Generalized nets in neurology : an example of mathematical modelling

S. K. Kim
Department of Statistics, Chonbuk National University,
Chonju, Chonbuk 561-756, Korea

K. T. Atanassov
Centre for Biomedical Engineering, Bulgaria Academy of Science,
Acad. G. Boncher, st B1, 105, Sofia-1113, Bulgaria

A. G. Shannon
School of Mathematical Sciences, University of Technology,
Sydney Po Box 124, Broadway, NSW, 2007, Australia

The purpose of this paper is to explain the prime theory of Generalized Nets(GNs) and their applications to the differential diagnosis of neurological diseases. We define formally the concepts of a GN and transitions of a GN and also outline some remarks on their theory. The work here construct the NGN45, which is the sample to trace the process of diagnosing different signs and symptoms in neurology.

1. Introduction to Generalized Nets

Processes in nature are complex and flow in parallel, while classical mathematical methods cannot reflect this facet in all details. The first tool for describing parallel processes were introduced in mathematics only thirty-five years ago - the so-called Petri nets. The second direction of GN-research in medicine is related to the diagnostic processes of diseases. The GNs are abstract mathematical objects for modelling, simulation, optimization and control of real processes. Also these are extensions of Petri nets, which provide the possibility of describing the time-parameter, the logical conditions, the capacities of the separate components, and the history of the modelled processes. In this chapter we define formally the prime concepts of a GN and transitions of a GN and outline some brief remarks on their theory, following Atanassov[1] and Shannon[3]

We begin with a transition. Every transition is described by a seven-tuple :

$$Z = \langle L^{'}, L^{''}, t_1, t_2, r, M, \rangle,$$

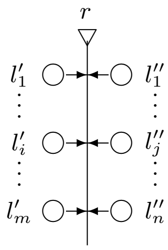
in which

(a) $L^{'}$ and $L^{''}$ are finite, non-empty sets of places (the transition's input and output places respectively) ; for the transition in Fig. 1. they are $L^{'} = l_1^{'}, l_2^{'}, \cdots, l_m^{'}$ and $L^{''} = l_1^{''}, l_2^{''}, \cdots, l_m^{''}$. Its places are marked with $\bigcirc$ and $\bigtriangledown$ indicates the transition's conditions.

(b) t_1 is the current time-moment of the process; firing

(c) t_2 is the current value of the duration of its activity;

(d) r is the transition's condition which determines these tokens which will transfer from the inputs of a transition to its outputs; it has the form of an index matrix:



$$r = \begin{array}{c|c} & l_1^{''} \cdots l_j^{''} \cdots l_n^{''} \\ \hline l_1^{'} & \\ \vdots & r_{ij} \\ l_i^{'} & (r_{ij} - \text{predicates}) \\ \vdots & \\ l_m^{'} & (1 \le i \le m, 1 \le j \le n) \end{array}$$

Fig. 1 A GN Transition

(i, j) denotes the element which corresponds to the i-th input and j-th output places; these elements are predicates and when the truth value of the (i, j)-th element is true, the token from i-th input place can be transferred to j-th output place; otherwise, it is not possible;

(e) M is an index matrix of the capacities of transition's arcs:

$$r = \begin{array}{c|c} & l_1^{''} \cdots l_j^{''} \cdots l_n^{''} \\ \hline l_1^{'} & \\ \vdots & m_{ij} \\ l_i^{'} & (m_{ij} \ge 0 - \text{natural numbers}) \\ \vdots & \\ l_m^{'} & (1 \le i \le m, 1 \le j \le n) \end{array}$$

(f) is an object with a form similar to that of a Boolean expression. In it the variables are all symbols which mark the transition's input names, and the Boolean operations $''\wedge''$ and $''\vee''$ determine the following conditions:
$(l_{i1}, l_{i2}, \cdots, l_{iu})-$ every place $l_{i1}, l_{i2}, \cdots, l_{iu}$ must contain at least one token,
$(l_{i1}, l_{i2}, \cdots, l_{iu})-$ in all places $l_{i1}, l_{i2}, \cdots, l_{iu}$ must contain at least one token,
where $(l_{i1}, l_{i2}, \cdots, l_{iu}) \subset L^{'}$.

