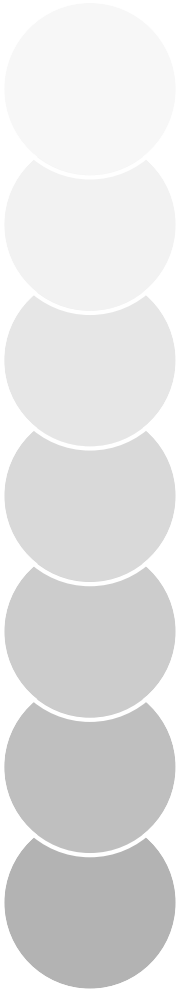
JUST IN TIME Math



JUST IN TIME

Math

Catherine V. Jeremko

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FORMULA CHEAT SHEET

PERCENT

$\frac{\text{part}}{\text{whole}} = \frac{\text{percent}}{100}$ $\frac{\text{is}}{\text{of}} = \frac{\text{percent}}{100}$ $\frac{\text{change}}{\text{original}} = \frac{\text{percent}}{100}$

DISTANCE FORMULA

$D = R \times T$

SIMPLE INTEREST FORMULA

$I = P \times R \times T$

RULES OF EXPONENTS

$x^0 = 1$ $x^{-a} = \frac{1}{x^a}$ $x^a \times x^b = x^{a+b}$ $x^a \div x^b = x^{a-b}$

$\frac{x^a}{x^b} = x^{a-b}$ $(x^a)^b = x^{a \times b}$ $x^{\frac{1}{a}} = \sqrt[a]{x}$

PROBABILITY

$P(E) = \frac{\text{\# favorable outcomes}}{\text{\# total outcomes}}$ $P(E_1) \text{ or } P(E_2) = P(E_1) + P(E_2)$

$P(E_1) \text{ and } P(E_2) = P(E_1) \times P(E_2)$

GEOMETRY

Pythagorean Theorem: $a^2 + b^2 = c^2$

Perimeter

Rectangle: $P = 2 \times l + 2 \times w$ Square: $P = 4 \times s$

Circumference of a Circle: $C = \pi \times d$ or $C = 2 \times \pi \times r$

Area

Triangle: $A = \frac{1}{2} \times b \times h$ Rectangle: $A = b \times h$

Trapezoid: $A = \frac{1}{2} \times h \times (b_1 + b_2)$

Volume

$V = b \times h$ (the area of the base times the height)

Rectangular Solid: $V = l \times w \times h$ Cylinder: $V = \pi \times r^2 \times h$

Introduction

You have to face a big exam that will test your mathematical skills. It is just a few weeks, perhaps even just a few days, from now. You haven't begun to study. Perhaps you just haven't had the time. We are all faced with full schedules and many demands on our time, including work, family, and other obligations. Or perhaps you have had the time, but procrastinated; topics in mathematics are topics that you would rather avoid at all costs. Computation and word problems have never been your strong suit. It is possible that you have waited until the last minute because you feel rather confident in your mathematical skills, and just want a quick refresher on the major topics. Maybe you just realized that your test included a mathematics section, and now you have only a short time to prepare.

If any of these scenarios sound familiar, then *Just in Time Math* is the right book for you. Designed specifically for last-minute test preparation, *Just in Time Math* is a fast, accurate way to build your essential computational and word problem skills. This book includes nine chapters of common mathematical topics, with an additional chapter on study skills to make your time effective. In just ten short chapters, you will get the essentials—just in time for passing your big test.

THE *JUST IN TIME* TEST-PREP APPROACH

At LearningExpress, we know the importance that is placed on test scores. Whether you are preparing for the PSAT, SAT, GRE, GMAT, or a Civil Service exam, or you simply need to improve your fundamental mathematical skills, our *Just in Time* streamlined approach can work for you. Each chapter includes:

- a ten-question benchmark quiz to help you assess your knowledge of the topics and skills in the chapter
- a lesson covering the essential content for the topic of the chapter
- sample problems imbedded in the lesson with full explanations

- calculator tips to make the most of technology on your exam
- specific tips and strategies to prepare for the exam
- a 25-question practice quiz followed by detailed answers and explanations to help you measure your progress

The *Just in Time* series also includes the following features:

-  *Extra Help:* refers you to other LearningExpress skill builders or other resources that can help you learn more about a particular topic.
-  *Calculator Tips:* offers hints on how your calculator can help you.
-  *Glossary*: provides critical definitions.
-  *Rule Book:* highlights the rules and procedures you really need to know.
-  *Shortcut:* offers tips for reducing your study and practice time—without sacrificing accuracy.
- *Formula Cheat Sheet:* a tear-out page preceding this introduction that lists important formulas from the chapters.

Of course, no book can cover every type of problem you may face on a given test. But this book is not just about recognizing specific problem types; it is also about building the essential skills, confidence, and processes that will ensure success when you are faced with a math problem. The math topics in this book have been carefully chosen to reflect not only what you are likely to see on an exam, but also what you are likely to come across regularly in books, newspapers, lectures, and other daily activities.

HOW TO USE THIS BOOK

While each chapter can stand on its own as an effective review of mathematical content, this book will be most effective if you complete each chapter in order, beginning with Chapter 1. Chapters 2, 3, and 4 review the basic mathematical knowledge of working with numbers. The remaining chap-

ters review higher mathematical knowledge that involves relationships among numbers. The chapters are arranged such that material covered in earlier chapters may be referenced in a later chapter.

Following is a brief outline of each chapter:

- **Chapter 1: Study Skills** reviews fundamental study strategies including how to budget your time, how to create a study plan, and how to use study aids such as flashcards.
- **Chapter 2: The Integers and Absolute Value** reviews how to work with positive and negative numbers.
- **Chapter 3: Properties of Numbers** reviews the fundamentals of working with numbers, a foundation for the rest of the book.
- **Chapter 4: Fractions, Decimals, and Ordering the Real Numbers** reviews the two most common ways to represent numbers and how the numbers are ordered.
- **Chapter 5: Ratio and Proportion** reviews how numbers are related to one another through comparisons.
- **Chapter 6: Percent** covers the most common use of ratio and proportion on tests and in daily life.
- **Chapter 7: Powers, Exponents, and Roots** reviews the use of repeated multiplication to represent rapid growth, the inverse operation, and roots.
- **Chapter 8: Geometry and Measurement Conversions** reviews the ways that mathematics is used in measurement and the use of common figures.
- **Chapter 9: Statistics and Probability** reviews the study of data and chance.
- **Chapter 10: Word Problems** covers processes and strategies used to solve mathematics in context.

Depending upon how much time you have before the exam, review as much as possible. If time is short, start with the chapters addressing your weak areas. The ten-question benchmark quizzes at the start of each chapter can help you assess your strengths and weaknesses.

Finally, remain calm and think positively. Your big test may be just a short while away, but you are taking the steps you need to prepare . . . *just in time*.

JUST IN TIME Math

Study Skills

If you have left studying for that big test until the last minute, you may be feeling that your only option is to cram. You might be feeling panicky that you will never have enough time to learn what you need to know. But the "Just in Time" solution is exactly that: just in time. This means that with the help of this book you can use your available time prior to your test effectively. First, to get ready for your test just in time, you need a plan. This chapter will help you put together a study plan that maximizes your time and tailors your learning strategy to your needs and goals.

There are four main factors that you need to consider when creating your study plan: what to study, where to study, when to study, and how to study. When you put these four factors together, you can create a specific plan that will allow you to accomplish more—in less time. If you have three weeks, two weeks, or even one week to get ready, you can create a plan that avoids anxiety-inducing cramming and focuses on real learning by following the simple steps in this chapter.

WHAT TO STUDY

Finding out what you need to study for your test is the first step in creating an effective study plan. You need to have a good measure of your

ability in math. You can accomplish this by looking over the Table of Contents to see what looks familiar to you and by answering the Benchmark Quiz questions starting in the next chapter. You also need to know what exactly is covered on the test you will be taking. Considering both your ability and the test content will tell you what you need to study.

Establish a Benchmark

In each chapter you will take a short, ten-question Benchmark Quiz that will help you assess your skills. This may be one of the most important steps in creating your study plan. Because you have limited time, you need to be very efficient in your studies. Once you take a chapter Benchmark Quiz and analyze the results, you will be able to avoid studying the material you already know. This will allow you to focus on those areas that need the most attention.

A Benchmark Quiz is only practice. If you did not do as well as you anticipated, do not be alarmed and certainly do not despair. The purpose of the quiz is to help you focus your efforts so that you can *improve*. It is important to carefully analyze your results. Look beyond your score, and consider *why* you answered some questions incorrectly. Here are some questions to ask yourself when you review your wrong answers:

- Did you get the question wrong because the material was totally unfamiliar?
- Was the material familiar but were you unable to come up with the right answer? In this case, when you read the right answer it will often make perfect sense. You might even think, "I knew that!"
- Did you answer incorrectly because you read the question carelessly?
- Did you make another careless mistake? For example, circling choice **a** when you meant to circle choice **b**.

Next, look at the questions you answered correctly and review how you came up with the right answer. Not all right answers are created equally.

- Did you simply know the right answer?
- Did you make an educated guess? An educated guess might indicate that you have some familiarity with the subject, but you probably need at least a quick review.
- Or did you make a lucky guess? A lucky guess means that you don't know the material and you will need to learn it.

Your performance on the Benchmark Quiz will tell you several important things. First, it will tell you how much you need to study. For example, if you got eight out of ten questions right (not counting lucky guesses!), you might only need to brush up on certain math topics. But if you got five out of ten questions wrong, you will need a thorough review. Second, it can tell you what you know well, that is which subjects you *don't* need to study. Third, you will determine which subjects you need to study in-depth and in which subjects you simply need to refresh your knowledge.

Targeting Your Test

For the "Just in Time" test-taker, it is important to focus your study efforts to match what is needed for your test. You don't want to waste your time learning something that will not be covered on your test. There are three important aspects that you should know about your test before developing your study plan:

- What material is covered?
- What is the format of the test? Is it multiple choice? Fill in the blank? Some combination? Or something else?
- What is the level of difficulty?

How can you learn about the test before you take it? For most standardized tests, there are sample tests available. These tests—which have been created to match the test that you will take—are probably the best way to learn what will be covered. If your test is non-standardized, you should ask your instructor specific questions about the upcoming test.

You should also know how your score will affect your goal. For example, if you are taking the SAT exam, and the median math score of students accepted at your college of choice is 550, then you should set your sights on achieving a score of 550 or better. Or, if you are taking the New York City Police Officer exam, you know that you need to get a perfect or near-perfect score to get a top slot on the list. Conversely, some exams are simply pass or fail. In this case, you can focus your efforts on achieving a passing score.

Matching Your Abilities to Your Test

Now that you understand your strengths and weaknesses and you know what to expect of your test, you need to consider both factors to determine what material you need to study. First, look at the subject area or question type with which you have the most trouble. If you can expect to find questions of this type on your test, then this subject might be your first priority. But be

sure to consider how much of the test will cover this material. For example, if there will only be a few questions out of a hundred that test your knowledge of a subject that is your weakest area, you might decide not to study this subject area at all. You might be better served by concentrating on solidifying your grasp of the main material covered on the exam.

The important thing to remember is that you want to maximize your time. You don't want to study material that you already know. And you don't want to study material that you don't need to know. You will make the best use of your time if you study the material that you know the least but that you most need to know.

WHERE TO STUDY

The environment in which you choose to study can have a dramatic impact on how successful your studying is. If you choose to study in a noisy coffee shop at a small table with dim lighting, it might take you two hours to cover the same material you could read in an hour in the quiet of the library. That is an hour that you don't have to lose! However, for some people the noisy coffee shop is the ideal environment. You need to determine what type of study environment works for you.

◗ *Consider Your Options*

Your goal is to find a comfortable, secure place that is free from distractions. The place should also be convenient and conform to your schedule. For example, the library might be ideal in many respects. However, if it takes you an hour to get there, and it closes soon after you arrive, you are not maximizing your study time.

For many people, studying at home is a good solution. Home is always open and you don't waste any time getting there, but it can have drawbacks. If you are trying to fit studying in between family obligations, you might find that working from home offers too many opportunities for distraction. Chores that have piled up, children or younger siblings who need your attention, or television that captures your interest, are just some of things that might interfere with studying at home. Or maybe you have roommates who will draw your attention away from your studies. Studying at home is a good solution if you have a room that you can work in alone and away from any distractions.

If home is not a good environment for quiet study, the library, a reading room, or a coffee shop are places you can consider. Be sure to pick a place that is relatively quiet and provides enough workspace for your needs.

Noise

Everyone has his or her own tolerance for noise. Some people need absolute silence to concentrate, while others will be distracted without some sort of background noise. Classical music can be soothing and might help you relax as you study. In fact, studies have shown that listening to Mozart actually enhances math performance. If you think you work better with music or the television on, you should be sure that you are not paying attention to what is on in the background. Try reading a chapter or doing some problems in silence, then try the same amount of work with noise. Which noise level allowed you to work the fastest?

Light

You will need to have enough light to read comfortably. Light that is too dim will strain your eyes and make you drowsy. Too bright and you will be uncomfortable and tense. Experts suggest that the best light for reading comes from behind and falls over your shoulder. Make sure your light source falls on your book and does not shine in your eyes.

Comfort

Your study place should be comfortable and conducive to work. While your bed might be comfortable, studying in bed is probably more conducive to sleep than concentrated learning. You will need a comfortable chair that offers good back support and a work surface—a desk or table—that gives you enough space for your books and other supplies. Ideally, the temperature should be a happy medium between too warm and too cold. A stuffy room will make you sleepy and a cold room is simply uncomfortable. If you are studying outside your home, you may not be able to control the temperature, but you can dress appropriately. For example, bring along an extra sweater if your local library is skimpy with the heat.

A Little Help

When you have settled on a place to study, you will need to enlist the help of your family and friends—especially if you are working at home. Be sure they know that when you go to your room and close the door to study that you do not want to be disturbed. If your friends all go to the same coffee shop where you plan to study, you will also need to ask them to respect your study place. The cooperation of your family and friends will eliminate one of the greatest potential distractions.

WHEN TO STUDY

Finding the time in your busy schedule may seem like the greatest hurdle in making your "just in time" study plan, but you probably have more time available than you think. It just takes a little planning and some creativity.

Analyze Your Schedule

Your first step in finding time to study is to map out your day-to-day schedule—*in detail.* Mark a piece of paper in fifteen-minute intervals from the time you get up to the time you generally go to bed. Fill in each fifteen-minute interval. For example, if you work from nine to five, do not simply block that time off as unavailable for study. Write down your daily routine at work and see when you might have some time to study. Lunch is an obvious time. But there may be other down times in your workday when you can squeeze in a short study session.

You will want to set aside a stretch of time when you plan to study in your designated study place. But you can also be creative and find ways to study for short bursts during your normal routine. For example, if you spend an hour at the gym on the stationary bike, you can read while you cycle. Or you can review flashcards on your bus ride. If you drive to work, you could record some study material on a tape or CD. You could also listen to this tape while you walk the dog.

When you look at your schedule closely, you will probably find you have more time than you thought. However, if you still don't have the time you need, you should rethink your routine. Can you ask your significant other to take on a greater share of the household chores for the few weeks you need to get ready for your test? Is there some activity that you can forgo for the next few weeks? If you normally go to the gym six days a week for an hour and a half, cut down to three days a week for forty-five minutes. You will add over six and a half hours to your schedule without completely abandoning your fitness routine. Remember any changes you make to your schedule are short-term and a small sacrifice, once you consider your goal.

Time Strategies

Now that you know when you have time available, you need to use that time to your best advantage. You will probably find that you can set aside one block of time during the day during which you will do the bulk of your studying. Use this time to learn new material or take a practice quiz and review your answers. Use the small spurts of time you have found in your schedule to review with flashcards, cheat sheets, and other tools.

Also consider your learning style and body rhythm when you make your

schedule. Does it take you some time to get into material? If so, you should build a schedule with longer blocks of time. Do you have a short attention span? Then you will do better with a schedule of several shorter study periods. No matter your style, avoid extremes. Neither very long study sessions nor very short sessions (except for quick reviews) are an efficient use of time. Whether you are a morning person or a night owl, plan to study when you are most energetic and alert.

Make sure your schedule allows for adequate rest and study breaks. Skipping sleep is not a good way to find time in your schedule. Not only will you be tired when you study, but you will also be sleep deprived by the time of the test. A sleep-deprived test-taker is more likely to make careless mistakes, lose energy and focus, and become stressed-out by the testing environment. If you plan to do most of your studying in one block of time, say four hours, be sure you leave time to take a study break. Experts have shown that students are more likely to retain material if they take some time to digest it. A five- or ten-minute break to stretch your legs or eat a snack will revive you and give your brain time to absorb what you have learned.

HOW TO STUDY

How you study is just as important as how long—especially if your time is limited. You will need to be in a good physical and mental state. And you will need to use the right tools for the job. You will also need to understand your learning style so that you can select the best study method. And, perhaps most important, you will need methods that will help you to remember not to memorize the material. All these techniques—using the right tools and methods—will help you make the most of your study time.

◗ *Sleep Well, Eat Right, and Relax*

Does your idea of studying hard include images of staying up into the wee hours and living on fast food and caffeine until the big test? Even though it may seem like you are working hard when you study around the clock and put aside good eating habits in order to save time, you are not working efficiently. If you have ever pulled an all-nighter you know that by four in the morning you can find yourself reading the same page several times without understanding a word. Adequate rest and good nutrition will allow you to be focused and energetic so you can get more work done in less time.

Most people need about eight hours of sleep a night. Do not sacrifice sleep in order to make time to study. Hunger can be a distraction, so don't skip meals. Eat three nutritious meals a day, and keep healthy snacks on hand during a long study session. The key word is healthy. Sugary snacks

might make you feel energized in the short term, but that sugar rush is followed by a crash that will leave you feeling depleted. Caffeine can have a similar effect. A little caffeine—a morning cup of coffee, for example—can give you a boost, but too much caffeine will make you feel jittery and tense. Tension can affect your ability to concentrate.

Being over-caffeinated is not the only potential source of tension. Pre-exam anxiety can also get in the way of effective studying. If your anxiety about the upcoming test is getting the better of you, try these simple relaxation techniques:

- **Breathe!** Sounds simple, and it is. Taking long, deep breaths can drain the tension from your body. Place one hand on your stomach and the other on your chest. Sit up straight. Inhale deeply through your nose and feel your stomach inflate. Your chest should remain still. Exhale slowly through your mouth and feel your stomach deflate. It is the slow exhalation that helps you relax, so make sure you take your time releasing your breath. Pausing during a study session to take three deep breaths is a quick way to clear your mind and body of tension so that you can better focus on your work.
- **Tense and relax your muscles.** You may not even notice it, but as anxiety mounts your muscles tense. You may tense your neck and shoulders, your toes, or your jaw. This tension can interfere with your concentration. Release the tension held in your muscles by purposely tensing, then relaxing, each muscle. Work from your toes to your head systematically.
- **Visualize a soothing place.** Taking a break to mentally visit a place that you find relaxing can be reinvigorating. Close your eyes and conjure up the sights, smells, and sounds of your favorite place. Really try to feel like you are there for five uninterrupted minutes and you will return from your mini vacation ready to study.

◗ *The Right Tools for the Job*

If you follow the steps above, you will have a rested, energized, and relaxed brain—the most important tool you need to prepare for your exam. But there are other tools that you will need to make your study session the most productive. Be sure that you have all the supplies you need on hand before you sit down to study. To help make studying more pleasant, select supplies that you enjoy using. Here is a list of supplies that you will need:

- a notebook or legal pad dedicated to studying for your test
- graph paper
- pencils

- pencil sharpener
- highlighter
- index or other note cards
- paper clips or sticky note pads for marking pages
- a calendar or personal digital assistant (which you will use to keep track of your study plan)
- a calculator

Break It Down

You may be feeling overwhelmed by the amount of material you have to cover in a short time. This seeming mountain of work can generate anxiety and even cause you to procrastinate further. Breaking down the work into manageable chunks will help you plan your studying and motivate you to get started. It will also help you organize the material in your mind. When you begin to see the large topic as smaller units of information that are connected, you will develop a deeper understanding of the subject. You will also use these small chunks of information to build your study plan. This will give you specific tasks to accomplish each day, rather than simply having time set aside to study for the test.

For example, if you have difficulty working with fractions, you could study a different fractions topic each day for a week: On Monday, practice adding fractions; on Tuesday, work on subtracting fractions; on Wednesday, try multiplying fractions; and so on. "Learn fractions" might seem like an overwhelming task, but if you divide the work into smaller pieces, you will find that your understanding of fractions improves with practice and patience.

Your Learning Style

Learning is not the same for everyone. People absorb information in different ways. Understanding how you learn will help you develop the most effective study plan for your learning style. Experts have identified three main types of learners: visual, auditory, and kinesthetic. Most people use a combination of all three learning styles, but one style might be more dominant. Here are some questions that will help you identify your dominant learning style:

1. If you have to remember an unusual word, you most likely
   a. picture the word in your mind.
   b. repeat the word aloud several times.
   c. trace out the letters with your finger.

2. When you meet new people, you remember them mostly by
   a. their actions and mannerisms.
   b. their names (faces are hard to remember).
   c. their faces (names are hard to remember).

3. In class you like to
   a. take notes, even if you don't reread them.
   b. listen intently to every word.
   c. sit up close and watch the instructor.

A visual learner would answer **a**, **c**, and **c**. An auditory learner would answer **b**, **b**, and **b**. A kinesthetic learner would answer **c**, **a**, and **a**.

Visual learners like to read and are often good spellers. When visual learners study, they often benefit from graphic organizers such as charts and graphs. Flashcards often appeal to them and help them learn, especially if they use colored markers, which will help them form images in their minds as they learn words or concepts.

Auditory learners, by contrast, like oral directions and may find written materials confusing or boring. They often talk to themselves and may even whisper aloud when they read. They also like being read aloud to. Auditory learners will benefit from saying things aloud as they study and by making tapes for themselves and listening to them later. Oral repetition is also an important study tool. Making up rhymes or other oral mnemonic devices will also help them study, and they may like to listen to music as they work.

Kinesthetic learners like to stay on the move. They often find it difficult to sit still for a long time and will often tap their feet and gesticulate a lot while speaking. They tend to learn best by doing rather than observing. Kinesthetic learners may want to walk around as they practice what they are learning, because using their bodies helps them remember things. Taking notes is an important way of reinforcing knowledge for the kinesthetic learner, as is making flashcards.

It is important to remember that most people learn in a mixture of styles, although they may have a distinct preference for one style over the others. Determine which is your dominant style, but be open to strategies for all types of learners.

◗ *Remember–Don't Memorize*

You need to use study methods that go beyond rote memorization to genuine comprehension in order to be fully prepared for your test. Using study methods that suit your learning style will help you to *really* learn the material you need to know for the test. One of the most important learning strategies is to be an active reader. Interact with what you are reading by

asking questions, making notes, and marking passages instead of simply reading the words on the page. Choose methods of interacting with the text that match your dominant learning style.

- **Ask questions.** When you read a lesson, ask questions such as, "What is the main idea of this section?" Asking yourself questions will test your comprehension of the material. You are also putting the information into your own words, which will help you remember what you have learned. This can be especially helpful when you are learning math techniques. Putting concepts into your own words helps you to understand these processes more clearly.
- **Make notes.** Making notes as you read is another way for you to identify key concepts and to put the material into your own words. Writing down important ideas and mathematical formulas can also help you memorize them.
- **Highlight.** Using a highlighter is another way to interact with what you are reading. Be sure you are not just coloring, but highlighting key concepts that you can return to when you review.
- **Read aloud.** Especially for the auditory learner, reading aloud can help aid in comprehension. Hearing mathematical information and formulas read aloud can clarify their meanings for you.
- **Make connections.** Try to relate what you are reading to things you already know or to a real world example. It might be helpful, for example, to make up a word problem, or draw a diagram or table, to clarify your understanding of what a problem is asking you to do.

Reading actively is probably the most important way to use your study time effectively. If you spend an hour passively reading and retaining little of what you have read, you have wasted that hour. If you take an hour and a half to actively read the same passage, that is time well spent. However, you will not only be learning new material; you will also need methods to review what you have learned:

- **Flashcards.** Just making the cards alone is a way of engaging with the material. You have to identify key concepts, words, or important information and write them down. Then, when you have made a stack of cards, you have a portable review system. Flashcards are perfect for studying with a friend and for studying on the go.
- **Mnemonics.** These catchy rhymes, songs, and acronyms are tools that help us remember information. Some familiar mnemonics are "i before e except after c" or ROY G. BIV, which stands for Red Orange Yellow Green Blue Indigo Violet—the colors of the rain-

bow. Developing your own mnemonics will help you make a personal connection with the material and help you recall it during your test. Mnemonics are also useful when you personalize your "cheat sheet."

- **Personalize your cheat sheet.** Of course, you aren't really going to cheat, but take the Formula Cheat Sheet found on page ix and add to it. Or, highlight the formulas you really need and don't yet know well. This will help them to stand out more than the ones you already know. You can then use the sheet to review—perfect for studying on the go.
- **Outlines and Maps.** If you have pages of notes from your active reading, you can create an outline or map of your notes to review. Both tools help you organize and synthesize the material. Most students are familiar with creating outlines using hierarchical headings, but maps may be less familiar. To make a map, write down the main point, idea, or topic under consideration in the middle of a clean piece of paper. Draw a circle around this main topic. Next, draw branches out from that center circle on which to record subtopics and details. Create as many branches as you need—or as many as will fit on your sheet of paper.

◗ *Studying with Others*

Studying in a group or with another person can be a great motivator. It can also be a distraction, as it can be easy to wander off the subject at hand and on to more interesting subjects such as last night's game, or some juicy gossip. The key is to choose your study partners well and to have a plan for the study session that will keep you on track.

There are definite advantages to studying with others:

- **Motivation.** If you commit to working with someone else you are more likely to follow through. Also, you may be motivated by some friendly competition.
- **Solidarity.** You can draw encouragement from your fellow test takers and you won't feel alone in your efforts. This companionship can help reduce test anxiety.
- **Shared expertise.** As you learned from your practice questions, you have certain strengths and weaknesses in the subject. If you can find a study partner with the opposite strengths and weaknesses, you can each benefit from your partner's strengths. Not only will you get help, but by offering *your* expertise you will build your confidence for the upcoming test.

There are also some disadvantages to studying with others:

- **Stress of competition.** Some study partners can be overly competitive, always trying to prove that they are better in the subject than you. This can lead to stress and sap your confidence. Be wary of the overly competitive study partner.
- **Too much fun.** If you usually hate studying but really look forward to getting together with your best friend to study, it may be because you spend more time socializing than studying. Sometimes it is better to study with an acquaintance who is well-matched with your study needs and with whom you are more likely to stay on task.
- **Time and convenience.** Organizing a study group can take time. If you are spending a lot of time making phone calls and sending e-mails trying to get your study group together, or if you have to travel a distance to meet up with your study partner, this may not be an efficient strategy.

Weigh the pros and cons of studying with others to decide if this is a good strategy for you.

JUST THE FACTS . . . JUST IN TIME

You have thought about the what, where, when, and how; now you need to put all four factors together to build your study plan. Your study plan should be as detailed and specific as possible. When you have created your study plan, you then need to follow through.

◗ *Building a Study Plan*

You will need a daily planner, a calendar with space to write, or a personal digital assistant to build your plan. You have already determined the time you have free for study. Now you need to fill in the details. You have also figured out what you need to study, and have broken the material down into smaller chunks. Assign one chunk of material to each of the longer study sessions you have planned. You may need to combine some chunks or add some review sessions depending on the number of long study sessions you have planned in your schedule.

You can also plan how to study in your schedule. For example, you might write for Monday 6:00 P.M. to 9:00 P.M.: Read Chapter 4, make notes, map notes, and create a set of flashcards. Then for Tuesday 8:30 A.M. to 9:00 A.M. (your commute time): Study Chapter 4 flashcards. The key to a successful study plan is to be as detailed as possible.

Staying on Track

Bear in mind that nothing goes exactly as planned. You may need to stay late at work, you may get a nasty cold, soccer practice may go late, or your child might need to go to the doctor: any number of things can happen to your well-thought-out study plan—and some of them probably will. You will need strategies for coping with life's little surprises.

The most important thing to remember when you get off track is not to panic or throw in the towel. You can adjust your schedule to make up the lost time. You may need to reconsider some of your other commitments and see if you can borrow some time for studying. Or you may need to forego one of your planned review sessions to learn new material. You can always find a few extra minutes here and there for your review.

Minimizing Distractions

There are some distractions, such as getting sick, that are unavoidable. Many others can be minimized. There are the obvious distractions such as socializing, television, and the telephone. There are also less amusing distractions such as anxiety and fear. They can all eat up your time and throw off your study plan. The good news is you can do a lot to keep these distractions at bay.

- **Enlist the help of your friends and family.** Just as you have asked your friends and family to respect your study space, you can also ask them to respect your study time. Make sure they know how important this test is to you. They will then understand that you don't want to be disturbed during study time, and will do what they can to help you stick to your plan.
- **Keep the television off.** If you know that you have the tendency to get pulled into watching TV, don't turn it on, even *before* you plan to study. This way you won't be tempted to push back your study time to see how a program ends or see "what's coming up next."
- **Turn off your cell phone and the ringer on your home phone.** This way you won't eat up your study time answering phone calls—even a five-minute call can cause you to lose focus and waste precious time.
- **Use the relaxation techniques discussed earlier in the chapter if you find yourself becoming anxious while you study.** Breathe, tense and relax your muscles, or visualize a soothing place.
- **Banish negative thoughts.** Negative thoughts—such as, "I'll never get through what I planned to study tonight," "I'm so mad all my friends are at the movies and I'm stuck here studying," "Maybe I'll just study for an hour instead of two so I can watch the season finale

of my favorite show"—interfere with your ability to study effectively. Sometimes just noticing your negative thoughts is enough to conquer them. Simply answer your negative thought with something positive—"If I study the full two hours, I can watch the tape of my show," "I want to study because I want to do well on the test so I can . . . " and so on.

Staying Motivated

You can also get off track because your motivation wanes. You may have built a rock-solid study plan and set aside every evening from 6:00 to 9:00 to study. And then your favorite team makes it to the playoffs. Your study plan suddenly clashes with a very compelling distraction. Or you may simply be tired from a long day at work or school or from taking care of your family and feel like you don't have the energy for three hours of concentrated study. Here are some strategies to help keep you motivated:

- **Visualization.** Remind yourself of what you will gain from doing well on the test. Take some time to visualize how your life will be positively changed if you accomplish your goal. Do not, however, spend time visualizing how awful your life will be if you fail. Positive visualization is a much more powerful motivator than negative imagery.
- **Rewards.** Rewards for staying on track can be a great motivator, especially for flagging enthusiasm. When you accomplish your study goal, perhaps watch your favorite TV program or have a special treat—whatever it is that will motivate you.
- **Positive feedback.** You can use your study plan to provide positive feedback. As you work toward the test date, look back at your plan and remind yourself of how much you have already accomplished. Your plan will provide a record of your steady progress as you move forward. You can also enlist the help of study partners, family, and friends to help you stay motivated. Let the people in your life know about your study plan and your progress. They are sure to applaud your efforts.

At the end of the day, you will be your prime motivator. The fact that you bought this book and have taken the time to create a well-thought out study plan shows that you are committed to your goal. Now all that is left is to go for it! Imagine yourself succeeding on your test and let the excitement of meeting your goal carry you forward.

2

The Integers and Absolute Value

Before you begin learning and reviewing the integers and absolute value, take a few minutes to take this ten-question *Benchmark Quiz*. These questions are similar to the type of questions that you will find on important tests. When you are finished, check the answer key carefully to assess your results. Your Benchmark Quiz analysis will help you determine how much time you need to spend on the integers and absolute value, and the specific areas in which you need the most careful review and practice.

BENCHMARK QUIZ

1. $-5 + 17 =$
- **a.** 22
- **b.** 12
- **c.** –22
- **d.** –12
- **e.** 10

2. $-12 - -14 =$
a. –26
b. 26
c. 0
d. –2
e. 2

3. $-6 + 6 =$
a. –12
b. 0
c. 36
d. 12
e. –36

4. $|4 - 10| =$
a. 6
b. –14
c. 14
d. –6
e. 7

5. $|16 - 3 + 7| =$
a. 26
b. 6
c. 20
d. –26
e. –20

6. $120 \div -5 =$
a. –4
b. 40
c. –40
d. –24
e. –60

7. Find a value for y in the following: $|-7 + y| = 3$
a. –10
b. –4
c. 4
d. 0
e. –3

8. Find the value of y in the following: $16 - y = 20$
a. 4
b. 36
c. –36
d. –4
e. 16

9. $-72 \div -9 =$
a. 8
b. –8
c. 12
d. –12
e. –81

10. Find the value of y in the following: $-6 \times y = -48$
a. –8
b. 16
c. 8
d. –42
e. 42

BENCHMARK QUIZ SOLUTIONS

How did you do on the integers and absolute value Benchmark Quiz? Check your answers here, and then analyze your results to figure out your plan of attack to master these topics.

1. b. –5 plus 17 is the same as taking 17 minus 5. Since 17 is the number with the largest absolute value, the answer is positive, resulting in 12.

2. e. When you subtract, you add the opposite of the second number. This makes the problem –12 plus +14. This is the same as subtracting 12 from 14. The number 14 has the largest absolute value, so the answer is +2.

3. b. When you add any number and its opposite, the result is always 0.

4. a. First, evaluate what is inside the absolute value symbol. When you subtract you add the opposite, so this is 4 plus –10. Adding these two numbers with different signs requires subtracting 4 from 10, resulting in –6. Finally, the absolute value symbol makes the answer +6.

5. c. First, evaluate what is inside the absolute value symbol. Working from left to right, first evaluate 16 minus 3, which results in 13. Then add 13 to 7, resulting in 20. The absolute value symbol, which makes the result positive, has no effect on the answer of 20.

6. d. When you divide a positive number by a negative number, the answer is negative; $120 \div -5 = -24$.

7. c. Since the expression on the left is enclosed in the absolute value symbol, there are two possible answers for the result of the left, that is –3 or 3. Try the answer choices to find a choice that works. Choice **a** becomes $-7 + -10$ resulting in –17, which becomes +17. Choice **b** becomes $-7 + -4$, resulting in –11, which becomes +11. Choice **c** becomes $-7 + 4$, resulting in –3, which becomes +3. Note that a second possible answer would have been +10, which is not a given choice.

8. d. The problem asks what you can subtract from +16 to result in +20. When you subtract you add the opposite. The value for y must be –4, since $16 - -4$ becomes $16 + 4 = 20$.

9. a. A negative number divided by a negative number results in a positive number, which is +8.

10. c. The question asks what you can multiply –6 by to get –48. A negative number times a positive number results in a negative answer, so the choice is +8.

BENCHMARK QUIZ RESULTS

If you answered 8–10 questions correctly, you have a good grasp of integers and absolute value. Read over the chapter, concentrating on the areas where your knowledge is weak. Then, proceed to the quiz at the end of this chapter for additional confirmation of your success.

If you answered 4–7 questions correctly, you need to refresh yourself on these topics. Carefully read through the lesson in this chapter for review and skill building. Pay attention to the sidebars that refer you to more in-depth practice, hints, and shortcuts. Work through the quiz at the end of the chapter to check your progress.

If you answered 1–3 questions correctly, you need help and clarification on the topics in this section. First, carefully read this chapter and concentrate on the sidebars and visual aids that will help with comprehension. Perhaps

you learned this information and forgot; take the time now to refresh your skills and improve your knowledge. Conquering integer arithmetic requires practice. Go to the suggested website in the Extra Help sidebar in this chapter, and do extended practice. You may also want to refer to *Visual Math: See How Math Makes Sense*, Chapter 1: Number Concepts and Properties, published by LearningExpress.

JUST IN TIME LESSON—THE INTEGERS AND ABSOLUTE VALUE

This lesson covers the basics of working with the integers and absolute value.

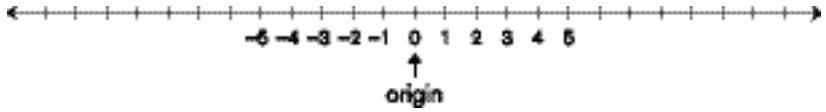
Topics in this chapter include:

- the integers
- absolute value
- operations with signed numbers

GLOSSARY

INTEGERS the set of whole numbers and their opposites, that is . . . −4, −3, −2, −1, 0, 1, 2, 3, 4, . . .

Integers can be shown on a number line where 0, called the origin, is in the middle:



The sign of an integer can be thought of as its direction on the number line. The integer –2 is read as "negative two" and is two units to the left of the origin. The integer 2 is read as "positive two" or simply "two," and is two units to the right of the origin. Occasionally, positive two is shown as +2. It is important to note that the number 0 is neither positive nor negative. Any pair of integers that are the same distance from zero, but are in opposite directions, are called opposites. So the opposite of –4 is 4, and the opposite of 14 is –14.

GLOSSARY

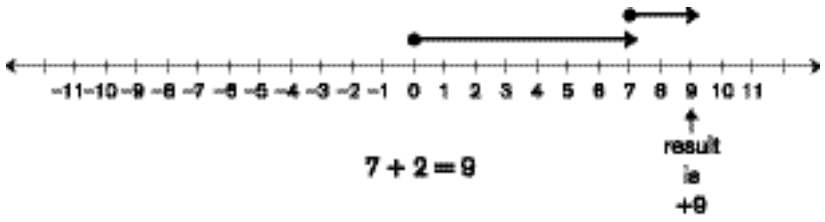
ABSOLUTE VALUE of a number is the distance, or number of units from the origin, on a number line. Absolute value is the magnitude (the size) of the number and is always positive.

The symbol for absolute value is two straight lines surrounding an expression. For example, |–4|, the absolute value of negative 4 is equal to positive 4. Likewise, |4| is also equal to positive 4. Both of these numbers are four units from the origin. Absolute value is concerned with the magnitude (the size) of a number, and ignores its direction. Absolute value is always positive.

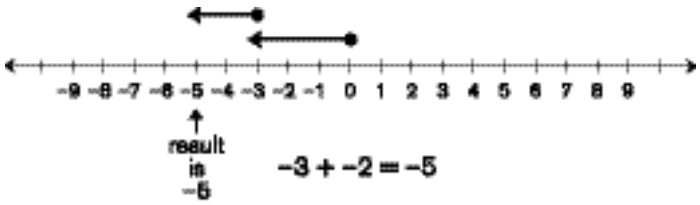
The integers, and all the real numbers, are ordered; keep in mind that –8 is smaller than –6 because –8 is to the left of –6 on a number line. However, the absolute value of –8, |–8|, is larger than |–6| because –8 is eight units from the origin and –6 is only six units from the origin.

ADDITION OF SIGNED NUMBERS

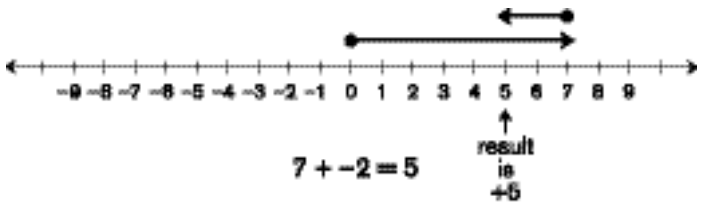
You can understand the addition of integers, or any signed numbers, by using a number line. When you add a positive integer you move to the right, and when you add a negative integer you move to the left. The answer to the problem 7 + 2 is 9, because you start at the origin, move 7 units right, and then move 2 more units right, ending on +9.



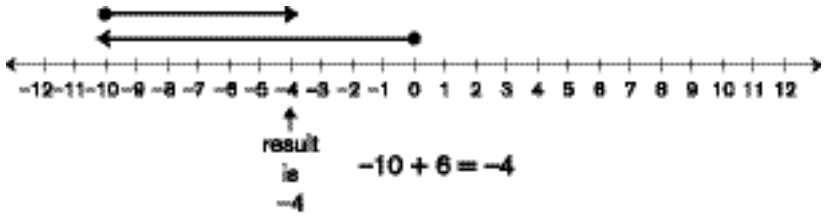
The answer to the problem – 3 + –2 is –5, because you start at the origin, move 3 units to the left, and then move 2 more units to the left, ending on –5.



The answer to the problem 7 + –2 is 5, because you start at the origin, move 7 units to the right, and then move 2 units to the left, ending on +5.



And the answer to the problem –10 + 6 is –4, because you start at the origin, move 10 units to the left, and then move 6 units to the right, ending on –4.



