of the model inclusive of a risk premium in which market corrections are endogenously generated by the bond-stock yield difference. The model accommodates both cases of prolonged yield deviations leading to a long series of small declines in the equity market and the case, peculiar of recent speculative bubbles, of a series of corrections over limited time periods. The inclusion of the yield differential as a key driver of the market correction process is tested and the model is validated with market data

Many of the critics focus on: 1) short term predicability that we know is weak as does Giot and Petitjean (2008), 2) simply do not focus on the long run value of the measure, or 3) dismiss it outright because of the nominal versus real minor flaw as does Montier (2011). Consigli, MacLean, Zhao and Ziemba (2009) use the model to estimate the current fair value of the S\&P500. Of course, market and fair value can diverge for long periods. However, our concern is whether or not the model actually predicts stock market crashes, stock market rallies and good times to be in and out of stock markets. Berge, Consigli and Ziemba (2008) discuss the latter issue and found for five countries (US, Germany, Canada, UK and Japan) that the strategy stay in the market when it is not in the danger zone and move to cash otherwise provides about double the final wealth with less variance and a higher Sharpe ratio than a buy and hold strategy during 1975-2005 and 1980-2005. There is some limited predictability of stock market increases but the evidence supports the good use of the model to predict crashes. In this paper we study the period 2007-2009 for the US, China and Iceland,
all of which had large crashes. Shiller (2006) observes, as we have in the past, that low PE periods seem to lead to higher future stock prices and high PE periods to lower future prices. But the evidence is that PE levels by themselves are not enough to call the crashes. The argument here is that it is usually the interplay of interest rates measured by the long bond with the PE ratios that gives the crash signal.

## 3. The Chinese Shanghai Stock Market Crash

Figure 2 shows the rise of the Shanghai stock index from January 2000 to May 2011. The market bottomed at 1011.50 on July 11, 2005. It then rose six-fold to peak at 6092.06 on October 16, 2007. Then there was a crash of $11.98 \%$ from 5180.51 to 4559.75 over the two day period January 21 and 22 followed by another $7.19 \%$ fall from 4761.69 to 4419.29 on January 28, 2008. Ultimately the index fell to 1706.70 on November 4, 2008, a decline of over two thirds from the peak and $23.09 \%$ and $29.93 \%$ from the December 12 and 25, 2006 BSEYD danger signals at 2218.95 and 2435.76. Table 2 discusses highlights for the Shanghai index from 2005-11.

Did the BSEYD model predict this crash? First, Figure 3 shows that the BSEYD measure is not normally distributed with fat right and even fatter left tails. The biggest declines are much larger than the biggest increases.

Figures 4(a) and 4(b) show that the model did in fact predict the crash. See also the signals hitting the danger level in Figures 5 and 6. It is a typical application of the model. The signal goes into the danger zone, then the market continues higher but within four to twelve months there is a crash of $10 \%+$ from the value at the initial signal. In this case, the decline is


Figure 2: The Shanghai stock exchange composite index, January 2000 to May 2011
much higher than $10 \%$. Figure 4(a) uses a $95 \%$ confidence one sided moving average interval using prior data out of sample. The danger signal occurred on December 12, 2006, some ten months before the stock market peak on October 16, 2007 with the index at 6092.06. Figure 4(b) uses a $95 \%$ one sided confidence interval and gives the first danger signal on December 12, 2006 with the index at 2218.95 . With a $99 \%$ one sided confidence interval, the danger signal was reached on December 25, 2006 with the index at 2435.76. The ultimate fall was to 1706.70 on November 4, 2008 about $30 \%$ below the index value of 2218.76 or 2435.76 of the crash signal. This signal and decline were a bit different than the usual case as it took almost two years to get the $10 \%$ plus crash and in the meantime the market almost tripled in value before the ultimate crash.

Figure 4 also illustrates the importance of the confidence level in relation to the rolling horizon of the moving average and the shape of the spread distribution. As pointed out in Section 2, a short time horizon of one year, combined with a lower confidence level of $95 \%$ and the non-Gaussian nature

Figure 3: Spread distribution of the BSEYD measure on the Shanghai Stock Exchange Composite

of the spread distribution may result in false positives. An ex-post analysis reveals that the Gaussian-based $95 \%$ confidence spread for the Shanghai Stock Exchange Composite index over the entire period equals $1.78 \%$. However, the actual $95 \%$ confidence level of the empirical distribution is $2.18 \%$. In fact, a full $9.32 \%$ of all actual observations occur at or above $1.78 \%$, a marked contrast from the $5 \%$ predicted by the Gaussian distribution. Raising the confidence level to $99 \%$ or increasing the rolling horizon does help reduce the impact of the shape of the distribution on the signal.

(a) First signal occurs on December 12, 2006. The market reaches its peak on October 16, 2007 and ultimate low on November 4, 2008 based on $95 \%$ confidence.