The ordered four-tuple
$E = \prec\prec A, \pi_A, \pi_L, c, f, \theta_1, \theta_2 \succ, \prec K, \pi_K, \theta_K \succ, \prec T, t^0, t^* \succ, \prec X, \Phi, b \succ\succ$ is called a Generalized Net (GN), if:

(a) A is a set of transition;

(b) π_A is a function which gives the priorities of the transitions; that is, $\pi_A : A \to N$, where $N = \{0, 1, 2, \cdots\} \bigcup \{\infty\}$;

(c) π_L is a function which gives the priorities of the places; that is, $\pi_L : L \to N$, in which $L = pr_1 \bigcup pr_2 A$ and $pr_i X$ is the i-th projection of the n-dimensional set X,
where $n \in N, n \geq 1$ and $1 \leq k \leq n$ (obviously, L is the set of all GN-places);

(d) c is a function which gives the capacities of the places; that is, $c : L \to N$;

(e) f is a function which calculates the truth values of the predicates of the transition's conditions (for the GN described here, let the function f have the values "false" or "true", that is, a value from the set $\{0, 1\}$). Other types of nets are described in Atanassov [1] where this function has a value in the interval $[0, 1]$ or the set $[0, 1] \times [0, 1]$);

(f) θ_1 is a function which yields the next time-moment when a given transition can be activated; that is, $\theta_1(t) = t'$, where $t \in [T, T + t*]$ and $t' \geq 0$. The value of this function is calculated at the moment when the transition ceases to function;

(g) θ_2 is a function which gives the duration of the activity of a given trasition ; that is, $\theta_2(t) = t'$, where $t \in [T, T + t*]$ and $t' \geq 0$. The value of this function is calculated at the moment when the transition starts to function;

(h) K is the set of the GN's tokens. In some cases, it is convenient to consider this set in the form

$$K = \bigcup_{l \in Q^I} K_l,$$

where K_l is the set of tokens which enter the net from place l, and Q^I is the set of all the net's input places;

(i) π_K is a function which gives the priorities of the tokens, that is, $\pi_K : K \to N$;

(j) θ_K is a function which yields the time-monent when a given token can enter the net, that is, $\theta_K(\alpha) = t$, where $\alpha \in K$ and $t \in [T, T + t*]$;

(k) T is the time-monent when the GN starts to function. This moment is determined within a fixed (global) time-scale;

(l) t^0 is an elementary time-step, related to fixed (global) time-scale;

(m) t^* is the duration of the functioning of a net;

(n) X is the set of all initial characteristics which can receive the tokens when they enter the net;

(o) Φ is a characteristic function which gives a new characteristic to every token when it makes a transfer from an input to an output place of a given transition. Like some of the functions above this function can also be represented in another from which is shown below.

(p) b is a function which gives the maximum number of characteristics which can receive a given token; that is, $b : K \to N$. If for a certain token, $\alpha, b(\alpha) = 1$, then the token will enter the net with an initial characteristic (as a zero-characteristic). After this, it will receive only the previous characteristic. When $b(\alpha) = \infty$, the token α will receive all possible characteristics. When $b(\alpha) = k < \infty$, except for the zero-characteristic, the token α will keep the last k as its characteristics (and previous characteristics will be "forgotten"). Hence, in general, every token α has $b(\alpha) + 1$ characteristics.

It is convenient to assume that the functions f, θ_1, θ_2 and also have the following forms, also:

$f = \bigcup_{i=1}^{|A|} f_i$, where f_i calculates the truth-values of the predicates of the i-th GN transition;

$\theta_1 = \bigcup_{i=1}^{|A|} {\theta_1}^i$, where θ^{i_1} calculates the next time-moment of the activation of the i-th GN transition;

$\theta_2 = \bigcup_{i=1}^{|A|} {\theta_2}^i$, where θ_2^i calculates the duration of the active state of the i-th GN transition;

$\Phi = \bigcup_{i=1}^{|L|} \Phi_i$, where Φ_i calculates the tokens' characteristics, which they will receive in the i-th GN place.

The static part of a given GN is determined by the elements of the set $pr_1, 2, 6, 7A$, where for a given n-dimensional set $X (u \geq 2)$

$$pr_{i_1, i_2, \cdots, i_k} X = \prod_{j=1}^{k} pr_{i_j} X$$

$(1 \leq i_j \leq n, 1 \leq j \leq k, i_{j'} \neq i_{j''} for j' \neq j'')$; that is, the static part of a GN is determined by input and output transition places, by the index matrix of the arcs and by the type of transition. The dynamic character of the net comes from the GN's tokens and the transition conditions ($pr_5 A$).

The temporal character comes from the components T, t^0, t^* and from the elements of the set $pr_{3,4}A$.

Finally, the components Φ, X and b play the part of the GN's memory.

A given GN may not have some of the components and such GNs we shall call "Reduced GNs". Also eight different types of GN-extensions are defined by modifications of transition-components and/or function and memory of GNs.