Notice in the examples that adding two numbers with the same sign yields the *sum* of the magnitudes, the absolute values, and the sign stays the same. Adding two numbers with different signs yields the *difference* between the absolute values, and the sign of the number with the largest absolute value will dictate the sign of the answer. You actually subtract absolute values when you add two numbers with different signs. This leads to the special case of when you add two opposites, which always results in an answer of zero.

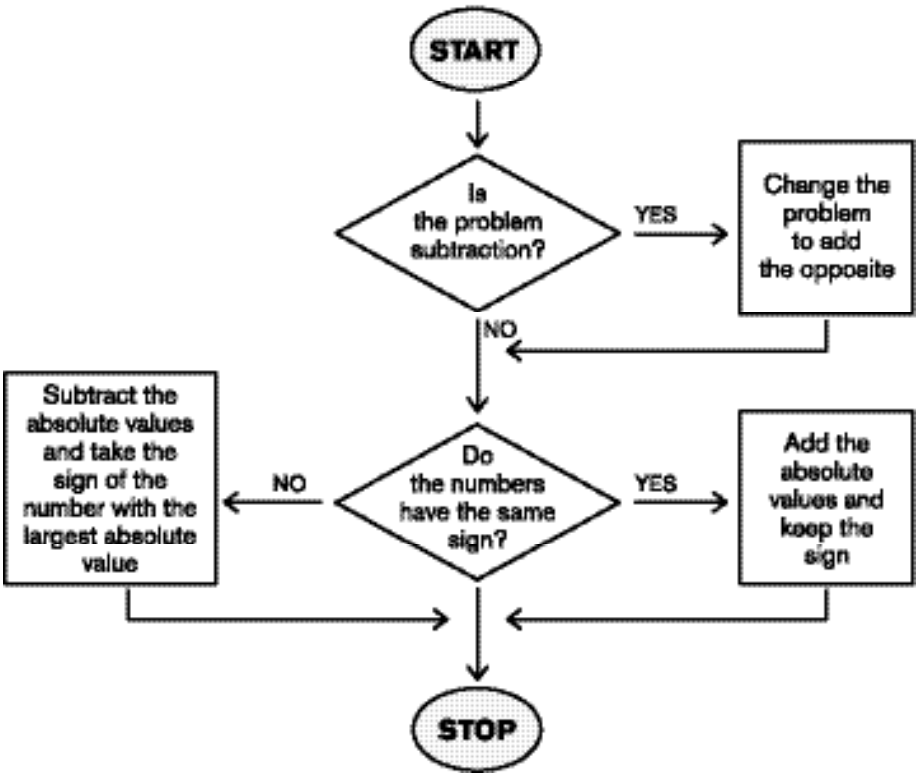
RULE BOOK

Use the flowchart on the next page to learn the rules for adding and subtracting signed numbers.

SUBTRACTION OF SIGNED NUMBERS

It is best to think of subtraction of integers, or subtraction of any signed numbers, as adding the opposite of the second number. For example, if the problem states –10 – 6 you will change this problem to be –10 + –6, and follow the rules for addition of signed numbers. Likewise, 5 – –8 will be changed to 5 + +8, or simply 5 + 8, and again, follow the rules for addition of signed numbers. Learn the rule for adding signed numbers. You will change all subtraction problems to adding the opposite, so all problems involving addition or subtraction of signed numbers become addition problems.

Adding and Subtracting Positive and Negative Numbers



Following are some examples of subtraction using the flowchart:

- Since the problem –7 – 4 is subtraction, you change it to –7 + –4 by adding the opposite. The numbers have the same sign, so you add the absolute values and keep the sign, resulting in –11.
- The problem 4 – –3 is subtraction, so change the problem to add the opposite, that is 4 + +3. This is simply 4 + 3, resulting in 7.

SHORTCUT

When you see a double negative, immediately change that double negative to one positive sign. For example, 6 – –8 simplifies to 6 + 8.

To add or subtract a string of signed numbers, you can pick up pairs of numbers, working from left to right. Then follow the rules given for each pair. For example, to evaluate –5 + 7 – 3, first evaluate –5 + 7. By following

the flowchart, you are adding two numbers with different signs. Subtract the absolute values and take the sign of the number with the larger absolute value, resulting in +2. Then evaluate this result with the next number in the string, that is +2 – 3. Subtract 2 from 3, and take the sign of the –3, since –3 has a larger absolute value. The answer is –1.

MULTIPLICATION AND DIVISION OF SIGNED NUMBERS

To understand the multiplication of integers, recall that 3×4 can be thought of as three groups of +4, which is $4 + 4 + 4 = 12$. So 3×-4 is three groups of –4, or $-4 + -4 + -4$, which is –12. The problem -5×2 is two groups of –5, or $-5 + -5$, which is –10. To understand -4×-5, think of this as the *opposite* of four groups of –5. Four groups of –5 is $-5 + -5 + -5 + -5$, which is –20, and the opposite of –20 is +20. So $-4 \times -5 = 20$.

RULE BOOK

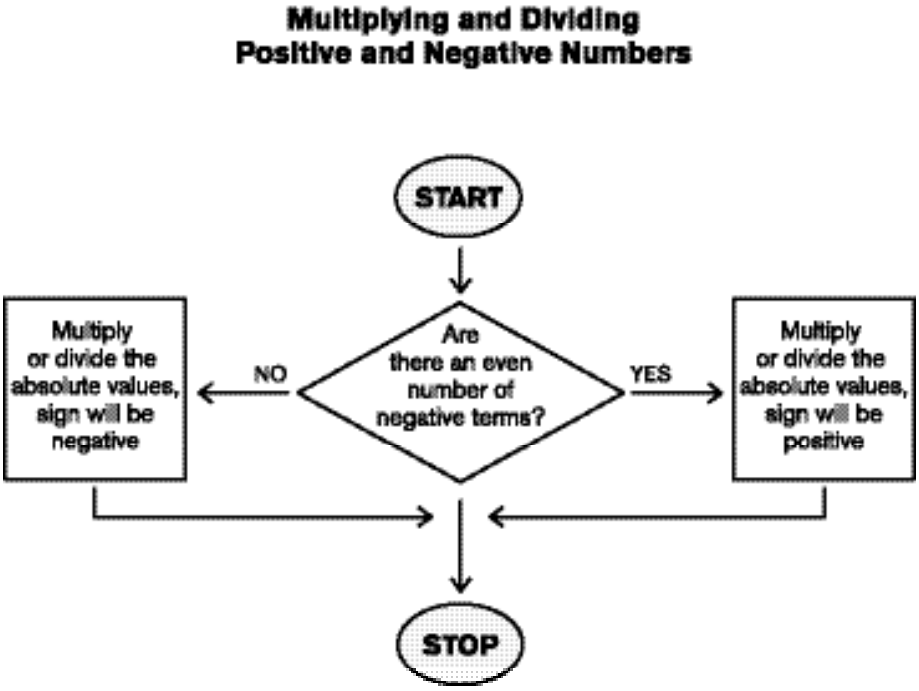
This leads to the rules for multiplying or dividing signed numbers:

positive × positive = positive	**positive ÷ positive = positive**
negative × negative = positive	**negative ÷ negative = positive**
positive × negative = negative	**positive ÷ negative = negative**
negative × positive = negative	**negative ÷ positive = negative**

When you multiply more than two signed numbers together, it is helpful to think of multiplying signed numbers by counting the amount of negative terms in the problem. If there are an even number of negative terms (remember that zero is an even number), the result is positive. If there are an odd number of negative terms, the result is negative. This can save you valuable time on a multiple choice test, in that you do not have to apply the above rule over and over for pairs of numbers. For example, if you are given the problem $-12 \times -5 \times 3 \times 10$, you count the number of negative terms. In this case there are two negative terms, so the result must be positive and you can eliminate any negative answer choices. If you are given the problem $9 \times -7 \times -100 \times -2$, you count the number of negative terms. There are three negative terms, so the answer must be negative, and you can eliminate any positive answer choices. In both examples, the sign of the result is obtained just by inspection. All that remains to solve the problem is to do the multiplication without regard to the sign, then to take the sign obtained by inspection. When you divide signed numbers, you follow the same reasoning that you use for multiplication.

RULE BOOK

Use this flowchart to learn the rules for multiplying or dividing signed numbers:



Take a moment to look at the flowcharts. As you can see at a glance, the multiplication and division rules are much easier to remember than addition and subtraction. It is a good idea to be proficient in integer arithmetic, in both speed and accuracy. The best way is to practice. It is just like learning to ride a bicycle. At first it seems so difficult, and then with practice you are riding without even thinking. As you are starting out with your review, follow the flowchart with each problem. Soon the flowchart will become second nature to you.

EXTRA HELP

The website *www.aaamath.com* is a good source for practicing speed and accuracy with the basic integer operations. Under Math Topics choose *Addition*, and then scroll down to select *One-Digit Integers*, or *Adding Three Integers*. Then click on *Play* and *Countdown*. Electronic flashcards will appear, and the program will keep track of your accuracy.

EXTRA HELP

Another interesting website, *http://matti.usu.edu/nlvm/nav/index.html*, has a game, *Circle 0,* for practicing one-digit integer arithmetic. Click on *Virtual Library*. Then, click on the *9–12* box in the *Numbers & Operations* row. Click on *Circle 0,* and play the integer game.

Many calculators can do positive and negative arithmetic. It is a good idea for you to be able to do this arithmetic mentally, but following is the key sequence for most calculators when doing positive and negative arithmetic.

CALCULATOR TIPS

If your calculator has the positive/negative key, then it will perform positive and negative arithmetic. Most calculators have you enter the sign of a negative number after you enter the magnitude. So −8 would be entered as [8] [+/−]. To perform the problem −3 − −6, you would use the key sequence: [3] [+/−] [−] [6] [+/−] [=] to get the result of 3.

Check your calculator with this tip sheet.

OPERATIONS WITH ABSOLUTE VALUE

When you evaluate expressions involving absolute value, first perform all arithmetic inside the absolute value symbol and then make the answer positive. The absolute value function is performed last. For example, $|-6 - 2|$ becomes $|-6 + -2|$ which becomes $|-8| = 8$. Another example is $|-10 + 4 - 3| = |-6 - 3| = |-6 + -3| = |-9| = 9$.

MISSING TERMS PROBLEMS

Sometimes, you will be asked to fill in a missing term in the form of a variable, as in the problem: What is the value of *y:* $-5 + y = 5$? Think: "What would you add to negative 5 to yield an answer of positive 5?" The variable *y* must be a positive number greater than 5 such that the difference between the number and 5 yields 5. The answer is 10. Another example is with absolute value: What is the value of *y:* $|y + -14| = 10$? The expression, enclosed in the absolute value symbol, must be either –10 or +10. Find a number from the answer choices that when added to –14 yields either –10 or +10. The answer would be either +4 or +24, since $4 + -14 = -10$ and $24 + -14 = 10$.

EXTRA HELP

This type of practice can also be done at the website *www.aaamath.com* by choosing, under Math Topics, *Addition* and then scroll down to select *One-Digit Integer Equations.* Click on *Play* and then *Countdown* for electronic flashcards.

TIPS AND STRATEGIES

When working with signed numbers, remember:

- Absolute value is always positive.
- When you subtract, you add the opposite.
- To add two numbers with the same sign, add the magnitudes and keep the sign.
- To add two numbers with different signs, find the difference of the numbers, and take the sign of the number with the largest absolute value.
- To multiply or divide two numbers with the same sign, the answer is positive.
- To multiply or divide two numbers with different signs, the answer is negative.
- Memorize and work with the flowcharts until they are automatic.

When you are taking a multiple-choice test, remember these tips to improve your score:

- If the problem is all numbers enclosed by the absolute value symbol, immediately eliminate any negative answer choices.
- If multiplying a string of integers, count the number of negative terms. If there are an even number of negative terms, the result will be positive. If there are an odd number of negative terms the answer will be negative.

PRACTICE

Following is additional practice on the integers and absolute value. When you have finished, check to see if you have mastered this concept.

1. $-9 + 22 =$
- **a.** 31
- **b.** 30
- **c.** –13
- **d.** –30
- **e.** 13

2. $-7 - -36 =$
- **a.** 43
- **b.** –43
- **c.** 29
- **d.** 48
- **e.** –29

3. $-16 + 16 =$
- **a.** 32
- **b.** –32
- **c.** 1
- **d.** 0
- **e.** 256

4. $180 \div -60 =$
- **a.** –30
- **b.** 3
- **c.** 30
- **d.** –3
- **e.** 120

5. $15 \times -3 =$
- **a.** –45
- **b.** –15
- **c.** 45
- **d.** –12
- **e.** 12

6. $-225 \div -25 =$
a. 8
b. 9
c. –8
d. –9
e. 5

7. $13 - -5 =$
a. 18
b. 8
c. 65
d. –8
e. –18

8. $-56 \div 8 =$
a. 9
b. 7
c. –8
d. –9
e. –7

9. $-9 \times 8 =$
a. 1
b. 72
c. –63
d. –72
e. –1

10. $|12 - 23| =$
a. –11
b. 35
c. 11
d. –35
e. 1

11. $|7 - 19 + 5| =$
a. 31
b. –7
c. –31
d. 7
e. 21

12. $|-16 \times 3 \times 2|$ =
a. –11
b. –24
c. 96
d. 24
e. –96

13. $|-20 + 3 - -6|$ =
a. 23
b. –29
c. 11
d. 29
e. –11

14. $|-2 - -7|$ =
a. 9
b. 5
c. –9
d. 14
e. –5

15. $|-13 - 12 + 25|$ =
a. 0
b. 1
c. 50
d. –50
e. 40

16. $|-12 \times 10|$ =
a. 120
b. 2
c. –120
d. –2
e. 22

17. Find the value of *y:* $20 + y = 12$
a. 32
b. 8
c. –18
d. –32
e. –8

18. Find a value for y: $|-7 + y| = 1$
a. –8
b. 7
c. 6
d. –7
e. –17

19. Find the value of y: $-49 \div y = 7$
a. 7
b. 8
c. –8
d. –343
e. –7

20. Find the value of y: $16 - y = 30$
a. 14
b. –46
c. 18
d. –14
e. 46

21. Find a value of y: $|y - 17| = 12$
a. –30
b. 5
c. –5
d. –29
e. 30

22. Find the value of y: $-12 \times y = 144$
a. 132
b. 12
c. –11
d. –12
e. 156

23. Find the value of y: $8 + y = -19$
a. –11
b. –152
c. –27
d. 11
e. 27

24. Find the value of *y*: $y \div 11 = -11$
a. –1
b. 22
c. 121
d. –22
e. –121

25. Find a value of *y*: $|20 - y| = 6$
a. 26
b. 13
c. 15
d. –13
e. –26

ANSWERS

Here are the answers and explanations to the chapter quiz. Read over the explanations carefully for any problems that you answered incorrectly. For more information and practice on working with integers and absolute value, see *Visual Math: See How Math Makes Sense*, Chapter 1: Number Concepts and Properties, published by LearningExpress.

1. e. When adding two numbers with different signs, subtract the numbers, and take the sign of the number with the larger absolute value. The difference between 22 and 9 is 13.

2. c. When you subtract, first change the problem to add the opposite. The problem becomes –7 + +36. When adding two numbers with opposite signs, take the difference of the numbers, which results in 29. The result is +29, since +36 was the number with the largest absolute value.

3. d. The sum of any number and its opposite is always zero.

4. d. When you divide a positive by a negative, the answer is negative, resulting in –3.

5. a. When you multiply a positive by a negative, the answer is negative, resulting in –45.

6. b. Dividing a negative by a negative yields a positive result; –225 divided by –25 is +9.

7. a. When you subtract, you add the opposite. The problem becomes 13 + 5, which is 18.

8. e. A negative number divided by a positive number yields a negative number; –56 divided by 8 is –7.

9. d. A negative number multiplied by a positive number has a negative result; –9 times 8 is –72.

10. c. First, evaluate what is inside the absolute value symbol. When you subtract, you add the opposite, which will be 12 + –23. Take the difference between 23 and 12, which is 11, and the sub answer is |–11|, since –23 is the number with the largest absolute value. Finally, the absolute value symbol makes the answer +11.

11. d. Simplify the expression within the absolute value symbol, and then make the final answer positive. Evaluate from left to right; 7 – 19 is the same as 7 + –19. Subtract 7 from 19 to get 12, which will be –12, since the 19 is negative. Now evaluate –12 + 5. This difference is –7, but remember the final answer will be positive because of the absolute value symbol.

12. c. Perform all arithmetic within the symbols first, and then make the final answer positive. Since there are an odd number of negative factors, namely one, the answer inside the symbols will be negative. This will not really matter, however since at the end you will make the answer positive because of the absolute value symbol. It is enough to just multiply 16 times 3 times 2, resulting in 96.

13. c. Perform all operations within the absolute value symbols first, and then make the final answer positive; –20 + 3 results in –17, since 20 – 3 is 17, and the answer is negative since –20 has the largest absolute value. Now evaluate –17 – –6. Change the problem to –17 + 6, which yields –11. Find the absolute value of –11. The final answer is +11 or simply 11.

14. b. First, evaluate the expression inside the absolute value symbols. Since this is a subtraction problem, first change it to add the opposite. The problem becomes +2 plus +7. Adding two numbers with different signs dictates that we take the difference, which is 5, and keep the sign of the number with the largest absolute value. In any case, the final answer is +5, since the expression is surrounded by the absolute value symbol.

15. a. First, simplify the expression that is inside the absolute value symbols. Work from left to right, and start by doing the subtraction –13 minus 12. This changes to –13 + –12. When you add two numbers with the same sign, just add the numbers and keep the sign. Now the problem is reduced to –25 + 25. Recall that adding any number and its opposite results in zero.

16. a. Since the entire problem is surrounded by the absolute value symbol, the answer must be positive. Just multiply the two numbers, and the answer is the positive product of these numbers.

17. e. To solve this problem, think *what can be added to positive 20 to result in 12?* It must be a negative number, and must be the difference between 20 and 12. The answer is –8.

18. c. The expression inside the absolute value must be either 1 or –1 for the problem to be true. Check the answer choices to find one that works. Choice **a** results in |–15|, which would be 15, and it does not work. Choice **b** would result in 0, and does not work. Choice **c** works, since –7 plus 6 results in –1, the difference between the numbers. The absolute value of –1 is 1. You may have realized that a second possible solution would be +8, which is not a choice.

19. e. To answer this problem, think *what can I divide negative 49 by to get positive 7?* The answer must be negative, since a negative times a negative will yield the positive result; –49 divided by –7 is +7.

20. d. Think: *what number can I subtract from positive 16 to get positive 30?* The answer cannot be a positive number, since 16 is less than 30. The answer will be –14, since the difference between 30 and 16 is 14.

21. b. To tackle this problem, the best approach is to try the answer choices, knowing that you are looking for a result inside the absolute value symbols of either 12 or –12. Trying choice **a** yields |–47|, since you would be adding two negative numbers. This cannot give the needed answer of 12. Trying choice **b** will give a desired answer of |–12| since the difference between 5 and 17 is –12. The absolute value will force the answer to be +12, which equals the right side of this equation. Note that a second solution of +29 would also work, but this number is not one of the choices.

22. d. Think: *what number can I multiply negative 12 by to get positive 144?* The correct answer must be negative, since you must multiply a negative by a negative to get a positive result; –12 times –12 equals 144.

23. c. To solve this problem, consider: *what can I add to a positive 8 to result in negative 19?* It must be a negative number, such that the difference between this number and 8 is 19. That would be 27, so the correct choice is –27.

24. e. One way to solve this problem is to think: *what number*, *when divided by positive 11*, *gives an answer of negative 11?* Remember that a negative divided by a positive is a negative result. Recall that 11 times 11 is 121, so the answer is –121.

25. a. The easiest way to solve this type of problem is to realize that the result inside the absolute value symbols must be either a positive 6 or a negative 6. Try the answer choices to find one that works. Choice **a** works, since 20 minus 26 is the same as 20 plus –26, and the difference between 20 and 26 is 6. Note that a second solution could have been 14, which is not a choice.

Properties of Numbers

Before you begin learning and reviewing the properties of numbers, take a few minutes to take this ten-question *Benchmark Quiz*. These questions are similar to the type of questions that you will find on important tests. When you are finished, check the answer key carefully to assess your results. Your Benchmark Quiz analysis will help you determine how much time you need to spend on the properties of numbers, and the specific areas in which you need the most careful review and practice.

BENCHMARK QUIZ

1. 10 – 5 + 2 =
a. 3
b. –3
c. –7
d. 7
e. 10

2. $32 - 40 \div 4 =$
a. –2
b. –18
c. 22
d. 12
e. 42

3. $7 + \frac{4+52}{4+3} =$
a. 15
b. 14
c. $24\frac{1}{3}$
d. 44
e. $\frac{63}{7}$

4. Which choice shows the prime factorization of 60?
a. $3 \times 4 \times 5$
b. 6×10
c. $6 \times (8 + 2)$
d. 1×60
e. $2 \times 2 \times 3 \times 5$

5. Find the greatest common factor of 20 and 30.
a. 60
b. 20
c. 5
d. 600
e. 10

6. Find the least common multiple of 40 and 50.
a. 10
b. 5
c. 100
d. 200
e. 2,000

7. Which choice shows an example of the distributive property?
a. $7 \times (8 \times 6) = (7 \times 8) \times 6$
b. $7(40 + 8) = 7 \times 40 + 7 \times 8$
c. $7 \times 48 = 48 \times 7$
d. $7(40 + 8) = 7 \times 20 + 7 \times 28$
e. $7 \times 40 + 8 = 7 \times (40 + 8)$

8. $100 \div 5 + |-5 \times 3| =$
a. 35
b. 75
c. 5
d. 45
e. $33\frac{1}{3}$

9. Which choice shows an example of the associative property?
a. $12 + (49 + 51) = (49 + 51) + 12$
b. $49 + (51 + 12) = (49 + 51) + 12$
c. $49 + (51 + 12) = 49 + 51 + (49 + 12)$
d. $12 + 49 + 51 = 51 + 49 + 12$
e. $12(49 + 51) = 12 \times 49 + 12 \times 51$

10. Which example shows the commutative property?
a. $29 \times 7 \times 21 = (29 \times 7) \times 7 \times 21$
b. $(29 \times 7) \times 21 = 29 \times (7 \times 21)$
c. $29 \times 7 \times 21 = 29 \times 21 \times 7$
d. $29 \times 7 \times 21 = 203 \times 21$
e. $29(7 + 21) = 29 \times 7 + 29 \times 21$

BENCHMARK QUIZ SOLUTIONS

How did you do on the properties of numbers Benchmark Quiz? Check your answers here, and then analyze your results to figure out your plan of attack to master these topics.

1. d. For the order of operations, addition and subtraction are done left to right. First subtract 5 from 10, and then add 2 to get a result of 7.

2. c. Division is done before subtraction for the order of operations. First, divide 40 by 4, and then subtract this number from 32.

3. a. A fraction bar acts as a grouping symbol, so this problem becomes $7 + (4 + 52) \div (4 + 3)$. Evaluate the parentheses first: $7 + 56 \div 7$. Next, perform the division, and $56 \div 7 = 8$. Finally, $7 + 8 = 15$.

4. e. All of the choices show examples of factors that produce 60, but choice **e** is the only factorization showing all prime numbers.

5. e. $20 = 2 \times 2 \times 5$, and $30 = 2 \times 3 \times 5$. Two and five are prime factors of both 20 and 30, so the greatest common factor is two times five, which is ten.

6. d. List the multiples of 40 and 50 until one is found in common. Multiples of 50 are: 50, 100, 150, 200, 250 . . . and multiples of 40 are: 40, 80, 120, 160, 200, 240, and so on. A common multiple is found, namely 200. An alternate approach to finding the least common multiple is explained later in the chapter.

7. b. The distributive property states that multiplication distributes over addition or subtraction. An illustration of this is choice **b.**

8. a. The absolute value symbol serves as a grouping symbol, and grouping symbols are evaluated first; $|-5 \times 3| = 15$. Now divide 5 into 100 to get 20. Finally, add $20 + 15 = 35$.

9. b. Choice **b** shows an example of the associative property, which states that when all operations in an expression are addition (or multiplication) you can change the grouping symbols to get the same result.

10. c. The commutative property states that when all operations are multiplication (or addition) you can change the order of the operands to get the same result.

BENCHMARK QUIZ RESULTS

If you answered 8–10 questions correctly, you have a good understanding of the properties of numbers. Perhaps the questions you answered incorrectly deal with one specific area in this chapter. Read over the chapter, concentrating on those areas of weakness. Proceed to the chapter assessment to try to improve your score.

If you answered 4–7 questions correctly, there are several areas you need to review. Carefully read through the lesson in this chapter for review and skill building. Work carefully through the examples and pay attention to the sidebars that refer you to definitions, hints, and shortcuts. Get additional practice on the properties of numbers by taking the quiz at the end of the chapter.

If you answered 1–3 questions correctly, you need to spend some time on the topics in this chapter. You may be tested specifically on these topics for your upcoming examinations or, more important, you need a solid under-

standing of these properties in order to solve various kinds of math problems. First, carefully read this chapter and concentrate on the sidebars and visual aids that will help with comprehension. Go to the suggested website in the Extra Help sidebar in this chapter, which will help with understanding and will provide extended practice. You may also want to refer to *Visual Math: See How Math Makes Sense*, Chapter 1: Number Concepts and Properties, published by LearningExpress.

JUST IN TIME LESSON—PROPERTIES OF NUMBERS

The topics in this chapter are:

- the order of operations
- basic properties of numbers
- factors and multiples of numbers

THE ORDER OF OPERATIONS

The basic operations of real numbers include addition, subtraction, multiplication, division, and exponentiation (discussed in Chapter 7 of this book). Often, in expressions, there are grouping symbols—usually shown as parentheses—which are used to make a mathematical statement clear. In math, there is a pre-defined order in which you perform operations. This agreed-upon order that must be used is known as the order of operations.

RULE BOOK: THE ORDER OF OPERATIONS

- **First, perform all operations enclosed in parentheses.**
- **Second, evaluate all exponents.**
- **Third, perform any multiplication and division, in order, working from left to right.**
- **Finally, evaluate any addition or subtraction, in order, working from left to right.**

SHORTCUT

The order of operations rules can be remembered by the visual aid:

P **parentheses**
E **exponents**
MD **multiplication and division**
AS **addition and subtraction**
or by the verbal aid that you may recall from your school days: Please Excuse My Dear Aunt Sally

Examples:

$8 + 15 \times 3$	There are no parentheses or exponents, so evaluate multiplication first.
$8 + 45$	Now perform the addition.
53	

$7 + 24 \div 6 \times 10$	There are no parentheses or exponents, so evaluate multiplication and division from left to right. First, do the division.
$7 + 4 \times 10$	Next, perform multiplication.
$7 + 40$	Finally, perform addition.
47	

$(36 + 64) \div (18 - 20)$	First, evaluation parentheses, from left to right.
$100 \div -2$	Now, do the division.
-50	

In Chapter 2, the absolute value operation was reviewed. For order of operations, the absolute value symbol is treated at the same level as parentheses.

Example:

$5 \times \lvert -13 + 3 \rvert$	First, evaluate the expression inside the absolute value symbol.
$5 \times \lvert -10 \rvert$	Second, evaluate the absolute value.
5×10	Now, perform the multiplication.
50	

One important distinction to note is that a fraction bar, which serves as a division symbol, is also treated as a grouping symbol.

Example:

$\frac{7 \times 10}{2 + 8}$ Evaluate the numerator, then evaluate the denominator, and then perform the division at the end.

$(7 \times 10) \div (2 + 8)$ Evaluate the parentheses from left to right.

$70 \div 10$ Now divide.

7

EXTRA HELP

For further information on the order of operations, refer to *Practical Math Success in 20 Minutes a Day,* Lesson 20, Miscellaneous Math, published by LearningExpress.

CALCULATOR TIPS

If your calculator has parentheses keys, then it most likely will perform the correct order of operations. Check your calculator with these examples to see if it performs the correct order of operations. To evaluate 16 − 100 ÷ 5, enter: [1] [6] [−] [1] [0] [0] [÷] [5] [=]. Your calculator should show a result of −4.

Here is how to enter an expression with parentheses. To evaluate 48 ÷ (4 + 2), enter: [4] [8] [÷] [(] [4] [+] [2] [)] [=]. Your calculator should show a result of 8.

THE PROPERTIES OF NUMBERS

Sometimes it is convenient to change the order of operations. The real numbers share some properties with which you should be familiar. These properties allow you to change the rules for the order of operations. They can be used to increase speed and accuracy when doing mental arithmetic. These properties are also used extensively in algebra, when solving equations.

Two properties, the commutative and associative properties, deal with expressions that involve a string of all addition operations, or a string of all multiplication operations. These properties are for *addition* and *multiplication* only.

GLOSSARY

COMMUTATIVE PROPERTY states that when performing a string of addition operations, or a string of multiplication operations, the *order* does not matter. In other words, ***a + b = b + a.***

Recall that the order of operations directs you to add or multiply working from left to right. When you balance your checkbook, and have to add up a string of outstanding checks, list them all and use the commutative property to arrive at the total. Then, change the order of addends to add pairs whose unit (ones) digit adds to ten.

Example:

To add	17 + 64 + 35 + 43 + 96	Change the order.
	17 + 43 + 64 + 96 + 35	Add 17 + 43 first, since 7 + 3 = 10.
	60 + 64 + 96 + 35	Now, change the order.
	64 + 96 + 60 + 35	Add 64 + 96 next, since 4 + 6 = 10.
	160 + 60 + 35	Work left to right.
	220 + 35	
	255	

In the same way, the commutative property is helpful when multiplying several numbers or terms. Change the order to find pairs of numbers whose product would be 10, 100, or 1,000.

Example:

To multiply:	$4 \times 2 \times 70 \times 50 \times 25$	Change the order.
	$4 \times 25 \times 2 \times 70 \times 50$	Multiply 4 and 25 first, since $4 \times 25 = 100$.
	$100 \times 2 \times 70 \times 50$	Now change the order.
	$2 \times 50 \times 100 \times 70$	Multiply 2 and 50 together, since $2 \times 50 = 100$.
	$100 \times 100 \times 70$	Finish left to right.
	700,000	

GLOSSARY

ASSOCIATIVE PROPERTY is used when grouping symbols are present. This property states that when you perform a string of addition operations, or all multiplication operations, you can change the *grouping*. In other words, $(a \times b) \times c = a \times (b \times c)$.

Example:

19 + (7 + 16) + 34	Change grouping to add 16 and 34 first, since 6 + 4 = 10.
19 + 7 + (16 + 34)	Evaluate parentheses.
19 + 7 + 50	Finish, working left to right.
26 + 50	
76	

Example with multiplication:

$15 \times (8 \times 20) \times 5$	Change grouping to multiply 20 and 5 first, since $20 \times 5 = 100$.
$15 \times 8 \times (20 \times 5)$	Evaluate parentheses first.
$15 \times 8 \times 100$	Finish, working left to right.
12,000	

Another useful property is the distributive property. This property deals with two operations, multiplication and addition, or multiplication and subtraction. Recall that 5(12 + 8) means "five times the quantity twelve plus eight."

GLOSSARY

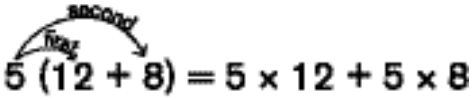
DISTRIBUTIVE PROPERTY states that multiplication distributes over addition or subtraction.

Example:
The expression $5(12 + 8) = 5(20) = 100$ could also be evaluated as $5(12 + 8) = 5 \times 12 + 5 \times 8 = 60 + 40 = 100$.

Example:
$9(x - 2) = 9x - 18$

SHORTCUT

Use this visual image to remember how to work with the distributive property:



This property can make mental multiplication easier.

Example:

17×5	Knowing that $17 = 7 + 10$:
$5(10 + 7)$	Use the distributive property.
$5 \times 10 + 5 \times 7$	Follow the order of operations.
$50 + 35$	
85	

These properties are used extensively in algebra when you solve equations, so an understanding of how the properties work is essential to understanding mathematics.

FACTORS AND MULTIPLES

GLOSSARY

WHOLE NUMBERS the counting numbers and 0, that is 0, 1, 2, 3, 4, . . .
FACTOR of a number is any whole number that divides evenly, without remainder, into the given number.
MULTIPLE of a number is the product of any whole number multiplied by the given number.
PRIME NUMBER a whole number, excluding 1 and 0, whose only factors are 1 and the number itself.
COMPOSITE NUMBER a whole number, excluding 1 and 0, that has more factors than 1 and the number itself.

It is helpful to remember that factors are less than, or equal to, the given number. Multiples start at 0, followed by the number itself, and then all other multiples are greater than the number. There are a finite number of factors, but an infinite number of multiples. To find the greatest common factor, list out all of the factors and find the largest one in common. For the least common multiple, list all of the multiples, starting with the number in question, until you find the first multiple that the numbers have in common.

Examples:
Find the greatest common factor of 63 and 81.
Factors of 63: 1, 3, 7, 9, 21, 63
Factors of 81: 1, 3, 9, 27, 81
All factors have been listed. The largest factor in common between 63 and 81 is 9.

Find the least common multiple of 63 and 81.
Multiples of 63: 63, 126, 189, 252, 315, 378, 441, 504, 567
Multiples of 81: 81, 162, 243, 324, 405, 486, 567
Multiples are listed until one is found in common. The least common multiple of 63 and 81 is 567.

SHORTCUT

The greatest common factor is abbreviated as the GCF, and the least common multiple is abbreviated as the LCM, or when working with fractions, the least common (multiple of the) denominators, the LCD.

It can be time consuming to list all multiples until one is found in common. There is a more efficient way to find the least common multiple and greatest common factor. This method is based on the most important and basic idea about whole numbers: The Fundamental Theorem of Arithmetic.

GLOSSARY

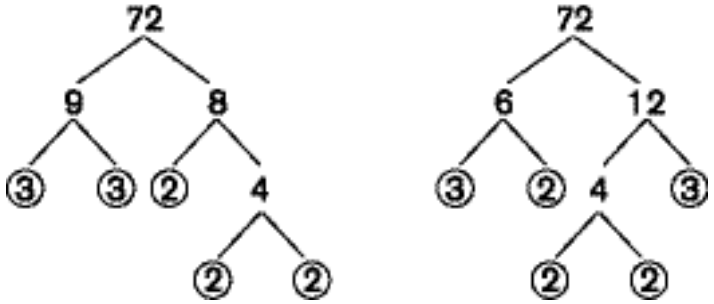
THE FUNDAMENTAL THEOREM OF ARITHMETIC states that every whole number greater than 1 is the product of prime factors. Furthermore, these prime factors are unique, and there is exactly one set of prime factors.

Because any number can be broken down into prime factors in exactly one unique way, you can use this fact to find the GCF and the LCM quickly, using these prime factors. It is helpful to use a tree diagram to find the prime factorization.

Examples:

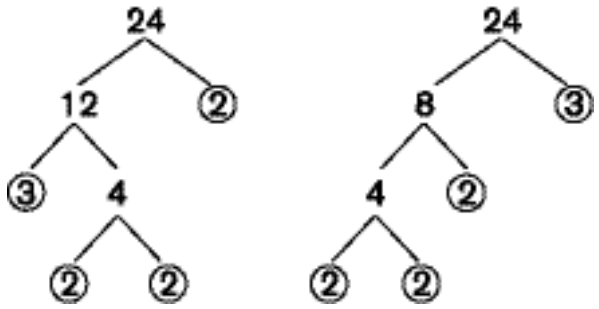
Find the greatest common factor and the least common multiple for 72 and 24.

Find the prime factorization of 72. Circle all prime factors.



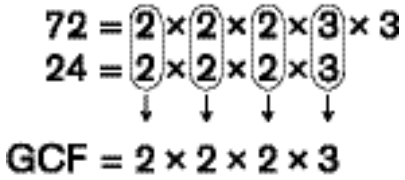
Notice that even though two factor trees are illustrated, the resultant set of prime factors is the same. The prime factorization of 72 is: $2 \times 2 \times 2 \times 3 \times 3$.

Likewise, find the prime factorization of 24:



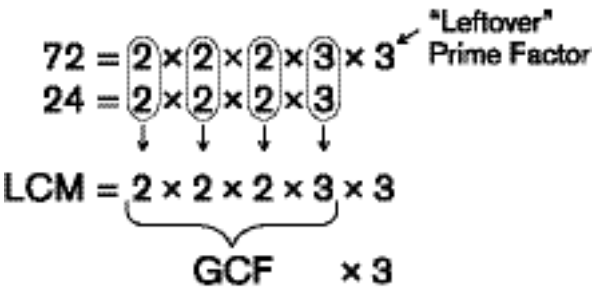
The prime factorization of 24 is: $2 \times 2 \times 2 \times 3$.

To find the greatest common factor of 72 and 24, list the prime factorizations of each number and then pair up common prime factors. Multiply these common prime factors:



From the illustration, the greatest common factor of 72 and 24 is $2 \times 2 \times 2 \times 3 = 24$.

After all common primes have been paired, multiply all "leftover" unpaired primes to the greatest common factor. This will be the least common multiple.

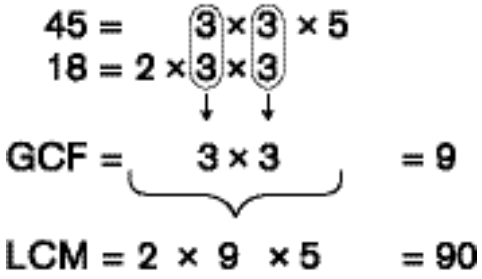


The least common multiple is the GCF $\times$ 3, which is $24 \times 3 = 72$.

Find the greatest common factor and the least common multiple for 45 and 18:

Find the prime factorization for 45 and 18:

Pair up the common prime factors. Multiply them together for the greatest common factor. Now, multiply the GCF with all of the unpaired primes left over to find the least common multiple:



The GCF of 45 and 18 is 9, and the LCM is 90.

You may need to answer questions directly about the GCF or the LCM, or you will need to find these numbers in your work with fractions and algebra.

EXTRA HELP

The website *http://matti.usu.edu/nlvm/nav/index.html* is very instructive when working with the GCF and the LCM. You can see interactively how to produce a factor tree, and to then find the GCF and the LCM. Click on *Virtual Library*. Click on the *9–12* box in the row labeled *Numbers & Operations.* Click on *Factor Tree.* Follow the instructions for the factor tree game.

TIPS AND STRATEGIES

Here are some highlights of the properties of numbers:

- The order of operations is to first evaluate parentheses, then exponents, then multiplication and division, left to right, and finally addition and subtraction, left to right.
- The order of operations can be remembered as PEMDAS, or Please Excuse My Dear Aunt Sally.
- The properties of numbers enable you to change the order of operations.
- The commutative property deals with the ORDER of the terms.
- The associative property deals with the GROUPING of the terms.
- The distributive property deals with multiplication and addition, or multiplication and subtraction.
- To find the prime factorization of a number, use a factor tree.
- To find the GCF or the LCM of a set of numbers, use the prime factorization.

PRACTICE

For additional practice, take this final assessment of the properties of numbers.