(b) First signal occurs December 25, 2006.

The market reaches its peak on October 16, 2007 and ultimate low on November 4, 2008 based on $99 \%$ confidence.

Figure 4: BSEYD danger signals for the Shanghai Stock Exchange Composite, $95 \%$ and $99 \%$ confidence

(a) Signal occurs on December 12, 2006 based on $95 \%$ confidence

(b) Signal occurs on December 25, 2006 based on $99 \%$ confidence

Figure 5: BSEYD danger signals for the Shanghai Stock Exchange Composite, Signals at $95 \%$ and $99 \%$ confidence

(a) First signal occurs on December 25, 2006. The market reaches its peak on October 16, 2007.

(b) Signal occurs on December 25, 2006
(based on Cantelli's inequality, $\mathrm{a}=15 \%$ )
Figure 6: BSEYD Cantelli danger signals for the Shanghai Stock Exchange Composite, Chart,

Figure 7: Spread distribution of the BSEYD measure on the Shanghai Stock Exchange Composite using Cantelli inequality.


Table 1: S\&P500 index, PE ratios, government bond yields and the yield premium over stocks, January 1984 to August 1988. Source: Ziemba and Schwartz (1991)

|  |  | S\&P |  | (a) | (b) |  |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
|  |  | Index | PER | 30 Yr G bd | 1/pe,\% | (a)-(b) |
| 1986 | Jan | 208.19 | 14.63 | 9.32 | 6.84 | 2.48 |
|  | Feb | 219.37 | 15.67 | 8.28 | 6.38 | 1.90 |
|  | Mar | 232.33 | 16.50 | 7.59 | 6.06 | 1.53 |
|  | Apr | 237.98 | 16.27 | 7.58 | 6.15 | 1.43 |
|  | May | 238.46 | 17.03 | 7.76 | 5.87 | 1.89 |
|  | Jun | 245.30 | 17.32 | 7.27 | 5.77 | 1.50 |
|  | Jul | 240.18 | 16.31 | 7.42 | 6.13 | 1.29 |
|  | Aug | 245.00 | 17.47 | 7.26 | 5.72 | 1.54 |
|  | Sep | 238.27 | 15.98 | 7.64 | 6.26 | 1.38 |
|  | Oct | 237.36 | 16.85 | 7.61 | 5.93 | 1.68 |
|  | Nov | 245.09 | 16.99 | 7.40 | 5.89 | 1.51 |
|  | Dec | 248.60 | 16.72 | 7.33 | 5.98 | 1.35 |
| 1987 | Jan | 264.51 | 15.42 | 7.47 | 6.49 | 0.98 |
|  | Feb | 280.93 | 15.98 | 7.46 | 6.26 | 1.20 |
|  | Mar | 292.47 | 16.41 | 7.65 | 6.09 | 1.56 |
|  | Apr | 289.32 | 16.22 | 9.56 | 6.17 | 3.39 |
|  | May | 289.12 | 16.32 | 8.63 | 6.13 | 2.50 |
|  | Jun | 301.38 | 17.10 | 8.40 | 5.85 | 2.55 |
|  | Jul | 310.09 | 17.92 | 8.89 | 5.58 | 3.31 |
|  | Aug | 329.36 | 18.55 | 9.17 | 5.39 | 3.78 |
|  | Sep | 318.66 | 18.10 | 9.66 | 5.52 | 4.14 |
|  | Oct | 280.16 | 14.16 | 9.03 | 7.06 | 1.97 |
|  | Nov | 245.01 | 13.78 | 8.90 | 7.26 | 1.64 |
|  | Dec | 240.96 | 13.55 | 9.10 | 7.38 | 1.72 |
| 1988 | Jan | 250.48 | 12.81 | 8.40 | 7.81 | 0.59 |
|  | Feb | 258.10 | 13.02 | 8.33 | 7.68 | 0.65 |
|  | Mar | 265.74 | 13.42 | 8.74 | 7.45 | 1.29 |
|  | Apr | 262.61 | 13.2418 | $8 \quad 9.10$ | 7.55 | 1.55 |
|  | May | 256.20 | 12.92 | 9.24 | 7.74 | 1.50 |
|  | Jun | 270.68 | 13.65 | 8.85 | 7.33 | 1.52 |
|  | Jul | 269.44 | 13.59 | 9.18 | 7.36 | 1.82 |
|  | Aug | 263.73 | 13.30 | 9.30 | 7.52 | 1.78 |