2. Generalized Net Models in Neurology

Medical decision analysis is the process by which neurologists analyze the clinical problem systematically and sequentially and determine the most likely diagnostic possibilities. The process is defined as decision trees or clinical algorithms. Here we will construct NGN45 shown in Fig. 2, the GN-model with "pupillary abnormalities", based on books [6,7] and our research [3,4,5].

NGN45: Pupillary abnormalities

The tokens enter GN with an initial characteristic "anisocoria".

$$Z_1 = < \{l_1\}, \{l_2, l_3\}, \begin{array}{c|cc} & l_2 & l_3 \\ \hline l_1 & W_{1,2} & W_{1,3} \end{array} >,$$

$W_{1,2}$ = "the anisocoria is episodic", $W_{1,3}$ = "the anisocoria is constant".

The tokens obtain a characteristic "pupil size is examined" in place l_2 and "assessment of the pupillary light reaction is accomplished" in place l_3.

$$Z_2 = < \{l_2\}, \{l_4, l_5\}, \begin{array}{c|cc} & l_4 & l_5 \\ \hline l_2 & W_{2,4} & W_{2,5} \end{array} >,$$

$W_{2,4}$ = "there is an unilateral midriasis", $W_{2,5}$ = "there is an unilateral miosis".

The tokens do not obtain any characteristics in place l_4 and they obtain characteristics "the miosis is associated with ptosis and headache" in place l_5.

$$Z_3 = < \{l_3\}, \{l_6, l_7\}, \begin{array}{c|cc} & l_6 & l_7 \\ \hline l_3 & W_{3,6} & W_{3,7} \end{array} >,$$

$W_{3,6}$ = "the pupillary light reaction is normal", $W_{3,7} = W_{3,6}$.

The tokens do not obtain any characteristics in places l_6 and l_7.

$$Z_4 = < \{l_4, l_{16}\}, \{l_8, l_9\}, \begin{array}{c|cc} & l_8 & l_9 \\ \hline l_4 & W_{4,8} & W_{4,9} \\ l_{16} & W_{16,8} & W_{16,9} \end{array} >,$$

$W_{4,8} = W_{16,8}$ = "the unilateral mydriasis is related to seizure", $W_{4,9} = W_{16,9} = \neg W_{4,8}$.

The tokens obtain the characteristics "EEG is necessary" in place l_8 and "there is contralateral hemiparesis; consider transtentorial hermiation" in place l_9.

$$Z_5 = < \{l_6\}, \{l_{10}, l_{11}\}, \begin{array}{c|cc} & l_{10} & l_{11} \\ \hline l_6 & W_{6,10} & W_{6,11} \end{array} >,$$

$W_{6,10}$ = "there is ptosis with miosis", $W_{6,11}$ = "there is ptosis without miosis".

The tokens obtain the characteristics "Horner's syndrome is present; 1 % hidroxyamphetamine test is performed" in place l_{10} and "the anisocoria is physiologic" in place l_{11}.

$$Z_6 = < \{l_7\}, \{l_{12}, l_{13}, l_{14}\}, \begin{array}{c|ccc} & l_{12} & l_{13} & l_{14} \\ \hline l_7 & W_{7,12} & W_{7,13} & W_{7,14} \end{array} >,$$

$W_{7,12}$ = "the near-target response is normal", $W_{7,13}$ = "there is not a near-target response" $W_{7,14}$ = "there is a tonic near-target response"

The tokens obtain the characteristics “there is pletectal lesion; CT/MRI is necessary” in place l_{12}, “slit lamp examination is performed” in place l_{13} and “there is pupillary construction in response of 0.125 % pilocarpine; Adie’s pupil is considered” in place l_{14}.

$$Z_7 =< \{l_8\}, \{l_{15}, l_{16}\}, \begin{array}{c|cc} & l_{15} & l_{16} \\ \hline l_8 & W_{8,15} & W_{8,16} \end{array} >,$$

$W_{8,15}$ = “the EEG is abnornal”, $W_{8,16} = \neg W_{8,15}$.

The tokens obtain the “the diagnosis of seizure phenomena is confirmed” in place l_{15} and “the patient must be re-examined” in place l_{16}.

$$Z_8 =< \{l_{10}\}, \{l_{17}, l_{18}\}, \begin{array}{c|cc} & l_{17} & l_{18} \\ \hline l_{10} & W_{10,17} & W_{10,18} \end{array} >,$$

$W_{10,17}$ = “there is a dilation of the pupil”, $W_{10,18} = \neg W_{10,17}$.