1. $25 + 15 \times 3 =$
a. 120
b. 30
c. 15
d. 70
e. $13\frac{1}{3}$

2. $8 - 10 \div 2 =$
a. –1
b. 1
c. 3
d. –3
e. 9

3. $5 \times (6 + 19) =$
a. 49
b. 125
c. 115
d. 5
e. –5

4. $12 + \frac{144}{8 + 4} =$
a. 24
b. 16
c. 144
d. 32
e. 34

5. $120 \div 5 \times -2 =$
a. 12
b. –50
c. –48
d. 48
e. 50

6. Which example shows the prime factorization of 90?
a. 9×10
b. 90×1
c. $2 \times 3 \times 3 \times 5$
d. $2 \times 5 \times 9$
e. $3 \times 3 \times 10$

7. Which choice shows an example of the distributive property?
a. $5 \times 27 = 27 \times 5$
b. $5(20 + 7) = 5 \times 20 + 5 \times 7$
c. $5 \times (9 \times 3) = (5 \times 9) \times 3$
d. $5(20 + 7) = 2 \times 15 + 5 \times 7$
e. $5 \times 20 + 7 = 5 \times (20 + 7)$

8. What is the greatest common factor of 48 and 120?
a. 12
b. 2
c. 240
d. 3
e. 24

9. What is the least common multiple of 20 and 30?
a. 20
b. 30
c. 600
d. 60
e. 10

10. $35 - 5 + 7 =$
a. 23
b. 14
c. 47
d. 22
e. 37

11. What is the GCF of 72 and 180?
a. 36
b. 9
c. 72
d. 360
e. 4

12. What is the LCM of 72 and 180?
a. 12,960
b. 72
c. 9
d. 360
e. 36

13. Which choice below shows an example of the commutative property?
a. $17 + 24 + 3 = 17 + (20 + 4) + 3$
b. $17 + 24 + 3 = 17 + 3 + 24$
c. $(17 + 24) + 3 = 17 + (24 + 3)$
d. $17 + 24 + 3 = 41 + 3$
e. $17(24 + 3) = 17 \times 24 + 17 \times 3$

14. $48 \div 4 - (4 \times 3) =$
a. –6
b. 24
c. 60
d. –24
e. 0

15. Which choice below shows an example of the associative property?
a. $20 \times (50 \times 8) = (20 \times 50) \times 8$
b. $20 \times (50 \times 8) = (50 \times 8) \times 20$
c. $20 \times (50 \times 8) = 20 \times 50 + 20 \times 8$
d. $20 \times 50 \times 8 = 50 \times 8 \times 20$
e. $20 \times (50 + 8) = 20 \times 50 + 20 \times 8$

16. Which example shows the prime factorization of 78?
a. 1×78
b. 2×39
c. $78 \times 2 = 156$
d. $2 \times 3 \times 13$
e. $70 + 8$

17. $\frac{10 + 7 \times 2}{2 + 4} =$
a. 24
b. 4
c. $4\frac{1}{4}$
d. 6
e. $5\frac{2}{3}$

18. $17 - |-10 + 3| =$
a. 30
b. 24
c. 4
d. 10
e. –13

19. What is the LCM of 27 and 90?
a. 90
b. 270
c. 9
d. 27
e. 3

20. $10 + 48 \div 8 \times 5 =$
a. 80
b. 11.2
c. 36.25
d. 35
e. 40

21. $3 \times |-16 - -3| =$
a. –45
b. 51
c. 39
d. –39
e. 57

22. What is the greatest common factor of 120 and 240?
a. 60
b. 240
c. 120
d. 2
e. 480

23. $10 \times (2 + 12 \div 3) =$
a. $\frac{32}{3}$
b. 24
c. 60
d. 46.666667
e. 10.666667

24. Which is NOT a factor of 60?
a. 6
b. 60
c. 2
d. 3
e. 8

25. What is the least common multiple of 54 and 81?
a. 81
b. 9
c. 3
d. 162
e. 1

ANSWERS

Here are the answers and explanations to the chapter quiz. Read them over carefully for explanations of any problems that you answered incorrectly. For more information and practice on the properties of numbers, see LearningExpress's *Practical Math Success in 20 Minutes a Day*, Lesson 20.

1. d. Multiplication is done before addition in the order of operations. First multiply 15 by 3 and then add 25 to get the result of 45 + 25 = 70.

2. c. For the order of operations, you must perform division first. This sub-result is 5; 8 – 5 = 3.

3. b. Expressions within parentheses are evaluated first; 6 + 19 = 25. Now perform the multiplication: $5 \times 25 = 125$.

4. a. The fraction bar stands for division of the numerator by the denominator; it also stands for a grouping symbol. This expression is 12 + 144 ÷ (8 + 4). First, evaluate parentheses to get 8 + 4 = 12. Now division is performed before addition; 144 ÷ 12 = 12. The final step is to add: 12 + 12 = 24.

5. c. Multiplication and division are done left to right, so first evaluate 120 divided by 5, which is 24. To evaluate multiplication, remember that a positive times a negative produces a negative result, and $24 \times -2 = -48$.

6. c. All of the choices show examples of factors that produce 90, but choice **c** is the only factorization showing all prime numbers.

7. b. The distributive property states that multiplication distributes over addition or subtraction. An illustration of this is choice **b**.

8. e. First find the prime factorization of both 48 and 120, and then look for all common prime factors; $48 = 2 \times 2 \times 2 \times 2 \times 3$ and $120 = 2 \times 2 \times 2 \times 3 \times 5$. The common prime factors are $2 \times 2 \times 2 \times 3 = 24$. The greatest common factor is 24.

9. d. With these numbers, it is easy to just list the multiples of 20 and 30 to find the first one in common. Multiples of 20 are: 20, 40, 60, 80 . . . and multiples of 30 are: 30, 60, 90 . . . The least common multiple is 60.

10. e. Addition and subtraction are done left to right. Subtraction must be evaluated first; $35 - 5 = 30$. The answer is $30 + 7 = 37$.

11. a. The GCF stands for the greatest common factor. Find the prime factorization of the numbers. The prime factorization of 72 is $2 \times 2 \times 2 \times 3 \times 3$. The prime factorization of 180 is $2 \times 2 \times 3 \times 3 \times 5$. The common prime factors are 2, 2, 3 and 3; $2 \times 2 \times 3 \times 3 = 36$.

12. d. The LCM is the least common multiple. Find the prime factorization of 72 and 180. The prime factorization of 72 is $2 \times 2 \times 2 \times 3 \times 3$. The prime factorization of 180 is $2 \times 2 \times 3 \times 3 \times 5$. The greatest common factor is 36. Multiply this number by the leftover primes; $36 \times 2 \times 5 = 360$. You may have also started listing multiples of 180, that is 180, 360, 540, and then tested to see that 72 divides evenly into 360; 360 is a multiple of 72 and is the least common multiple of 72 and 180.

13. b. The commutative property states that when all operations are addition (or multiplication) you can change the order of the operands to get the same result.

14. e. Parentheses are evaluated first, and $4 \times 3 = 12$. Next, division is performed; $48 \div 4 = 12$. Finally, $12 - 12 = 0$.

15. a. Choice **a** shows an example of the associative property. This property states that when all operations in an expression are multiplication (or addition) you can change the grouping symbols to get the same result.

16. d. All of the choices show examples of factors that produce 78, but choice **d** is the only factorization showing all prime numbers.

17. b. Be aware that a fraction bar means division and the bar is also a grouping symbol. So evaluate the expression as $(10 + 7 \times 2) \div (2 + 4)$. Evaluate parentheses left to right. In the first grouping symbol, do multiplication first; $10 + 14 = 24$. The second grouping symbol expression is $2 + 4 = 6$. Finally, evaluate division; $24 \div 6 = 4$.

18. d. The absolute value must be evaluated first. The expression now becomes $17 - |-7|$ because when you add two numbers with opposite signs you subtract the numbers and keep the sign of the number with the larger absolute value (see Chapter 2). The absolute value of -7 is $+7$. The final step is $17 - 7 = 10$.

19. b. The LCM is an acronym for the least common multiple. Find the prime factorization of the two numbers; $27 = 3 \times 3 \times 3$ and $90 = 2 \times 3 \times 3 \times 5$. Nine is the greatest common factor since both numbers have 3 times 3 in common. Multiply this GCF by the unpaired leftover primes; $9 \times 3 \times 2 \times 5 = 270$; 270 is the least common multiple.

20. e. There are no parentheses or exponents, therefore multiplication and division will be evaluated first, from left to right; $48 \div 8 = 6$. Next, $6 \times 5 = 30$. Finally, $10 + 30 = 40$.

21. c. Absolute value must be evaluated first; –16 minus –3 is $-16 + 3$, because when you subtract you add the opposite (see Chapter 2). So, $-16 + 3 = -13$. The absolute value symbol makes this term +13. Finally, $3 \times 13 = 39$.

22. c. If you notice that 120 divides evenly without a remainder into 240, then it is obvious that 120 is the greatest common factor. If you did not notice this, you could find the prime factorization of both 120 and 240; $120 = 2 \times 2 \times 2 \times 3 \times 5$ and $240 = 2 \times 2 \times 2 \times 2 \times 3 \times 5$. The common factors are 2, 2, 2, 3, and 5. When multiplied all together, these factors equal 120.

23. c. Parentheses must be evaluated first. Inside the parentheses, division is done before addition; $12 \div 3 = 4$, and $2 + 4 = 6$. Finally, $10 \times 6 = 60$.

24. e. A factor of 60 is a whole number less than or equal to 60 that divides evenly into 60. The only number that does not divide evenly without a remainder into 60 is 8.

25. d. One approach is to list multiples of 54 and 81 to find the first one in common. Start by listing multiples of 81, checking to see if they are also multiples of 54. The multiples of 81 are: 81, 162, 243 . . . and the multiples of 54 are: 54, 108, 162 . . . The least common multiple is 162.

4

Fractions, Decimals, and Ordering the Real Numbers

This chapter reviews fractions and decimals and how to order real numbers. Fractions and decimals are the most common ways that numbers are represented. An understanding of these representations, and how to perform operations on these types of numbers is essential to your success at math. Before you study the lessons in this chapter, take a few minutes to take the following ten-question *Benchmark Quiz*. These questions are similar to the type of questions that you will find on important tests. When you are finished, check the answer key carefully to assess your results. The quiz will help you assess your prior knowledge of fractions and decimals. You may find that you are successful with one type and need additional help with another. You can then proceed to the lessons with focus.

BENCHMARK QUIZ

1. $\frac{4}{9} - \frac{7}{9} =$

a. $\frac{3}{9}$

b. $-\frac{1}{3}$

c. $-\frac{3}{18}$

d. $-\frac{11}{9}$

e. $-\frac{11}{18}$

2. $\frac{8}{15} + \frac{9}{30} =$

a. $\frac{7}{30}$

b. $\frac{26}{30}$

c. $\frac{5}{6}$

d. $\frac{17}{45}$

e. $\frac{6}{15}$

3. $2\frac{3}{4} - 3\frac{2}{4} =$

a. $-\frac{3}{4}$

b. $-1\frac{1}{4}$

c. $\frac{3}{4}$

d. $5\frac{1}{4}$

e. $6\frac{1}{4}$

4. $\frac{7}{9} \times \frac{3}{4} =$

a. $\frac{27}{28}$

b. $\frac{21}{9}$

c. $\frac{28}{27}$

d. $\frac{7}{12}$

e. $\frac{36}{21}$

5. $4\frac{2}{3} \div 6 =$

a. $\frac{14}{6}$

b. $\frac{84}{3}$

c. $24\frac{2}{3}$

d. 28

e. $\frac{7}{9}$

6. $\frac{\frac{3}{4}}{\frac{7}{8}} =$

a. $\frac{7}{6}$

b. $\frac{21}{32}$

c. $\frac{6}{7}$

d. $\frac{32}{21}$

e. $\frac{21}{24}$

7. 1,036.09 + 2.4 + 17 =

a. 1,036.50

b. 1,055.49

c. 103.650

d. 105.549

e. 1,077.09

8. $26.19 \times 0.3 =$

a. 7.857

b. 7,857

c. 7.837

d. 7,837

e. 0.7857

9. Which is greatest?

a. $\frac{7}{12}$

b. $\frac{5}{7}$

c. 0.079

d. 0.63

e. 0.0108

10. $540 \div 2.7 =$
a. 2
b. 20
c. 2,000
d. 0.2
e. 200

BENCHMARK QUIZ SOLUTIONS

How did you do on the Benchmark Quiz? Check your success in working with fractions and decimals here, and then analyze your results to figure out your plan of attack to master these topics. The answer explanations will give you an indication of what is required to solve these types of problems.

1. b. The two fractions already have a common denominator, so combine the numerators, and keep the denominator. Then simplify; $\frac{4}{9} - \frac{7}{9} = \frac{4-7}{9} = -\frac{3}{9} \div \frac{3}{3} = -\frac{1}{3}$, in lowest terms.

2. c. First, find a common denominator, the LCM of 15 and 30. The LCM is 30. Convert the first fraction to have a denominator of 30; $\frac{8}{15} \times \frac{2}{2} = \frac{16}{30}$. Now combine to get $\frac{16}{30} + \frac{9}{30} = \frac{25}{30} \div \frac{5}{5} = \frac{5}{6}$.

3. a. Change each mixed number to an improper fraction; $2\frac{3}{4} - 3\frac{2}{4} = \frac{11}{4} - \frac{14}{4}$. The fractions have the same denominator, so combine the numerators and keep the denominator; $\frac{11}{4} - \frac{14}{4} = \frac{11-14}{4} = -\frac{3}{4}$.

4. d. When you multiply fractions, multiply straight across and then simplify; $\frac{7}{9} \times \frac{3}{4} = \frac{21}{36} \div \frac{3}{3} = \frac{7}{12}$. Alternately, you can cancel as follows

$$\frac{7}{\cancel{9}_{3}} \times \frac{\cancel{3}^{1}}{4} = \frac{7 \times 1}{3 \times 4} = \frac{7}{12}.$$

5. e. Change $4\frac{2}{3}$ and 6 to improper fractions. Then change the operation to multiply by the reciprocal of 6; $\frac{14}{3} \div \frac{6}{1} = \frac{14}{3} \times \frac{1}{6}$. Multiply the numerators and the denominators straight across and simplify; $\frac{14}{3} \times \frac{1}{6} = \frac{14 \times 1}{3 \times 6} = \frac{14}{18} \div \frac{2}{2} = \frac{7}{9}$.

6. c. When you have a complex fraction, change it to show division; $\frac{3}{4} \div \frac{7}{8}$. To divide fractions, change the problem to multiply by the reciprocal. Multiply straight across and then simplify; $\frac{3}{4} \div \frac{7}{8} = \frac{3}{4} \times \frac{8}{7} = \frac{24}{28} \div \frac{4}{4} = \frac{6}{7}$.

7. b. Line up the decimal points, and add vertically:

$$\begin{array}{r} 1{,}036.09 \\ 2.40 \\ +17.00 \\ \hline 1{,}055.49 \end{array}$$

8. a. Multiply as you would with whole numbers, then count the number of digits to the right of the decimal point in the factors. There are three digits, so the decimal place is moved in the product three places to the left:

$$\begin{array}{r} {}^{1}{}^{2}26.19 \\ \times \quad 0.3 \\ \hline 7.857 \end{array}$$

9. b. The fractions are changed to decimals by dividing; $\frac{7}{12} = 7 \div 12 = 0.58$ and $\frac{5}{7} = 5 \div 7 = 0.71$. To divide 5 by 7, use long division, by adding trailing zeros and using the division algorithm. Follow the diagrams for converting the fractions using long division.

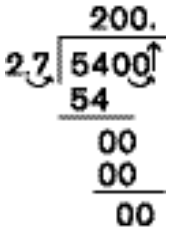
$$\begin{array}{r} .714 \\ 7\overline{)\,5.000} \\ \underline{4\,9}\downarrow \\ 10 \\ \underline{7}\downarrow \\ 30 \\ \underline{28} \\ 2 \end{array}$$

This is sufficient to compare the fractions. Treat $\frac{5}{7} \approx 0.71$

Change all numbers to decimal numbers with the same number of digits after the decimal point: 0.5800, 0.7100, 0.6300, 0.0790, 0.0108. The decimal 0.71, which equals $\frac{5}{7}$, is the greatest.

10. e. Set up the problem as a long division problem. Since the divisor (2.7) has one digit to the right of the decimal point, you must move the decimal place one place to the right in both the divisor and the dividend (540). You must add a trailing zero onto the

dividend as a placeholder. Then move the decimal place straight up to the quotient.



BENCHMARK QUIZ RESULTS

If you answered 8–10 questions correctly, you most likely have a solid understanding of performing basic arithmetic with fractions and decimals. Read over the chapter; there may be alternative ways you can learn to work with decimals and fractions, and the shortcuts and sidebar information may improve your performance.

If you answered 4–7 questions correctly, you need to study these topics. Carefully read through the lesson in this chapter for review and skill building. Take note of the various sidebars that refer you to more in-depth practice, hints, and shortcuts. Work through the quiz at the end of the chapter to check your progress.

If you answered 1–3 questions correctly, you need extended clarification on the topics in this section. A solid understanding of fractions and decimals is crucial to your success in any type of mathematics test. First, carefully read this chapter and concentrate on the sidebars and visual aids that will help with comprehension. Perhaps you learned this information and forgot; take the time now to refresh your skills and improve your knowledge. Go to the suggested website in the Extra Help sidebar in this chapter, and do extended practice. You may also want to refer to *Practical Math Success in 20 Minutes a Day*, Lessons 1–8, published by LearningExpress. This book provides more extensive lessons on fractions and decimals.

JUST IN TIME LESSON—FRACTIONS, DECIMALS, AND ORDERING THE REAL NUMBERS

This lesson covers the basics of working with fractions, decimals and ordering the real numbers.

The topics in this chapter are:

- equivalent fractions and simplifying
- operations with fractions

- ordering fractions
- equivalent decimals and ordering decimals
- operations with decimals
- converting between fractions and decimals
- common fraction and decimal equivalents

GLOSSARY

COMMON FRACTION a rational number expressed in the form $\frac{a}{b}$, where a and b are integers, and $b \neq 0$

PROPER FRACTION a fraction whose numerator is less than its denominator

IMPROPER FRACTION a fraction whose numerator is greater than or equal to its denominator

MIXED NUMBER a number with an integer part and a fractional part. Mixed numbers can be converted into improper fractions.

EQUIVALENT FRACTIONS AND SIMPLIFYING

RULE BOOK

Any number, when multiplied or divided by 1, remains unchanged.

This rule is used to make equivalent fractions, by multiplying or dividing by a special form of 1, that is $\frac{a}{a}$, where $a \neq 0$.

Examples:

$\frac{50}{100} \div \frac{5}{5} = \frac{10}{20}$, so $\frac{50}{100}$ and $\frac{10}{20}$ are equivalent fractions.

$\frac{3}{4} \times \frac{6}{6} = \frac{18}{24}$, so $\frac{3}{4}$ and $\frac{18}{24}$ are equivalent fractions.

To simplify a fraction, divide the numerator and the denominator by the greatest common factor (GCF). The greatest common factor was reviewed in Chapter 3 of this book.

GLOSSARY

SIMPLIFIED FRACTION a fraction whose numerator and denominator are relatively prime

RELATIVELY PRIME any numbers whose only common factor is 1

Dividing the numerator and denominator of a fraction by the GCF simplifies the fraction.

Example:
Simplify $\frac{24}{72}$.
The GCF of 24 and 72 is 24:

$$24 = 2 \times 2 \times 2 \times 2$$
$$72 = 2 \times 2 \times 2 \times 3 \times 3$$
$$\text{GCF} = 2 \times 2 \times 2 \times 2 = 24$$

Divide by the GCF: $\frac{24}{72} \div \frac{24}{24} = \frac{1}{3}$.
$\frac{24}{72} = \frac{1}{3}$, and $\frac{1}{3}$ is in simplest form because 1 and 3 are relatively prime.

CALCULATOR TIPS

Many calculators do fractional operations and automatically simplify fractions. If your calculator has the a b/c key, it does fractional arithmetic. Most calculators show fractions as $\frac{3}{4}$ shown as 3 _ 4. Enter the fraction $\frac{24}{72}$ with the following key sequence: 2 4 a b/c 7 2 =. The calculator will automatically show this fraction simplified, as 1 _ 3. Some fractional calculators may not automatically simplify fractions. Read over the user guide provided with your specific calculator and practice using the fraction keys.

OPERATIONS WITH FRACTIONS—ADDING AND SUBTRACTING

To add or subtract fractions, a common denominator is necessary. If the denominators are already the same, add or subtract the numerators and keep the denominator. Then simplify if required.

Example:
$\frac{2}{9} - \frac{7}{9} = \frac{2-7}{9} = \frac{-5}{9}$, or simply $-\frac{5}{9}$.

If the denominators are different, first convert the fractions into equivalent fractions that have the same denominators. To find the common denominator, it is best to use the least common multiple (LCM) of the given denominators. This is often called the least common denominator (LCD). The least common multiple is reviewed in Chapter 3 of this book. Once you have equivalent fractions with a common denominator, add or subtract as already described.

Examples:

To add $\frac{3}{8} + \frac{5}{24}$, you need a common denominator. The LCM of 8 and 24 is 24. Convert the first fraction to have a denominator of 24; $\frac{3}{8} \times \frac{3}{3} = \frac{9}{24}$. Now perform the addition; $\frac{9}{24} + \frac{5}{24} = \frac{14}{24}$. Finally, if needed, simplify $\frac{14}{24}$ to $\frac{7}{12}$.

To add $\frac{4}{9} + \frac{7}{48}$, you again need a common denominator. The LCM of 9 and 48 is 144. Convert both fractions to equivalent fractions with a denominator of 144. Now, $\frac{4}{9} = \frac{64}{144}$, and $\frac{7}{48} = \frac{21}{144}$. So perform the addition: $\frac{64}{144} + \frac{21}{144} = \frac{85}{144}$. This answer is already simplified, as 85 and 144 are relatively prime.

OPERATIONS WITH FRACTIONS—MULTIPLYING

To multiply fractions, simply multiply the numerators and then multiply the denominators straight across. Finally, simplify if needed.

Example:

$\frac{2}{3} \times \frac{4}{7} = \frac{2 \times 4}{3 \times 7} = \frac{8}{21}$

SHORTCUT

To make multiplication easier, look for common factors shared in a numerator with any of the denominators, and cancel out these common factors.

Example:

$$\frac{7}{\cancel{48}_{8}} \times \frac{\cancel{6}^{1}}{5} = \frac{7 \times 1}{8 \times 5} = \frac{7}{40}$$

$$\frac{\cancel{9}^{1}}{\cancel{17}_{1}} \times \frac{\cancel{17}^{1}}{\cancel{18}_{2}} = \frac{1 \times 1}{1 \times 2} = \frac{1}{2}$$

Be careful; canceling can only be performed when all terms are being multiplied. You cannot cancel if there is addition or subtraction in the numerator. This is a common error made when canceling with fractions. For example,

$$\frac{5+7}{5} = \frac{12}{5}$$

$$\text{NOT } \frac{\cancel{5}^{1}+7}{\cancel{5}_{1}} \neq \frac{8}{1}.$$

OPERATIONS WITH FRACTIONS—DIVIDING

GLOSSARY

RECIPROCAL the multiplicative inverse of a fraction. Any number multiplied by its reciprocal equals 1. To find the reciprocal of a fraction, "flip" the numerator and denominator, and keep the sign of the fraction. For example, $\frac{2}{1}$ is the reciprocal of $\frac{1}{2}$.

To divide fractions, recall that dividing by a fraction is equivalent to multiplying the first given fraction by the reciprocal, the flip, of the second fraction. Then multiply as shown previously.

Example:

$$\frac{5}{8} \div \frac{1}{5} = \frac{5}{8} \times \frac{5}{1} = \frac{5 \times 5}{8 \times 1} = \frac{25}{8}$$

RULE BOOK

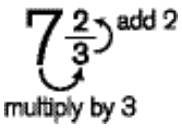
To perform positive and negative arithmetic with fractions, just follow the rules as outlined in Chapter 2 of this book.

OPERATIONS WITH FRACTIONS—MIXED NUMBERS

When working with mixed numbers it is usually easiest to change the mixed number to an improper fraction and then perform the given operations. This advice is especially helpful when working with positive and negative arithmetic. To convert a mixed number to an improper fraction, multiply the whole number part by the denominator. Add this product to the numerator. This sum is the numerator of the improper fraction. The denominator stays the same.

Example:

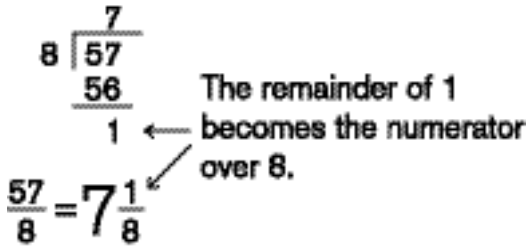
Convert $7\frac{2}{3}$ to an improper fraction.



$7 \times 3 + 2 = 23$, so the improper fraction is $\frac{23}{3}$.

To convert an improper fraction to a mixed number, divide the numerator by the denominator. Find the whole number part, and the remainder becomes the numerator of the fractional part of the mixed number. The denominator stays the same.

Example:
Convert $\frac{57}{8}$ to a mixed number.



$\frac{57}{8} = 7\frac{1}{8}$

To perform mixed number arithmetic, convert to improper fractions:

Example:
$2\frac{3}{8} - 5\frac{3}{4}$

Changing to improper fractions yields $2\frac{3}{8} - 5\frac{3}{4} = \frac{19}{8} - \frac{23}{4}$.

Find the common denominator $\frac{19}{8} - \frac{46}{8}$.

Perform the subtraction $\frac{19-46}{8} = \frac{-27}{8} = -\frac{27}{8}$, or $-3\frac{3}{8}$.

OPERATIONS WITH FRACTIONS—COMPLEX FRACTIONS

GLOSSARY

COMPLEX FRACTION a fraction that has a fractional numerator or fractional denominator, or both. For example, $\frac{3}{\frac{2}{5}}$ or $\frac{\frac{4}{5}}{\frac{6}{7}}$.

To perform operations with complex fractions, it is usually easiest to recall that the fraction bar means division and rewrite the complex fraction as one fraction divided by another fraction.

Example:

$$\frac{\frac{2}{3}}{\frac{5}{8}} = \frac{2}{3} \div \frac{5}{8} = \frac{2}{3} \times \frac{8}{5} = \frac{2 \times 8}{3 \times 5} = \frac{16}{15}$$

EXTRA HELP

Working with fractions is a lengthy topic. If you feel you need further clarification and more extensive lessons on fractions and fractional operations, refer to *Practical Math Success in 20 Minutes a Day*, Lessons 1–5, published by LearningExpress.

The website *http://matti.usu.edu/nlvm/nav/index.html* has some helpful interactive exercises that explore the concept of fractional ordering and addition. Upon entering the website, click on *Virtual Library*. Click on the *6–8* box in the row entitled *Numbers & Operations*. There are several activities to choose from that deal with fractions, including: *Fraction Pieces*, *Fractions—Adding*, *Fractions—Comparing*, and *Fractions—Equivalent*.

CALCULATOR TIPS

If the test you will be taking allows the use of a fractional calculator, you may want to invest in one. This can save precious time and increase accuracy when taking a timed test. If your calculator has a key that looks like a/b, then it does fractional arithmetic. Most calculators will follow the key sequence as described below. Check your calculator with this example:

To perform the operation $4\frac{2}{9} - 7\frac{5}{9}$, enter the key sequence: 4 a/b 2 a/b 9 − 7 a/b 5 a/b 9 =. Your calculator should show the result $-\frac{10}{3}$ or $-3\frac{1}{3}$.

Consult the user guide that came with your calculator for specific differences. It is suggested that if you do use a calculator, double check by performing the operation a second time. A calculator is only as accurate as the operator!

ORDERING FRACTIONS

To compare and order fractions, it is best to find a common denominator and compare the numerators as you would compare integers.

Example:
Arrange these fractions from least to greatest: $\frac{2}{3}, \frac{5}{6}, \frac{15}{24}, \frac{3}{4}, \frac{7}{12}$.
Find the least common multiple of all the denominators, which is 24. Convert all fractions to equivalent fractions with the denominator of 24 and then compare numerators.
$\frac{2}{3} = \frac{16}{24}$ $\frac{5}{6} = \frac{20}{24}$ $\frac{3}{4} = \frac{18}{24}$ $\frac{7}{12} = \frac{14}{24}$ $\frac{15}{24} = \frac{15}{24}$

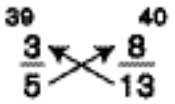
Now, it is easy to order the fractions from the least to the greatest:

$\frac{7}{12}, \frac{15}{24}, \frac{2}{3}, \frac{3}{4}, \frac{5}{6}$

Another convenient way to order fractions is to convert the fractions into decimal equivalents and compare. Decimals are reviewed in the forthcoming sections.

A third way to quickly compare two fractions is often called the "bowtie" method. Write the two fractions next to each other and multiply on the diagonals from the bottom to the top. Then, write the product above the fraction. The fraction below the larger product is the larger fraction.

Example:



$\frac{8}{13}$ is the larger fraction because 40 > 39.

EQUIVALENT DECIMALS AND ORDERING DECIMALS

The decimal system is a way to name numbers based on the powers of 10. The numbers to the right of the decimal point are fractional equivalents with denominators that are powers of ten.

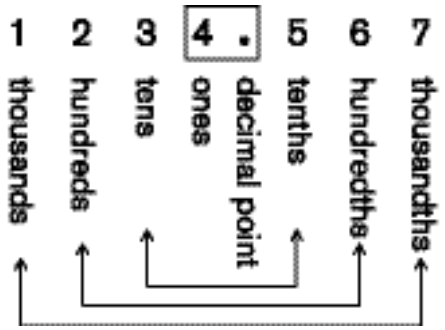
$$0.1 = \frac{1}{10^1} = \frac{1}{10} \text{ one tenth}$$

$$0.01 = \frac{1}{10^2} = \frac{1}{100} \text{ one hundredth}$$

$$0.001 = \frac{1}{10^3} = \frac{1}{1000} \text{ one thousandth}$$

RULE BOOK

Decimals are based on the place value of our number system where the position from the decimal point has meaning:



Note the pattern of how the names of the decimal places to the right of the decimal are similar to the names of the place values to the left of the decimal point. Keep the mental image of the decimal point pairing up with the "ones" place to easily remember the place value names.

The decimal number 1.52 is read as "one and fifty-two hundredths," or $1\frac{52}{100}$. The number .05 is read as "five hundredths," or $\frac{5}{100}$.

Decimal numbers are easy to compare and order, when you remember that the place value has meaning. In mathematics, 2.4 is the same number as 2.400 because both numbers represent "two and four tenths." A whole number is understood to have a decimal point to the right of the number. For example, 12 = 12. = 12.0 = 12.000. Each expression represents twelve with no remainder. To compare decimals, it is best to change each decimal into an equivalent decimal with the same number of decimal places.

Example:

Order the numbers from least to greatest: .016, 0.7, .203, .75.

Because some of the numbers have three places to the right of the decimal point, change each decimal to an equivalent decimal with three decimal places to the right of the decimal point. One of the numbers shows a leading zero; also include this leading zero in all of the numbers; 0.016, 0.700, 0.203, 0.750. Now the decimals can be compared in the same manner as whole numbers, and 16 < 203 < 700 < 750, so the answer is: .016, .203, 0.7, .75.

GLOSSARY

TERMINATING DECIMAL a decimal that terminates. Eventually, when performing long division, the divisor divides evenly into one of the sub-dividends. Example: 4.5, 23.6003.
REPEATING DECIMAL a decimal whose fractional part follows a repeating pattern. The divisor never divides evenly into one of the sub-dividends, but a pattern emerges. Examples: 8.99999 . . . , 0.121212 . . . ., 4.567777. . . .
RATIONAL DECIMAL NUMBER any decimal number that terminates or repeats

OPERATIONS WITH DECIMALS—ADDING AND SUBTRACTING

To add or subtract decimal numbers, *line up* the decimal points and add or subtract as usual. The decimal point will be in the same place as it is in the terms. If the numbers are written across the page, set up the problem one number below the other, lining up the decimal points. You can add trailing zeros as placeholders.

Example:
Add 17.8 + 5.06 + 127.432.

$$\begin{array}{r} 17.800 \\ 5.060 \\ +\ 127.432 \\ \hline 150.292 \end{array}$$

SHORTCUT

Remember, when you add or subtract decimals, you must line up the decimal points in a straight line. You can use the following memory aid to help you remember that decimal points must be aligned when adding or subtracting decimals:

A (Add) **All**
S (Subtract) **Straight**

OPERATIONS WITH DECIMALS—MULTIPLYING

When multiplying with decimals, multiply as you would for whole numbers, and ignore the decimal points until after the product is found. After performing the multiplication, count the number of digits after the decimal points (to the right of the decimal point), in both factors being multiplied. This count is the number of decimal places (to the right of the decimal point) that will be in the answer. Start at the side furthest to the right of the product (the answer) and count to the left the number of digits (the amount

of digits to the right of the decimal point in both terms) in order to place the decimal point.

Examples:
Multiply 2.48 × 1.7.

Multiply 248 × 17 to get 4,216. There are three digits to the right of the decimal points in the factors, namely 4, 8, and 7. Starting to the right of the 6 in the answer, move three digits to the left, and place the decimal point between the 4 and the 2 in the answer.

$$\begin{array}{r} 2.48 \\ \times\ 1.7 \\ \hline 1736 \\ 248 \\ \hline 4.216 \end{array}$$

Therefore, 2.48 × 1.7 = 4.216.

Multiply 0.067 × 0.04.

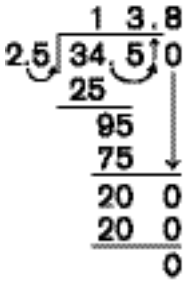
Multiply 67 × 4 to get the product of 268. There are five decimal places to the right of the decimal points in the factors: 0, 6, 7, and then 0 and 4. Starting to the right of the 8 in the answer, move five digits to the left. There are only three digits in this product, so two zeros must be added onto the left as placeholders.

$$\begin{array}{r} .067 \\ \times 0.04 \\ \hline .00268 \end{array}$$

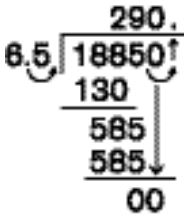
OPERATIONS WITH DECIMALS—DIVIDING

To divide with decimal numbers, first change the problem to division by a whole number. It may be necessary to move the decimal point in the divisor (the number you are dividing *by*) to make it a whole number. Move the decimal in the dividend (the number you are dividing *into*) the same number of places, and copy the new decimal place holder straight up into the quotient (the *answer* to the division problem). Once the decimal point is placed, divide as you normally would with long division.

Examples:
Divide 34.5 ÷ 2.5.



Divide 1,885 ÷ 6.5.



SHORTCUT

Remember, when you multiply or divide decimals:
M (Multiply) Move
D (Divide) Decimal

EXTRA HELP

For in-depth coverage of decimals and decimal operations, refer to *Practical Math Success in 20 Minutes a Day*, Lessons 6–8, published by LearningExpress. There are also lessons, examples, and online quizzes on the various aspects of decimal operations at *www.math.com*. Click on *Decimals* under *Hot Subjects*. Then click on one of the following: *Decimal Numbers, Adding/Subtracting Decimals, Multiplying Decimals,* or *Dividing Decimals.*

DECIMAL OPERATIONS—MULTIPLYING OR DIVIDING BY THE POWERS OF 10

The decimal number system is based on the powers of 10. This makes multiplication and division by 10, 100, 1,000 . . . very easy. It is simply a matter of moving the decimal point the number of places dictated by the

number of zeroes in 10, 100, or 1,000. This is because once you add or remove the zeroes, you are essentially multiplying or dividing by one.

SHORTCUT

For decimal numbers:

To MULTIPLY a number by 10	**Move the decimal point one place to the RIGHT**
by 100	**two places**
by 1,000	**three places**
To DIVIDE a number by 10	**Move the decimal point one place to the LEFT**
by 100	**two places**
by 1,000	**three places**

CONVERTING BETWEEN FRACTIONS AND DECIMALS

To convert a fraction to a decimal, recall that $\frac{3}{5}$ means "three divided by five." Divide 3 by 5 to get the decimal equivalent of 0.6. To convert a decimal to a fraction, use the place value names for decimals as listed in the sidebar at the start of the decimal section of this chapter. Rewrite the decimal as the named fraction, and then simplify the fraction. For example, 0.018 is read as "eighteen thousandths" which is $\frac{18}{1{,}000}$. Now simplify, $\frac{18}{1{,}000} \div \frac{2}{2} = \frac{9}{500}$.

COMMON FRACTION AND DECIMAL EQUIVALENTS

It is helpful to remember some of these common equivalents:

$\frac{1}{2} = 0.5$	$\frac{1}{5} = 0.2$	$\frac{2}{3} = 0.66\overline{6}$
$\frac{1}{3} = 0.33\overline{3}$	$\frac{1}{8} = 0.125$	$\frac{3}{4} = 0.75$
$\frac{1}{4} = 0.25$	$\frac{1}{10} = 0.1$	$\frac{4}{5} = 0.8$

RULE BOOK

To perform decimal operations with positive and negative numbers, just follow the rules as outlined in Chapter 2 of this book.

TIPS AND STRATEGIES

To summarize your study of fractions and decimals:

- Simplify fractions by dividing the numerator and the denominator by the GCF of these numbers.
- To add and subtract fractions, you need a *common denominator*. Add the numerators, keeping the common denominator, and finally simplify.
- To multiply fractions, multiply straight across for numerator and denominator, then simplify.
- Canceling is an effective shortcut when multiplying fractions.
- To divide fractions, change the operation to multiply the original first fraction by the reciprocal of the second fraction.
- Obtain a fraction calculator if allowed on your particular test. Learn and practice how to do fractional operations on the calculator.
- When **A**dding or **S**ubtracting decimal numbers, remember to keep the decimal points **A**ll **S**traight.
- When **M**ultiplying and **D**ividing decimals, remember you need to **M**ove the **D**ecimal point.
- Know the shortcuts for multiplying or dividing decimals by 10, 100, 1,000.
- Memorize some of the common fraction to decimal equivalents.

PRACTICE

Do you feel more confident about fractions and decimals? Take the following quiz to see if you have mastered these concepts.

1. $\frac{5}{8} + \frac{1}{8} =$

a. $\frac{6}{16}$

b. $\frac{3}{4}$

c. $\frac{1}{2}$

d. $\frac{5}{16}$

e. $\frac{5}{64}$

2. $\frac{7}{24} - \frac{3}{12} =$

a. $-\frac{4}{24}$

b. $\frac{1}{12}$

c. $\frac{4}{24}$

d. $\frac{1}{24}$

e. $\frac{5}{12}$

3. $\frac{2}{7} - \frac{6}{7} =$

a. $-\frac{4}{7}$

b. $-\frac{8}{7}$

c. $\frac{4}{7}$

d. $\frac{8}{7}$

e. $\frac{12}{49}$

4. $2\frac{3}{4} + \frac{3}{8} =$

a. $2\frac{1}{2}$

b. $2\frac{3}{8}$

c. $\frac{5}{2}$

d. $2\frac{9}{16}$

e. $\frac{25}{8}$

5. $4\frac{2}{6} - 5\frac{5}{12} =$

a. $-1\frac{3}{12}$

b. $1\frac{1}{12}$

c. $-9\frac{7}{12}$

d. $-1\frac{1}{12}$

e. $-9\frac{3}{12}$

6. $\frac{1}{2} \times \frac{3}{4} =$

a. $\frac{6}{4}$

b. $\frac{4}{6}$

c. $\frac{3}{8}$

d. $\frac{8}{3}$

e. 8

7. $2\frac{2}{3} \times 7 =$

a. $\frac{56}{21}$

b. $\frac{56}{3}$

c. $\frac{8}{21}$

d. $2\frac{2}{3}$

e. $14\frac{2}{3}$

8. $\frac{14}{5} \div -\frac{7}{8} =$

a. $-3\frac{1}{5}$

b. 1

c. $-\frac{4}{5}$

d. $-\frac{8}{5}$

e. $\frac{16}{5}$

9. $\frac{9}{11} \div \frac{7}{22} =$

a. $4\frac{1}{2}$

b. $\frac{9}{2}$

c. $2\frac{4}{7}$

d. $2\frac{11}{7}$

e. $\frac{2}{11}$

10. $\frac{1\frac{1}{5}}{\frac{1}{3}}$ =

a. $\frac{18}{5}$

b. $\frac{6}{15}$

c. $\frac{15}{6}$

d. $2\frac{1}{2}$

e. $\frac{2}{5}$

11. $\frac{\frac{2}{3}}{\frac{5}{6}}$ =

a. $\frac{10}{18}$

b. $\frac{18}{10}$

c. $\frac{15}{18}$

d. $\frac{20}{36}$

e. $\frac{4}{5}$

12. Which is greatest?

a. $\frac{5}{8}$

b. $\frac{17}{20}$

c. $\frac{1}{2}$

d. $\frac{7}{10}$

e. $\frac{4}{5}$

13. Which is smallest?

a. $\frac{1}{3}$

b. $\frac{2}{6}$

c. $\frac{5}{12}$

d. $\frac{1}{2}$

e. $\frac{1}{4}$

14. 34.7 + 4.1 + 0.03 =
a. 391
b. 3.91
c. 38.83
d. 39.1
e. 41.8

15. 125.05 – 11.4 =
a. 123.91
b. 114.1
c. 113.01
d. 113.65
e. 114.65

16. 16.8×0.2 =
a. 3.36
b. 336
c. 0.336
d. 33.6
e. 3.28

17. 5.34×10 =
a. 50.34
b. 15.34
c. 5.44
d. 53.4
e. 534

18. 42.19×0.4 =
a. 168.19
b. 16.876
c. 168.19
d. 16,876
e. 1.6876

19. $1.95 \div 0.03$ =
a. 6.5
b. 0.65
c. 65
d. 650
e. 68

20. $245 \div 4.9 =$
- **a.** 5
- **b.** 0.5
- **c.** 500
- **d.** 0.05
- **e.** 50

21. Convert $\frac{7}{8}$ to a decimal.
- **a.** 0.78
- **b.** 7.8
- **c.** 0.875
- **d.** 8.7
- **e.** 87.5

22. Convert 0.12 to a fraction.
- **a.** $\frac{3}{25}$
- **b.** $1\frac{1}{5}$
- **c.** $\frac{12}{50}$
- **d.** $\frac{1}{2}$
- **e.** $\frac{12}{500}$

23. Which is greatest?
- **a.** 0.100
- **b.** 0.07
- **c.** 0.25
- **d.** 0.4
- **e.** 0.009

24. Which is the smallest?
- **a.** $\frac{2}{5}$
- **b.** 0.5
- **c.** 0.07
- **d.** 0.0099
- **e.** 0.071

25. Change $\frac{3}{4}$ to a decimal.
- **a.** 3.4
- **b.** 0.75
- **c.** 4.3
- **d.** 0.34
- **e.** 75

ANSWERS

Here are the answers and explanations to the chapter quiz. Read them over carefully to understand any problems that you answered incorrectly. For more information and practice on working with fractions, decimals, and ordering the real numbers, see LearningExpress's *Practical Math Success in 20 Minutes a Day*, Lessons 1–8.