Table 2: Highlights on the Shanghai Stock Index, 2005-2011

| Date | Index Value | Comment |
| :---: | :---: | :---: |
| Dec 31, 2004 | 1266.50 |  |
| May 23, 2005 | 1070.84 | The index was down $15.4 \%$ year-to-date |
| July 11, 2005 | 1011.50 | Market bottom |
| Dec 31, 2005 | 1161.06 | The index was down 8.3\% in 2005 |
| Dec 12, 2006 | 2218.95 | 95\% confidence BSEYD crash signal occurs |
| Dec 25, 2006 | 2435.76 | 99\% confidence BSEYD crash signal occurs |
| Dec 31, 2006 | 2675.47 | The index was up $130.4 \%$ in 2006 |
| May 29, 2007 | 4334.92 | Local high of the market. The market was up $62 \%$ year-to-date |
| July 5, 2007 | 3615.87 | Local low of the market. The market was still up $35.1 \%$ year-to-date |
| Oct 16, 2007 | 6092.06 | Highest historical market close. The market was up 127.7\% year-to-date |
| Nov 28, 2007 | 4803.39 | Local low of the market. The market was still up $79.5 \%$ year-to-date |
| Dec 31, 2007 | 5261.56 | Index was up 96.7\% in 2007, but down $13.6 \%$ from its peak in October |
| Jan 14, 2008 | 5497.90 | Local high of the market |
| Jan 18, 2008 | 5180.51 | The index closes the week at 5180.51, which is $5.8 \%$ lower than than its local high on Jan 14 |
| Jan 21, 2008 | 4914.43 | The index experiences a one-day drop of $5.1 \%$ from 5180.51 to 4914.43 |
| Jan 22, 2008 | 4559.75 | The index experiences declines by $7.2 \%$ on this day, opening at 4914.43 to close at 4559.75 |
| Jan 25, 2008 | 4761.69 | The index recovers slightly to close the week at 4761.69 |
| Jan 28, 2008 | 4419.29 | The index drops by $7.2 \%$ from 4761.69 to 4419.29 |
| Apr 18, 2008 | 3094.67 | Local low of the market |
| May 5, 2008 | 3761.01 | Local high of the market |
| Nov 4, 2008 | 1706.70 | Global market minimum. The market was down by $72 \%$ peak to trough and $23.09 \%$ and $29.93 \%$ from the December 12 and 25 danger signal levels |
| Dec 31, 2008 | 1820.81 | The market was down $65.4 \%$ in 2008 |
| Aug 3, 2009 | 3471.44 | Local high of the market. The market was up $103.4 \%$ from the trough |
| Aug 31, 2009 | 2667.75 | Local low of the market |
| Nov 23, 2009 | 3338.66 | Local high of the market |
| Dec 31, 2009 | 3277.14 | The market was up $80 \%$ in 2009 |
| Jul 5, 20100 | 2363.95 | Local low of the market |
| Dec 31, 2010 | 2808.08 | The market was down $14.3 \%$ in 2009 |
| Jun 30, 2011 | 2762.08 | The market was down about $1 \%$ year-to-date |
| August 31, 2011 | 2567.34 | The market was down about $8.6 \%$ year-to-date |

## 4. The Iceland stock market crash

Iceland is a small country with only about 300,000 people. From 2002 to 2007, the economy and asset prices rose dramatically with much leveraging of investments, especially by the banks. This led to high interest rates of about $10 \%$ long term and $16 \%$ short term. Eventually, it all collapsed in the wake of the 2007-2009 worldwide financial crisis. And the decline was a massive crash of $-95 \%$ in the equity index and a currency collapse. The equity index, see Table 3, had 15 stocks in it with three of the banks having very high weighting: Kaupthing (26.5\%), Lansbanki (13.0\%) and Glitnir (12.3\%) were more than half the market capitalization and Actavis Banki had 9.9\%, and FL Banki another $6.7 \%$. So the banks were close to two thirds of the index value. And index funds that tracked the market actually slightly over weight these banks to yield higher returns.

Figure 8, shows the dramatic rise of the stock market particularly since 2003 and, similarly, Figure 9 shows how quickly the crash occurred. However, the notable sharp sell-offs, were, to a large extent, blips before the big crash and there was a question whether these investments could continue to produce similar returns, and if not, whether that would prompt investors to seek other markets.

Figure 10 shows that in the fall of 2007, the long bond interest rates did get above $10 \%$ and that with an increase in the PE ratio respectively to 10.91 for Glitnir as of October 10, 11.09 for Kaupthing as of October 11 and 9.94 for Lansbanki as of October 17 the BSEYD spread signal did predict the massive crash. The market peaked at 8174.28 on July 18, 2007 starting in 2002 at 1180.75 . Then it fell to 5803.55 by the end of December 2007. The
complete collapse came in 2008 with the market falling about $90 \%$ to end 2008 at 581.76 and about -95\% at the ultimate bottom on February 2, 1010 at 491.58. As of August 31, 2011 the market had fallen to 596.58. Table 4 summarizes the year by year story and Figure 9 shows the index values from 1998 to August 31, 2011.