The tokens pbtain the characteristics “there is preganglionic lesion” in place l_{17} and “there is postganglionic lesion; skin radiography, orbital CT/MRI, angiography are necessary” in place l_{18}.

$$Z_9 =< \{l_{13}\}, \{l_{19}, l_{20}\}, \begin{array}{c|cc} & l_{19} & l_{20} \\ \hline l_{13} & W_{13,19} & W_{13,20} \end{array} >,$$

$W_{13,19}$ = “the iris is abnormal”,
$W_{13,20} = W_{13,19}$.

The tokens obtain the characteristics “ophthalmologic evaluation is necessary” in place l_{19} and “the dilated pupil is tested with 1 % Pilocarpine” in place l_{20}.

$$Z_{10} =< \{l_{17}\}, \{l_{21}, l_{22}\}, \begin{array}{c|cc} & l_{21} & l_{22} \\ \hline l_{17} & W_{17,21} & W_{17,22} \end{array} >,$$

$W_{17,21}$ = “there are neurologic abnormalities”, $W_{17,22} = \neg W_{17,21}$.

The tokens obtain the characteristic “the probable diagnosis is spinal cord lesion, brachial plexus lesion” in place l_{21} and “mediastinal lesion is consider” in place l_{22}.

$$Z_{11} =< \{l_{20}\}, \{l_{23}, l_{24}\}, \begin{array}{c|cc} & l_{23} & l_{24} \\ \hline l_{20} & W_{20,23} & W_{20,24} \end{array} >,$$

$W_{20,23}$ = “the pupil is constricted”. $W_{20,24} = \neg W_{20,23}$.

The tokens obtain the characteristics “the diagnosis is: third nerve paresis; CT/MRI and angiography are needed” in place l_{23} and “there is pharmacologic neuromuscular blockade” in place l_{24}.

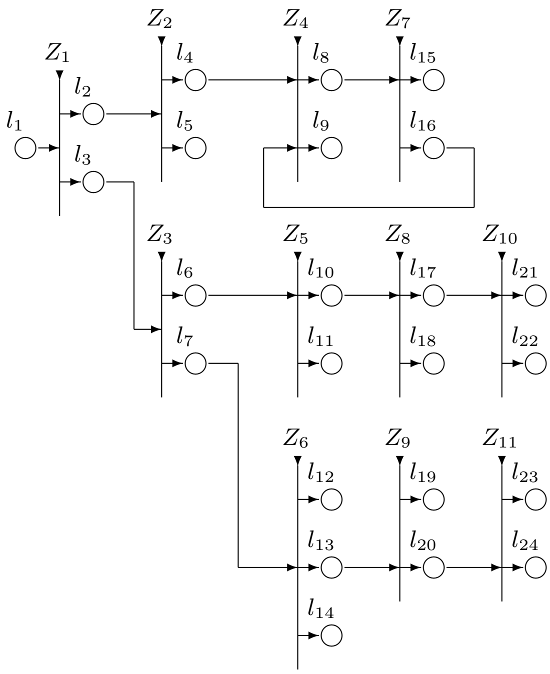


Fig. 2 NGN45

We need to bear in mind that GN models can be corrected and adopted on the basis of the difference between expected and observed data in relation to certain fixed criteria. and also constructed on a computer with a saitable program tool, one of which, " A Program Package for GNs", is available from one(KTA) of the authors.

Acknowledgements

The author Kim would like to thank Korea Sciences and Engineering Foundation and University of Technology, Sydney for their finacial aid and facilities.

References

[1] Atanassov, K. 1991 Generalized Nets, Singapore: World Scientific.

[2] Atanassov, K. 1993 Applications of Generalized Nets, Singapore: World Scientific.

[3] Shannon, A. and Sorsich, J. and Atanassov, K. 1996 Generalized Nets in Medicine, Publishing House of the Bulgarian Academy of Sciences.

[4] Kim, S. K. and Shannon, A. and Atanassov, K. and Sorsich, J. and Bustine, H. and Daskalov, M. 1995 preprint of MRL-MFAIS-2-95, 20-29, NGN84: Guillain-barre syndrome suspected.

[5] Kim, S. K. and Shannon, A. and Atanassov, K. and Sorsich, J. and Bustine, H. and Daskalov, M. 1995 Preprint of MRL-MFAIS-2-95, 26-29, NGN100: Findings of chorea and athetoid movement.

[6] Weisberg, L. and Strub, R. and Garcia, C. 1993 Decision Making in Adult Neurology, B. C. Decker Mosby-Year Book Inc. Second edition.

[7] Weisberg, L. and Strub, R. and Garcia, C. 1993 Decision Making in Adult Neurology, B. C. Decker Toronto, First edition.