1. b. When you add fractions with the same denominator, add the numerators and keep the denominator. Simplify if needed; $\frac{5}{8} + \frac{1}{8} = \frac{5+1}{8} = \frac{6}{8} \div \frac{2}{2} = \frac{3}{4}$.

2. d. To subtract fractions, first find a common denominator. In this problem, the common denominator (LCD) is 24. Change $\frac{3}{12} \times \frac{2}{2} = \frac{6}{24}$. Now subtract the numerators and keep the denominator; $\frac{7}{24} - \frac{6}{24} = \frac{7-6}{24} = \frac{1}{24}$.

3. a. These fractions have the same denominator. Subtract the numerators and keep the denominator; $\frac{2}{7} - \frac{6}{7} = \frac{2-6}{7} = -\frac{4}{7}$.

4. e. Change the mixed number to an improper fraction; $2\frac{3}{4} = \frac{11}{4}$. In order to add these numbers, a common denominator of 8 is needed; $\frac{11}{4} \times \frac{2}{2} = \frac{22}{8}$. Now the fractions can be added; add the numerators and keep the denominator; $\frac{22}{8} + \frac{3}{8} = \frac{25}{8}$.

5. d. Change the mixed numbers to improper fractions; $4\frac{2}{6} = \frac{26}{6}$ and $5\frac{5}{12} = \frac{65}{12}$. To add these improper fractions, you need to get a common denominator, which is 12; $\frac{26}{6} \times \frac{2}{2} = \frac{52}{12}$. Subtract the numerators and keep the denominators; $\frac{52}{12} - \frac{65}{12} = \frac{52-65}{12} = -\frac{13}{12}$, which is $-1\frac{1}{12}$.

6. c. To multiply fractions, multiply the numerator and denominator straight across; $\frac{1}{2} \times \frac{3}{4} = \frac{1 \times 3}{2 \times 4} = \frac{3}{8}$.

7. b. Change each number into an improper fraction; $2\frac{2}{3} = \frac{8}{3}$ and $7 = \frac{7}{1}$. Multiply the improper fractions, multiplying the numerators and denominators straight across; $\frac{8}{3} \times \frac{7}{1} = \frac{56}{3}$.

8. a. Recall that a positive times a negative equals a negative, so the answer will be negative. To divide, change the operation to multiplication, and take the reciprocal of the second fraction; $\frac{14}{5} \times -\frac{8}{7}$. One approach is then to use canceling to simplify before multi-

plying. Then multiply the numerators and denominators straight across.

$$\frac{\overset{2}{\cancel{14}}}{5} \times -\frac{8}{\cancel{7}_1} = \frac{2 \times -8}{5 \times 1} = \frac{-16}{5} = -3\frac{1}{5}$$

9. c. To divide fractions, change the operation to multiply and take the reciprocal of the second fraction. Multiply the numerators and denominators straight across; $\frac{9}{11} \times \frac{22}{7} = \frac{198}{77}$. Change this to an improper fraction by dividing 198 by 77. What you get is 2 with remainder 44; $\frac{198}{77} = 2\frac{44}{77}$. Simplify the fractional part: $\frac{44}{77} \div \frac{11}{11} = \frac{4}{7}$.

10. a. First change the numerator to an improper fraction; $1\frac{1}{5} = \frac{6}{5}$. Then, change the problem to be a fraction divided by a fraction; $\frac{6}{5} \div \frac{1}{3}$. Dividing fractions is the same as multiplying by the reciprocal of the second fraction. Multiply straight across the numerators and denominators; $\frac{6}{5} \times \frac{3}{1} = \frac{18}{5}$.

11. e. This complex fraction means a fraction divided by a fraction. Change the problem to multiply by the reciprocal of the second fraction; $\frac{2}{3} \times \frac{6}{5} = \frac{12}{15}$. Put the fraction in lowest terms; $\frac{12}{15} \div \frac{3}{3} = \frac{4}{5}$.

12. b. You can convert each fraction to an equivalent fraction, all of which have a common denominator. Find the LCM of the denominators; 40 is the least common multiple; $\frac{5}{8} \times \frac{5}{5} = \frac{25}{40}$, $\frac{17}{20} \times \frac{2}{2} = \frac{34}{40}$, $\frac{1}{2} \times \frac{20}{20} = \frac{20}{40}$, $\frac{7}{10} \times \frac{4}{4} = \frac{28}{40}$, and $\frac{4}{5} \times \frac{8}{8} = \frac{32}{40}$. Now the numerators can be inspected for the greatest common multiple, which is $\frac{17}{20} = \frac{34}{40}$. An alternative approach would be to change each fraction to a decimal and compare as described in the section on ordering decimals.

13. e. Convert all fractions to have a common denominator, which is the LCM = 12; $\frac{1}{3} \times \frac{4}{4} = \frac{4}{12}$, $\frac{2}{6} \times \frac{2}{2} = \frac{4}{12}$, $\frac{5}{12}$, $\frac{1}{2} \times \frac{6}{6} = \frac{6}{12}$, $\frac{1}{4} \times \frac{3}{3} = \frac{3}{12}$. It is sufficient to compare the numerators to find the smallest because the denominators are the same. An alternative approach would be to change each fraction to a decimal and compare as described in the section on ordering decimals.

14. c. Add the numbers vertically; make sure to line up the decimal points and add trailing zeroes when necessary.

$$\begin{array}{r} 34.70 \\ 4.10 \\ +\ 0.03 \\ \hline 38.83 \end{array}$$

15. d. When subtracting decimals, it is easiest to do the problem vertically, remembering to line up the decimal points.

$$\begin{array}{r} 125.05 \\ -11.40 \\ \hline 113.65 \end{array}$$

16. a. Multiply without regard to the decimal points; $168 \times 2 = 336$. Since there are two digits to the right of the decimal points in the factors, move the decimal point two places left in the product.

$$\begin{array}{r} {\scriptstyle 1\,1}16.8 \\ \times\ \ 0.2 \\ \hline 3.36 \end{array}$$

17. d. Use the shortcut when multiplying by 10. Move the decimal point one place to the right, to get the product of 53.4.

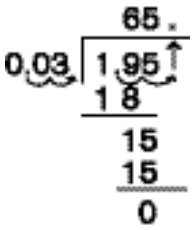
18. b. Multiply without regard to the decimal point:

$$\begin{array}{r} 42.19 \\ \times 0.4 \\ \hline 16{,}876 \end{array}$$

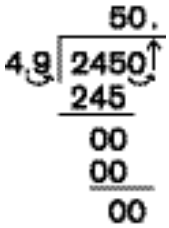
There are three digits to the right of the decimal points in the factors, namely 1, 9, and then 4. Move the decimal point three places to the left:

16.876

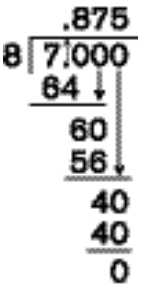
19. c. Set up the problem as a long division problem. Since the divisor (0.03) has two digits to the right of the decimal point, you must move the decimal point two places to the right in both the divisor and the dividend (1.95). Then move the decimal place straight up to the quotient.



20. e. Set up the problem as a long division problem. Since the divisor (4.9) has one digit to the right of the decimal point, you must move the decimal point one place to the right in both the divisor and the dividend (245). You must add a trailing zero onto the dividend as a placeholder. Then move the decimal point straight up to the quotient.



21. c. The fraction $\frac{7}{8}$ means "seven divided by eight." Divide 7 by 8, using long division:



22. a. 0.12 is read as "twelve hundredths"; $\frac{12}{100} \div \frac{4}{4} = \frac{3}{25}$.

23. d. Change each decimal to an equivalent decimal, all of which have the same number of digits to the right of the decimal point; 0.4 =

0.400, 0.07 = 0.070, 0.25 = 0.250, 0.100, 0.009. Now 400 > 250 > 100 > 70 > 7; 0.4 is the greatest.

24. d. Change all numbers to decimals with four digits to the right of the decimal point. The fraction $\frac{2}{5} = \frac{4}{10}$, is read as "four tenths." The decimal equivalent of $\frac{4}{10}$ = 0.4,000; 0.5 = 0.5,000, 0.07 = 0.0700, 0.0099, 0.071 = 0.0710. Now 5,000 > 4,000 > 710 > 700 > 99; 0.0099 is the smallest.

25. b. It is helpful to memorize the decimal equivalent of this common fraction, namely 0.75. Also, $\frac{3}{4}$ is read as "three divided by four." Divide 3 by 4, using long division, to get 0.75.

Ratio and Proportion

Ratios are numbers that are used to compare things. Ratios play an important role in mathematics because they quantify all of the items that we compare on a day-to-day basis. Ratio and proportion are evident in numerous mathematical problems. Before you begin learning about ratios and proportions, take a few minutes to take this ten-question *Benchmark Quiz*. These questions are similar to the type of questions that you will find on important tests. When you are finished, check the answer key carefully to assess your results. Your Benchmark Quiz analysis will help you determine how much time you need to spend on ratios and proportions, and the specific areas in which you need the most careful review and practice.

BENCHMARK QUIZ

1. On the beach, the ratio of boogie boards to surf boards is 12 to 3. If there are 84 boogie boards, how many surfboards are there on the beach?
   a. 24
   b. 21
   c. 3
   d. 36
   e. 15

2. There are 48 people on a camping trip. Sixteen are female. What is the ratio of male to female?
a. 3 to 1
b. 16 to 48
c. 1 to 3
d. 32 to 48
e. 2 to 1

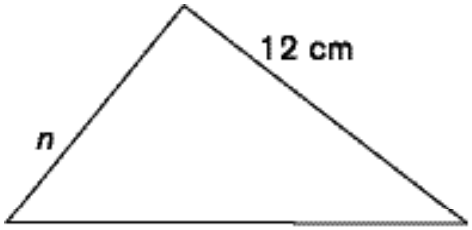
3. Two numbers are in the ratio of 5 to 8. If the larger number is 72, what is the smaller number?
a. 5
b. 13
c. 45
d. 117
e. 9

4. Find the missing term: $\frac{a}{14} = \frac{3}{7}$.
a. 6
b. 7
c. 42
d. 9
e. 21

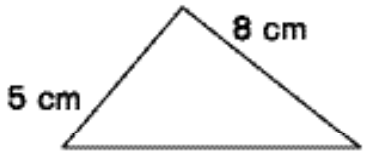
5. Find the missing term: $\frac{5}{6} = \frac{60}{x}$.
a. 50
b. 24
c. 70
d. 72
e. 80

6. A set of dollhouse furniture is made to scale and is $\frac{1}{72}$ the size of real furniture; it has a scale of 1 to 72. If a real table is 6 feet in length, what is the length of the corresponding dollhouse table?
a. $\frac{1}{6}$ foot
b. 6 inches
c. 1 inch
d. 0.5 foot
e. $\frac{1}{9}$ foot

7. The two triangles below are similar. What is the length of the missing side?



Triangle A



Triangle B

a. 6 cm
b. 10 cm
c. 8 cm
d. 16 cm
e. 7.5 cm

8. A family drove 390 miles at a constant speed. It took 6 hours of driving time. What was the speed of their car?
a. 55 miles per hour
b. 60 miles per hour
c. 234 miles per hour
d. 65 miles per hour
e. 35 miles per hour

9. Strawberries are 3 quarts for $4.98. How much will 10 quarts of strawberries cost?
a. $14.98
b. $16.60
c. $25.00
d. $49.80
e. $4.98

10. Driving 60 miles per hour, it takes one-half of an hour to drive to work. How much ADDITIONAL time will it take to drive to work if the speed is now 40 miles per hour?
a. 1 hour
b. 2 hours
c. 15 minutes
d. 30 minutes
e. 45 minutes

BENCHMARK QUIZ SOLUTIONS

How did you do on the ratio and proportion Benchmark Quiz? Check your answers here, and then analyze your results to figure out your plan of attack to master these topics.

1. b. Set up a proportion based on the ratio: $\frac{\text{boogie boards}}{\text{surfboards}}$.

$\frac{12}{3} = \frac{84}{x}$	Set up the proportion.
$3 \times 84 = 12 \times x$	Cross-multiply.
$252 = 12 \times x$	Divide by 12.
$21 = x$	There are 21 surfboards.

2. e. You are given the total number of people and the number of females. The question is what is the ratio of male to female. There are 48 – 16 = 32 males on the trip. The ratio of male to female is $\frac{\text{male}}{\text{female}} = \frac{32}{16} = \frac{2}{1}$. The ratio of male to female is 2 to 1.

3. c. Set up a proportion based on the ratio: $\frac{\text{smaller number}}{\text{larger number}}$.

$\frac{5}{8} = \frac{n}{72}$	Set up the proportion.
$8 \times n = 5 \times 72$	Cross-multiply.
$8 \times n = 360$	Divide by 8.
$n = 45$	The smaller number is 45.

4. a.

$14 \times 3 = a \times 7$	Cross-multiply.
$42 = a \times 7$	Divide by 7.
$6 = a$	The missing term is 6.

5. d.

$6 \times 60 = 5 \times x$	Cross-multiply.
$360 = 5 \times x$	Divide by 5.
$72 = x$	The missing term is 72.

6. c. The ratio of *dollhouse* to *real* is 1 to 72. The problem states that the real table is 6 feet in length. Knowing that the dollhouse table will be much smaller, first convert 6 feet to 72 inches. Since the ratio is 1 to 72, and the real table is 72 inches, the dollhouse table is 1 inch.

7. e. Notice on the figure that side AB corresponds to side DE, and side BC corresponds to side EF. One possible setup for the proportion is $\frac{\text{Triangle A}}{\text{Triangle B}}$.

$\frac{12}{8} = \frac{n}{5}$	Set up the proportion.
$8 \times n = 12 \times 5$	Cross-multiply.
$8 \times n = 60$	Divide by 8.
$n = 7.5$ cm	The missing side is 7.5 centimeters.

8. d. Use the formula $D = R \times T$. The distance, D, equals 390, and the time, T, equals 6 hours. The speed is the missing term, R.

$D = R \times T$

$390 = R \times 6$	Substitute in the known values and then divide by 6.
$65 = R$	The speed is 65 miles per hour.

9. b. Set up a proportion of: $\frac{\text{quarts of strawberries}}{\text{price}}$.

$\frac{3}{4.98} = \frac{10}{p}$

Set up the proportion.

$4.98 \times 10 = 3 \times p$	Cross-multiply.
$49.8 = 3 \times p$	Divide by 3.
$16.6 = p$	The price is \$16.60.

10. c. First, determine the distance to drive to work. Use the formula $D = R \times T$.

$D = 60 \times \frac{1}{2}$	Substitute in the known values and then multiply.
$D = 30$	The distance to work is 30 miles.

Now, determine the time to drive to work at a rate of 40 miles per hour. Use the formula $D = R \times T$.

$30 = 40 \times T$	Substitute in the known values and then divide by 40.
$.75 = T$	The new time is 0.75 hour, or three-quarters of an hour.

The problem asks how much extra time will it take to drive. This is the difference between one-half of an hour and three-quarters of an hour. Subtract the fractions, after changing one-half to two-fourths: $\frac{3}{4} - \frac{2}{4} = \frac{1}{4}$. One-fourth of an hour is 15 minutes.

BENCHMARK QUIZ RESULTS

If you answered 8–10 questions correctly, you have a good understanding of ratio and proportion. After reading through the lesson and focusing on the areas you need to refresh, try the quiz at the end of the chapter to ensure that all of the concepts are clear.

If you answered 4–7 questions correctly, you need to refresh yourself on some of the material. Read through the chapter carefully for review and skill building, and pay careful attention to the sidebars that refer you to more in-depth practice, hints, and shortcuts. Work through the quiz at the end of the chapter to check your progress.

If you answered 1–3 questions correctly, you need extended help in understanding this chapter. Make sure you have read through Chapter 3 on fractions and decimals before proceeding with this lesson. You will most likely encounter many problems dealing with ratio and proportion on your upcoming test. Take your time as you read through this lesson. Try the examples that are illustrated on a separate sheet of paper and compare your method of solution with that given in the text. Attend to the sidebars and visual aids that will help you to grasp the material. Go to the suggested website in the Extra Help sidebar in this chapter, and do extended practice. You may want to refer to *Practical Math Success in 20 Minutes a Day*, Lesson 12, published by Learning Express.

JUST IN TIME LESSON—RATIO AND PROPORTION

This lesson covers the basics of working with ratio and proportions.

The topics in this chapter are:

- ratios
- proportions
- rates
- scale
- other applications using ratio and proportion

RATIO

GLOSSARY

RATIO comparison of two things using numbers

In the community choral group, there are 25 women and 16 men. If you want to compare the number of women to the number of men, you can show this comparison in several different ways:

25 : 16
25 to 16
$\frac{25}{16}$

Regardless of which form is used, the meaning is the same: "There were 25 women for every 16 men." Notice that $\frac{25}{16}$ is a fractional form of a ratio. The fractional form of a ratio is often a convenient way to represent a ratio when solving problems.

In addition to comparing women to men, a comparison could also be made between women to total members. The total membership is 25 + 16 = 41 people. This ratio is $\frac{25}{41}$, or 25 to 41, or 25 : 41.

Ratios are usually shown in lowest terms and can be simplified in the same way that fractions are simplified. Refer to Chapter 3 of this book to review simplifying fractions. For example, in a choral group there are 24 women and 16 men. This ratio can be expressed as 3 : 2, since $\frac{24}{16} \div \frac{8}{8} = \frac{3}{2}$. In this group there are 3 women for every 2 men. We can also express the ratio of men to total choral members. This ratio is 2 : 5, because $\frac{16}{16+24} = \frac{16}{40} = \frac{2}{5}$.

RULE BOOK

When comparing similar measurements, such as one time to another, or one volume to another, most often it is necessary to have consistent units in order to have a meaningful ratio. The only exception to this rule is when working with scale drawings, explained in a later section of this chapter.

As an example of the above rule, suppose you want to compare the amount of time spent driving to work with the amount of time driving to vacation. It takes 24 minutes to drive to work and 5 hours to drive to vacation. It is not accurate to say that this ratio is $\frac{24}{5}$. The five hours must be converted to $5 \times 60 = 300$ minutes since there are 60 minutes in an hour. The correct ratio is $\frac{24}{300} = \frac{2}{25}$, which accurately gives the relationship between driving times.

PROPORTION

GLOSSARY

PROPORTION an equation that states that two ratios are equal

Often, in addition to comparing items, it is natural to want to compare ratios. For example, if you want to choose an amusement park, you may choose to compare the number of gift shops to the number of rides. Park A has 10 gift shops and 50 rides. Park B has 17 gift shops and 85 rides. For park A, the ratio is 1 to 5: $\frac{shops}{rides} = \frac{10}{50} \div \frac{10}{10} = \frac{1}{5}$. For park B, the ratio is 1 to 5: $\frac{shops}{rides} = \frac{17}{85} \div \frac{17}{17} = \frac{1}{5}$. For both parks, the ratios are exactly the same. By using a proportion you can compare these parks on this issue. The proportion is $\frac{10}{50} = \frac{17}{85}$, because both ratios equal $\frac{1}{5}$.

Proportions have an interesting relationship that is always true. In the previous example, notice that:

$$\frac{10}{50} = \frac{17}{85}$$

In the above example: $10 \times 85 = 50 \times 17$; $850 = 850$. This will always be true for a proportion. This illustrates the method of cross-multiplication, which is used to determine if two ratios are in proportion, and is used to solve a variety of mathematical problems.

RULE BOOK

In a proportion, the product of the means is equal to the product of the extremes.

$$\frac{a}{b} = \frac{c}{d}$$

means $b \times c$

extremes $a \times d$

$$b \times c = a \times d$$

To solve problems involving ratios and proportions, you can use the method of cross-multiplication.

Cross-multiplication is a very easy procedure. Follow these examples to practice the procedure.

Example:

Solve $\frac{6}{26} = \frac{n}{52}$. Use cross-multiplication.

$26 \times n = 6 \times 52$

Show that the product of the means equals the product of the extremes.

$6 \times 52 = 26 \times n$	Multiply 6 and 52.
$312 = 26 \times n$	Divide 312 by 26 to find the missing term.
$n = 12$	

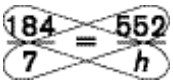
The procedure for cross-multiplication is straightforward and relatively easy. The challenge in solving word problems using ratios is in the set-up of the proportion. Take care to keep all terms in order. Remember that two ratios are being compared and that the order of the ratio set-up has meaning.

Example:

If a carpenter earns \$184 for every 7 hours of work, how many hours did he work if he earned \$552?

$\frac{dollars}{hour}$	First, decide on how you will set up the problem.
$\frac{184}{7} = \frac{552}{h}$	Set up a proportion that shows two equal ratios.

Take care to be consistent in the set-up. The variable h will stand for the number of hours. Use cross-multiplication.



Show that the product of the means equals the product of the extremes.

$7 \times 552 = 184 \times h$	Multiply 7 and 552.
$3{,}864 = 184 \times h$	Divide 3,864 by 184 to find h, the number of hours.
$h = 21$	The number of hours worked is 21.

When approaching a word problem involving ratios, in addition to a proper set-up, be clear on what the problem is asking for you to solve. Study the next example.

Example:
In a flower garden, all the flowers are either purple or pink petunias. The ratio of purple to pink petunias is 8 : 15. If there are 299 total petunias, how many are pink?

The ratio of purple : pink is given, in addition to the total number of flowers. The ratio set-up should therefore be $\frac{pink}{total}$. The ratio of pink : total is $\frac{15}{8+15} = \frac{15}{23}$.

$\frac{15}{23} = \frac{p}{299}$	Set up the proportion.
$23 \times p = 15 \times 299$	Cross-multiply.
$23 \times p = 4{,}485$	Multiply 15 and 299.
$p = 195$	Divide by 23.

There are 195 pink petunias.

SHORTCUT

When working on a multiple-choice test, the answer selection must be a multiple of the ratio given for the item in question. For example, in the preceding flower example, the number of pink flowers must be a multiple of 15. Eliminate any answer choices that are not a multiple of 15.

RATES

There are numerous examples of rates in everyday life. Here are some of the most common ones:

- speed, at $\frac{miles}{hour}$
- salary, at $\frac{dollars}{hour}$
- fuel efficiency, at $\frac{miles}{gallon}$
- cost, at $\frac{dollars}{item}$

GLOSSARY

RATE a ratio comparing two items with unlike units
UNIT RATE a rate with a 1 in the denominator

Rate problems can be solved just like any other type of proportion.

Example:
Oranges are selling at the price of $1.99 for 6 oranges. How much will 20 oranges cost?

$\frac{dollars}{oranges}$	Decide on a set-up for the problem.
$\frac{1.99}{6} = \frac{c}{20}$	Set up the proportion, where c stands for the cost of 20 oranges.
$6 \times c = 1.99 \times 20$	Cross-multiply.
$6 \times c = 39.80$	Multiply 1.99 by 20.
$c = 6.63$	Divide by 6, rounding to the nearest cent.

Twenty oranges will cost $6.63.

Rates are most often expressed as unit rates. The keyword for rates is "per." Speed is known as "miles per hour" and is actually a rate that means "miles per one hour." In the same way, unit price, "dollars per pound" means "dollars per one pound." Unit rates are easier to work with than other ratios, because the denominator is always 1, and so the denominator has no effect when performing cross-multiplication.

Example:
The speed limit on the highway is 65 miles per hour. At this constant speed, how many miles will be driven in 10.5 hours?

$\frac{65}{1} = \frac{m}{10.5}$	Set up the proportion, using $\frac{miles}{hour}$.
$m \times 1 = 65 \times 10.5$	Cross-multiply.
$m = 682.5$	Multiply 65 and 10.5. Note that there is no need to divide by 1, as dividing by 1 has no effect on the solution.

SHORTCUT

Speed is a unit rate. To solve problems involving distance, time, and speed, use the familiar formula $D = R \times T$, where D is the distance, R is the rate (the speed), and T is the time.

From the previous example, you can see that miles (D) did in fact equal the speed (R) multiplied by the time (T). The formula can also be used to find the time when given distance and speed.

Example:
If a car drives at a constant speed of 52 miles per hour, how long will it take to drive 442 miles?

$442 = 52 \times T$ Use the formula $D = R \times T$.
$8.5 = T$ Divide by 52 to find the time.

It will take 8.5 hours to drive the 442 miles.

SCALE

Scale is a special ratio used for models of real life items, such as model railroads and model airplanes, or scale drawings such as blueprints and maps. On model airplanes, you will often find the scale ratio printed on the model as *model* : *real*. For example, a toy car may have the ratio 1 : 62 printed on the bottom. This is the ratio of all of the dimensions of the actual toy to the corresponding dimensions of the real car. This scale ratio says that the real car is 62 times larger than the toy, since the ratio is 1 : 62.

Example:
A model locomotive measures 8.7 inches in length. If the scale given is 1 : 16, how long is the real locomotive?

Since the real train is 16 times as big as the model, the real train will be 8.7 times 16, which is 139.2 inches, or 11.6 feet.

On scale drawings, the scale will be a comparison of a small distance unit, like inches, to a large distance unit, like feet. So a scale on a map could read "3 inches = 10 miles." This means for every 3 inches on the map, it is 10 miles on the actual road. This ratio is $\frac{3}{10}$, but you should take care to remember that the units do not agree. On a scale drawing, if "1 inch = 10 feet," this does not mean that the real item is 10 times bigger, even though the ratio would be 1 : 10. Solve scale drawing problems as you would any type of ratio problem, keeping the units consistent and clear in your answer.

Example:
A scale drawing of the Statue of Liberty is said to be "$\frac{3}{4}$ inch = 12 feet." How tall is the statue, if the scale drawing height is 10 inches?

$\frac{\textit{drawing of statue}}{\textit{real statue}}$ Choose a set-up for the proportion.

$\frac{\frac{3}{4}}{12} = \frac{10}{h}$ Set up the proportion, where h stands for height.

Cross-multiply.

$12 \times 10 = \frac{3}{4} \times h$ Multiply 12 times 10; change $\frac{3}{4}$ to 0.75.

$120 = 0.75 \times h$ Divide by 0.75.

$160 = h$

According to this drawing, the height of the real statue is 160 feet.

OTHER APPLICATIONS

There are several applications of ratio in geometry. Geometry will be covered in Chapter 8 of this book. Following are some examples of geometry and ratios.

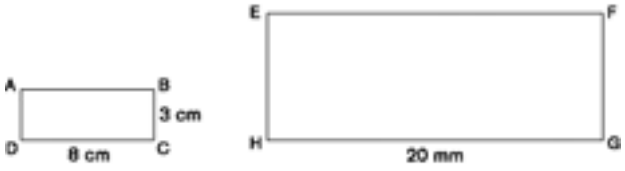
GLOSSARY

CONGRUENT FIGURES figures of equal measure

SIMILAR POLYGONS polygons with corresponding angles that are congruent, and corresponding sides that are in proportion

Similar polygons have the same shape (congruent angles) but can have different size. If you are told that two polygons are similar, then their corresponding sides are in proportion.

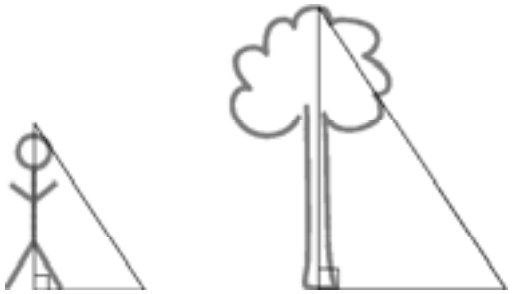
Example:
In the following picture, rectangle ABCD is similar to rectangle EFGH. Find side FG.



Side DC corresponds to side HG, and side BC corresponds to side FG.

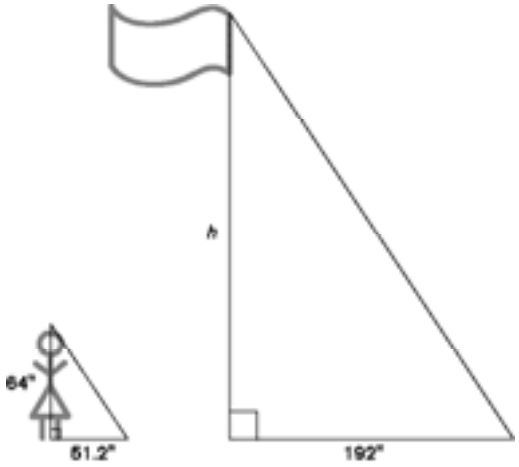
$\frac{\text{length of } \overline{FG}}{\text{length of } \overline{BC}} = \frac{\text{length of } \overline{GH}}{\text{length of } \overline{CD}}$	Set up the proportion, knowing that the sides are proportional.
$\frac{\overline{FG}}{3} = \frac{20}{8}$	Substitute in the values of the known sides.
$3 \times 20 = 8 \times \overline{FG}$	Cross-multiply.
$60 = 8 \times \overline{FG}$	Divide by 8.
$7.5 = \overline{FG}$	Side FG is 7.5 cm.

Right triangles are commonly used in conjunction with ratios to measure tall items such as trees or flagpoles. On a sunny day the sun hits all objects positioned at the same location at the same angle.



Notice from the above picture that two right triangles are formed, and each triangle has the same angle measures. The triangles are similar, since the angles are congruent. Therefore, the sides are in proportion. One way to solve this type of problem is to set up a proportion, such as $\frac{\text{shadow of big}}{\text{shadow of little}} = \frac{\text{height of big}}{\text{height of little}}$.

Example:
A woman, 64 inches tall, casts a shadow that is 51.2 inches long. If the length of the shadow cast by the flagpole is 192 inches, how tall is the flagpole?



$\frac{192}{51.2} = \frac{h}{64}$	Set up the proportion as suggested above; cross-multiply.
$51.2 \times h = 192 \times 64$	Multiply 192 times 64.
$51.2 \times h = 12{,}288$	Divide by 51.2.
$h = 240$ inches, or 20 feet.	

Another application of geometry and ratio is measurement conversions, which will be covered in Chapter 8, Geometry and Measurement Conversions.

EXTRA HELP

For further practice and extended lessons on ratio and proportions, refer to *Practical Math Success in 20 Minutes a Day*, Lesson 12, Ratio and Proportion. In addition, the website *www.math.com* has helpful mini lessons and further practice problems. Click on *Pre-Algebra* from the column entitled *Select Subject*. Then click on any of the following links, under the title *Ratios and Proportions: Ratios, Proportions, Distance, Rates and Time,* or *Similar Figures.*

TIPS AND STRATEGIES

- Ratios are numerical comparisons.
- You can compare a part to a whole, or you can compare parts.
- Ratios can be expressed as a to b, $a : b$, or $\frac{a}{b}$.
- Ratios can be simplified just as fractions are simplified.
- A proportion is an equation that states that two ratios are equal.

- In a proportion, the product of the means equals the product of the extremes.
- Solve proportions using cross-multiplication.
- Take care to ensure a proper set-up when solving problems with proportions.
- Unit rates are ratios with a 1 in the denominator.
- Remember that $D = R \times T$, where D is the distance, R is the speed, and T is the time.
- Similar polygons have the same shape, but usually not the same size.
- For similar polygons, the corresponding sides are in proportion.

PRACTICE

Now that you have studied these lessons, see how much you have learned and reviewed about ratio and proportion.

1. Find the missing term: $\frac{4}{3} = \frac{25}{y}$.
a. 20
b. 33
c. 18.75
d. 75
e. 188

2. Find the missing term: $\frac{8}{y} = \frac{4}{8}$.
a. 4
b. 64
c. 32
d. 16
e. 24

3. Find the missing term: $\frac{15}{7} = \frac{90}{y}$.
a. 112
b. 150
c. 45
d. 4.5
e. 42

4. Find the missing term: $\frac{x}{12} = \frac{9}{2}$.
a. 54
b. 5.4
c. 23
d. 108
e. 42

5. There are 45 registered voters for every 70 eligible voters. How many eligible voters would be expected if there were 135 registered voters in a township?
a. 65
b. 86.8
c. 115
d. 180
e. 210

6. A snack mix consists of peanuts and raisins in a ratio of 3 to 2. How many pounds of raisins are in a 20-pound snack mix?
a. 30
b. 8
c. 12
d. 32
e. 25

7. The ratio of lunch buyers to lunch packers is 7 to 2. How many people pack their lunch if 35 people buy their lunch?
a. 123
b. 42
c. 10
d. 119
e. 7

8. An 8-inch by 10-inch photograph is blown up to a billboard size that is in proportion to the original photograph. If 8 inches is considered the width of the photo, what would be the length of the billboard if the width is 5.6 feet?
a. 7 feet
b. 400 feet
c. 56 feet
d. 9 feet
e. 156 inches

9. The ratio of Spanish to French students is 5 to 2. If a total of 350 students took Spanish or French, how many students take French?
a. 140
b. 250
c. 357
d. 10
e. 100

10. For every three cabinets built, 24 holes need to be drilled. How many holes must be drilled for ten cabinets?
a. 240
b. 30
c. 8
d. 80
e. 720

11. A map has a scale that specifies $\frac{1}{4}$ inch = 1 mile. How wide is an island that measures 6.2 inches on the map?
a. 24.8 miles
b. 1.55 miles
c. 4.43 miles
d. 6.45 miles
e. 7.6 miles

12. A recipe that serves 6 people calls for 3 cups of uncooked rice. How much uncooked rice is needed to serve 15 people?
a. 5 cups
b. 30 cups
c. 7 cups
d. 7.5 cups
e. 18 cups

13. How far will a car travel if it is running at a constant speed of 53 miles per hour for 4 and one-half hours?
a. 2,385 miles
b. 238.5 miles
c. 23.85 miles
d. 11.8 miles
e. 118 miles

14. A printer can print 17 sheets in 3 minutes. How long will it take to print 68 sheets of paper?
a. 10 minutes
b. 12 minutes
c. 51 minutes
d. 4 minutes
e. 18 minutes

15. It takes five and one quarter hours to drive to the state capital at an average speed of 55 miles per hour. What is the average speed if it takes seven hours to travel home the same distance?
a. 40 miles per hour
b. 25 miles per hour
c. 41.25 miles per hour
d. 100 miles per hour
e. 45 miles per hour

16. How long will it take a train moving at a speed of 70 miles per hour to travel 560 miles?
a. 8 minutes
b. 80 minutes
c. 80 hours
d. 39.2 hours
e. 8 hours

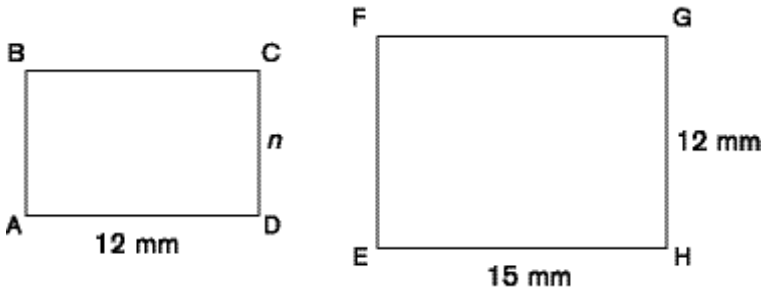
17. The height of the Eiffel Tower is 986 feet. A replica of the tower made to scale is 4 inches tall. What is the scale of the replica to the real tower?
a. 1 to 246.5
b. 1 to 3,944
c. 246.5 to 1
d. 1 to 2,958
e. 2,958 to 1

18. An action figure of King Kong made to scale is 10 inches high. An action figure of Godzilla made to the same scale is 11.5 inches high. If King Kong was 45 feet tall, how tall was Godzilla?
a. 51.75 feet tall
b. 5 feet tall
c. 55 feet tall
d. 517.5 feet tall
e. 51.75 inches tall

19. A data entry clerk can type 80 words per minute. How long will it take the clerk to type a manuscript of 160,000 words?
a. 200 minutes
b. 1,280 minutes
c. 2,000 minutes
d. 128,000 minutes
e. 20,000 minutes

20. At the grocery, 2.4 pounds of boneless chicken costs $6.00. At this unit cost, how much would 6 pounds of boneless chicken cost?
a. $2.40
b. $15.00
c. $5.00
d. $38.40
e. $86.40

21. Two similar rectangles are shown below. Find the missing length, *n*.



a. 9.6 mm
b. 15 mm
c. 159 mm
d. 96 mm
e. 15.9 mm

22. A man 6 feet tall casts a shadow that is 40 inches long. How tall is a black bear that casts a shadow of 48 inches long?
a. 94 inches
b. 7.2 inches
c. 720 feet
d. 7.2 feet
e. 240 feet

23. A recipe that serves 8 people requires one and one-half cups of cheddar cheese. How much cheddar cheese is needed to serve 6 people?
a. 3 cups
b. 9 cups
c. 2 cups
d. 0.75 cups
e. 1.125 cups

24. A faucet leaks at the rate of 2.5 gallons of water every 5 days. How many gallons are lost in a 365-day year?
a. 182,682.5 gallons
b. 12.5 gallons
c. 125 gallons
d. 18,250 gallons
e. 182.5 gallons

25. A car's fuel tank holds 15 gallons of gasoline. How far can the car travel on a full tank of gas, if the fuel efficiency is 23 miles per gallon?
a. 3,450 miles
b. 34.5 miles
c. 345 miles
d. 38 miles
e. 380 miles

ANSWERS

Here are the answers and explanations to the chapter quiz. Read them over carefully to understand any problems that you answered incorrectly. For more information and practice on working with fractions, decimals, and ordering the real numbers, see LearningExpress's *Practical Math Success in 20 Minutes a Day*, Lesson 12.

1. c. Cross-multiply to get $3 \times 25 = 4 \times y$. Multiply 3 times 25 to get $75 = 4 \times y$. Divide 75 by 4 to find the missing term, 18.75.

2. d. Cross-multiply to get $y \times 4 = 8 \times 8$. Multiply 8 times 8 to get $y \times 4 = 64$. Divide 64 by 4 to find the missing term, 16.

3. e. Cross-multiply to get $7 \times 90 = 15 \times y$. Multiply 7 times 90 to get $630 = 15 \times y$. Divide 630 by 15 to find the missing term, 42.

4. a. Cross-multiply to get $12 \times 9 = x \times 2$. Multiply 12 times 9 to get $108 = x \times 2$. Divide 108 by 2 to find the missing term, 54.

5. e. Set up a ratio of registered to eligible voters: $\frac{\text{registered voters}}{\text{eligible voters}}$. The proportion is $\frac{45}{70} = \frac{135}{v}$. Cross-multiply to get $70 \times 135 = 45 \times v$. Multiply 70 times 135 to get $9{,}450 = 45 \times$ v. Divide 9,450 by 45 to get 210 eligible voters.

6. b. We know the ratio of peanuts to raisins, but we are also told the total weight of the mixture. Set up a ratio of raisins to total mixture: $\frac{\text{raisins}}{\text{total mix}}$. The proportion is $\frac{2}{5} = \frac{r}{20}$. Cross-multiply to get $5 \times r = 2 \times 20$. Multiply 2 times 20 to get $5 \times r = 40$. Divide 40 by 5 to get 8 pounds of raisins.

7. c. Set up a ratio of lunch buyers to lunch packers: $\frac{\text{lunch buyers}}{\text{lunch packers}}$. The proportion is $\frac{7}{2} = \frac{35}{p}$. Cross-multiply to get $2 \times 35 = 7 \times p$. Multiply 2 times 35 to get 70; $70 = 7 \times p$. Divide 70 by 7 to get 10 lunch packers.