Figure 11 shows the BSEYD spreads for the three top banks and Figures 12 and 13 show the bond-stock earnings yield crash measure from June 2004 to December 2008 for the three largest banks. Like China, the BSEYD distributions of these three largest banks are not normally distributed and have very fat left tails.

Finally, the question of whether or not the bond-stock earnings yield model predicted the crash is studied in Figures 12ace which use $95 \%$ one sided confidence intervals using moving averages. These graphs show that the crash was predicted. For Kaupthing, the danger zone was penetrated on September 28, 2007, two months after the July 18 peak and less than a month before the November 11 crash. For Glitnir, the signal was much earlier on October 17, 2006, some thirteen months before the crash. Finally, for Lansbanki, the danger signal was on February 13, 2007. Figures 12bdf show the BSEYM using Cantelli's inequality to account for the non-normality of the BSEYD measure. Figure 13 shows the signal dates. We focus on the largest banks because they led the market into the collapse and they are a majority of the index weighting. The smallest cap stocks in the index were in the danger zone in 2006 but not the large banks and the overall index was not in the danger zone then as discussed in the book Ziemba and Ziemba (2007).

Table 3: Stock market index. Source: Glitnir (2006)

| Weights |  |  | Real rates of return |  |  |
| :---: | :---: | :---: | :---: | :---: | :---: |
| Company | Fund | Index |  | Fund | Index |
| 1 Kaupthing Banki hf. | 27.2\% | 26.5\% | 2005 | 56.5\% | 54.5\% |
| 2 Landsbanki Islands hf. | 13.1\% | 13.0\% | 2004 | 49.1\% | 47.2\% |
| 3 Glitnir Banki hf. | 12.5\% | 12.3\% | 2003 | 42.7\% | 40.7\% |
| 4 Straumur Buróarás Fjárfes | 8.9\% | 8.9\% | 2002 | 20.7\% | 19.2\% |
| 5 Actavis Banki hf. | 10.1\% | 9.9\% | 2001 | -16.4\% | -16.6\% |
| 6 FL Banki hf. | 6.3\% | 6.2\% | 2000 | -16.7\% | -17.2\% |
| 7 Bakkavör Group hf. | 4.0\% | 4.1\% | 1999 | 36.7\% | 36.4\% |
| 8 Avion Group hf. | 3.7\% | 3.7\% | 1998 | 6.3\% | 3.4\% |
| 9 Mosaic Fashions hf. | 2.7\% | 2.6\% | 1997 | 9.0\% | 10.2\% |
| 10 Ossur hf. | 2.2\% | 2.2\% | 1996 | 44.1\% | 57.3\% |
| 11 Tryggingamiətöðin hf. | 2.1\% | 2.0\% | 1995 | 33.4\% | 31.1\% |
| 12 Dagsbrúnhf. | 1.5\% | 1.5\% |  |  |  |
| 13 Alfresca hf. | 1.2\% | 1.2\% |  |  |  |
| 14 Fjárfestingafélagỉ Atorka | 1.0\% | 1.0\% |  |  |  |
| 15 Grandi hf. | 1.0\% | 1.0\% |  |  |  |



Figure 8: The 15 stocks in the Iceland equity index and their growth in real terms from 1997-2006. Source: Glitnir (2006).


Figure 9: OMX Iceland all share - price index, 1993 to November 2010


Figure 10: Iceland Treasury note (long bond) maturing May 17, 2013

Table 4: Highlights for the Iceland stock index ICEXI-OMZ 1998-2011 with start value $=$ 1000 on December 31, 1997

| Date | Index Value | Comment |
| :---: | :---: | :---: |
| Feb 11, 1998 | 977.58 |  |
| Dec 30, 1998 | 1046.58 |  |
| Dec 30, 1999 | 1511.86 | Gain of 44.5\% in 1999 |
| Dec 28, 2000 | 1303.31 | \} |
| Dec 28, 2001 | 1180.75 | \} Weak market during US stock market weak period |
| Dec 30, 2002 | 1436.22 | \} |
| Dec 29, 2003 | 2064.05 | Gain of $43.7 \%$ in 2003 |
| Dec 30, 2004 | 3173.91 | Gain of $53.8 \%$ in 2004 |
| Dec 30, 2005 | 5107.49 | Gain of $60.9 \%$ in 2005 |
| Feb 2,2006 | 6287.29 | Local high of market |
| Aug 2, 2006 | 4854.95 | Local low of market |
| Dec 30, 2006 | 5857.50 | Gain of $14.7 \%$ in 2006 |
| July 18, 2007 | 8174.28 | Global highest historical market close |
| Aug 16, 2007 | 6931.69 | Market falls to local lows on day of US stock market turmoil when long-short funds had heavy losses |
| Oct 11, 2007 | 7796.36 | Local market peak then complete market collapse |
| Dec 28, 2007 | 5803.35 | Market down just slightly ( $-0.9 \%$ ) in 2007 but $29 \%$ below the July 18, 2007 peak |
| Dec 30, 2008 | 581.76 | Complete depression including a collapse of over $90 \%$ from the peak |
| Dec 30, 2008 | 496.48 | Further fall in 2009 |
| Feb 2, 2010 | 491.58 | Global historical bottom of market |
| Nov 30, 2010 | 579.17 | The market rallied back to make the yearly return nearly positive |
| Dec 30, 2010 | 569.19 | Gain of $15.8 \%$ since global historical bottom but the index was still $93.0 \%$ below its historical high |
| Jan 31, 2011 | 624.97 | Gain of 9.8\% in January 2011 |
| Jun 30, 2011 | 604.95 | Gain of $23.0 \%$ since the global historical bottom but the index was still $92.5 \%$ below its historical high |
| August 31, 2011 | 596.58 | Gain of $21.4 \%$ since the global historical bottom but the index was still $92.7 \%$ below its historical high |