8. a. Set up a ratio of length to width: $\frac{\text{length}}{\text{width}}$. The proportion is $\frac{10}{8} = \frac{l}{5.6}$. Cross-multiply to get $8 \times l = 10 \times 5.6$. Multiply 10 times 5.6 to get $8 \times l = 56$. Divide 56 by 8 to get 7 feet long.

9. e. We know the ratio of Spanish to French students, but we are also told the total number of students. Set up a ratio of French students to total students: $\frac{\text{French}}{\text{total students}}$. The proportion is $\frac{2}{7} = \frac{f}{350}$. Cross-multiply to get $7 \times f = 2 \times 350$. Multiply 2 times 350 to get $7 \times f = 700$. Divide 700 by 7 to get 100 French students.

10. d. Set up a ratio of holes to cabinets: $\frac{\text{holes}}{\text{cabinets}}$. The proportion is $\frac{24}{3} = \frac{h}{10}$. Cross-multiply to get $3 \times h = 24 \times 10$. Multiply 24 times 10 to get $3 \times h = 240$. Divide 240 by 3 to get 80 holes.

11. a. Set up a ratio of map to real: $\frac{\text{map}}{\text{real}}$. The proportion is $\frac{\frac{1}{4}}{1} = \frac{6.2}{r}$. Cross-multiply to get $1 \times 6.2 = \frac{1}{4} \times r$; $6.2 = 0.25 \times r$. Divide 6.2 by 0.25 to get 24.8 miles long.

12. d. Set up a ratio of cups to people: $\frac{\text{cups}}{\text{people}}$. The proportion is $\frac{3}{6} = \frac{c}{15}$. Cross-multiply to get $6 \times c = 3 \times 15$. Multiply 3 times 15 to get $6 \times c = 45$. Divide 45 by 6 to get 7.5 cups.

13. b. Use the formula $D = R \times T$. Substitute in the known values: $D = 53 \times 4.5$. Multiply 53 times 4.5 to get 238.5 miles.

14. b. Set up a ratio of sheets to minutes: $\frac{\text{sheets}}{\text{minute}}$. The proportion is $\frac{17}{3} = \frac{68}{m}$. Cross-multiply to get $3 \times 68 = 17 \times m$. Multiply 3 times 68 to get $204 = 17 \times m$. Divide 204 by 17 to get 12 minutes.

15. c. First, find the distance to the state capital by using the formula $D = R \times T$. The distance is 55 times 5.25 = 288.75 miles. Now use

this distance again in the distance formula: $D = R \times T$; $288.75 = R \times 7$. Divide 288.75 by 7 to get the speed of 41.25 miles per hour.

16. e. Use the formula $D = R \times T$; $560 = 70 \times T$. Divide 560 by 70 to get the number of hours, which is 8.

17. d. A scale is a ratio of model to real, keeping the units consistent. The tower is 986 feet tall, and the replica is 4 inches. The height of 986 feet must be converted to inches, by multiplying by 12; 986 times 12 is 11,832 inches. Set up the ratio of replica to real and simplify: $\frac{4}{11{,}832} \div \frac{4}{4} = \frac{1}{2{,}958}$.

18. a. This problem is solved with a ratio, and can be set up as $\frac{\text{model kong}}{\text{real kong}} = \frac{\text{model godzilla}}{\text{real godzilla}}$. Substitute the known values; $\frac{10}{45} = \frac{11.5}{g}$. Cross-multiply to get $45 \times 11.5 = 10 \times g$. Multiply 45 by 11.5; $517.5 = 10 \times g$. Divide 517.5 by 10 to get 51.75 feet tall.

19. c. Set up a ratio of words to minutes: $\frac{\text{words}}{\text{minute}}$. The proportion is $\frac{80}{1} = \frac{160{,}000}{m}$. Cross-multiply to get $160{,}000 = 80 \times m$. Divide 160,000 by 80 to get 2,000 minutes.

20. b. Set up a ratio of pounds to dollars: $\frac{\text{pounds}}{\text{dollars}}$. The proportion is $\frac{2.4}{6} = \frac{6}{d}$. Cross-multiply to get $6 \times 6 = 2.4 \times d$. Divide 36 by 2.4 to get 15 dollars.

21. a. Set up a ratio of big to little: $\frac{\text{big}}{\text{little}}$. Determine the corresponding sides as being $\overline{AD}$ and $\overline{EH}$, and also $\overline{CD}$ and $\overline{GH}$. You need to find the length, n of $\overline{CD}$. The proportion is $\frac{15}{12} = \frac{12}{n}$. Cross-multiply to get $12 \times 12 = 15 \times n$. Divide 144 by 15 to get 9.6 millimeters.

22. d. Set up a proportion of figure to shadow: $\frac{\text{figure}}{\text{shadow}}$. The proportion is $\frac{6}{40} = \frac{b}{48}$. Cross-multiply to get $40 \times b = 6 \times 48$. Multiply 6 times 48, and then divide this answer by 40 to get the height of the bear: 7.2 feet.

23. e. Set up a ratio of cups to people: $\frac{\text{cups}}{\text{people}}$. The proportion is $\frac{1.5}{8} = \frac{c}{6}$. Cross-multiply to get $8 \times c = 1.5 \times 6$. Multiply 1.5 times 6 to get $8 \times c = 9$. Divide 9 by 8 to get 1.125 cups, which is one and one-eighth cups.

24. e. Set up a ratio of gallons to days: $\frac{\text{gallons}}{\text{days}}$. The proportion is $\frac{2.5}{5} = \frac{g}{365}$. Cross-multiply to get $5 \times g = 2.5 \times 365$. Multiply 2.5 times 365 to get $5 \times g = 912.5$. Divide 912.5 by 5 to get 182.5 gallons.

25. c. Set up a ratio of miles to gallons: $\frac{\text{miles}}{\text{gallons}}$. The proportion is $\frac{23}{1} = \frac{m}{15}$. Cross-multiply to get the miles as 23 times 15, which is 345 miles on a full tank of gasoline.

Percent

Now that you have reviewed fractions, decimals, and ratios in previous chapters, you are ready to tackle the most common ratio—the percent. The word "percent" means literally "out of one hundred"; percents compare numerical data to 100. Percent problems are so common that an entire chapter is devoted to this concept. Before you begin learning and reviewing percent, take a few minutes to take this ten-question *Benchmark Quiz*. These questions are similar to the type of questions that you will find on important tests. When you are finished, check the answer key carefully to assess your results. Your Benchmark Quiz analysis will help you determine how much time you need to spend on the specific areas of percent in which you need the most careful review and practice.

BENCHMARK QUIZ

1. What is 57% of 350?
a. 614
b. 16.29
c. 182
d. 199.5
e. 19.95

2. What percent of 200 is 68?
a. 68%
b. 2.94%
c. 34%
d. 136%
e. 0.34%

3. Nineteen is 76% of what number?
a. 76
b. 25
c. 14.44
d. 400
e. 250

4. Ravi borrowed $2,500.00 for three years at a simple annual interest rate of 8%. What is the TOTAL amount he will pay after the three years?
a. $200.00
b. $2,700.00
c. $600.00
d. $3,100.00
e. $6,000.00

5. Two out of every five members of the town board are male. What percentage of the board members is male?
a. 25%
b. 40%
c. 20%
d. 4%
e. 15%

6. Hockey sticks that normally sell for $89.00 are on sale for 35% off the regular price. There is also a 6% sales tax. How much will the stick cost after the sale and the sales tax?
a. $57.85
b. $33.02
c. $54.38
d. $29.28
e. $61.32

7. The book club attendance rose from 25 members to 30 members. What is the percent increase in membership, to the nearest percent?
a. 20%
b. 83%
c. 17%
d. 2%
e. 5%

8. Out of the 28 selections on the menu, four of them are desserts. What percentage, to the nearest tenth, of the menu are NOT desserts?
a. 14.3%
b. 12.5%
c. 85.7%
d. 75.0%
e. 7.0%

9. Twenty-five percent of the voters voted for the incumbent. How many voted for the incumbent, if there were 1,032 voters?
a. 258
b. 4,128
c. 25
d. 1,007
e. 2,422

10. How much will be paid for a $28.00 dinner, assuming a 15% tip?
a. $4.20
b. $43.00
c. $23.80
d. $30.80
e. $32.20

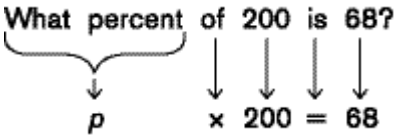
BENCHMARK QUIZ SOLUTIONS

How did you do on the percent Benchmark Quiz? Check your answers here, and then analyze your results to figure out your plan of attack to master this topic.

1. d. One method of solution is to set up a proportion; $\frac{\text{part}}{\text{whole}} = \frac{\text{percent}}{100}$. The "whole" is 350, and the "part" is what is being requested in the problem. Substitute in the given information; $\frac{n}{350} = \frac{57}{100}$. Cross-multiply to get $350 \times 57 = n \times 100$, and then multiply 350

times 57; 19,950 = $n \times 100$. Divide 19,950 by 100 to get 199.50 or the equivalent 199.5.

2. c. Set up an equation, recalling that "of" means *multiply* and "is" means *equals*, make a straight translation using the variable p for percent:



So, $p \times 200 = 68$. Divide 68 by 200 to get 0.34. This is the answer as a decimal. Change this answer to a percent by multiplying by 100 to get 34%.

3. b. Set up a proportion: $\frac{\text{part}}{\text{whole}} = \frac{\text{percent}}{100}$, which results by substitution in $\frac{19}{n} = \frac{76}{100}$. In the problem, 19 is the "part," 76 is the percent, and the "whole" is what you need to calculate. Cross-multiply to get $n \times 76 = 19 \times 100$. Multiply; $n \times 76 = 1{,}900$. Now divide 1,900 by 76 to get 25.

4. d. Use the formula Interest = Principal × Rate × Time, or simply $I = PRT$. Time is in years, and the rate is the percentage rate written as a decimal. Substitute in the given information to find the interest; $I = 2{,}500 \times 0.08 \times 3$. Multiply the terms on the right to get $I = 600$. This interest is added to the money borrowed to get the TOTAL repayment. The repayment is \$600.00 + \$2,500.00, which is \$3,100.00.

5. b. Two out of every five indicates a ratio, so use the proportion $\frac{\text{part}}{\text{whole}} = \frac{\text{percent}}{100}$. Substitute in the numbers; $\frac{2}{5} = \frac{p}{100}$. Cross-multiply to get $5 \times p = 2 \times 100$. Multiply; $5 \times p = 200$. Now divide 200 by 5 to get 40%.

6. e. This is a multi-step problem, because the sale percentage is a percent decrease, and the sales tax is a percent increase. There are several methods to solve this problem. The key word "is" means equals, "of" means multiply, and 35% is 0.35 written as a decimal. Set up the equation: discount = percent × original, or $d = 0.35 \times 89$. Multiply, to get the discount, which is \$31.15. The sale price is thus \$89.00 – \$31.15 = \$57.85. The sales tax is then calculated based on this sale price; sales tax = percent × sale price. The tax

will be $t = 0.06 \times 57.85$, or $t = \$3.47$, rounded to the nearest cent. Add this to the sales price to find the cost of the stick; \$57.85 + \$3.47 = \$61.32.

7. a. This is a percent increase problem, so set up the proportion; $\frac{\text{change}}{\text{original}} = \frac{\text{percent}}{100}$. The change in attendance is 30 – 25 = 5. The original attendance is 25 members. The proportion setup is: $\frac{5}{25} = \frac{p}{100}$. Cross-multiply to get $25 \times p = 5 \times 100$. Multiply 5 times 100 to get $25 \times p = 500$. Divide 500 by 25 and the percent is therefore 20%.

8. c. The problem asks what percentage are NOT desserts. Since four of the 28 selections are desserts, then 28 – 4 = 24 selections are not desserts. Set up the proportion; $\frac{\text{part}}{\text{whole}} = \frac{\text{percent}}{100}$, and substitute in the correct numbers: $\frac{24}{28} = \frac{p}{100}$. Cross-multiply; $28 \times p = 24 \times 100$, or $28 \times p = 2{,}400$. Divide 2,400 by 28 to get the percent, rounded to the nearest tenth, of 85.7%.

9. a. Twenty-five percent of the 1,032 voters voted for the incumbent. The key word "of" means multiply, and 25% is 0.25 written as a decimal; $0.25 \times 1{,}032 = 258$ voters.

10. e. The tip is a percent increase to the price of the dinner. Fifteen percent can be written as 0.15, and the key word "of" means multiply; $0.15 \times 28 = 4.2$ The tip is \$4.20, which is added to the \$28.00 to get \$28.00 + \$4.20 = \$32.20.

BENCHMARK QUIZ RESULTS:

If you answered 8–10 questions correctly, you have a good basic understanding of percent. Read through the lessons in this chapter to gain mastery of the different contexts in which percent problems are presented. The advice in Tips and Strategies may be especially helpful to you.

If you answered 4–7 questions correctly, you need to spend more effort in your study of this chapter. Carefully read through the lesson and try each example. Visit the suggested websites for extended practice. Work through the quiz at the end of the chapter to check your progress.

If you answered 1–3 questions correctly, give this chapter your concentrated effort. Make sure you have read and understood Chapters 4 and 5; they are the foundation for the concept of percent. Work through all examples, and try the different methods of solution with each problem. Go to the suggested websites in the Extra Help area in this chapter, and do extended

practice. You may want to refer to a more comprehensive book on the subject of percents, such as *Practical Math Success in 20 Minutes a Day*, Lessons 9, 10, and 11, published by LearningExpress.

JUST IN TIME LESSON—PERCENT

Pick up any newspaper or watch any news program; chances are that these media use percentages to make their information clear. Every time you shop for a sale, borrow money, choose a credit card, invest money, or pay a tip to a service worker, you use your understanding of percents. Percents are special ratios that compare a part to a whole; the whole is always one hundred. Percent problems can be solved just like any other type of ratio/proportion problem. This chapter will also review alternate methods to solve percent problems.

The topics in this chapter are:

- introduction to percentages: a part/whole relationship
- finding the percent of a number (find the part, given the percent and the whole)
- find the whole when the percent and the part are known
- given the part and the whole, find the percent
- percent increase/decrease and the various applications of this type of percent
- simple interest percentage problems

GLOSSARY

PERCENT a ratio that compares numerical data to one hundred. The symbol for percent is %.

INTRODUCTION TO PERCENTAGES—A PART/WHOLE RELATIONSHIP

Our number system is the decimal system, where digits are based on the powers of ten. It is natural to want to compare numbers to a common baseline, like one hundred, which is 10^2. Ratios that are comparisons of a part to a whole are percents when the whole is one hundred. The symbol for percent is %; 74% means 74 out of every one hundred. As an example, if two out of every five people live in a city, to find the percent you can use a proportion; $\frac{2}{5} = \frac{p}{100}$. Cross-multiply to get $5 \times p = 2 \times 100$, or $5 \times p = 200$. Divide 200 by five to get $p = 40$. So it is true that 40% of the people live in cities. A ratio such as 2 to 5 is perhaps more clear when expressed as 40%.

Another way to convert a ratio in fractional form into a percent is to first change the ratio to a decimal by long division and then multiply by 100. Changing a fraction into a decimal was covered in Chapter 4 of this book.

RULE BOOK

There are two ways to change a ratio in fractional form, such as $\frac{a}{b}$, to a percent:

- **Set up a proportion $\frac{a}{b} = \frac{c}{100}$, where the percent is c%.**
- **Change the ratio to a decimal and multiply by 100.**

For example, $\frac{7}{8} = 0.875$, so $\frac{7}{8}$ is 87.5% because 0.875 times 100 is 87.5.

SHORTCUT

To change a decimal to a percent, move the decimal point two places to the right.

Example 1:

To change 0.87 to a percent, move the decimal point two places to the right:

0.87 ➡ 87.

So 0.87 = 87%

Example 2:

To change 1.3 to a percent:

1.3 ➡ 130.

So 1.3 = 130%

It is helpful to know some common fraction/decimal/percent conversions:

$\frac{1}{2}$	0.5	50%	$\frac{1}{5}$	0.2	20%	$\frac{3}{5}$	0.6	60%
$\frac{1}{4}$	0.25	25%	$\frac{1}{8}$	0.125	12.5%	$\frac{1}{3}$	$0.\overline{3}$	$33\frac{1}{3}$%
$\frac{3}{4}$	0.75	75%	$\frac{1}{10}$	0.1	10%	$\frac{2}{3}$	$0.\overline{6}$	$66\frac{2}{3}$%

SHORTCUT

To change a percent to a decimal, divide the percent value by 100; move the decimal point two places to the left.

Example 1:

To change 17% to a decimal, move the decimal point two places to the left:

17. ➡ 0.17

Example 2:

To change 6% to a decimal, move the decimal point two places to the left. It is necessary in this instance to add a zero as a placeholder:

.6. ➡ 0.06

FINDING THE PERCENT OF A NUMBER

Percents can be solved in several ways. Two main ways are by using a proportion or by using an equation. To use a proportion, understand that percents are a part/whole relationship and set up $\frac{\text{part}}{\text{whole}} = \frac{\text{percent}}{100}$. To find the percent of a number, the "part" is the missing term. The percent is always over one hundred. Set up the proportion and cross-multiply.

Example:
What is 28% of 275?

The term 275 is the "whole" and the problem asks for 28% of this whole:

$\frac{n}{275} = \frac{28}{100}$	Set up the proportion and cross-multiply.
$275 \times 28 = n \times 100$	Multiply 275 times 28.
$7{,}700 = n \times 100$	Divide 7,700 by 100.
$77 = n$	28% of 275 is 77.

SHORTCUT

To set up the proportion, think: $\frac{\text{is}}{\text{of}} = \frac{\%}{100}$.

In all percent problems using proportions, the percent value is ALWAYS over the 100.

The term that immediately follows the key word "of" is the denominator. The term that either immediately precedes or immediately follows the key word "is" is the numerator.

To use the equation method, recall this important information when translating words into math: "is" means *equals* and "of" means *multiply*. Also, for equations, change the percent to a decimal by dividing by one hundred

(move the decimal point two places to the left). To use an equation, you take a problem and rewrite it as an equation with an equal sign and the multiplication operator. You can use a straight translation whose set up is "part is percentage of whole" or "percentage of whole is part."

Example:

What is 18% of 75?
↓ ↓ ↓ ↓ ↓
n = 0.18 × 75

So 18% of 75 is 13.5.

CALCULATOR TIPS

If your calculator has the % key, then it handles percents directly. Check your calculator carefully. Often the percent function (%) is a second function. Often, you must enter the percent as the last term in the calculator. Check with the user guide of your calculator for specific information.

Example:

What is 47% of 370?

Recall that the key word "of" means multiply, and enter into your calculator:

What is 47% of 370?
↓ ↓ ↓ ↓ ↓
n = 0.47 × 370

Before moving on in the chapter, take a few minutes to go back and attempt to solve the first example using an equation and the second example using a proportion. After trying both methods, you will naturally prefer one method, but it is a good idea to be proficient in using both methods. Some types of problems lend themselves to one or the other method of solution.

FIND THE WHOLE WHEN THE PERCENT AND THE PART ARE GIVEN

Example:
Fifty-six percent of what number is 63?

In this example, the percent, 56, is known and the "part" term, 63, is given. The "whole" is the missing term. Just as in the previous section, there are two approaches to this problem.

Proportion Method: Set up the proportion as $\frac{\text{is}}{\text{of}} = \frac{\%}{100}$.

$\frac{63}{w} = \frac{56}{100}$	Substitute in the given information and cross-multiply.
$w \times 56 = 63 \times 100$	Multiply 63 times 100.
$w \times 56 = 6{,}300$	Divide 6,300 by 56 to find the whole, w.
$w = 112.5$	

Now try this example using the equation method, and be sure your answer agrees with the above.

Example:
Ninety-eight is 14% of what number?

Equation Method: Write the equation, substituting in an equal sign for "is," the multiplication sign for "of," and change the percent to a decimal.

$98 = 0.14 \times w$	Divide 98 by 0.14 to find the whole, w.
$700 = w$	

Now, try this example using the proportion method. Your answer should be the same, 700.

GIVEN THE PART AND THE WHOLE, FIND THE PERCENT

Example:
Forty-two is what percent of 105?

Again, the term that precedes the key word "is" is the "part," and the term that immediately follows the key word "of" is the "whole." The percent, the missing term, is always over one hundred when setting up a proportion.

$\frac{42}{105} = \frac{p}{100}$	Set up the proportion; cross-multiply.
$105 \times p = 42 \times 100$	Multiply 42 times 100.
$105 \times p = 4{,}200$	Divide 4,200 by 105 to get the percent.
$p = 40\%$	

Try this example with the equation method.

Example:
What percent of 1,136 is 71?

Substitute in to make an equation, where the key word "is" becomes the equal sign, and the key word "of" is the multiplication sign.

$p \times 1{,}136 = 71$	Divide 71 by 1,136, and the percent will be written as a decimal.
$p = 0.0625$	Move the decimal point two places to the right.
$p = 6.25\%$	

Try this example with the proportion method.

PERCENT INCREASE AND DECREASE

Often, there are comparisons made from one condition to another; the population increases, the merchandise price decreases, and taxes are paid. These can all be expressed as percent increases or decreases. Like all percents, these are part/whole relationships. The "part" is the *change* and the "whole" is the *original* number. It is important to be aware of this distinction; note that the new condition is not part of the proportion to be set up. The new condition is used to calculate the *change*. The setup for percent increase or decrease is: $\frac{\text{change}}{\text{original}} = \frac{\%}{100}$.

In the same manner, when you use an equation to solve these problems, the new condition does not appear in the equation. The amount of change of the original condition (the whole) is the part. So change is some percent of the original number.

Example:
The class size dropped from 400 students in 2002 to 375 students in 2003. What is the percent decrease from 2002 to 2003?

Set up a proportion based on $\frac{\text{change}}{\text{original}} = \frac{\%}{100}$. Calculate the change, which is 400 – 375 = 25 students.

$\frac{25}{400} = \frac{p}{100}$	Substitute in the pertinent numbers.
$400 \times p = 25 \times 100$	Multiply 25 times 100.
$400 \times p = 2{,}500$	Divide 2,500 by 400 to get the percent decrease.
$p = 6.25\%$	

Now, try this same problem by setting up an equation as *change = percent of original.*

Percent increase and decrease has several applications in math problems. Sales tax and service tips are percent increases and merchandise sales are percent decreases.

Example:
The $79.00 picnic table came to a total price of $83.74 with sales tax. What was the sales tax percentage?

Sales tax is a percent increase, so calculate the change in price, $83.74 – $79.00 = $4.74. Set up an equation using "change is percent of original."

$4.74 = p \times 79$	Divide 79 by 4.74.
$p = 0.06$	This percent is in decimal form. Move the decimal point two places to the right to find the percent.
$p = 6\%$	

You can also work this problem out using proportions to get the same result.

Example:
Raincoats, normally selling for $46.00, are on sale at 20% off the regular price. What is the sale price?

In this example, the original price is $46.00 and the percent is 20. The change to be calculated will result in the *discount*. Solve for the discount and then subtract this from the original price to get the sale price.

$\frac{n}{46} = \frac{20}{100}$	Set up the proportion with the given information. Cross-multiply.
$46 \times 20 = n \times 100$	Multiply 46 times 20.
$920 = n \times 100$	Divide 920 by 100 to get the discount.
$9.20 = n$	Subtract this from the original price.
$\$46.00 - \$9.20 = \$36.80$	

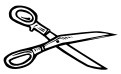
SHORTCUT

You can solve sales problems quicker by realizing that if there is 20% off, you will actually pay 100 − 20 = 80% of the original price. As another example, 35% off means 100 − 35 = 65% of the original price will be paid. So the last example can be solved directly as "80% of the original price is the sale price," and 0.80 times $46.00 is $36.80.

In a percent problem that involves both a sale and sales tax, these percents must be handled separately. It is a two-step problem.

Example:
Tents normally sell for $139.95 and are on sale at 25% off of the regular price. How much will the tent cost at this sale percentage, if there is also a 7% sales tax?

First, find the sale price. Using the shortcut, the tent will cost 100 – 25 = 75% of the regular price; 0.75 × 139.95 = 104.96, rounded to the nearest cent. Second, calculate the sales tax, which is based on the sale price, and add it to the sale price; 0.07 × 104.96 = 7.35, rounded to the nearest cent. The total price is therefore $104.96 + $7.35 = $112.31.



SHORTCUT

Total price with sales tax can be solved more quickly. The price will be one hundred percent of the sale price, plus seven percent of this price, or 100 + 7 = 107% of the sale price. The previous example can be solved directly as 1.07 × $104.96 = $112.31.

Service tips are another common use for percent-increase applications. They are treated the same as sales tax because they are calculated and then added to the cost.

SIMPLE INTEREST PERCENTAGE PROBLEMS

When you borrow money, you pay additional funds for the service. When you deposit money into savings you earn additional funds for your deposit. In either case, the additional funds are called *interest.* The money borrowed or deposited is called *principal;* the percentage used to calculate the additional funds is called the *interest rate.* Interest earned or owed is dependent not only on the principal and rate, but also on another variable, which is the length of *time in years.* To calculate interest for one year, you can set up a proportion. Because interest is such a common occurrence in the business world, there is a formula used to calculate interest: $I = PRT$. The interest is I, P is the principal, R is the rate written as a decimal, and T is the time in years.

Example:
What is the interest earned on $1,990.00 invested at 4% simple interest if it is invested for three years?

$I = PRT$	Using the simple interest formula, substitute in the given numbers.
$I = 1{,}990 \times 0.04 \times 3$	Multiply the values to find the interest.
$I = \$238.80$	

In these types of problems, be careful to pay attention to what is being asked for; sometimes the interest is requested, but other times the total return is wanted.

Example:
Rachel borrows $12,000 for her new car at a simple interest rate of 3.5% for five years. What is the total amount Rachel will repay for this loan?

$I = PRT$	Use the simple interest formula.
$I = 12{,}000 \times 0.035 \times 5$	Multiply the values to find the interest.
$I = \$2{,}100.00$	Add this to the amount borrowed for the total repayment.

Total = $12,000 + $2,100 = $14,100.00.

Take care when calculating time. Time is measured in years.

Example:
Tyrone purchased a certificate of deposit for $500.00 and it matured in 7 years, 6 months. If after the maturity time Tyrone was given $837.50 as his total return, what was the percent interest rate, assuming a simple interest formula?

Calculate the interest, which is the change: $837.50 – $500.00 = $337.50

Use $I = PRT$ to solve for the percentage rate. The time is 7 and one-half years.

$337.5 = 500 \times p \times 7.5$	Multiply 500 times 7.5.
$337.5 = 3{,}750 \times p$	Divide 337.5 by 3,750 to find the rate as a decimal.
$0.09 = p$	Move the decimal point two places to the right.
$9\% = p$	

EXTRA HELP

If you feel you need extended help with the concept of percent, *Practical Math Success in 20 Minutes a Day*, published by LearningExpress, has several chapters devoted to this topic: Lessons 9, 10, and 11.

In addition, the website *www.math.com* has helpful mini lessons and further practice problems. Click on *Percents* under *Hot Subjects*. Then click on *numbers* under the word *percent.* Follow along with the lesson explanation and then practice problems. To explore the part/whole relationship of percents and how they are related to decimals and fractions, try *http://matti.usu.edu/nlvm/nav/index.html*. Click on the *Virtual Library*. Then choose *Numbers and Operations* under the column 9–12. Scroll down, and then click on *Percentages*.

You will also find practice on percents at the website *www.aaamath.com*. Choose *Percent* under *Math Topics.* From there, there are several topics you can practice.

TIPS AND STRATEGIES

- Solve percent problems using a proportion: $\frac{\text{part}}{\text{whole}} = \frac{\text{percent}}{100}$.
- Percent can be thought of as $\frac{\text{is}}{\text{of}} = \frac{\%}{100}$, where the number associated with "is" is the part and the number associated with "of" is the whole.
- Proportions or equations can be used to solve percent problems.
- The key word "is" means *equals;* the key word "of" means *multiply*.
- Percent increase or decrease involves the proportion $\frac{\text{change}}{\text{original}} = \frac{\%}{100}$, where the change is the part and the original number is the whole.
- For sale percentage off, the change is the discount.
- An alternative to solving for the discount is to solve directly for the sale price.
- For sales tax or tips, the change in price is the tax or tip.
- An alternative to solving for the tax or tip is to solve directly for the total price.
- Simple interest problems can be solved using the formula $I = PRT$.

PRACTICE

Now that you have studied this lesson, try this set of practice problems. Carefully read over the answer explanations. Keep in mind that there are several ways to solve percent problems, so you may use an alternate method to arrive at the correct answer.

1. Change $\frac{15}{16}$ to a percent.
a. 0.9375%
b. 93.75%
c. 15.16%
d. 15%
e. 85%

2. Change 23.5% to a decimal.
a. 2.35
b. 23.5
c. 0.235
d. 235.00
e. 23,500.00

3. Change 1.8 to a percent.
a. 18%
b. 80%
c. 1.8%
d. 180%
e. 20%

4. What is 12.8% of 405, to the nearest hundredth?
a. 51.84
b. 0.03
c. 31.64
d. 0.52
e. 518.40

5. 272 is what percent of 400?
a. 1.47%
b. 147%
c. 0.68%
d. 10.88%
e. 68%

6. 533 is 82% of what number?
a. 15.38
b. 6.5
c. 149.24
d. 650
e. 437.06

7. Which of the following is 49% of 3,000?
a. 6,122.45.
b. 14.70.
c. 1,470.
d. 1,530.
e. 612.25

8. 4.25 is what percent of 25?
a. 5.88%
b. 0.17%
c. 106.25%
d. 1.06%
e. 17%

9. 91% of what number is 200.2?
a. 2.2
b. 220
c. 45.45
d. 454.5
e. 182.182

10. The number of customers in ABC department store rose from 1,200 on Saturday to 1,350 on Sunday. What is the percent increase?
a. 11.11%
b. 88.9%
c. 150%
d. 12.5%
e. 112.5%

11. The population of Trendsville fell from 25,670 in 1990 to 24,500 in 1998. What was the percent decrease during this time frame, to the nearest tenth?
a. 4.6%
b. 8%
c. 46%
d. 4.8%
e. 95.44%

12. Car prices rose from $5,200 to $10,200 from 1976 to 2002. What is the percent increase to the nearest percent?
a. 100%
b. 50%
c. 26%
d. 96%
e. 49%

13. Raisin Crunchy cereal advertises that 26% of its total volume is raisins. How many ounces of raisins are in an 18-ounce package?
a. 13.32 oz
b. 4.68 oz
c. 26 oz
d. 2.6 oz
e. 0.26 oz

14. Of the 44 items in the closet, 11 of the items are red. What percentage of the items is NOT red?
a. 25%
b. 11%
c. 44%
d. 75%
e. 33%

15. What percent of 170 is 40.8?
a. 4.2%
b. 12.92%
c. 24%
d. 69.36%
e. 76%

16. At a 15% tip rate, what is the cost of an $83.00 meal, including tip?
a. $98.00
b. $68.00
c. $70.55
d. $12.45
e. $95.45

17. Jeans whose regular price is $28.00 are on sale at 30% off. What is the sale price?
a. $8.40
b. $19.60
c. $2.00
d. $36.40
e. $26.00

18. What is the out-of-pocket expense for a blender whose price is $89.95, plus a 6% sales tax?
a. $95.35
b. $95.95
c. $84.55
d. $5.40
e. $143.92

19. The truck whose sticker price is $13,500 is on sale at 15% off. There is also an 8% sales tax. What will be the total price paid for this truck, including discount and sales tax?
a. $2,187.00
b. $3,645.00
c. $11,475.00
d. $13,800.00
e. $12,393.00

20. What is the interest due on a $462 loan at 18% simple interest for 2 years?
a. $83.16
b. $628.32
c. $166.32
d. $18.00
e. $36.00

21. $2,300.00 is placed in a savings account at 9% interest for 6 months. How much TOTAL money is in the account after the 6 months, assuming no additional deposits or withdrawals?
a. $2,403.50
b. $3,542.00
c. $1,242.00
d. $2,070.00
e. $103.50

22. Dinner at the XYZ restaurant includes an 18% tip automatically added to the bill. How much of the $49.56 bill is for the food?
a. $18.00
b. $31.56
c. $42.00
d. $7.56
e. $40.66

23. What is the simple interest rate on a $3,500.00 loan, taken out for 3 years, if the interest due after this time frame is $2,205.00?
a. 0.21%
b. 4.76%
c. 47.6%
d. 63%
e. 21%

24. The picnic table is sale priced at $35.40. What was the original price if the sale price reflects 40% off?
a. $14.16
b. $49.56
c. $88.50
d. $75.40
e. $59.00

25. Jeanne deposits $150.00 into a savings account that pays 4% interest per year. How long must she keep the money in this account to gain $42.00 in interest?
a. 4.2 years
b. 37.5 years
c. 3.75 years
d. 7 years
e. 10 years

ANSWERS

1. b. Change the fraction $\frac{15}{16}$ to a decimal by long division, to get 0.9375. To change this decimal to a percent, move the decimal point two places to the right, to get 93.75%.

2. c. To change a percent to a decimal, move the decimal point two places to the left; 23.5% = 0.235.

3. d. To change 1.8 to a percent, move the decimal two places to the right. It is necessary to add a trailing zero as a placeholder; 1.8 = 180%.

4. a. To solve this problem, remember that the key word "of" means *multiply*, and change the percent to a decimal; 12.8% = 0.128. Multiply 0.128 times 405 to get 51.84.

5. e. Set up a proportion, using $\frac{\text{is}}{\text{of}} = \frac{\%}{100}$. The term immediately preceding the key word "is" is 272, and the term following the key word "of" is 400. The setup is $\frac{272}{400} = \frac{p}{100}$. Cross-multiply to get $400 \times p = 272 \times 100$. Multiply 272 times 100; $400 \times p = 27{,}200$. Divide 27,200 by 400 to get 68%.

6. d. You can set up an equation, recalling that "is" means *equals* and "of" means *multiply*. For equations, the percent must also be converted to a decimal. A straight translation gives $533 = 0.82 \times n$. Divide 533 by 0.82 to get 650.

7. c. Change 49% to a decimal to get 0.49. Since the key word "of" means *multiply*, multiply 0.49 times 3,000 to get 1,470.

8. e. Set up a proportion; 4.25 is the part, since it precedes the key word "is," and 25 is the whole, as it follows the key word "of." Use $\frac{\text{is}}{\text{of}} = \frac{\%}{100}$, $\frac{4.25}{25} = \frac{p}{100}$. Cross-multiply to get $25 \times p = 4.25 \times 100$. Multiply 4.25 times 100; $25 \times p = 425$. Divide 425 by 25 to get 17%.

9. b. Set up an equation, changing 91% to a decimal, that is 0.91. The key word "of" means *multiply* and "is" means *equals*, so translate as $0.91 \times n = 200.2$. Divide 200.2 by 0.91 to get 220.

10. d. This is a percent-increase problem, so use the proportion $\frac{\text{change}}{\text{original}} = \frac{\%}{100}$. The change is 1,350 – 1,200 = 150. The original number is 1,200; $\frac{150}{1{,}200} = \frac{p}{100}$. Cross-multiply to get $1{,}200 \times p = 150 \times 100$.

Multiply 150 times 100; $1{,}200 \times p = 15{,}000$. Divide 15,000 by 1,200 to get 12.5%.

11. a. For percent decrease, set up a proportion: $\frac{\text{change}}{\text{original}} = \frac{\%}{100}$. The change is 25,670 – 24,500 = 1,170. The original population is 25,670; $\frac{1{,}170}{25{,}670} = \frac{p}{100}$. Cross-multiply to get $25{,}670 \times p = 1{,}170 \times 100$. Multiply 1,170 by 100; $25{,}670 \times p = 117{,}000$. Divide 117,000 by 25,670 to get approximately 4.5578, which rounds to the nearest tenth as 4.6%.

12. d. Calculate the change in price; 10,200 – 5,200 = 5,000. The original price is 5,200. Set up the proportion $\frac{\text{change}}{\text{original}} = \frac{\%}{100}$, which is $\frac{5{,}000}{5{,}200} = \frac{p}{100}$. Cross-multiply; $5{,}200 \times p = 5{,}000 \times 100$. Multiply 5,000 by 100; $5{,}200 \times p = 500{,}000$. Divide 500,000 by 5,200 to get approximately 96.1538, which is 96% rounded to the nearest percent.

13. b. The problem states that 26% of the total volume is raisins. Change 26% to a decimal, 0.26, and recall that the key word "of" means *multiply;* 0.26 times 18 equals 4.68 oz.

14. d. If eleven of the items are red, then 44 – 11 = 33 items are NOT red. Set up a proportion using $\frac{\text{part}}{\text{whole}} = \frac{\text{percent}}{100}$ which is $\frac{33}{44} = \frac{p}{100}$. Cross-multiply to get $44 \times p = 33 \times 100$. Multiply 33 times 100; $44 \times p = 3{,}300$. Divide 3,300 by 44 to get 75%.

15. c. This problem can be solved with an equation. The key word "of" means *multiply*, and the key word "is" means *equals*. Write the equation as a straight translation; $p \times 170 = 40.8$. Divide 40.8 by 170 to get 0.24. Remember that for equations, this term, 0.24, is the percent written as a decimal. Move the decimal point two places to the right to get the answer expressed as a percent.

16. e. Since the tip is an additional percentage added to the cost, use the shortcut that says the cost of the meal will be 100% + 15% = 115% of the price of the food. Write 115% as a decimal, that is 1.15; 1.15 of \$83.00 becomes $1.15 \times 83 = 95.45$.

17. b. Jeans that are on sale for 30% off will cost 100% – 30% = 70% of the regular price. Using the key word "of" to mean *multiply*, and change 70% to the decimal 0.7; 28 times 0.7 is \$19.60.

18. a. The blender will cost the price plus an additional 6% of this price as tax; 100 + 6 = 106% of the price will be the out-of-pocket

expense. Because the key word "of" means *multiply*, and 106% is 1.06 written as a decimal, 1.06 times \$89.95 equals 95.347, which is \$95.35 rounded to the nearest cent.

19. e. The truck is on sale for a percentage off, and there is also a sales tax percentage increase. This is a two-step problem. First find the sale price and then find the total price with tax. The discount can be calculated using a proportion $\frac{\text{change}}{\text{original}} = \frac{\%}{100}$; $\frac{n}{13{,}500} = \frac{15}{100}$. Cross-multiply; $13{,}500 \times 15 = n \times 100$. Multiply 13,500 by 15; $202{,}500 = n \times 100$. Divide 202,500 by 100 to get the DISCOUNT of 2,025. Subtract this from the sticker price to get the sale price \$13,500 – \$2,025 = \$11,475. Use this sale price to calculate the tax: $\frac{n}{11{,}475} = \frac{8}{100}$. Cross-multiply; $11{,}475 \times 8 = n \times 100$. Multiply 11,475 by 8; $91{,}800 = n \times 100$. Divide 91,800 by 100 to get the tax of \$918.00. Add this to the sale price to get the total price paid; \$11,475 + \$918 = \$12,393.00.

20. c. Use the simple interest formula $I = PRT$, where the principal is \$462; the rate is 18%, written as a decimal as 0.18; and the time is 2; $I = 462 \times 0.18 \times 2$. Multiply the three terms on the right to get \$166.32.

21. a. Use the interest formula to calculate interest. Then you must add this to the total placed in the account. Remember that 9% must be written as a decimal, which is 0.09, and time is in years, so the time is 0.5 years. Use $I = PRT$; $I = 2{,}300 \times 0.09 \times 0.5$. Multiply the three terms to get \$103.50 as the interest earned. The total in the account after the six months is \$2,300 + \$103.50 = \$2,403.50.

22. c. This problem gives the total and asks for the part of this bill that is the food, without the gratuity. To solve this you must realize that this \$49.56 represents 100% + 18% = 118% of the cost of the food. So 118% of the food is \$49.56. Write 118% as a decimal, which is 1.18. Set up the equation $1.18 \times n = 49.56$. Divide 49.56 by 1.18 to get the price of the food. This results in \$42.00.

23. e. In this problem you are given the interest paid, 2,205, the principal of 3,500, and the time (in years) of 3. Use the formula $I = PRT$ to solve for the rate, the percent; $2{,}205 = 3{,}500 \times R \times 3$. Multiply 3,500 by 3; $2{,}205 = 10{,}500 \times R$. Divide 2,205 by 10,500 to find the rate written as a decimal; $0.21 = R$, and move the decimal two places to the right to express this as a percent. The percentage rate is 21%.

24. e. In this problem you are given the sale price and the percentage off, and need to calculate the original price. This sale price is 100% – 40% = 60% of the original price. Set up an equation that shows the sale price is 60% of the original price. Change 60% to a decimal to get 0.6; $35.40 = 0.6 \times n$. Divide 35.40 by 0.6 to get the original price, which is $59.00.

25. d. Use the simple interest formula $I = PRT$, and solve for T. Substitute in the given information, changing 4% to the decimal 0.04; $42 = 150 \times 0.04 \times T$. Multiply 150 times 0.04; $42 = 6 \times T$. Divide 42 by 6 to get 7 years.

Powers, Exponents, and Roots

In Chapter 3, the properties of numbers were reviewed. Recall that a factor of a number is a whole number that divides evenly, without a remainder, into the given number. Frequently, you multiply the same factor by itself several times. In math, there is a special notation for this idea: exponents and the inverse operation, roots. How much do you recall about exponents, roots, and scientific notation? Take a few minutes now to take this ten-question *Benchmark Quiz*. These questions are similar to the type of questions that you will find on important tests. When you are finished, check the answer key carefully to assess your results. Your Benchmark Quiz analysis will help you determine how much time you need to spend on the specific areas in which you need the most careful review and practice.

BENCHMARK QUIZ

1. Simplify 7^2.
a. 14
b. 7.2
c. 3.5
d. $\frac{1}{49}$
e. 49

2. Simplify 2^{-3}.
a. 6
b. –8
c. –6
d. $\frac{1}{8}$
e. $\frac{1}{6}$

3. Simplify $\sqrt[3]{27}$.
a. 5
b. 3
c. 9
d. $\frac{27}{3}$
e. $\frac{1}{3}$

4. Simplify $25^{\frac{1}{2}}$.
a. 12.5
b. 625
c. –625
d. $\frac{1}{5}$
e. 5

5. $2^2 \times 2^3 =$
a. 2^5
b. 10
c. 4^5
d. 64
e. 2

6. What is 7.206×10^{-4} written in standard form?
a. 72,060
b. 0.00007206
c. 0.0007206
d. 7,206
e. 72,060,000

7. What is 567,090,000 written in scientific notation?
a. $567{,}090 \times 10^3$
b. 5.6709×10^8
c. 5.679×10^7
d. 567×10^9
e. 0.56709×10^9

8. $\frac{5.4 \times 10^{16}}{9 \times 10^{14}}$ =
a. 600
b. 0.6
c. 60
d. 6
e. 0.006

9. Simplify -5^2.
a. –10
b. $\frac{1}{25}$
c. 25
d. –25
e. 10

10. $700 + 25 - 5^2 \times 2$ =
a. 725
b. 1,036,800
c. 675
d. 1,400
e. 705

BENCHMARK QUIZ SOLUTIONS

How did you do on the powers, exponents, and roots Benchmark Quiz? Check your answers here, and then analyze your results to figure out your plan of attack to master these topics.

1. e. $7^2 = 7 \times 7$, and $7 \times 7 = 49$.

2. d. When there is a negative exponent, take the reciprocal of the base and raise it to the positive power; $2^{-3} = \left(\frac{1}{2}\right)^3$ and $\left(\frac{1^3}{2^3}\right) = \frac{1}{8}$.

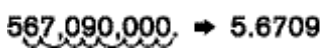
3. b. $\sqrt[3]{27}$ is the cube root of 27. What number multiplied by itself three times equals 27? Since $3 \times 3 \times 3 = 27$, the answer is 3.

4. e. An exponent of $\frac{1}{2}$ means the second root of 25, or the square root of 25. Since $5 \times 5 = 25$, the answer is 5.

5. a. When you multiply two numbers with the same base, you keep the base and add the exponents; $2^2 \times 2^3 = 2^{2+3} = 2^5$.

6. c. The negative exponent of 4 dictates that you move the decimal point four places to the left.

7. b. Change the large number to a decimal number between 1 and 10, followed by a multiplication by a power of 10. By doing this, you have moved the decimal point eight places to the right.



8. c. Divide $\frac{5.4}{9} = 0.6$ and use the law of exponents; $\frac{10^{16}}{10^{14}} = 10^{16-14} = 10^2$. So this is $0.6 \times 10^2 = 60$, by moving the decimal point two places to the right.

9. d. Evaluate the exponent first and then evaluate the negative sign last; $5^2 = 5 \times 5$, which is 25, so the answer is –25.

10. c. For order of operations, parentheses would have been evaluated first, but there are no parentheses. Evaluate exponents next to get $700 + 25 - 25 \times 2$. Multiplication is performed next; $700 + 25 - 50$. Now, addition and subtraction are done left to right for a result of $725 - 50 = 675$.

BENCHMARK QUIZ RESULTS

If you answered 8–10 questions correctly, you are on your way to mastering this concept. Study the lessons in this chapter to refresh your knowledge. The Tips and Strategies may be especially helpful to you.

If you answered 4–7 questions correctly, careful study is needed. Work through the lessons, attempting all examples. Visit the suggested websites for extended practice. Find the one that will help you to grasp these concepts. When you feel that you have gained a better understanding, try the practice quiz at the end of the chapter to test your newly acquired knowledge.

If you answered 1–3 questions correctly, give this chapter your concentrated effort. Take your time in reading over each section, and try the given examples on a separate piece of paper. There are several websites suggested that give additional explanation and further practice. Visit these sites and work through the examples and problems.

JUST IN TIME LESSON—POWERS, EXPONENTS, AND ROOTS

Recall that when whole numbers are multiplied together, each of these numbers is a factor of the result. When all of the factors are identical, the result is called a power of that factor. For example, because $6 \times 6 \times 6 \times 6 \times 6 = 7{,}776$, we say that 7,776 is the "fifth power of 6."

The topics in this chapter are:

- powers and exponents
- negative exponents
- square roots and cube roots
- fractional exponents
- scientific notation
- laws of exponents
- exponents and the order of operations

POWERS AND EXPONENTS

In the previous example, 7,776 was said to be the fifth power of six, since $6 \times 6 \times 6 \times 6 \times 6 = 7{,}776$. There is a shorthand notation used to indicate repeated multiplication by the same factor. This is called *exponential form*. In exponential form, 7,776 can be written as 6^5, and we say that "six to the fifth power is 7,776." In the expression 6^5, the six is called the *base* and the five is called the *exponent*.

GLOSSARY

BASE the number that is used as a repeated factor in an exponential expression. It is the "bottom" number in an exponential expression. For example, in the expression 5^3, the five is the base number.

EXPONENT the number of times that a repeated factor is multiplied together to form a power; it is the superscript in an exponential expression. For example, in the expression 5^3, the three is the exponent.

POWER a product that is formed from a repeated factor multiplied together. For example, 27 is the third power of 3 since $3 \times 3 \times 3 = 27$.

Any base number raised to the second power is called the *square of the base*. So 4^2 is said to be "four squared." Since $4^2 = 4 \times 4$, which is 16, 16 is called a *perfect square*. Any base number raised to the third power is called the *cube of the base*. So 4^3 is said to be "four cubed." Since $4^3 = 4 \times 4 \times 4$, which is 64, 64 is called a *perfect cube*. It is helpful to know some of the perfect squares and cubes, both for raising to an exponent, and taking roots, discussed in a later section.

BASE	SQUARE	CUBE	BASE	SQUARE
1	$1^2 = 1$	$1^3 = 1$	7	$7^2 = 49$
2	$2^2 = 4$	$2^3 = 8$	8	$8^2 = 64$
3	$3^2 = 9$	$3^3 = 27$	9	$9^2 = 81$
4	$4^2 = 16$	$4^3 = 64$	10	$10^2 = 100$
5	$5^2 = 25$	$5^3 = 125$	11	$11^2 = 121$
6	$6^2 = 36$	$6^3 = 216$	12	$12^2 = 144$

SHORTCUT

The exponent for the positive powers of 10 equals the number of zeroes in the number in standard form. For example, one million, 1,000,000 = 10^6 since there are six zeroes in 1,000,000.

NEGATIVE EXPONENTS

Observe the pattern in the table for the base of three:

EXPONENT	EXPONENTIAL FORM	STANDARD FORM
1	3^1	3
2	3^2	9
3	3^3	27
4	3^4	81
5	3^5	243

Look at the last column, standard form. As you go down this column, each entry is multiplied by three to get the next entry below, such as $81 \times 3 = 243$. Notice that as you go UP this column, you would divide each entry by three to get the previous entry above. Now let's extend the table to include an exponent of zero and some negative exponents:

EXPONENT	EXPONENTIAL FORM	STANDARD FORM
−3	3^{-3}	$\frac{1}{9} \div 3 = \frac{1}{27} = \frac{1}{3^3}$
−2	3^{-2}	$\frac{1}{3} \div 3 = \frac{1}{9} = \frac{1}{3^2}$
−1	3^{-1}	$1 \div 3 = \frac{1}{3} = \frac{1}{3^1}$
0	3^0	$3 \div 3 = 1$
1	3^1	3
2	3^2	9
3	3^3	27

From the patterns noted above, the rules for zero exponents and negative exponents follow:

RULE BOOK

1. **Any number (except zero) to the zero power is 1; $x^0 = 1$, where x is any number, not equal to zero. For example, $7^0 = 1$, $29^0 = 1$ and $2{,}159^0 = 1$.**
2. **A negative exponent is equivalent to the reciprocal of the base, raised to that positive exponent; $x^{-a} = \frac{1^a}{x^a} = \frac{1}{x^a}$, where x is not equal to zero. Recall that the reciprocal of 4 is $\frac{1}{4}$. So, for example, $4^{-2} = \frac{1}{4^2} = \frac{1}{16}$, and $2^{-3} = \frac{1}{2^3} = \frac{1}{8}$, and $\left(\frac{1}{3}\right)^{-3} = \frac{3^3}{1^3} = 27$.**

CALCULATOR TIPS

The x^2 key on the calculator is used to find the square of a number. To calculate 13^2 on the calculator, enter: 1 3 x^2. On most calculators, the exponent key looks like y^x or x^y. In either case, to calculate 5^3, enter into the calculator 5 y^x 3 =. You can also enter negative exponents, but be aware that the calculator will give the result as a decimal instead of a fraction. To calculate 3^{-2}, enter 3 y^x 2 − = to get 0.1111111 . . . , which is $\frac{1}{9}$.

SQUARE ROOTS AND CUBE ROOTS

Earlier, perfect squares and cubes were described. For example, 64 is a perfect square because 8×8, or 8^2 equals 64. This factor, 8, is called the square root of 64. Likewise, since $3 \times 3 \times 3$, or 3^3 equals 27, the cube root of 27 is 3.

GLOSSARY

SQUARE ROOT of a number is one of the two identical factors whose product is the given number.

CUBE ROOT of a number is one of the three identical factors whose product is the given number.

The number 64 has another square root, namely –8, since $-8 \times -8 = (-8)^2 = 64$. If we want to indicate the positive square root, use the $\sqrt{n}$ radical symbol to denote square root. In an expression such as $\sqrt{25}$, 25 is called the *radicand*, and the expression is the *radical*. So $\sqrt{64} = 8$, and $-\sqrt{64} = -8$. The notation for cube root is $\sqrt[3]{n}$.

WORKING WITH SQUARE ROOTS

You can simplify square roots by expressing the radicand (the number under the radical symbol) as the product of other numbers, where one of the factors is a perfect square. For example, $\sqrt{288} = \sqrt{144 \times 2} = \sqrt{144} \times \sqrt{2} = 12\sqrt{2}$, which means "twelve times the square root of two." Likewise, when you are given a problem of two radicals multiplied together, you can combine by multiplying the radicands; $\sqrt{3} \times \sqrt{27} = \sqrt{3 \times 27} = \sqrt{81} = 9$.

The same rule holds for division; $\frac{\sqrt{192}}{\sqrt{3}} = \sqrt{\frac{192}{3}} = \sqrt{64} = 8$, and $\sqrt{\frac{3}{9}} = \frac{\sqrt{3}}{\sqrt{9}} = \frac{\sqrt{3}}{3}$.

You can add or subtract radicals only if they have the same radicand; $5\sqrt{3} + 7\sqrt{3} = 12\sqrt{3}$, and $7\sqrt{2} - 10\sqrt{2} = -3\sqrt{2}$, but $6\sqrt{2} + 7\sqrt{3}$ CANNOT be combined since the radicands are different.

CALCULATOR TIPS

Use the [√] key to find the square root of a number. To find $\sqrt{289}$, enter [2] [8] [9] [√]. Check your particular calculator. Another method for using the calculator is to use the $\sqrt[x]{y}$ key or it may be $\sqrt[y]{x}$ key. To find $\sqrt[3]{125}$, enter [1] [2] [5] [$\sqrt[x]{y}$] [3] [=].

FRACTIONAL EXPONENTS

When an exponent is a fraction, the denominator of this fractional exponent means the root of the base number, and the numerator means a raise of the base to that power; $\frac{\text{numerator}}{\text{denominator}} \Rightarrow \frac{\text{raise base to this power}}{\text{take the root of the base}}$. For example, $8^{\frac{1}{3}}$ is the same as $\sqrt[3]{8^1} = 2$.

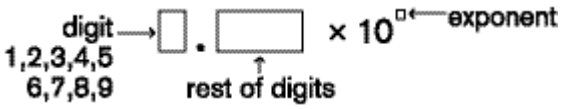
Another example: $8^{\frac{2}{3}}$ means $\sqrt[3]{8^2}$ which is $\sqrt[3]{64} = 4$, or alternatively, $(\sqrt[3]{8})^2$ which is $2^2 = 4$.

CALCULATOR TIPS

If your calculator has the fraction key $a\frac{b}{c}$ and the parentheses keys ( ), then you can calculate fractional exponents on your calculator. To find $32^{\frac{1}{5}}$, enter 3 2 y^x (1 $a\frac{b}{c}$ 5) = to get a result of 2.

SCIENTIFIC NOTATION

Scientists can measure very large numbers, such as the distance from the earth to the sun, or very small numbers, such as the diameter of an electron. Because these numbers involve a lot of digits as placeholders, a special notation was invented as a shorthand for these numbers. This notation is very specific in the way it is expressed:



The exponent of 10 tells whether the number is really big (a positive exponent) or really small (negative exponent). The absolute value of the exponent denotes how far the decimal point was moved to fit the pattern:

For example, to express 6,403,500,000 in scientific notation, first change the number to a decimal between 1 and 10, that is 6.4035. Now, multiply this decimal by a power of 10, determined by the number of placeholders the decimal was moved. This is a large number, so the power of 10 will be positive.

$$6{,}403{,}500{,}000 \rightarrow 6.4035 \times 10^{9}$$

Since the decimal point was moved nine places to the left, the power of 10 is nine, and the number written in scientific notation is 6.4035×10^{9}. Note that even though 64.035×10^{8} is another form of the same number, this is NOT scientific notation, since the decimal number is not between one and ten. As an example of a very small number, consider changing 0.000006007 to a number expressed in scientific notation. Write the num-

ber as a decimal between 1 and 10: 6.007. The number is very small, so the power of 10 will be negative.

$$0.000006007 \rightarrow 6.007 \times 10^{-6}$$

Since the decimal point was moved six places to the right, the power of 10 is –6 and the number written in scientific notation is 6.007×10^{-6}. You can also convert a number written in scientific notation to a number in standard form by going through a similar process. For example, to change 5.02×10^{7} to standard form, first recognize that the number is a large number since the exponent is positive. The decimal point will be moved to the right to create a large number. Take the decimal number and move the decimal point seven places to the right:

$$5.0200000 \rightarrow 50,200,000$$

To change 1.09×10^{-5}, the number is a very small number since the exponent is negative. The decimal point will be moved five places to the left to create a small number. Take the decimal number and move the decimal point five places to the left:

$$1.09 \rightarrow 0.0000109$$

Note that in both of these last two examples, either leading zeroes or trailing zeroes were added as placeholders.

LAWS OF EXPONENTS

RULE BOOK

When working with exponents there are certain rules that can be helpful.

1. **When you multiply powers with the same base, keep the base and add the exponents. For example, $4^2 \times 4^3 = 4^{2+3} = 4^5$.**
2. **When you divide powers with the same base, keep the base and subtract the exponents. For example, $3^7 \div 3^4 = 3^{7-4} = 3^3$, or $\frac{2^6}{2^4} = 2^{6-4} = 2^2$.**
3. **When you raise a power to a power, you keep the base and multiply the exponents: $(7^3)^2 = 7^{3\times2} = 7^6$.**
4. **If two powers are being multiplied together and the bases are not the same, check to see if you can convert the numbers to have the same base to use the above laws. For example, to simplify $27^2 \times 3^2$, recognize that 27 can be written as 3^3. Change the problem to $(3^3)^2 \times 3^2$. Use law #3 to get $3^{3\times2} \times 3^2 = 3^6 \times 3^2$. Now, you can use law #1 to get $3^{6+2} = 3^8$.**

You can use these laws of exponents with scientific notation.

Example:

$\frac{1.44 \times 10^8}{1.2 \times 10^6}$

For this problem, handle the decimal portions separately from the powers of 10 portion. Divide 1.44 by 1.2, which is 1.2. Then use the laws of exponents for the powers of 10: $\frac{10^8}{10^6} = 10^{8-6} = 10^2$. The problem becomes 1.2×10^2, which is 120 in standard form, when you move the decimal point two places to the right.

Example:

$(2.0 \times 10^{-3}) \times (1.8 \times 10^{-2})$

First, multiply the decimal portions, to get 2.0 times 1.8, which is 3.6. Then use the laws of exponents on the powers of 10, $10^{-3} \times 10^{-2} = 10^{-3+-2} = 10^{-5}$. The problem becomes 3.6×10^{-5}, or 0.000036 in standard form, by moving the decimal point five places to the left.

EXTRA HELP

There are several useful web sites that deal with the topics of exponents, roots, and scientific notation. Visit these sites if you feel you need further clarification on these concepts. Each one has a unique method of presentation.

1. ***www.purplemath.com***
   Click on *"How do you REALLY do this stuff?"* which is under Purplemath's algebra lessons.
   Scroll down to the Beginning Algebra Topics section.
   Click on any of these topics: *Exponents*, *Radicals*, or *Scientific Notation*.
2. ***http://library.thinkquest.org/20991/home.html***
   Beneath Math for Morons like Us, click click here to view this site.
   Under the heading Learn, click on Algebra.
   Scroll down a bit on the left and then click on any of these topics: *Exponents*, *Square Roots*, or *Scientific Notation*.
3. ***www.math.com***
   Under Hot Subjects, click on *Exponents.*
   Then, under *Exponents*, click on *Numbers.*

EXPONENTS AND THE ORDER OF OPERATIONS

The order of operations was covered in Chapter 3, Properties of Numbers. Be aware of some distinctions when working with the order of operations and exponents. Exponents are done after parentheses and before any other operations, including the negative sign. For example, $-3^2 = -(3 \times 3) = -9$ because you first take the second power of three and then the answer is negative. However, $(-3)^2 = -3 \times -3 = 9$, since –3 is enclosed in parentheses. Following are some examples of order of operations with exponents.

Examples:

$200 - 6^2 \div (8 + 4)$	First, evaluate parentheses.
$200 - 6^2 \div 12$	Then handle the exponent.
$200 - 36 \div 12$	Now, division is performed.
$200 - 3$	And finally, subtraction.
197	
$-20 + (-2 + 5)^3 \div (10 - 7) \times 2$	Evaluate the parentheses, left to right.
$-20 + 3^3 \div 3 \times 2$	Now, evaluate the exponent.
$-20 + 27 \div 3 \times 2$	Division will be done next.
$-20 + 9 \times 2$	Evaluate multiplication.
$-20 + 18$	Finally, perform addition.
–2	

TIPS AND STRATEGIES

- Exponential expressions are in the form x^a. The number x is multiplied by itself "a" times.
- Learn the common perfect squares and perfect cubes.
- Learn the particulars of your specific calculator when working with exponents and roots.
- $\sqrt{x}$ means the positive square root of x, where x is called the radicand and the expression is called the radical.
- $\sqrt[3]{x}$ means the cube root of x.
- You can express the radicand as the product of its factors and separate the factors to simplify.
- You can combine two roots multiplied together by multiplying the radicands and then simplifying.
- For division, you can combine radicands, or separate radicands into numerator and denominator, to simplify radicals.
- You can add or subtract two radicals only if they have the same radicand.
- Fractional exponents are another way to express a root.
- Scientific notation is a shorthand way to express very large or very small numbers. The notation expresses a number as a decimal (between one and ten), multiplied by an appropriate power of ten.
- When you multiply two powers with the same base, keep the base and add the exponents.
- When you divide two powers with the same base, keep the base and subtract the exponents.
- When you raise a power to a power, keep the base and multiply the exponents.

PRACTICE

Try this set of twenty-five questions to see if your knowledge of powers, exponents, and roots has improved.

1. 6^3 =
a. 18
b. 36
c. 2
d. 9
e. 216

2. $-11^2 =$
a. -22
b. -121
c. 121
d. 22
e. 101

3. $4^{-2} =$
a. -8
b. -16
c. $-\frac{1}{16}$
d. $\frac{1}{16}$
e. 16

4. $81^{\frac{1}{4}} =$
a. 20.25
b. 3
c. 324
d. $\frac{1}{3}$
e. $\frac{1}{324}$

5. $\sqrt[3]{64} =$
a. 4
b. 192
c. $21\frac{1}{3}$
d. $\frac{1}{192}$
e. $\frac{1}{3}$

6. $2^{-3} =$
a. -6
b. $\frac{1}{8}$
c. -8
d. $\frac{1}{6}$
e. $-\frac{1}{8}$

7. $\sqrt{169}$ =
a. 13
b. 16
c. 16.9
d. 84.5
e. –16

8. Simplify $\sqrt{75}$.
a. $25\sqrt{3}$
b. 7.5
c. 150
d. 37.5
e. $5\sqrt{3}$

9. $\sqrt{48} \times \sqrt{3}$
a. $\sqrt[4]{144}$
b. –16
c. 12
d. $\frac{1}{12}$
e. $3\sqrt{48}$

10. $\frac{\sqrt{75}}{\sqrt{3}}$ =
a. 25
b. –25
c. –0.5
d. 5
e. $\frac{\sqrt{75}}{3}$

11. Simplify $\sqrt{48}$.
a. $4\sqrt{3}$
b. $16\sqrt{3}$
c. 24
d. 96
e. –24

12. $\sqrt{\frac{3}{9}} =$

a. $\frac{1}{3}$
b. $3\sqrt{3}$
c. $\frac{3}{9}$
d. $-3\sqrt{3}$
e. $\frac{\sqrt{3}}{3}$

13. $36^{\frac{1}{2}} =$

a. 18
b. –18
c. 6
d. $\frac{1}{6}$
e. $\frac{1}{18}$

14. Write 2.701×10^7 in standard form.

a. 27×10^6
b. 27,010,000
c. 27,010,000,000
d. 0.0002701
e. 0.0000002701

15. Write 4.09×10^{-5} in standard form.

a. 409,000
b. 0.00000409
c. 0.0000409
d. 40,900,000
e. 409×10^{-7}

16. Write 5,063,000,000 in scientific notation.

a. $5{,}063 \times 10^6$
b. 5.063×10^9
c. 5.063×10^6
d. 5.063×10^{-6}
e. 5.063×10^{-9}

17. $\frac{7.2 \times 10^{20}}{1.8 \times 10^{17}} =$

a. 4,000
b. 4
c. 40
d. 400
e. 0.004

18. $(2.5 \times 10^{-4}) \times (3.0 \times 10^{8}) =$

a. 0.00075
b. 7.5
c. 75,000
d. 7,500,000,000,000
e. 750

19. $\frac{3^{\frac{3}{2}}}{3^{\frac{1}{2}}} =$

a. –3
b. 3
c. –9
d. 9
e. $\frac{\sqrt{3}}{3}$

20. $7^5 \times 7^{-2} =$

a. 7^7
b. 7^3
c. -7^3
d. $\frac{1}{7^3}$
e. $7^{\frac{5}{2}}$

21. Simplify $(3^2)^6$.

a. 3^8
b. 3^4
c. 3^{12}
d. 3^{36}
e. 3^6

22. $\frac{2^5}{2^2} =$

a. 6
b. 1^7
c. 2^7
d. 8
e. 2^{10}

23. $24 - 4^2 \div 2 =$
a. 16
b. 4
c. 80
d. 8
e. 20

24. $(8 + 4)^2 - (13 - 3)^2 \div 4 =$
a. 11
b. 19
c. 31
d. 119
e. 169

25. $5 - (-17 + 7)^2 \times 3 =$
a. –135
b. 315
c. –295
d. –45
e. 75

ANSWERS

Now check your answers. Be sure to read over these explanations carefully to aid in your learning.

1. e. Six to the third power is $6 \times 6 \times 6 = 216$.

2. b. For this problem, the exponent is handled first. Eleven squared is 11 times 11, which is 121, and then the negative sign is evaluated to get the answer of –121.

3. d. The exponent of –2 requires that you take the reciprocal of the base and then square that number. The reciprocal of 4 is $\frac{1}{4}$, and $\left(\frac{1}{4}\right)^2 = \frac{1}{16}$.

4. b. The exponent of $\frac{1}{4}$ means the fourth root of 81; what factor, multiplied by itself four times will yield 81? Trial and error will show that $3 \times 3 \times 3 \times 3 = 81$. The root is 3.

5. a. The problem is asking for the cube root of 64; what number, multiplied by itself three times will give the product of 64? Since $4 \times 4 \times 4 = 64$, the cube root is 4.

6. b. The exponent of –3 requires that you take the third power of the reciprocal of 2, which is $\left(\frac{1}{2}\right)^3 = \frac{1^3}{2^3} = \frac{1}{8}$.

7. a. The problem is asking for the square root of 169. Because $13 \times 13 = 169$, the root is 13.

8. e. To simplify $\sqrt{75}$, think of a perfect square factor of 75, namely 25. Rewrite the radical as $\sqrt{25 \times 3} = \sqrt{25} \times \sqrt{3}$. The square root of 25 is 5, and the square root of 3 has no perfect square factors so cannot be simplified any further. The answer is therefore $5\sqrt{3}$.

9. c. Combine these radicals by multiplying the radicands together; $\sqrt{48 \times 3} = \sqrt{144}$. The square root of 144 is 12, since $12 \times 12 = 144$.

10. d. Combine these radicals together under one radical sign: $\sqrt{\frac{75}{3}} = \sqrt{25}$. The square root of 25 is 5.

11. a. The square root of 48 can be simplified by knowing that there are perfect square factors of 48, the largest being 16. Break up 48 into factors of 16 and 3; $\sqrt{16 \times 3} = \sqrt{16} \times \sqrt{3}$. The square root of 16 simplifies to 4 and the square root of 3 cannot be simplified. The answer is therefore $4\sqrt{3}$.

12. e. Break this fractional radicand up into two separate radicals and then simplify what can be simplified; $\sqrt{\frac{3}{9}} = \frac{\sqrt{3}}{\sqrt{9}}$. The $\sqrt{3}$ is simplified, and the $\sqrt{9} = 3$.

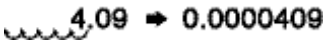
So the answer is $\frac{\sqrt{3}}{3}$.

13. c. The exponent of one-half means the square root. The square root of 36 is 6, since $6 \times 6 = 36$.

14. b. The exponent of 7 on the power of 10 dictates that you move the decimal point in the decimal 2.701 seven places to the right. Three of the places will be taken up by the digits 7, 0 and 1 and then four more zeroes will follow to result in 27,010,000.

2.7010000 ➡ 27,010,000

15. c. The negative exponent, –5, on the power of 10 means that you must move the decimal point five places to the left. The number 4.09 has only one digit to the left of the decimal point. Four leading zeroes must be added as placeholders; 0.0000409.



16. b. Write the number as a decimal between 1 and 10, then multiply by the appropriate power of 10. You move the decimal point nine places to the left to go from 5,063,000,000 to 5.063. The answer is 5.063×10^9.

17. a. Break the problem up, dividing the decimal portions, and then using the laws of exponents on the powers of 10. For the powers of 10, when you divide two powers with the same base, you keep the base and subtract the exponents; $7.2 \div 1.8 = 4$, and $\frac{10^{20}}{10^{17}} = 10^{20-17} = 10^3$. So the answer is 4.0×10^3, which is 4,000 because you move the decimal point three places to the right.

18. c. For this problem you use the commutative and associative properties of multiplication and change the order of the factors to get $(2.5 \times 3.0) \times (10^{-4} \times 10^8)$; $2.5 \times 3 = 7.5$. For the powers of 10, when you multiply two powers with the same base, you keep the base and add the exponents, which results in $10^{-4} \times 10^8 = 10^{-4+8} = 10^4$. Now, $7.5 \times 10^4 = 75{,}000$ because you move the decimal point four places to the right, one place being the 5 (in the tenths place) followed by three trailing zeroes for placeholders.

19. b. When you divide two powers with the same base, you keep the base and subtract the exponents; $\frac{3^{\frac{3}{2}}}{3^{\frac{1}{2}}} = 3^{\frac{3}{2}-\frac{1}{2}} = 3^{\frac{2}{2}} = 3^1 = 3$.

20. b. When you multiply two powers with the same base, you keep the base and add the exponents; $7^5 \times 7^{-2} = 7^{5+-2} = 7^3$.

21. c. When you raise a power to a power, you keep the base and multiply the exponents; $(3^2)^6 = 3^{2\times6} = 3^{12}$.

22. d. For this problem, you can just simplify the numerator, simplify the denominator and then divide; $\frac{2^5}{2^2} = \frac{2\times2\times2\times2\times2}{2\times2} = \frac{32}{4}$. Thirty-two divided by 4 is 8.

23. a. Evaluate the exponent first. Four squared is 16, so the problem becomes $24 - 16 \div 2$. Next, evaluate division; $16 \div 2 = 8$. Finally, subtract; $24 - 8 = 16$.

24. d. First evaluate parentheses from left to right, to get $12^2 - 10^2 \div 4$. Now evaluate the exponents; $144 - 100 \div 4$. Next, divide 100 by 4 to get $144 - 25$. Finally, subtract to get the answer of 119.

25. c. First perform the addition enclosed in the parentheses; $5 - (-10)^2 \times 3$. Now, take –10 and square it, which is $-10 \times -10 = 100$, so the problem becomes $5 - 100 \times 3$. Now, multiply; $5 - 300$. Finally, subtract to get –295.

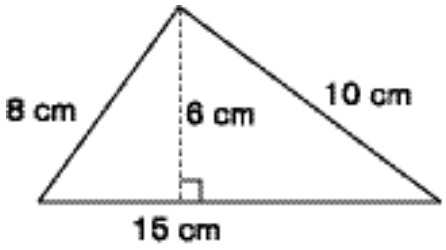
8

Geometry and Measurement Conversions

Almost every type of math test includes problems dealing with geometry. The word *geometry* means "measure of the earth." We constantly measure things in real life. We measure for fences and carpets; we measure the amount of water in the swimming pool. A basic understanding of geometry is crucial to success on your upcoming test. Right now, take a few minutes to take the following ten-question *Benchmark Quiz*. These questions are similar to the type of questions that you will find on important tests. When you are finished, check the answer key carefully to assess your results. Your Benchmark Quiz analysis will help you determine how much time you need to spend on geometry and measurement conversions, and the specific areas in which you need the most careful review and practice.

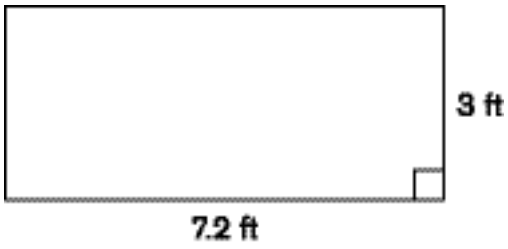
BENCHMARK QUIZ

1. Find the area.



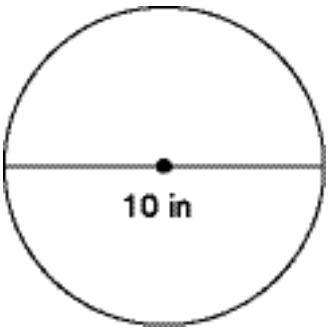
a. 45 cm^2
b. 33 cm^2
c. 60 cm^2
d. 75 cm^2
e. 90 cm^2

2. Find the perimeter.



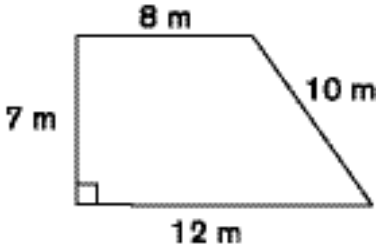
a. 10.2 ft
b. 21.6 ft
c. 20.4 ft
d. 43.2 ft
e. 15 ft

3. Find the area.



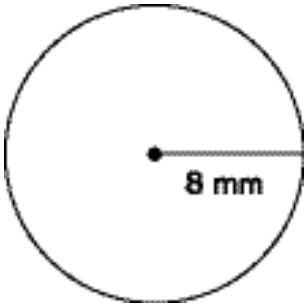
a. 5π in^2
b. 100π in^2
c. 25π in^2
d. 100 in^2
e. 25 in^2

4. Find the area.



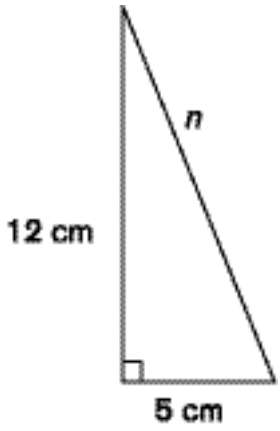
a. 37 m^2
b. 56 m^2
c. 120 m^2
d. 70 m^2
e. 140 m^2

5. Find the circumference.



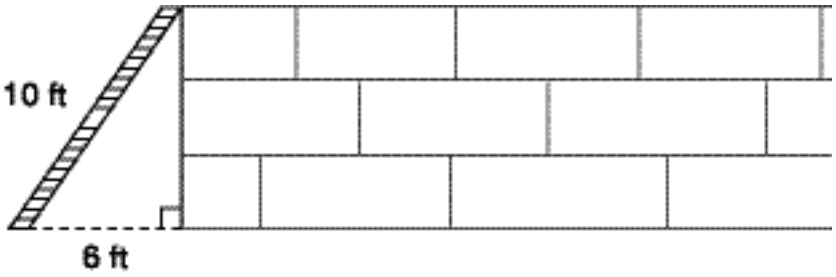
a. 16 mm
b. 8π mm
c. 64π mm
d. 256π mm
e. 16π mm

6. Find n.



a. $\sqrt{34}$ cm
b. $\sqrt{119}$ cm
c. 7 cm
d. 13 cm
e. $\sqrt{17}$ cm

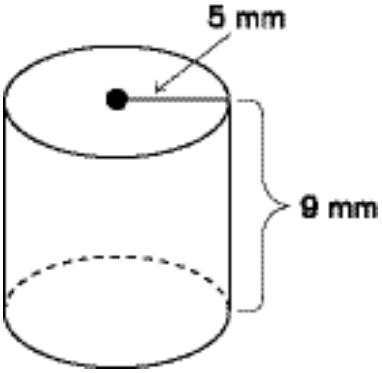
7. A ten-foot ladder rests on a building as shown.



If the base of the ladder is 6 feet away from the building, how high up the building does the ladder touch?

a. 4 ft
b. 8 ft
c. 6 ft
d. $\sqrt{136}$ ft
e. $\sqrt{8}$ ft

8. Find the volume.

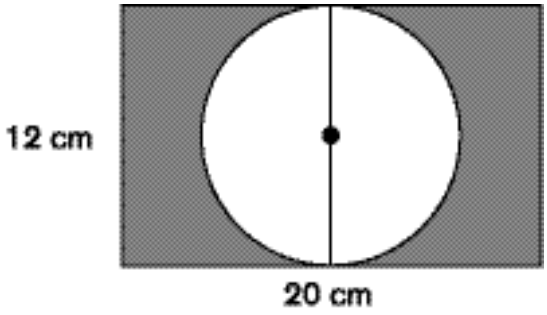


a. 900 mm^2
b. 45 mm^2
c. 225 mm^2
d. 90π mm^2
e. 225π mm^2

9. If a faucet drips 1 quart of water per week, how many gallons does the faucet drip per 52-week year?

a. 13
b. 208
c. 52
d. 26
e. 104

10. Find the area of the shaded region. Use 3.14 for π.



a. 240 cm^2
b. 126.96 cm^2
c. 13.92 cm^2
d. 136.96 cm^2
e. 202.32 cm^2

BENCHMARK QUIZ SOLUTIONS

How did you do on the Benchmark Quiz? Check your answers here, and then analyze your results to figure out your plan of attack to master these topics.

1. a. The area of a triangle is $A = \frac{1}{2}bh$, where A stands for area, b stands for the base length, and h stands for the height (the length of the segment that is perpendicular to the base). In this triangle, the base is 15 cm and the height is 6 cm; $A = \frac{1}{2} \times 15 \times 6$ and multiplying all together yields an area of 45 cm^2.

2. c. To find perimeter, you add up all the lengths of the sides of the polygon. Because this is a rectangle, opposite sides are congruent. The perimeter is 7.2 + 3 + 7.2 + 3 = 20.4 ft.

3. c. The area of a circle is $A = \pi r^2$, where π is a constant, and r is the radius of the circle. The problem gives the diameter to be 10 inches. The radius, 5 inches, is one-half of the length of the diameter. Using the formula, $A = \pi \times 5 \times 5$, the area is 25π in^2.

4. d. The area of a trapezoid is $A = \frac{1}{2}h(b_1 + b_2)$, where A stands for area, b_1 and b_2 are the lengths of the parallel bases, and h is the height (the length of the segment perpendicular to the bases). In this trapezoid, the side of length 7 m is the height, since it is perpendicular to the bases. Substitute the given information into the formula:

$A = \frac{1}{2} \times 7 \times (8 + 12)$ to get $A = \frac{1}{2} \times 7 \times (20)$. Multiply all terms on the right together to yield an area of 70 m^2.

5. e. Circumference is found by the formula $C = \pi d$, where C is the circumference, π is a constant and d is the diameter. The radius of 8 mm is given, and diameter is twice the radius. The diameter is 16 mm; $C = 16\pi$ mm.

6. d. This is a right triangle, so use the Pythagorean theorem to find the missing side, which in this case is the hypotenuse. Use the formula $a^2 + b^2 = c^2$, where a and b are the legs, whose lengths are 5 and 12, and c is "n." Substitute into the formula; $5^2 + 12^2 = n^2$. Evaluate the exponents; $25 + 144 = n^2$, or the equivalent $169 = n^2$. Since n^2 is 169, take the square root of 169 to find n; $\sqrt{169} = 13$ cm.

7. b. In the diagram, a right triangle is formed, where the ladder is the hypotenuse (value of 10 ft) and the length up the wall is one of the legs; the distance from the bottom of the ladder to the bottom of the wall is the other leg (value of 6 ft). Use the Pythagorean Theorem, $a^2 + b^2 = c^2$, and substitute in 6 for a and 10 for c. $6^2 + b^2 = 10^2$. Evaluate the exponents to yield $36 + b^2 = 100$. Subtract 36 from 100 to get $b^2 = 64$. So, $b = \sqrt{64} = 8$ ft.

8. e. The volume of a cylinder is $V = \pi r^2 h$, where V is the volume, π is a constant, r is the radius of the base circle (value of 5 mm), and h is the height (value of 9 mm). Substitute in the values; $V = \pi \times r \times r \times h$; $V = \pi \times 5 \times 5 \times 9$. Multiply the numbers together on the right to get the volume of $V = 225\pi$ mm^3.

9. a. You can use dimensional analysis to solve this problem. Set up the fractional multiplication; $\frac{1 \text{ quart}}{1 \text{ week}} \times \frac{1 \text{ gallon}}{4 \text{ quarts}} \times \frac{52 \text{ weeks}}{1 \text{ year}}$. The unit dimensions will cancel as follows: quarts will cancel with quarts, weeks will cancel with weeks, and gallons will remain as the unit on the top; year will remain as the unit on the bottom. Fifty-two divided by 4 is 13 gallons per year.