Figure 11: BSEYM Spread Distributions, Iceland

Figure 12: BSEYD Crash Indicators, Iceland

(a) BSEYD Chart (Indicator at $95 \%$ confidence): Glitnir. First crash signal occurs around October 17, 2006. The market reaches its peak on July, 20, 2007

(b) BSEYD Chart (Indicator using Cantelli's Inequality, 20\%): Glitnir. First crash signal occurs on October 17, 2006. The market reaches its peak on July, 20, 2007

(c) BSEYD Chart (Indicator at $95 \%$ confidence): Kaupthing. First signal occurs around October 2, 2007. The market reaches its peak on July 18, 2007 and a crash occurred on November 11, 2007.

(d) BSEYD Chart (Indicator using Cantelli's Inequality, 20\%): Kaupthing. First crash signal occurs on October 2, 2007. The market reaches its peak on July 18, 2007 and a crash occurred on November 11, 2007.

(e) BSEYD Chart (Indicator at 95\% confidence): Lansbanki. First crash signal occurs around February 13, 2007. The market reaches its peak on October 17, 2007.

(f) BSEYD Chart (Indicator using Cantelli's Inequality, 20\%): Lansbanki. Faint crash signal throughout the first half of 2007. The market reaches its peak on October 17, 2007.

Figure 13: Crash Indicators, Iceland

(a) Crash Signal Indicator (based on $95 \%$ confidence): Glitnir. First crash signal occurs around October 17, 2006. The market reaches its peak on July, 202007

(b) Signal Chart (Indicator using Cantelli's Inequality, 20\%): Glitnir. First crash signal occurs on October 17, 2006. The market reaches its peak on July, 202007

(c) Signal Chart ( $95 \%$ confidence): Kaupthing. First crash signal occurs around October 2, 2007. The market reaches its peak on July 18, 2007 and a crash occurred on November 11, 2007

(d) Signal Chart (Indicator using Cantelli's Inequality, 20\%):

Kaupthing. First crash signal occurs on October 2, 2007. The market reaches its peak on July 18, 2007 and a crash occurred on November 11, 2007.

(e) Signal Chart ( $95 \%$ confidence): Lansbanki. First crash signal occurs around February 13, 2007. The market reaches its peak on October 17, 2007

(f) Signal Chart (Indicator using Cantelli's Inequality, 20\%): Lansbanki.

Faint crash signal throughout the first half of 2007. The market reaches its peak on October 17, 2007.

Comments on the crash signals for the three banks
Glitnir:

- Signal from September 15, 2003 until December 23, 2003: on October 12, 2004, the share price reached 11.90. By November 2, 2004, the share had gone down to 10 , a $15.97 \%$ drop.
- Faint signal from November 17, 2004 until November 19, 2004 and December 6, 2004: on February 18, 2005, the share price fell to 11.60. By March 3, 2005, the share had gone down to 11.60, a $7.20 \%$ drop.
- Signal from November 22, 2005 until February 24, 2006 (with some interruptions): on October 12, 2004, the share price reached 22.60. By April 19, 2006, the share had gone down to 16.50 , a $26.99 \%$ drop.
- Signal from October 17, 2006 (with PE ratio of 22.30) until December 27, 2006: announces the market crash.
- Signal from December 28, 2007 until January 8, 2008.
- Signal from March 26, 2008 until April 10, 2008.

The crash signal analysis is basically according to the script except for the faint signal between November 17, 2004 until November 19, 2004 and December 6, 2004 for which we do not have a better explanation.

Kaupthing:

- Signal from February 7, 2006 until February 24, 2006: on October 26, 2006, the share price reached 868. By November 28, 2006, the share had fallen to 785 , a $9.56 \%$ drop.
- The market reaches its peak on July 18, 2007 and a crash occurred on November 11, 2007.
- Signal from September 28, 2007 until November 5, 2007.
- Signal from March 26, 2008 until April 4, 2008.

Lansbanki:

- Signal from January 15, 2004 until March 4, 2004: on October 11, 2004, the share price reached 15.28 . By November 2, 2004, the share has fallen to 10.85 , a $28.98 \%$ drop.
- Signal from August 30, 2005 until December 29, 2005 (with minor interruptions): on February 16, 2006, the share price reached 30.56 . By May 3, 2006, the share has fallen to 20.05 , a $34.39 \%$ drop.
- Signal from February 13, 2007 until March 13, 2007 and from March 30, 2007 until the June 28, 2007: market crash signal.
- The market reaches its peak on October 17, 2007.

The crash model works out well for Lansbanki: the signal identifies the market crashes and two large declines. Although the signal could be clearer for Glitnir and Kaupthing, we do not observe any false positives.

## 5. The US 2007-2009 crash

In this section we investigate whether or not the bond-stock measure did predict the US 2007-2009 crash, including the devastating September 2008
to March 2009 period. Table 6 shows the BSEYM at various periods from 2006 to 2011. In 2006 the weak economy led the Fed to dramatically reduce short term interest rates which tended to drop the long term rates that we use in the BSEYM calculations.

Table 5 considers the measure in five major countries on July 12, 2006. None of these major markets were in the danger zone then. Table 6 for the S\&P500 has the calculations for the years from 2006 to 2011 on trailing priceearnings ratios that are usually used for these calculations and 2011 which uses Shiller's average inflation-adjusted earnings from the previous ten years. Figures 16ab show the S\&P500 PE ratios (Shiller's calculation method) and the ten-year Treasury bond yield.

Ex post it is clear that this stock market crash had a lot of components such as the first decline in aggregate US housing prices in more than thirty years, a subprime market collapsing because home buyers could not cover their mortgages, lots of suspect AAA rated packages of these mortgages and then a credit squeeze with much counter party risk with firms unwilling to lend money to others including supposedly sound financial institutions and the collapse of many large and previously sound financial institutions such as Bear Stearns, Freddie Mac, Fannie May and, the killer for the market, Lehman Brothers.

Table 5: Long-bond (10 yr) versus earning yield differentials for major countries, July 12, 2006. Source: Ziemba and Ziemba (2007).

|  | S\&P500 | FTSE200 | Nikkei225 | CAC40 | DAX30 |
| :--- | ---: | ---: | ---: | ---: | ---: |
| Index | 1259 | 5861 | 15249 | 4942 | 5638 |
| A) PE ratio | 16.86 | 16.61 | 36.26 | 13.82 | 13.33 |
| B) Stock Return (1/A) | $5.93 \%$ | $6.02 \%$ | $2.76 \%$ | $7.24 \%$ | $7.50 \%$ |
| C) Bond Return (10 yr) | $5.10 \%$ | $4.67 \%$ | $1.94 \%$ | $4.10 \%$ | $4.09 \%$ |
| D) Crash Signal (C-B) | -0.83 | -1.35 | -0.82 | -3.14 | -3.41 |

Table 6: Bond-stock yield model calculations leading up to the 2007-2009 crisis in the S\&P500.

Long Bond Trailing

| Date | (10 years) | PE | $1 / \mathrm{PE}, \%$ | B-Y(pe) |
| :--- | ---: | ---: | ---: | ---: |
| Feb-06 | 4.49 | 20.00 | 5.00 | $(0.56)$ |
| Jun-07 | 5.15 | 17.00 | 5.98 | $(0.74)$ |
| Jun-08 | 4.14 | 18.00 | 5.55 | $(1.41)$ |
| May-09 | 3.70 | 33.30 | 3.00 | 0.70 |
| May-10 | 3.41 | 20.47 | 4.89 | $(1.48)$ |
| Feb-11 | 3.59 | 23.83 | 4.20 | $(0.61)$ |

Table 7 lists some of the main events regarding the S\&P500 from 2006 to 2011. There are numerous books concerning this period plus many articles and columns. Ziemba has several in Wilmott. Starting in June 2007, he designed strategies and traded for an offshore BVI based hedge fund for a group headed by a top trader Nikolai Battoo. Battoo and his hedge funds had investments in Bear Stearns and in June 2007 asked for his money back. That took three months and gave him a strong signal of danger. As an astute trader, he hedged and studied carefully the market situation through technical indicators that he has developed. Ziemba remembers his words to him starting in the summer of 2007 "this is the big one" ..."eventually the market will go to 660 on the S\&P500". In the fall of 2007 the S\&P500 was about 1500, see Figure 14. So this was a rather bold call but a private one and it turned out to be very accurate. Nouriel Roubini was predicting very boldly a serious financial meltdown starting in 2006 when the housing market was beginning its decline; see Figure 15 which gives the Case Shiller Home Price Index as of July 24, 2008. There was a sharp decline from 2005 to 2008. He and other bears such as Yale Professor Robert Shiller are still (September 2011) pessimistic about the economy, real estate and financial markets. Dropping real estate has several depressive effects such as homeowners can no longer use house price gains to fund consumption, foreclosures, etc. The March 2009 low closing was 676.53 with an intraday low of 660 on March 6. The subsequent rally has doubled the S\&P500 to 1320.64 as of the end of June 2011. There is considerable discussion regarding whether or not this rally is low interest rate related to the Fed quantitative easing, or only game in town since real estate, bonds and cash look unattractive. This
is a case when the BSEYM signaled the rise in stock prices.