10. b. The area of the shaded region is found by taking the area of the outer figure, the rectangle, and subtracting out the area of the inner figure, the circle; $A = A_{\text{rectangle}} - A_{\text{circle}}$. Using the area formulas, you get $A = bh - \pi r^2$. Notice that the diameter of the circle is the same length as the height of the rectangle. So the radius is one-half of 12. The radius is 6 cm. Substitute in the given lengths and the

value of π to get $A = 20 \times 12 - 3.14 \times 6 \times 6$. Using order of operations, you multiply first, from left to right; $A = 240 - 113.04$, or $A = 126.96$ cm^2.

BENCHMARK QUIZ RESULTS

If you answered 8–10 questions correctly, you have a good understanding of geometry. Perhaps the questions you answered incorrectly deal with one specific area in this chapter. Read over the chapter, concentrating on those areas of weakness. Proceed to the chapter assessment to try to improve your score.

If you answered 4–7 questions correctly, there are several areas you need to review. Carefully read through the lesson in this chapter for review and skill-building. Work carefully through the examples and pay attention to the sidebars that refer you to definitions, hints, and shortcuts. Get additional practice on geometry by visiting the suggested websites and taking the quiz at the end of the chapter.

If you answered 1–3 questions correctly, you need to spend some time on the topics in this chapter. First, carefully read this chapter and concentrate on the sidebars and visual aids that will help with comprehension. Go to the suggested websites in the Extra Help sidebar in this chapter, which will help with understanding and will provide extended practice. You may also want to refer to *Practical Math Success in 20 Minutes a Day*, Lessons 17, 18, and 19, published by LearningExpress. There are several chapters devoted to geometry.

JUST IN TIME LESSON—GEOMETRY AND MEASUREMENT CONVERSIONS

There are many applications of geometry in mathematical problems. This chapter will review basic polygons and solid figures, with an emphasis on three- and four-sided figures. It will also describe the various ways that geometry is used for measurement.

The topics in this chapter are:

- polygons
- classifying triangles
- classifying quadrilaterals
- perimeter
- area
- circles

- area of shaded regions
- right triangle measurements: Pythagorean theorem
- three-dimensional geometry
- volume
- measurement conversions

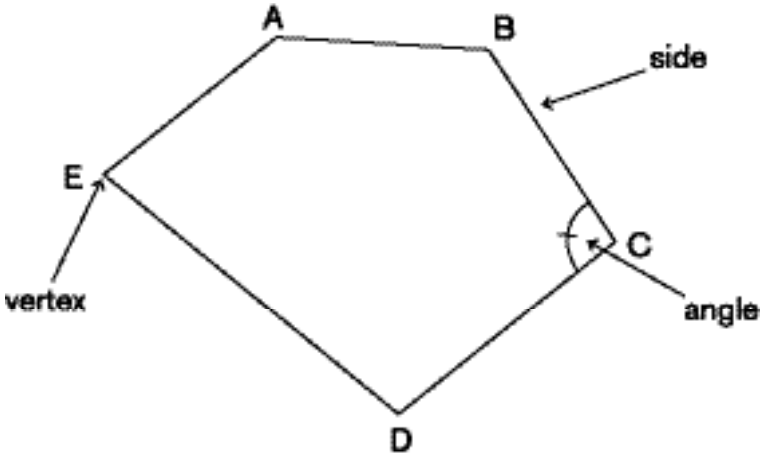
POLYGONS

Plane figures are two-dimensional objects that reside on a plane. You can think of a plane like a sheet of paper that extends forever in all directions. Special figures are called *polygons*, several of which are defined on this page.

GLOSSARY

POLYGON a closed-plane figure made up of line segments
TRIANGLE a polygon with three sides
QUADRILATERAL a polygon with four sides
PENTAGON a polygon with five sides
HEXAGON a polygon with six sides
OCTAGON a polygon with eight sides

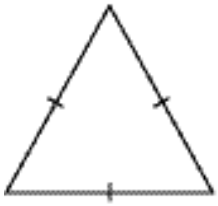
Polygons are made up of angles and line segments called *sides*. Each angle is made up of two sides and the point at which they meet is called the *vertex*.



This figure has 5 sides, 5 angles, and 5 vertexes.

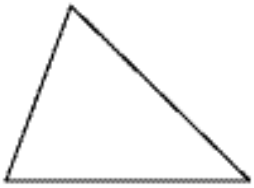
TRIANGLES

Triangles are three-sided polygons. Recall that Chapter 5 showed you how to use similar triangles to measure heights of tall objects using shadows. Triangles are classified, or grouped, in two different ways. One classification distinguishes among the sides, and another by the angles. For a triangle, you can have all three sides congruent (equal measure), or two sides congruent, or no sides congruent. Shown below are the classification names when grouping by sides.



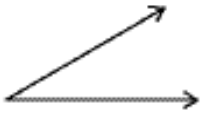
equilateral
All sides congruent

isosceles
2 sides congruent



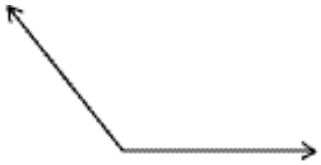
scalene
no sides congruent

To classify triangles by angles, first recall how angles are classified:



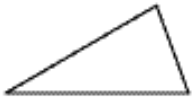
acute
0 < angle < 90°

right
exactly 90°

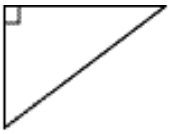


obtuse
90° < angle < 180°

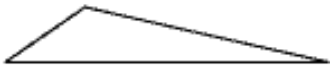
In triangle figures, the little box drawn inside an angle stands for a right angle. Below is the classification for triangles when grouped by angle:



acute
3 acute angles



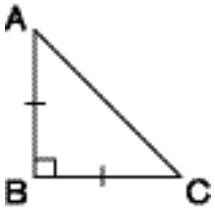
right
1 right angle



obtuse
1 obtuse angle

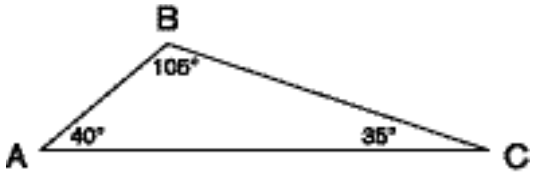
Note that even though right triangles and obtuse triangles each have two acute angles, their classification is not affected by these angles. Acute triangles have all *three* acute angles.

Example:
Classify this triangle by sides and angles.



To group by sides, notice that there are two sides ($\overline{AB}$, $\overline{BC}$) that are congruent. The side classification is isosceles. To group by angles, note that there is a right angle in this triangle. So the classification is right isosceles.

Example:
Classify this triangle by sides and angles

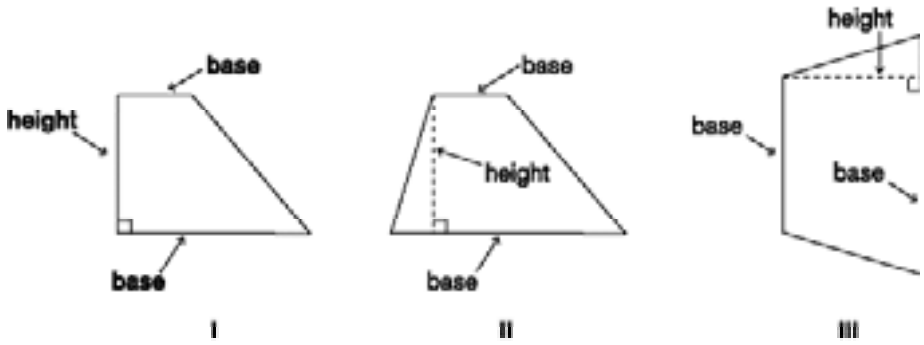


Because no sides are marked as congruent in this figure, the classification by sides is scalene. There is one angle greater than 90° (angle *b* is 105°); therefore, the angle classification is obtuse. This triangle is obtuse scalene.

QUADRILATERALS

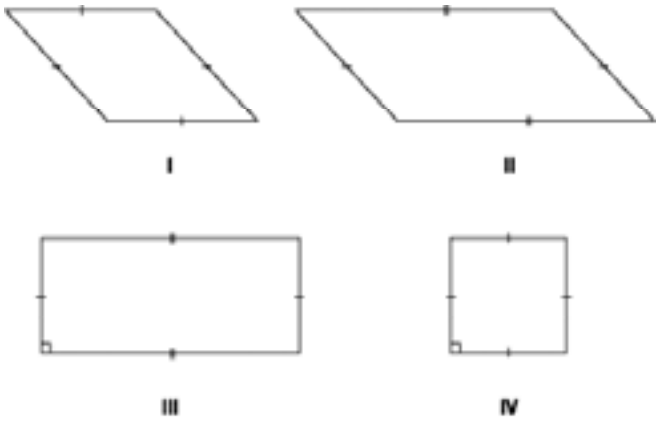
Four-sided polygons are called *quadrilaterals* and like triangles, there are classifications for quadrilaterals.

A quadrilateral with one pair of parallel sides (bases) is called a *trapezoid*.



In an *isosceles trapezoid*, the sides that are not bases are congruent. An example can be found in figure III in the above graphic. Because the parallel bases are not the same length in a trapezoid, we call these bases b_1 and b_2.

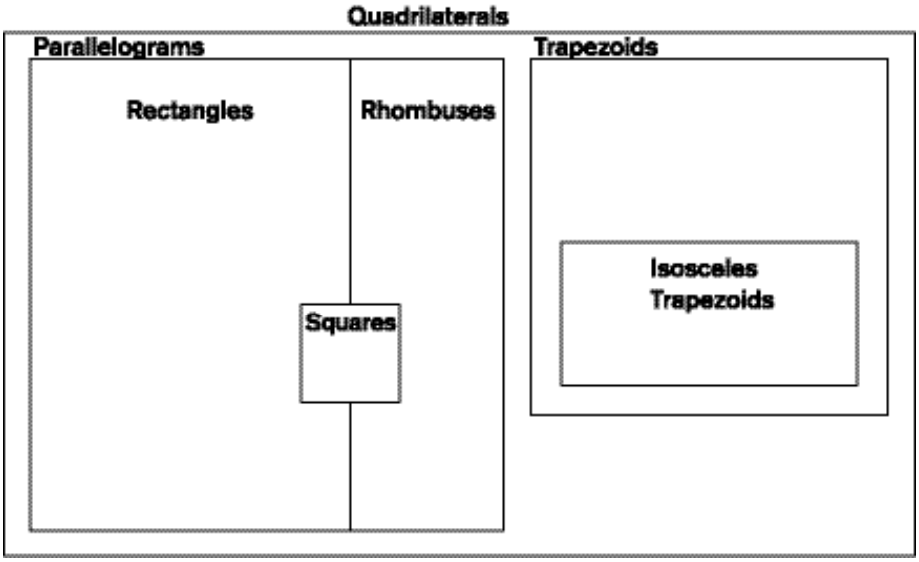
A quadrilateral with two pairs of parallel sides is called a *parallelogram*. The two sets of opposite sides that are parallel are congruent in a parallelogram, as shown in the figures:



Parallelograms are broken down into further sub-groups.

- A *rectangle* is a parallelogram with four right angles. Refer to figure III, above.
- A *rhombus* is a parallelogram with four congruent sides. Refer to figure I, above.
- A *square* is a parallelogram with both four right angles and four congruent sides. A square is a rhombus, a rectangle, a parallelogram, and a quadrilateral. Refer to figure IV, above.

This diagram may help you to understand the classification of four sided figures:



Example: Which of the following is NOT true?
a. All squares are rectangles.
b. Trapezoids have one set of parallel sides.
c. All squares are rhombuses.
d. All rhombuses are squares.
e. A square is a parallelogram.

The statement that is NOT true is **d**, "All rhombuses are squares." While it is true that SOME rhombuses are squares, there are rhombuses that do not have four right angles, and are therefore not squares. All the other statements are true.

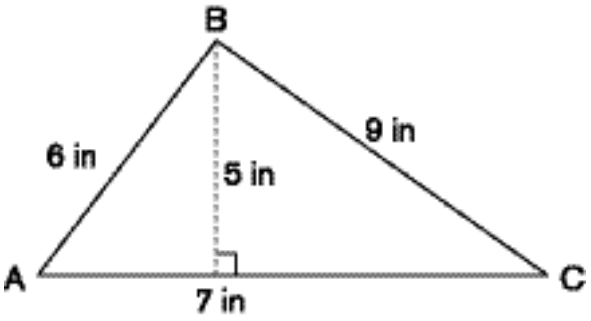
PERIMETER

Perimeter is the measure *around* a polygon. Perimeter is an addition concept; it is a linear, one-dimensional measurement.

RULE BOOK

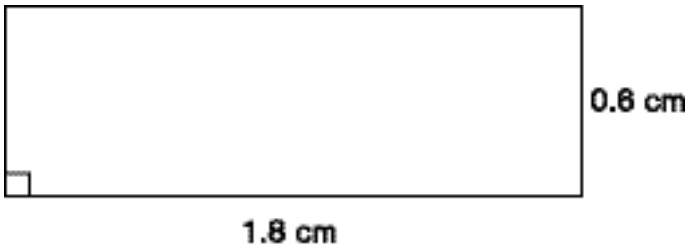
To find the perimeter of a polygon, add up all of the lengths of the sides of the figure. Be sure to name the units.

Example:
Find the perimeter.



Add up the lengths of all sides, that is, $\overline{AB}$, $\overline{BC}$ and $\overline{AC}$. Substitute in to get 6 + 9 + 7 = 22 in.

Example:
Find the perimeter of the rectangle.



In a rectangle, like all parallelograms, the opposite sides are parallel and congruent. In this example, there are two sides of length 1.8 cm and two sides of length 0.6 cm. Add up all the sides; 1.8 + 1.8 + 0.6 + 0.6 = 4.8 cm.

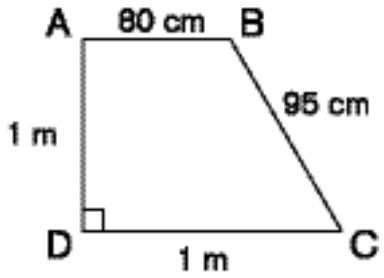
SHORTCUT

For a rectangle, perimeter can be found using the formula $P = 2l + 2w$, where P is the perimeter, l is the length and w is the width.

For a square, or any rhombus, the perimeter can be found by $P = 4s$, where P is the perimeter and s is the length of one of the sides.

Be alert when working with geometry problems to make sure that the units are consistent. If they are different, a conversion must be made before calculating perimeter or area.

Example:
Find the perimeter of the trapezoid.



Change the lengths of the sides given in meters to be centimeters; 1 m = 100 cm; length $\overline{AD}$ = length $\overline{DC}$; both will be 100 cm. Now, add up the four sides: 80 + 95 + 100 + 100 = 375 cm.

AREA

Area is a measure of how many square units it takes to *cover* a closed figure. Area is measured in square units. You may recall the term "square" from Chapter 7. Area is a multiplication concept, where two measures are multiplied together. You can also think of units being multiplied together: cm × cm = cm^2, or the words "centimeters squared." There are formulas to use for the area of common polygons:

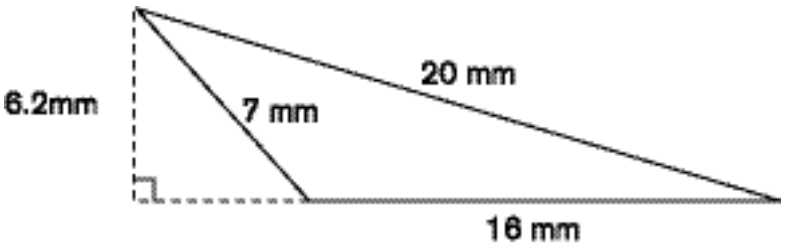
RULE BOOK: COMMON POLYGON FORMULAS

A stands for area, b stands for base, h stands for height (which is perpendicular to the base), and b_1 and b_2 are the parallel sides of a trapezoid.

Area of a triangle	$A = \frac{1}{2}bh$
Area of a parallelogram	$A = bh$
Area of a trapezoid	$A = \frac{1}{2}h(b_1 + b_2)$

Be sure to include square units in your answer.

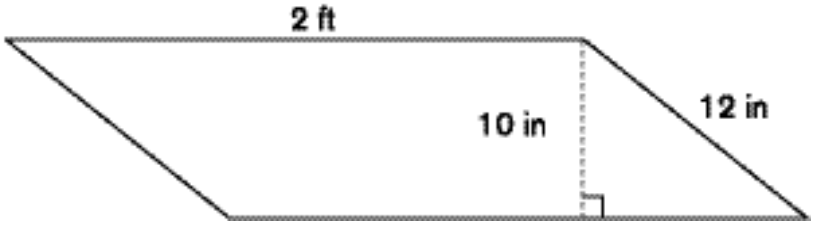
Example:
Find the area of the triangle.



Note that the height must be perpendicular to the base, so the height is 6.2 mm and the base is 16 mm.

$A = \frac{1}{2}bh$	Substitute in the given information.
$A = \frac{1}{2} \times 16 \times 6.2$	Multiply the terms on the right together.
$A = 49.6$	Include the square units.
$A = 49.6 \text{ mm}^2$	

Example:
Find the area of the parallelogram.



Because the figure is a parallelogram, the height is the length that is perpendicular to the base, not a side of the figure. The base is two feet, and the height is 10 inches. Before using the area formula, all units need to be consistent. Change two feet into inches before proceeding. There are 12 inches in a foot; therefore there are 12 times 2 inches, which is 24 inches in the base.

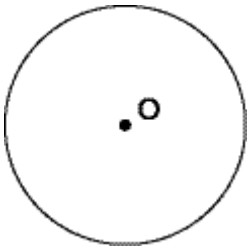
$A = bh$	Use the area formula and substitute in the given lengths.
$A = 24 \times 10$	Multiply the base times the height.
$A = 240$	Include the square units.
$A = 240 \text{ in}^2$	

CIRCLES

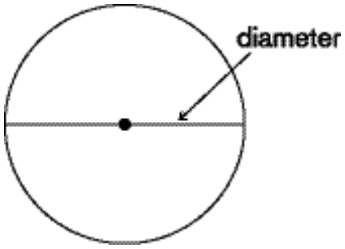
Circles are another common plane figure.

GLOSSARY

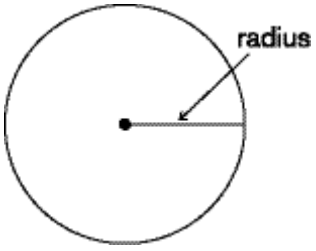
CIRCLE the set of all points equidistant from one given point, called the center. The center point defines the circle but is not on the circle.



DIAMETER of a circle is a line segment that passes through the center of the circle whose endpoints are on the circle. The diameter is twice the radius of the circle; $d = 2r$.



RADIUS of a circle is the line segment whose one endpoint is at the center of the circle and whose other endpoint is on the circle. The radius is one-half the length of the diameter; $r = \frac{1}{2}d$.



RULE BOOK

Circumference of a circle is the distance *around* the circle (the perimeter).

$C = \pi d$, where π is a constant, and d is the length of the diameter.

OR

$C = 2\pi r$, where r is the length of the radius.

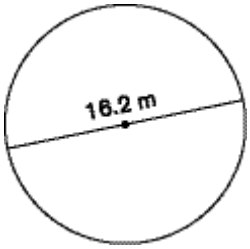
Area of a circle is the number of square units it takes to cover the circle.

$A = \pi r^2$, where π is a constant and r is the radius.

π, called "pi," is a special ratio that is a constant value of approximately 3.14. Pi compares circumference to diameter in the following ratio: $\pi = \frac{C}{d}$. It is the same value for every circle. Often, in math tests, answers will be given in terms of π, such as 136π square units. If answers are not given in terms of π, use the π key on your calculator unless otherwise instructed. Sometimes, a problem will direct you to use either $\pi = 3.14$, or $\pi = \frac{22}{7}$, which are approximations for pi.

Using the preceding formulas, you can calculate the circumference and area of circles. Take care and check if the problem gives the radius or diameter. If the problem asks for the area of the circle, for example, and gives the length of the diameter, you must first calculate the length of the radius. The radius can be found by dividing the diameter by two. Just like all area calculations, the units will be square units. The units for circumference will be linear (single) units.

Example:
Find the area of the circle.



The problem gives the diameter, which is 16.2 meters. The first step is to calculate the radius.

$r = \frac{1}{2}d$	Substitute in 16.2 for d.
$r = 8.1$ m	Now, use the formula for the area of a circle.
$A = \pi r^2$	Substitute in for the radius.

$A = \pi \times 8.1 \times 8.1$	Multiply 8.1 times 8.1, and include the square units.
$A = 65.61\pi \text{ m}^2$	

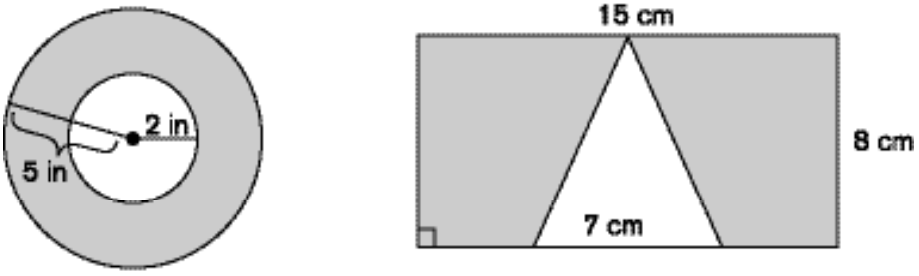
As in this example, answers are frequently left in terms of π.

Example:
Given that the circumference $C \approx 81.64$ feet, find the radius.
Use $\pi = 3.14$.

$C = \pi d$	Substitute in for C and π.
$81.64 = 3.14 \times d$	Divide 81.64 by 3.14 to find diameter.
$26 = d$	To find radius, use the radius formula.
$r = \frac{1}{2}d$	Substitute in the diameter.
$r = \frac{1}{2} \times 26$	Multiply one-half times 26, and include units.
$r = 13$ ft.	

AREA OF SHADED REGIONS

Often on math tests, you are asked to find the area of a shaded region, such as these:



I II

To solve this type of problem, you must identify the figures in the diagram. There is an outer figure and an inner figure. The area of the shaded region will be $\text{Area}_{\text{outer figure}}$ minus the $\text{Area}_{\text{inner figure}}$.

In figure I above, the outer figure is a circle of radius 5 inches, and the inner figure is a circle of radius 2 inches. To find the area of the shaded region, perform the following:

$A_{shaded} = A_{outer} - A_{inner}$	Substitute in the correct formulas.
$A_{shaded} = \pi r^2 - \pi r^2$	Now, substitute in the radius lengths.
$A_{shaded} = \pi \times 5 \times 5 - \pi \times 2 \times 2$	The order of operations directs multiplication to be done next, left to right. The answer will be left in terms of π.
$A_{shaded} = 25\pi - 4\pi$	Now, combine the π terms, and include the square units.
$A_{shaded} = 21\pi \text{ in}^2$	

In figure II above, the outer figure is a rectangle and the inner figure is a triangle. The height of both the rectangle and the triangle is 8 cm. The base of the rectangle is 15 cm and the base of the triangle is 7 cm.

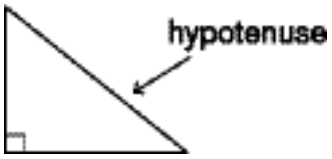
$A_{shaded} = A_{outer} - A_{inner}$	Substitute in the correct formulas.
$A_{shaded} = bh - \frac{1}{2}bh$	Substitute in the given lengths.
$A_{shaded} = 15 \times 8 - \frac{1}{2} \times 7 \times 8$	Multiplication is done next, working left to right.
$A_{shaded} = 120 - 28$	Now, evaluate the subtraction, and include the units.
$A_{shaded} = 92 \text{ cm}^2$	

RIGHT TRIANGLE MEASUREMENTS—PYTHAGOREAN THEOREM

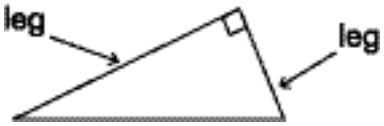
Right triangles are special triangles used for measuring. In a right triangle, the base and one side are perpendicular.

GLOSSARY

HYPOTENUSE of a right triangle is the side of the right triangle that is opposite the right angle.



LEGS of a right triangle are the two sides of the right triangle that make up the right angle.



In right triangles, there is a special relationship between the hypotenuse and the legs of the triangle. This relationship is always true and it is known as the *Pythagorean theorem*.

RULE BOOK

The Pythagorean theorem states that in all right triangles, the sum of the squares of the two legs is equal to the square of the hypotenuse; $leg^2 + leg^2 = hypotenuse^2$.

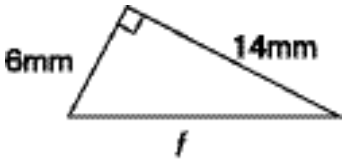
The converse of the Pythagorean theorem is also true: In a triangle, if the sum of the squares of the legs is equal to the square of the hypotenuse, then the triangle is a right triangle.

SHORTCUT

You can remember the Pythagorean theorem as the well-known formula: $a^2 + b^2 = c^2$, where a and b are the two legs of the right triangle, and c is the hypotenuse.

Special note: Be careful! There is nothing special about the letters a, b and c. A test question could be tricky and could call one of the legs "c."

Example:
Find f to the nearest hundredth.

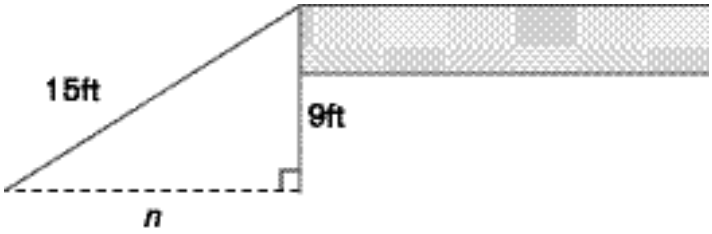


Use the Pythagorean theorem: $a^2 + b^2 = c^2$, where a and b are 6 mm and 14 mm, and c is the unknown, "f." Note that it does not matter whether you set $a = 6$ and $b = 14$, or $a = 14$ and $b = 6$, due to the commutative property of addition covered in Chapter 3.

$a^2 + b^2 = c^2$	Substitute in the given lengths.
$6^2 + 14^2 = f^2$	Evaluate the exponents, left to right.
$36 + 196 = f^2$	Now perform addition.
$232 = f^2$	Take the square root of 232 to find f.
$\sqrt{232} = f$	The value of f will be approximate, to the nearest hundredth, and will include the units.

$15.23 \text{ mm} \approx f$

Example:
A volleyball net is staked to the ground as shown in the diagram:



The cord on the stake is 15 feet, and the pole is 9 feet tall. How far from the bottom of the pole is the stake (value *n* in the diagram)?

The cord, the pole, and the distance from the bottom of the pole to the stake form a right triangle. The cord in this diagram is the hypotenuse of the right triangle. The height of the pole, 9 feet, is one of the legs. The unknown is the other leg.

$a^2 + b^2 = c^2$	Substitute in the given lengths.
$9^2 + b^2 = 15^2$	Evaluate the exponents, left to right.
$81 + b^2 = 225$	Subtract 81 from 225.
$b^2 = 144$	Take the square root of 144 to find *b*.
$b = \sqrt{144}$	The value of *b* is 12, and includes units.
$b = 12$ ft	

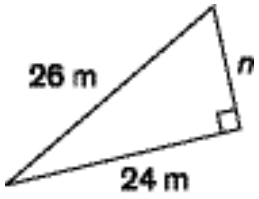
SHORTCUT

There are three sets of Pythagorean triples that appear over and over again in math test problems. Knowing these three common triples will save you valuable time in working problems of this type.

	a	*b*	*c*
One set is:	**3**	**4**	**5**
and multiples thereof:	**6**	**8**	**10**
	9	**12**	**15**
	12	**16**	**20**
Another set is:	**5**	**12**	**13**
and multiples thereof:	**10**	**24**	**26**
	15	**36**	**39**
The third set is:	**8**	**15**	**17**
	16	**30**	**34**

Memorize these sets: {3, 4, 5}, {5, 12, 13}, and {8, 15, 17}. If a right triangle problem is given and two of the three numbers in one set appear (or multiples of the two numbers), you can avoid all the substituting and calculating and save precious test time.

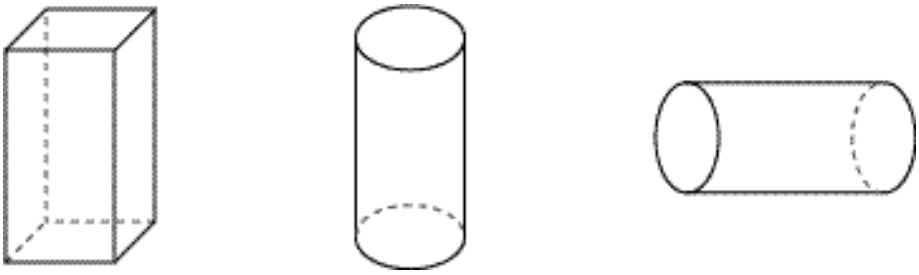
Example:
Find the value of n.



This is a right triangle, where the hypotenuse is 26 m and one of the legs is 24 m. This is a multiple of the common Pythagorean triple {5, 12, 13}, so the Pythagorean triple is {10, 24, 36} by multiplying each length by two. The unknown side is therefore 10 m.

THREE-DIMENSIONAL GEOMETRY

Solid figures are three-dimensional entities. The most common ones are the rectangular prism and the cylinder:



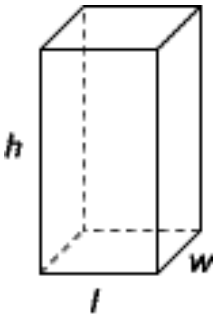
VOLUME

Volume is a measure of how many cubic units it takes to *fill* a solid figure. Volume is measured in cubic units. You may recall the term "cube" from Chapter 7. Volume, like area, is a multiplication concept, where three measures are multiplied together. The units can also be thought of as multiplied together: cm × cm × cm = cm^3, or the words "centimeters cubed."

RULE BOOK

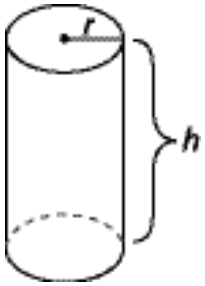
Volume is the area of the base of a solid figure, multiplied by the height of the figure. This can be expressed as $V = bh$, where V is the volume, b is the area of the base, and h is the height.

Volume of a rectangular prism $V = lwh$



In this case, $b = lw$, where l is the length and w is the width.

Volume of a cylinder $V = \pi r^2 h$



In this case, $b = \pi r^2$, where π is the constant, and r is the radius of the cylindrical base.

Example:
Given the volume of a rectangular prism to be 362.1 mm^3, find the height if the length is 7.1 mm and the width is 5 mm.

Use the formula for the volume of a rectangular prism.

$V = lwh$	Substitute in the given information.
$362.1 = 7.1 \times 5 \times h$	Multiply 7.1 times 5.
$362.1 = 35.5 \times h$	Divide 362.1 by 35.5 and include units.
$10.2 \text{ mm} = h$	The units are linear (single) because this is a height measurement.

Example:
Find the volume of a cylinder whose base diameter is 12.4 inches and height is 15 inches.

First, recognize that while diameter is given, the radius is needed to calculate volume. Use the formula to find the radius.

$r = \frac{1}{2}d$	Substitute in the given value for diameter.
$r = \frac{1}{2} \times 12.4$	Multiply one-half times 12.4.
$r = 6.2$ in	Now use the formula for the volume of a cylinder.
$V = \pi r^2 h$	Substitute in the given information.
$V = \pi \times 6.2 \times 6.2 \times 15$	Multiply all the number terms together on the right.
$V = 576.6\pi$	Include the cubic units.
$V = 576.6\pi$ in^3	

EXTRA HELP

If you feel you need extended help in working with geometry, *Practical Math Success in 20 Minutes a Day*, published by Learning Express, has several chapters devoted to this topic: Lessons 17, 18, and 19.

There are several useful websites that deal with the various topics of geometry. Visit these sites if you feel you need further clarification on these concepts. Each one has a unique method of presentation.

1) The website *www.math.com* has extensive lessons on geometry. Once at the site, click on *Geometry*, which you will find on the left under *Select Subject*. From this page, select any topic of interest. Each topic has a lesson, followed by an interactive quiz. Answers to all quizzes are provided.

2) The website *www.aaamath.com* is another good resource for practice with geometry. Once on the home page, click on *Geometry*. You will find this on the right under *Math Topics*. The topics are well organized, and there is a brief description of the topic followed by an interactive quiz. Answers are provided.

3) A third site to visit is *http://rock.uwc.edu/galexand/baw/toc.htm.* You will see the heading *Table of Contents* on the left. Click on *Lesson #10*, Geometric Figures. Again, there is a lesson followed by a quiz with answers provided.

MEASUREMENT CONVERSIONS

When working with measurements, you often have to convert units before performing other calculations. There are two methods of converting measurements. One is using proportions and the other is using a scientific method called dimensional analysis.

RULE BOOK

Conversion Factors

1 foot = 12 inches	1 cup = 8 ounces
3 feet = 1 yard	1 pint = 2 cups
1 mile = 5,280 feet	1 quart = 2 pints
1 minute = 60 seconds	1 gallon = 16 cups
1 hour = 60 minutes	1 gallon = 4 quarts
1 inch = 2.54 centimeters	

Metric Conversions

1 meter = 1,000 millimeters
1 meter = 100 centimeters
1 meter = 10 decimeters
1,000 meters = 1 kilometer

The metric prefixes and their meanings are:

Prefix	Meaning	Example
kilo	1,000 times	1 kilometer is 1,000 meters.
hecto	100 times	1 hectogram is 100 grams.
deca	10 times	1 decaliter is 10 liters.
deci	$\frac{1}{10}$ times	1 decigram is $\frac{1}{10}$ of a gram.
centi	$\frac{1}{100}$ times	1 centimeter is $\frac{1}{100}$ of a meter.
milli	$\frac{1}{1{,}000}$ times	1 milliliter is $\frac{1}{1{,}000}$ of a liter.

The prefixes can be used with any of the following: *meters* measure length, *liters* measure volume, and *grams* measure mass.

To use the proportion method to convert units, set up a proportion as described in Chapter 5. Keep the units consistent on both sides of the proportion. For example, if you want to convert 50.8 centimeters to inches, set up a proportion, such as $\frac{\text{inch}}{\text{centimeter}} = \frac{\text{inch}}{\text{centimeter}}$, and substitute in the given values on one side, the conversion factor on the other; $\frac{1}{2.54} = \frac{n}{50.8}$. Cross-multiply to get $2.54 \times n = 1 \times 50.8$. Now, divide 50.8 by 2.54 to get $n = 20$ inches.

The dimensional analysis method involves multiplying a series of fractions that are all equal to a value of one, with unwanted units alternately on

top and bottom. This way, the units "cancel" and what is left is the needed unit. Again, to convert 50.8 centimeters to inches, set up the analysis: $\frac{1 \text{ inch}}{2.54 \text{ centimeter}} \times \frac{50.8 \text{ centimeter}}{n}$. The set-up for this analysis is the opposite of using a proportion; notice that in this set-up the centimeter units are in opposite positions in the fraction. The centimeter units, which we do not want, will cancel, and all that remains is the needed inch unit.

$$\frac{1 \text{ inch}}{2.54 \text{ centimeters}} \times \frac{50.8 \text{ centimeters}}{n}$$

After canceling the centimeter units and multiplying the fractions straight across, the problem becomes $\frac{50.8 \text{ inches}}{2.54} = 20$ inches.

You may be wondering: "*Why bother with dimensional analysis at all?*" There is a very good reason to know this method. It is the method scientists prefer because it allows you to do several conversions at once. Study the next example. Both methods of converting units will be used.

Example:
A toy car travels a distance of 7,620 cm in four minutes. What is the speed in inches per second?

First, use the proportion method. To use this method, centimeters must be converted into inches in one step and then minutes must be converted to seconds in another step. Set up the proportion for centimeters to inches by using $\frac{\text{inch}}{\text{centimeter}} = \frac{\text{inch}}{\text{centimeter}}$, which is $\frac{1}{2.54} = \frac{n}{7{,}620}$. Cross-multiply to yield $2.54 \times n = 1 \times 7{,}620$. Divide 7,620 by 2.54 to get 3,000 inches. Now, use the proportion $\frac{\text{seconds}}{\text{minutes}} = \frac{\text{seconds}}{\text{minutes}}$, to convert minutes to seconds. The proportion becomes $\frac{60}{1} = \frac{n}{4}$. Cross-multiply to get $1 \times n = 60 \times 4$. Sixty times 4 is 240, so $n = 240$ seconds. Finally, calculate inches per second, which means inches per one second; $\frac{3{,}000}{240} = 12.5$ inches per second.

Now, solve the problem using the dimensional analysis method. We want the answer to be in inches per second. Set up the fractions with inches on the top and seconds on the bottom, so that the centimeter and minute units cancel.

$$\frac{1 \text{ inch}}{2.54 \text{ centimeters}} \times \frac{7{,}620 \text{ centimeters}}{4 \text{ minutes}} \times \frac{1 \text{ minutes}}{60 \text{ seconds}}$$

Now, cancel out units:

$$\frac{1 \text{ inch}}{2.54 \text{ centimeters}} \times \frac{7{,}620 \text{ centimeters}}{4 \text{ minutes}} \times \frac{1 \text{ minutes}}{60 \text{ seconds}}$$

Finally, multiply straight across and leave the units that did not cancel: $\frac{7{,}620 \text{ inches}}{2.54 \times 4 \times 60 \text{ seconds}} = \frac{7{,}620}{609.6}$.

Divide 7,620 by 609.6 to get 12.5 inches per second. As you can see from this example, dimensional analysis is an efficient way to convert measurement units when there are several conversions to be made.

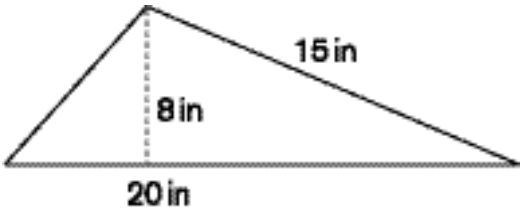
TIPS AND STRATEGIES

- A polygon is a closed plane figure made up of line segments. They are named according to the number of sides in the polygon.
- Triangles are classified in two ways, by side measurement and by angles.
- Quadrilaterals are classified by parallel sides, angles, and side measurements.
- Perimeter is a linear measurement that measures around a plane figure.
- Area is a square measurement that measures how many square units it takes to cover a figure.
- Circles can be measured for circumference and area.
- Know the formulas for the area of a circle and common polygons.
- Calculate the area of a shaded region by subtraction: $A_{\text{shaded}} = A_{\text{outer}} - A_{\text{inner}}$.
- The Pythagorean theorem is used to make measurements with right triangles.
- Know the common Pythagorean triples and how to find their multiples.
- Volume is a cubic measurement that measures how many cubic units it takes to fill a solid figure.
- Know the volume formulas for the common solid figures.
- There are two methods for converting measurement units: the proportion method and dimensional analysis.

PRACTICE

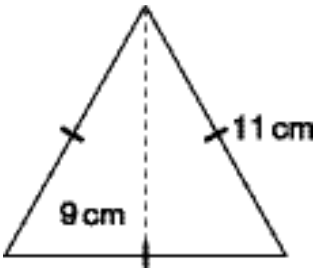
Now that you have studied these lessons, try this set of practice problems to gauge your success. Carefully read over the answer explanations. Keep in mind that there are often several ways to solve geometry problems, and you may use an alternate method.