Figure 14: The S\&P500, January 1, 2006 to April 30, 2011

Table 7: Highlights of the S\&P500, January 1, 2006 to April 30, 2011

| Date | Index Value | Comment |
| :---: | :---: | :---: |
| December 31, 2005 | 1248.29 |  |
| August 31, 2006 | 1303.82 | Gain of 4.4\% year-to-date |
| December 31, 2006 | 1418.30 | Gain of $13.6 \%$ in 2006 |
| February 26, 2007 | 1437.50 | Local high of market |
| March 1, 2007 | 1374.12 | Local low of market |
| June 7, 2007 | 1480.72 | Bear Stearns suspends redemptions from one of its hedge funds |
| June 14, 2007 | 1522.97 | BSEYD crash signal occurs |
| July 13, 2007 | 1552.50 | Local high of market |
| July 31, 2007 | 1455.27 | Bear Stearns liquidates two hedge funds |
| August 16, 2007 | 1411.27 | Local low of market |
| October 9, 2007 | 1565.15 | Market peak. Gain of $10.4 \%$ year-to-date |
| December 31, 2007 | 1468.35 | Gain of $3.5 \%$ in 2007 |
| March 17, 2008 | 1276.60 | Local low of market |
| September 15, 2008 | 1192.70 | Lehman Brothers files for bankruptcy protection |
| September 30, 2008 | 1166.36 | Market down by $20.6 \%$ year-to-date |
| October 10, 2008 | 899.22 | Local low of market |
| October 31, 2008 | 968.75 | Market down by $16.9 \%$ in October |
| December 31, 2008 | 903.25 | Market down by $38.5 \%$ in 2008 |
| March 9, 2009 | 683.38 | Lowest intraday: 666.79. |
| March 9, 2009 | 676.53 | Market trough. The market was down by $56.8 \%$ peak to trough |
| July 28, 2009 | 979.62 | Date Shepherd claimed that the S\&P500 had a 723 PE ratio based on reported (real earnings) to the SEC on 10Q forms |
| December 31, 2009 | 1115.10 | Market was down by $23.5 \%$ in 2009, but up by $64.8 \%$ since trough |
| April 23, 2010 | 1217.28 | Local high |
| July 2, 2010 | 1022.58 | Local low |
| December 31, 2010 | 1257.64 | Gain of $12.8 \%$ in 2010 , and of $23 \%$ since the local low of July 2, 2010 |
| May 10, 2011 | 1357.16 | Local high. Gain of $7.91 \%$ year-to-date and of $32.72 \%$ since the local low of July 2, 2010 |
| August 31, 2011 | 1218.89 | Loss of $3.1 \%$ year-to-date and of $10.2 \%$ since the local high of May 10, 2010 |



Figure 15: Case Shiller Index as of July 29, 2008.


Figure 16: S\&P500 and ten-year Treasury bond yields. Source: Robert Shiller data.

Did the BSEYD model predict the US crash? Figures 17 and 18 show that it did on June 14, 2007. As with China and Iceland, the BSEYD measure is not normally distributed but rather has fat tails especially on the downside as shown in Figure 19. The 95\% confidence graphs with the crash danger signal is shown in Figure 17(a). The Cantelli's inequality version of the model, Figure 17(b), gives the danger signal on the same day, namely, June 14, 2007, see also Figures 18ab.

(a) Crash Indicator (95\% confidence: (b) Crash Indicator (Cantelli's Inequality): S\&P500 MA. Signal occurs on June 14, S\&P500, MACI. Signal occurs on June 14, 2007. The market reaches its peak on Octo- 2007. The market reaches its peak on October 9, 2007 and crashes in September 2008) ber 9, 2007 and crashes in September 2008

Figure 17: Crash Indicators, US

Lets go back to the BSEYM and consider Table 8 which was published in the Maudlin weekly newsletter which has 1.5 million subscribers in May 2009, and discuss whether or not it called the September 2008 to March 2009 crash. Figure 20 shows the S\&P500 during this 2007-2009 period. Table 8 has the S\&P500 2008 estimated earnings and 2009 forecasted earnings. On July 25, 2008, the S\&P500 earnings for 2008 were estimated to be 72.00

(a) Based on a standard 1-tail $95 \%$ confi- (b) Based on a standard 1-tail $95 \%$ confidence level, moving average normal distri- dence level, moving average fat tail distribution assumption bution Cantelli's inequality assumption

Figure 18: Signal occurs on June 14, 2007

Figure 19: Spread distribution of the BSEYD measure on the S\&P500.

with the S\&P500 at 1257.76 which gives a PE ratio of 17.47 which is not high enough to signal the September 2008 to March 2009 crash. But by February 20, 2009, the 2008 earnings were estimated to be only 26.23 . With the $\mathrm{S} \& \mathrm{P} 500$ at 770.05 on that day the trailing PE ratio was 29.36 which gives a BSEYD value of $2.78-(100 / 29.36)=-0.626$.

Table 8: Earnings revisions for 2008 and 2009, analysts estimates of earnings in dollars. Source: Maudlin, 2009

| Date | Earnings | Date | Earnings |
| :--- | ---: | :--- | ---: |
|  |  | $\ldots$ and estimates for 2009 |  |
| March 2007 | 92.00 | March 20, 2008 | 81.52 |
| December 2007 | 84.00 | April 9, 2008 | 72.60 |
| February 2008 | 71.20 | June 25, 2008 | 70.13 |
| June 1, 2008 | 68.93 | September 10, 2008 | 48.52 |
| July 25, 2008 | 72.00 | August 29, 2008 | 64.44 |
| September 30, 2008 | 60.00 | February 1, 2009 | 42.00 |
| October 15, 2008 | 54.82 | February 20, 2009 | 32.41 |
| February 20, 2009 | 26.23 | April 10, 2009 | 28.51 |
| April 10, 2009 | 14.88 |  |  |
|  |  |  |  |

Shepherd (2009) has the S\&P500 PE ratio at 723 on July 28, 2009 four months into the rally that began in March 2009! The S\&P500 was then 979.62, up nearly $50 \%$ from the March lows. This high PE ratio was based on reported real earnings from SEC 10Q filings. So what do we conclude here? Our conclusion is in Figure 21. The BSEYD model did not give any


Figure 20: The S\&P500, January 1, 2007 to December 31, 2009
additional sell signals during 2008. The signal was on June 14, 2007 with the index at 1522.97 and the crash occurred in various phases with closing peak of 1565.15 on October 9, 2007 and a closing low of 676.53 on March 9, 2009, down some $56.8 \%$ from the peak. The conclusions are similar: over the period January 1, 1995 to April 6, 2011, two BSYED crash signals occurred: one in June 1999 and the other in June 2007.

## 6. Logarithmic Model

Koivu, Pennanen and Ziemba (2005) use a logarithmic version of the BSEYD model. The question is does that model suggest anything new for our analysis? The logarithmic model is based on the Fed model bond stock yield ratio

$$
\ln B S E Y R(t)=\ln \left(\frac{r(t)}{\gamma(t)}\right)=\ln r(t)-\ln \gamma(t)
$$



Figure 21: Crash Indicator ( $95 \%$ confidence) : S\&P500.

For China and Iceland, both measures produce similar results which are available from the authors. The pattern and timing of the crash signals nearly coincide for all three Icelandic banks. In China, the $\ln B S E Y R$ generates a slightly earlier signal than the BSEYD.

In the US, the result of the $\ln B S E Y R$ and of the BSEYD measures are also broadly similar. A signal precedes both the internet-related crash of 2000 and the credit crunch crash of 2008. In addition, the logarithmic $\operatorname{lnBSYR}(\mathrm{t})$ measure generates a signal in April 1998, ahead of a $19 \%$ decline from July 17th to August 31st. However, neither measure predicted the market decline of 2002. This is a combined result of the relatively low level reached by the two measures in 2001 compared to 1999, and of the increase in the confidence level starting in 2000.

## 7. Final remarks

The bond stock earnings yield model has been shown to be useful in a number of contexts. First, using it for being in or out of the market over
long investment periods has been shown to produce about double the returns of buy and hold with lower risk in five major countries. Secondly, over the years, the model has predicted many significant stock market crashes such as those in China, Iceland and the US during 2007-2009. Finally it has possible, but less clear, use concerning when to re-enter markets after a crash.

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[^1]:    ${ }^{1}$ Using a different index rather than the S\&P500 has the same conclusion but slightly different results. Berge, Consigli and Ziemba (2008) used the MSCI index. The danger zone was entered in May 1987 and the correction occurred in October, four months later. During June, July and August investors kept rebalancing their portfolios from the bond to the equity market (MSCI TRI $+13.87 \%$ over the quarter) then the equity market fell $31.80 \%$ in the following quarter (September to November 1987) with the major decline in