1. Find the area.



a. 160 in^2
b. 80 in^2
c. 120 in^2
d. 90 in^2
e. 240 in^2

2. Find the perimeter.

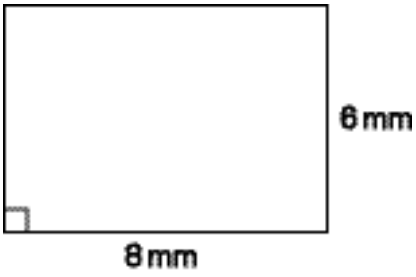


a. 121 cm
b. 99 cm
c. 47.5 cm
d. 22 cm
e. 33 cm

3. Find the perimeter.

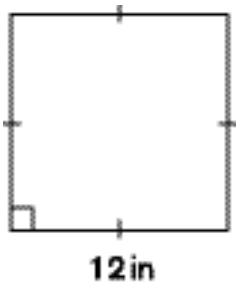
a. 100 in
b. 42 in
c. 34 ft
d. 17 ft
e. 50 in

4. Find the area.



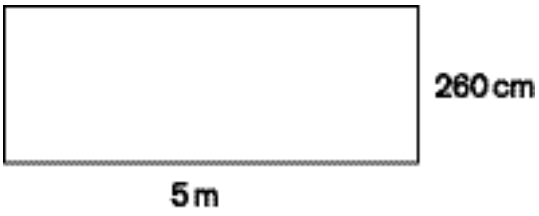
a. 14 mm^2
b. 48 mm^2
c. 24 mm^2
d. 64 mm^2
e. 28 mm^2

5. Find the area.



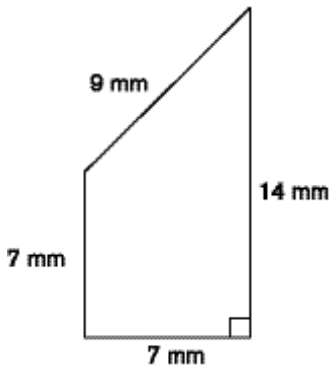
a. 24 in^2
b. 48 in^2
c. 144 in^2
d. 72 in^2
e. 24 in^2

6. Find the perimeter.



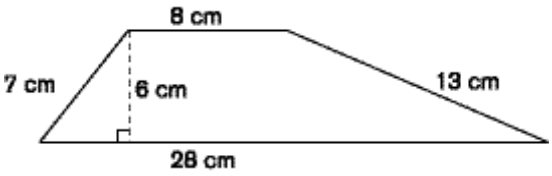
a. 1,520 cm
b. 1,520 m
c. 265 cm
d. 265 m
e. 1,300 cm

7. Find the area.



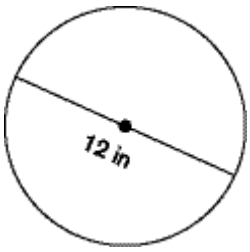
a. 37 mm^2
b. 98 mm^2
c. 63 mm^2
d. 126 mm^2
e. 73.5 mm^2

8. Find the perimeter.



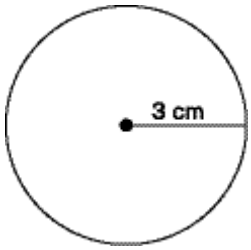
a. 55 cm
b. 62 cm
c. 56 cm
d. 196 cm
e. 61 cm

9. Find the circumference.



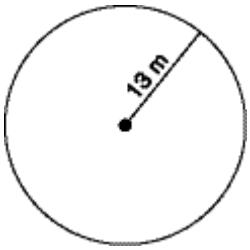
a. 6 in
b. 6π in
c. 12 in
d. 12π in
e. 36π in

10. Find the circumference.



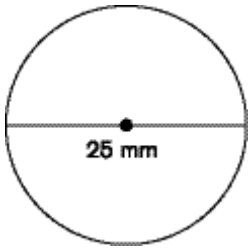
a. 6 cm
b. 6π cm
c. 3π cm
d. 9π cm
e. 36π cm

11. Find the area.



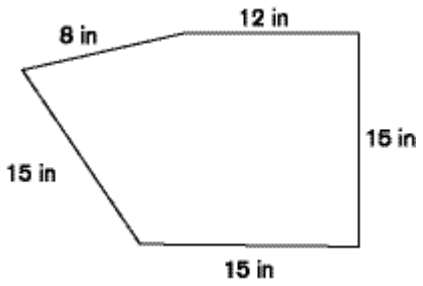
a. 13π m^2
b. 39π m^2
c. 169π m^2
d. 676 m^2
e. 169 m^2

12. Find the area.



a. 156.25π mm^2
b. 625π mm^2
c. 50π mm^2
d. 25 mm^2
e. 156.25 mm^2

13. Find the perimeter.

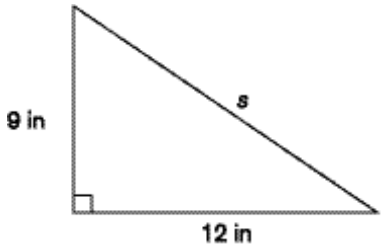


a. 225 in^2
b. 65 in
c. 225 in
d. 45 in
e. 65 in^2

14. An ant travels nine inches every three seconds. How fast does the ant travel in centimeters per minute?
a. 3 cm/min
b. 0.15 cm/min
c. 0.125 cm/min
d. 22.86 cm/min
e. 457.2 cm/min

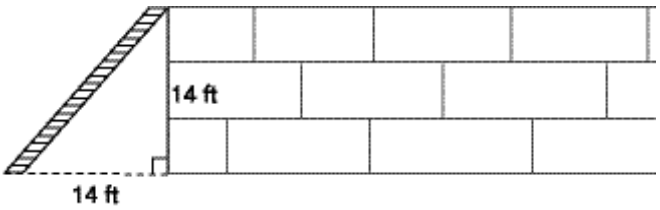
15. A garden hose leaks four cups of water per day. How many gallons are leaked per week?
a. 1.75 gal/week
b. 3.5 gal/week
c. 28 gal/week
d. 14 gal/week
e. 0.25 gal/week

16. Find *s*.



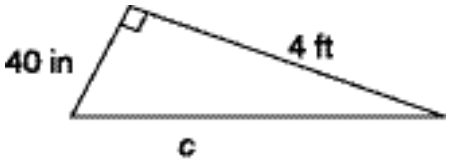
a. 108 in
b. 42 in
c. 15 in
d. 6.48 in
e. 225 in

17. A ladder rests on the side of a building, as shown in the figure below



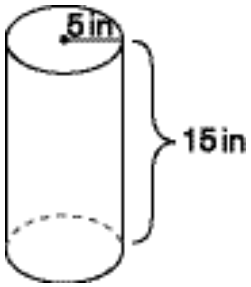
How tall is the ladder, to the nearest foot?
a. 14 ft
b. 20 ft
c. 7 ft
d. 28 ft
e. 392 ft

18. Find *c* to the nearest tenth of an inch.



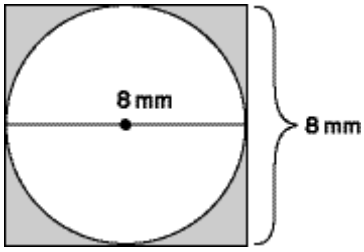
a. 3,904.0 in
b. 40.2 in
c. 22.0 in
d. 62.5 in
e. 5.0 in

19. Find the volume.



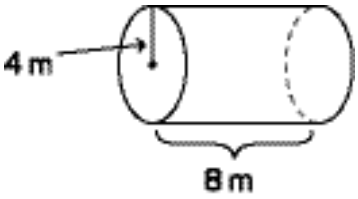
a. 75 in^3
b. 75π in^3
c. 1,500 in^3
d. 1,500π in^3
e. 375π in^3

20. Find the area of the shaded region. Use 3.14 for π.



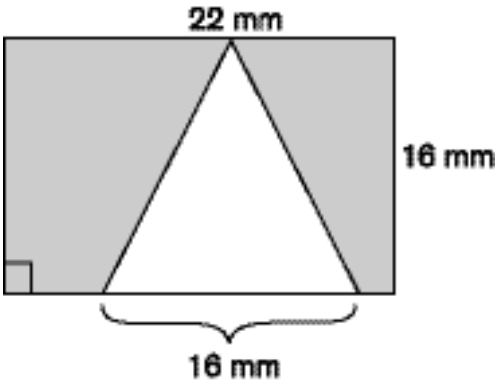
a. 64 mm^2
b. 13.76 mm^2
c. 32 mm^2
d. 128 mm^2
e. 50.24 mm^2

21. Find the volume.



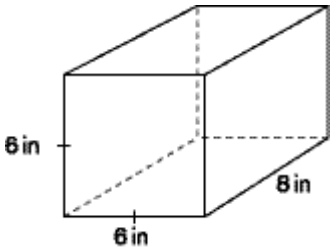
a. 128π m^3
b. 32 m^3
c. 128 m^3
d. 512 m^3
e. 512π m^3

22. Find the area of the shaded region.



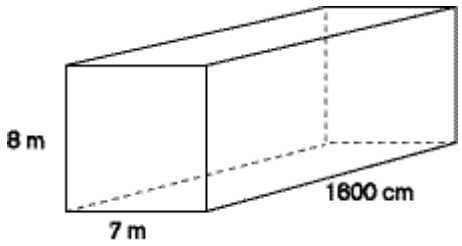
a. 352 mm^2
b. 96 mm^2
c. 76 mm^2
d. 224 mm^2
e. 60 mm^2

23. Find the volume.



a. 48 in^3
b. 288 in^3
c. 36 in^3
d. 144 in^3
e. 20 in^3

24. Find the volume



a. 89,600 m^3
b. 89,600 cm^3
c. 1,656 m^3
d. 896 cm^3
e. 896 m^3

25. A steady wind blows a leaf two meters every five seconds. At this speed, how far is the leaf blown in centimeters per hour?
a. 144,000 cm/hour
b. 14.4 cm/hour
c. 40 cm/hour
d. 3.6 cm/hour
e. 3,600 cm/hour

ANSWERS

1. b. The formula for the area of a triangle is $A = \frac{1}{2}bh$. Substitute in the base of 20 inches and the height of 8 inches; $A = \frac{1}{2} \times 20 \times 8$. Multiply to yield 80 in^2.

2. e. The figure shown is an equilateral triangle. Therefore each side is 11 cm; 11 + 11 + 11 = 33 cm.

3. a. Note that the units are not consistent; the base is measured in feet and the height is measured in inches. Convert 3 feet to inches by multiplying by 12, which yields 36 inches for the measure of the base. The opposite sides in a rectangle are congruent. The perimeter is 36 + 36 + 14 + 14 = 100 inches.

4. b. A rectangle is a parallelogram. The formula for the area of a parallelogram is $A = bh$. Substitute in 8 mm for the base and 6 mm for the height; $A = 8 \times 6$. The area is 48 mm^2.

5. c. The figure shown is a square, and all sides are congruent. A square is a parallelogram and the area of a parallelogram is found by the formula $A = bh$. In this square, the base and the height are 12 inches; $12 \times 12 = 144$ square inches.

6. a. The units in this figure are not consistent. Change the base measure from meters to centimeters. There are 100 centimeters in a meter and therefore the measure of the base is 500 cm. Opposite sides of a rectangle are congruent. The perimeter is $500 + 500 + 260 + 260 = 1{,}520$ cm.

7. e. Use the formula for the area of a trapezoid, $A = \frac{1}{2}h(b_1 + b_2)$. Identify the parallel sides as the bases. The bases are 14 mm and 7 mm. The height is perpendicular to the base sides, and the height is one of the sides in this figure. The height is 7 mm. Substitute the given values in the formula; $A = \frac{1}{2} \times 7 \times (14 + 7)$. Evaluate the parentheses first to yield $A = \frac{1}{2} \times 7 \times (21)$. Multiply from left to right to find the area in square units.

8. c. To find the perimeter you add up all of the sides. The height, 6 cm, is not a side; $7 + 8 + 13 + 28 = 56$ cm.

9. d. Use the formula $C = \pi d$, where the diameter is 12 inches; $C = 12\pi$ in.

10. b. Use the formula $C = \pi d$. Radius is given, and diameter is $d = 2r$; 2×3 cm = 6 cm. The circumference is 6π cm.

11. c. Use the formula $A = \pi r^2$, where r is given to be 13 meters. Substitute in to yield $A = \pi \times 13^2$. Thirteen squared is 13 times 13, or 169. The area is 169π square meters.

12. a. Use the formula $A = \pi r^2$, and note that since diameter is given, you must first solve for the radius, which is one-half of the diameter. One-half of 25 is 12.5 mm. Substitute in to yield $A = \pi \times 12.5 \times 12.5$.

13. b. Add up all of the sides to find the perimeter; $8 + 12 + 15 + 15 + 15 = 65$ inches.

14. e. Use dimensional analysis.

$$\frac{9 \text{ in}}{3 \text{ sec}} \times \frac{2.54 \text{ cm}}{1 \text{ in}} \times \frac{60 \text{ sec}}{1 \text{ min}}$$

The second units cancel and the inch units cancel as shown above. Now multiply the fractions; $\frac{9}{3} \times \frac{2.54}{1} \times \frac{60}{1} = \frac{9 \times 2.54 \times 60}{3} = \frac{1{,}371.6}{3}$; 1,371.6 divided by 3 is 457.2 cm/min.

15. a. Use dimensional analysis.

$$\frac{4 \text{ cup}}{1 \text{ day}} \times \frac{7 \text{ day}}{1 \text{ week}} \times \frac{1 \text{ gal}}{16 \text{ cup}}$$

The day units and the cup units cancel as shown above. Multiply the fractions; $\frac{4}{1} \times \frac{7}{1} \times \frac{1}{16} = \frac{4 \times 7}{16} = \frac{28}{16}$; 28 divided by 16 is 1.75 gallons per week.

16. c. This is a common Pythagorean triple, namely {9, 12, 15}. The hypotenuse is 15 inches. Note that 9, 12, and 15 are found by multiplying the Pythagorean triple {3, 4, 5} by 3.

17. b. Use the Pythagorean theorem:

$a^2 + b^2 = c^2$	Substitute in the given lengths.
$14^2 + 14^2 = c^2$	Evaluate the exponents, left to right.
$196 + 196 = c^2$	Add.
$392 = c^2$	Take the square root of 392 to find c.
$\sqrt{392} = c$	The value of c to the nearest foot is 20 ft.
$20 = c$	

18. d. First make all units consistent. Change 4 feet to 48 inches. Now, use the Pythagorean theorem:

$a^2 + b^2 = c^2$	Substitute in the given lengths.
$40^2 + 48^2 = c^2$	Evaluate the exponents, left to right.
$1{,}600 + 2{,}304 = c^2$	Add.
$3{,}904 = c^2$	Take the square root of 3,904 to find c.
$\sqrt{3{,}904} = c$	The value of c to the nearest foot is 62.5 inches.
$62.5 = c$	

19. e. Use the formula for the volume of a cylinder, which is $V = \pi r^2 h$. In the figure, the radius is 5 inches and the height is 15 inches. Substitute into the formula to yield $V = \pi \times 5 \times 5 \times 15$. Multiply 5 times 5 times 15, and leave the answer in terms of pi. The volume is 375π cubic inches.

20. b. The figure shown is an outer square and an inner circle. The circle has a radius of one-half of the diameter of 8 mm, which is 4 mm. The base and height of the square are both 8 mm.

$A_{shaded} = A_{outer} - A_{inner}$	Substitute in the correct formulas.
$A_{shaded} = bh - \pi r^2$	Now, substitute in the given measures.
$A_{shaded} = 8 \times 8 - 3.14 \times 4 \times 4$	Order of operations directs multiplication to be done next, left to right.
$A_{shaded} = 64 - 50.24$	Now subtract and include the units.
$A_{shaded} = 13.76 \text{ mm}^2$	

21. a. Use the formula for the volume of a cylinder, which is $V = \pi r^2 h$. In the figure, the radius is 4 meters and the height is 8 meters. Substitute into the formula to yield $V = \pi \times 4 \times 4 \times 8$. Multiply 4 times 4 times 8 and leave the answer in terms of pi. The volume is 128π cubic meters.

22. d. The figure shown is an outer rectangle and an inner triangle. The base and height of the rectangle are 22 mm and 16 mm respectively. The base of the triangle is 16 mm. The height of the triangle is the same as the height of the rectangle.

$A_{shaded} = A_{outer} - A_{inner}$	Substitute in the correct formulas.
$A_{shaded} = bh - \frac{1}{2}bh$	Now, substitute in the given measures.
$A_{shaded} = 22 \times 16 - \frac{1}{2} \times 16 \times 16$	Order of operations directs multiplication to be done next, left to right.
$A_{shaded} = 352 - 128$	Now subtract and include the units.
$A_{shaded} = 224 \text{ mm}^2$	

23. b. Use the formula for the volume of a rectangular solid, that is $V = lwh$. Substitute in the given values to yield $V = 6 \times 6 \times 8 = 288$ cubic inches.

24. e. Note that the units are not consistent. Change 1,600 cm to 16 m by dividing by 100. Use the formula for the volume of a rectangular solid; $V = lwh$. Substitute in the given values to yield $V = 8 \times 7 \times 16 = 896 \text{ m}^3$.

25. a. Use dimensional analysis.

$$\frac{2\ m}{5\ sec} \times \frac{60\ sec}{1\ min} \times \frac{60\ min}{1 hour} \times \frac{100\ cm}{1\ m}$$

The meter, second, and minute units cancel as shown above. Multiply the fractions; $\frac{2}{5} \times \frac{60}{1} \times \frac{60}{1} \times \frac{100}{1} = \frac{2 \times 60 \times 60 \times 100}{5} = \frac{720{,}000}{5}$; 720,000 divided by 5 is 144,000 centimeters per hour.

Statistics and Probability

Statistics is a branch of mathematics that involves the study of data. Probability is the study of chance. This chapter will refresh your understanding of common statistical measures, graphs, and probability. Before proceeding to the lesson, take this ten-item *Benchmark Quiz* to see how much you remember about statistics and probability. These questions are similar to the type of questions that you will find on important tests. When you are finished, check the answer key carefully to assess your results. Your Benchmark Quiz analysis will help you determine how much time you need to spend on statistics and probability, and the specific areas in which you need the most careful review and practice.

BENCHMARK QUIZ

1. Find the mean for the set of data: {96, 90, 78, 90, 92}.
a. 18
b. 89.2
c. 90
d. 89.5
e. 71.2

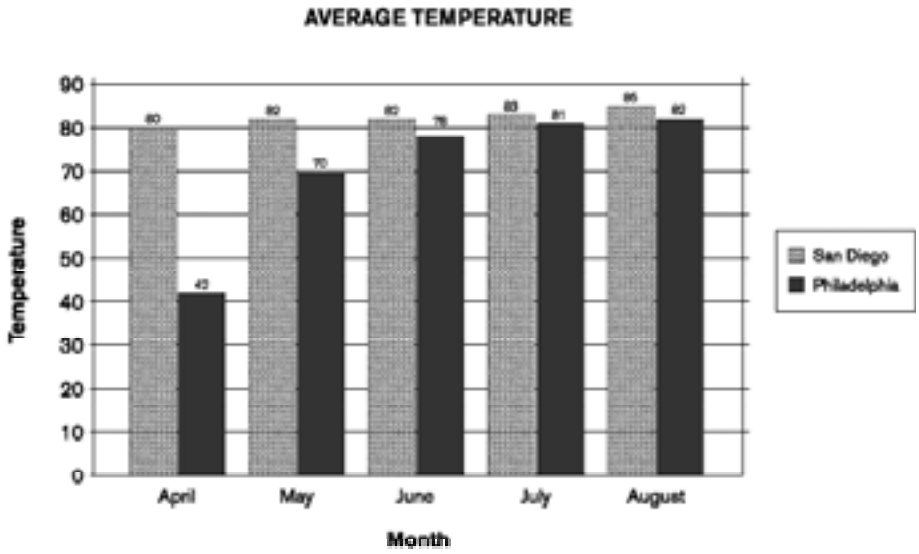
2. The fuel efficiency for a truck varies, depending on whether the truck is traveling uphill or downhill. The following efficiencies were recorded for one hour at ten-minute intervals, in miles per gallon: 16, 22, 14, 28, 16, 12. What is the mean fuel efficiency for the truck in this hour?
a. 16 miles per gallon
b. 12 miles per gallon
c. 18 miles per gallon
d. 6 miles per gallon
e. 20 miles per gallon

3. Given the set of numbers {26, 27, 29, 27, 29, 27, 30, 30}, which of the following is true?
a. The mean equals the median.
b. There are three modes.
c. The median is greater than the mode.
d. The median is 27.
e. The median equals the mode.

4. The graph shows how Nathan spends his take-home pay. What percent of his money is allocated for rent?

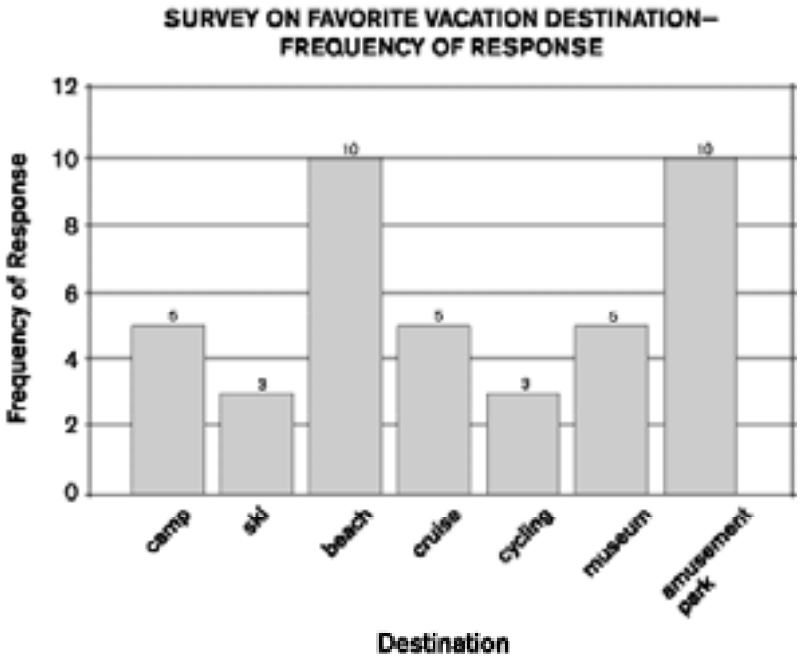
a. 125%
b. 0.24%
c. 72%
d. 24%
e. 90%

5. The bar chart compares average temperature for a six-month period. Which city has the biggest range of temperatures for this period, and what is the range?



a. San Diego, 85°
b. San Diego, 80°
c. Philadelphia, 82°
d. Philadelphia, 42°
e. Philadelphia, 40°

6. For the histogram, how many total people participated in the survey?



a. 36
b. 10
c. 7
d. 41
e. 38

7. On a standard die, what is the probability of rolling a 3?

a. $\frac{1}{6}$
b. $\frac{3}{6}$
c. $\frac{3}{8}$
d. $\frac{1}{3}$
e. $\frac{5}{6}$

8. When rolling two dice, what is the probability of rolling a sum of eight?

a. $\frac{5}{12}$

b. $\frac{3}{36}$

c. $\frac{3}{12}$

d. $\frac{2}{36}$

e. $\frac{5}{36}$

9. When randomly picking from a standard deck of playing cards, what is the probability of picking a heart or a jack?

a. $\frac{16}{52}$

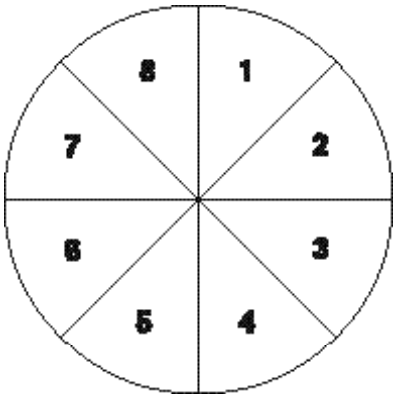
b. $\frac{13}{52}$

c. $\frac{17}{52}$

d. $\frac{4}{52}$

e. $\frac{1}{52}$

10. Using the spinner, what is the probability of spinning an odd number and then a two?



a. $\frac{4}{16}$

b. $\frac{1}{16}$

c. $\frac{5}{8}$

d. $\frac{1}{2}$

e. $\frac{1}{4}$

BENCHMARK QUIZ SOLUTIONS

How did you do on the Benchmark Quiz? Check your answers here, and then analyze your results to figure out your plan of attack to master these topics.

1. b. To find the mean of a set of data, add up all the numbers and divide by the number of data items; 96 + 90 + 78 + 90 + 92 = 446. There are five data items; 446 ÷ 5 = 89.2.

2. c. To find the mean fuel efficiency, add up the six numbers and divide by 6; 16 + 22 + 14 + 28 + 16 + 12 = 108, and 108 ÷ 6 = 18 miles per gallon.

3. c. To answer this question, calculate the mean, median, and mode of the set of numbers. First, arrange the numbers in ascending order: 26, 27, 27, 27, 29, 29, 30, 30. There are eight numbers; add them together and divide by 8 for the mean: 26 + 27 + 27 + 27 + 29 + 29 + 30 + 30 = 225, and 225 divided by 8 is 28.125, which is the *mean*. The *mode* is the number that occurs most often, which is 27. The *median* is the middle number; since there are eight entries, it is the average of the two middle numbers; 27 + 29 = 56, and 56 divided by 2 is 28. Knowing the three measures can lead to the only right conclusion, which is that the median is greater than the mode, choice **c**.

4. d. The amount spent on rent is $478.80 out of the total money of $1,995.00. Percent is a ratio of $\frac{\text{part}}{\text{whole}}$, which is $\frac{478.80}{1{,}995} = 0.24$. This is the percent written as a decimal; multiply by 100 (move the decimal two places to the right) to get 24%.

5. e. The range is the difference between the highest and lowest values in the temperatures. The range for San Diego is 5 degrees, that is 85 – 80 = 5 degrees. The range for Philadelphia is 82 – 42 = 40 degrees. Philadelphia has a larger range of temperatures.

6. d. Histograms show the frequency on the vertical, *y*-axis. In this case, frequency is the number of people that gave an opinion. The total number surveyed is found by adding up all of the frequencies; 5 + 3 + 10 + 5 + 3 + 5 + 10 = 41 respondents.

7. a. Probability is a ratio of $\frac{\text{number of favorable outcomes}}{\text{number of total outcomes}}$.

There are six sides to a die, so there are six total outcomes, one of which is a three. The probability is therefore $\frac{1}{6}$.

8. e. Use a table to find all of the possible sum outcomes when rolling two dice. The table is:

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	(8)
3	4	5	6	7	(8)	9
4	5	6	7	(8)	9	10
5	6	7	(8)	9	10	11
6	7	(8)	9	10	11	12

From the table, there are 36 total outcomes, five of which generate a sum of 8. The sums of eight are circled on the chart; 6 and 2, 2 and 6, 3 and 5, 5 and 3, and 4 and 4. The probability is, therefore, $\frac{\text{number of favorable outcomes}}{\text{number of total outcomes}} = \frac{5}{36}$.

9. a. Probability is a ratio of $\frac{\text{number of favorable outcomes}}{\text{number of total outcomes}}$.

In this example, there are two separate conditions, a heart, P(h) OR a jack, P(j). The probability of P(h) OR P(j) is equal to P(h) + P(j). Out of 52 playing cards, 13 are hearts and 4 are jacks. One of these is the jack of hearts, which would be counted twice, so you must subtract one. The probability of a heart OR a jack is $\frac{13}{52} + \frac{4}{52} - \frac{1}{52} = \frac{16}{52}$.

10. b. The questions asks for the probability of spinning an odd number, P(odd) AND then spinning a two, P(2). AND means to multiply. There are eight total outcomes, four of which are odd and one of which is a two. Probability is a ratio of $\frac{\text{number of favorable outcomes}}{\text{number of total outcomes}}$, so P(odd) AND then P(2) = $\frac{4}{8} \times \frac{1}{8} = \frac{4}{64}$, which in lowest terms is $\frac{1}{16}$.

BENCHMARK QUIZ RESULTS

If you answered 8–10 questions correctly, you are well on your way to success with the topics of statistics and probability. Read over the lesson, concentrating on any areas of weakness. Be sure to try all practice problems at the end of the chapter.

If you answered 4–7 questions correctly, spend some time reading and understanding the lessons in this chapter. Pay attention to the various sidebars. Try all given examples, to ensure that you understand how to solve various types of problems.

If you answered 1–3 questions correctly, give the lessons in this chapter your full concentration. Read carefully through all explanations. Try all examples on your own. Visit the suggested websites and work through any problems given. You may need to reference a suggested book that will present the material in an alternate form. After reviewing the lessons, retry the benchmark questions and then proceed to the practice problems.

JUST IN TIME LESSON—STATISTICS AND PROBABILITY

This chapter focuses on problems that deal with data and chance.

The topics in this chapter include:

- mean
- median
- mode
- range
- graphs that represent data
- probability
- mutually exclusive events
- probability of one event OR another
- probability of one event AND THEN another

When dealing with sets of numbers, there are measures used to describe the set as a whole. These are called *measures of central tendency* and they include *mean*, *median*, *and mode*.

MEAN

GLOSSARY

MEAN the average of a set of data

RULE BOOK

To calculate the mean of a set of data, add up all of the numbers in the set and divide by how many entries are in the set.

Example:
Find the mean of the following set: {17, 22, 18, 31, 27, 17}.
Add up the six numbers in the set, 17 + 22 + 18 + 31 + 27 + 17 = 132.
Now divide 132 by 6, the number of entries in the set; 132 ÷ 6 = 22.
The mean (or average) of the set is 22.

Example:
The temperature, in degrees Fahrenheit, for the first week of July is as follows: 84, 88, 86, 87, 80, 84, and 86. What is the average temperature for the week?
Add up the seven temperatures: 84 + 88 + 86 + 87 + 80 + 84 + 86 = 595; 595 divided by 7, the number of days measured, is 595 ÷ 7 = 85.
The average temperature is 85 degrees Fahrenheit.

SHORTCUT

If you are asked to find the mean of a set of numbers, and the set is evenly spaced apart such as 2, 4, 6, 8, 10, 12, 14, the mean is the middle number in this set, because there is an odd number of data items. In this example, the mean is 8. If there is an even number of data items, there are two middle numbers; 4, 8, 12, 16, 20, and 24. In this case the mean is the average of the two middle numbers; 12 + 16 = 28, and 28 divided by 2 is 14.

MEDIAN

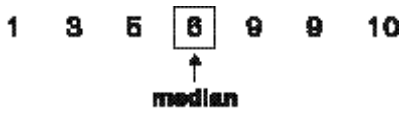
GLOSSARY

MEDIAN the middle value in a set of numbers that are arranged in increasing or decreasing order. If there are two middle numbers, it is the average of these two.

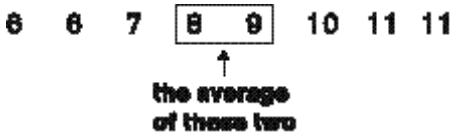
RULE BOOK

To calculate the median of a set of numbers, first arrange the data in increasing or decreasing order. Find the middle value in a set of an odd number of entries. The median is the mean of the two middle numbers in a set of an even number of entries.

Example:
Find the median of this set: {1, 3, 5, 6, 9, 9, 10}.
The data is already in increasing order. Since there are seven entries in this set, the median is the fourth entry, namely 6.



Example:
What is the median for this data: {7, 6, 9, 6, 8, 11, 10, 11}?
First arrange the data in increasing order: {6, 6, 7, 8, 9, 10, 11, 11}.
There are eight entries in this set, so the median is the average of the fourth and fifth entries.



So, 8 + 9 = 17. Seventeen divided by 2 is 8.5.

MODE

GLOSSARY

MODE the value in the set that occurs most often. There can be one mode, several modes, or no mode.

Example:
Find the mode in the following set: {13, 13, 14, 15, 15, 16, 13, 18, 17, 17}.
Thirteen is the mode of the set. The value occurs three times in the set, and the only other repetitions are 15 and 17, which each occur twice.

Example:
Find the mode: {24, 25, 25, 24, 26, 28, 29, 28, 30, 22}.
There are three modes, namely 24, 25, and 28, which each occur two times in the set.

Example:
Find the mode in the following: {50, 52, 54, 58, 60, 62, 64, 66}.
There is no mode in this set. Note that we do not specify the mode as 0, we say that there is "no mode" or "none."

RANGE

The *range* indicates how close together the given values are to one another in a set of data.

GLOSSARY

RANGE the spread of the data, which indicates how close together the data points are

RULE BOOK

To calculate the range, find the difference between the largest and the smallest values in the set of data. Subtract the smallest value from the largest value in the set.

Example:
Find the range of ages in the community play, given these ages in years: 68, 54, 49, 40, 39, 39, 24, 22, 20, 10, and 10.
The range of ages is 68 – 10 = 58 years.

Example:
Find the range of this set: {42, 40, 45, 43, 43, 40, 45}.
Find the largest and smallest values in the set. In this example, these are 45 and 40 respectively. The difference between 45 and 40, the range, is 45 – 40 = 5.

GRAPHS THAT REPRESENT DATA

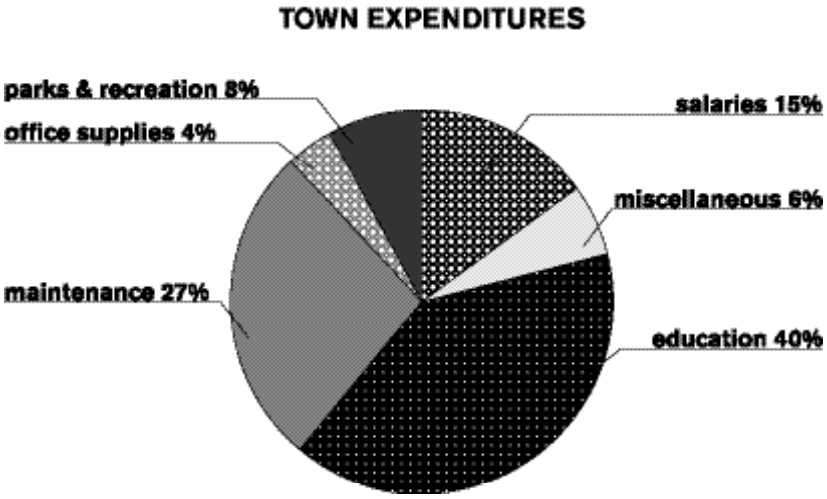
A *statistical graph* is a picture of a set of data. There are several different types of statistical graphs, three of which are reviewed in this lesson:

- circle graph (or pie chart)
- bar graph
- histogram

CIRCLE GRAPHS

Circle graphs, or *pie charts*, are used to represent parts of a whole. Often, they are shown as percentages of the whole. Percents were reviewed in Chapter 6 of this book.

Use this diagram for the two examples following:



Examples:

If the total town expenditures (budget) are $15,000,000.00, how much of the town expenditures are represented by salaries?

This is a percent problem, where salaries are 15% of the total expenditures, by reading off of the graph. So 15% of the total is salary:

$$\underbrace{15\%}\ \underbrace{\text{of}}\ \underbrace{\text{the total}}\ \underbrace{\text{is}}\ \underbrace{\text{salary}}$$

$$0.15 \times 15{,}000{,}000 = \text{salary}$$

$0.15 \times 15{,}000{,}000 = 2{,}250{,}000$

If the miscellaneous expenses are $600,000.00, how much is the total town expenditure budget?

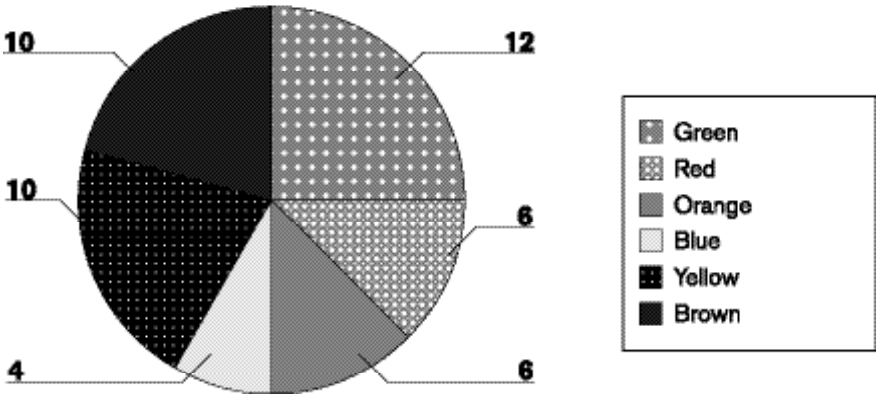
By reading the graph, 6% of the total is miscellaneous expenses. Using T to stand for total expenditures, set up the equation as follows:

$$\underbrace{6\%}_{0.6} \underbrace{\text{ of }}_{\times} \underbrace{\text{ the total}}_{T} \underbrace{\text{ is}}_{=} \underbrace{\text{ miscellaneous}}_{600{,}000}$$

$0.06 \times T = 600{,}000$; 600,000 divided by 0.06 is $10,000,000.00.

Example:
The circle graph shows the number of candies in a package, arranged by color.

CANDY COLOR IN PACKAGE



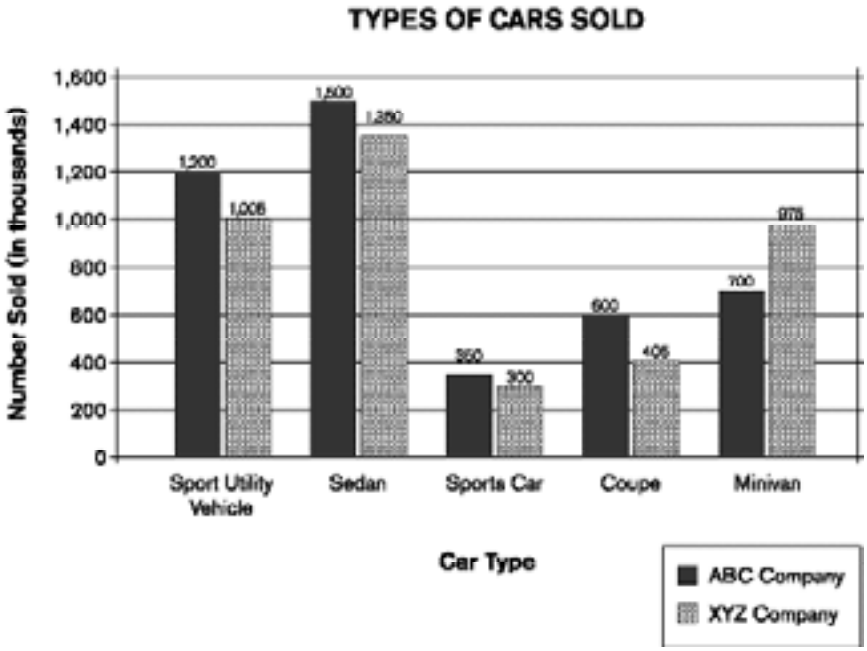
What percent of the candies are green?

Add up the amounts of each color to obtain the total number of candies in the package; 12 + 6 + 6 + 4 + 10 + 10 = 48. The percent of green is the ratio of $\frac{\text{green}}{\text{total}} = \frac{12}{48}$. Change $\frac{12}{48}$ to a decimal to get 0.25. Multiply the decimal by 100 to get the percent; move the decimal point two places to the right. Green candies are 25% of the total.

BAR GRAPHS

Bar graphs show data organized in bar format. Double bar graphs are often used to compare two items over several categories. When reading graphs, it is important to be careful to read the graph carefully. Often there are keys that denote categories, and units are written on the axes.

Use the bar graph that follows to answer the first two examples:



Examples:
Which car manufacturer sold the most minivans?

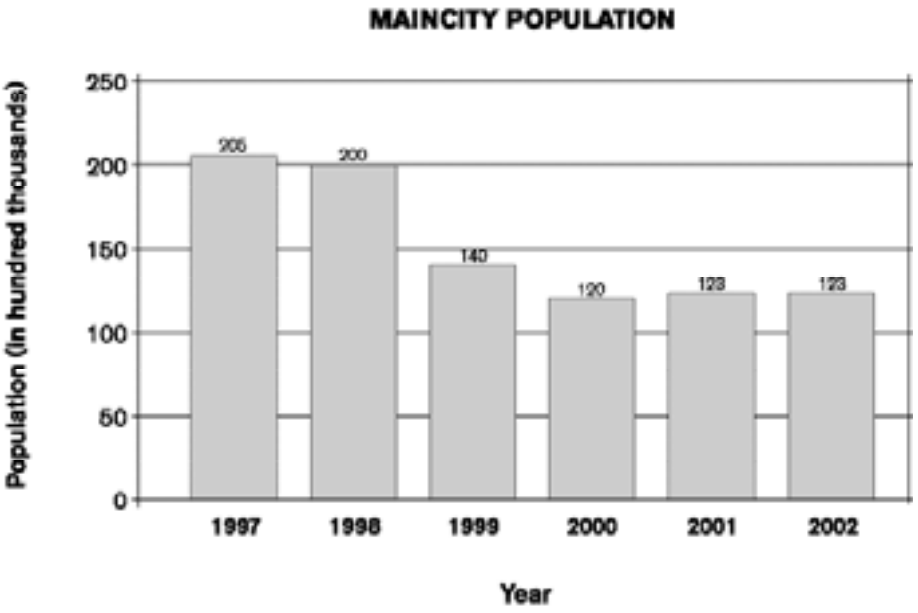
Reading the key on the graph, a solid bar represents ABC manufacturer, and a patterned bar represents XYZ manufacturer. The bar for XYZ is taller. Manufacturer XYZ sold the most minivans.

What is the total number of sport utility vehicles sold by both manufacturers?

Note carefully that on the vertical axes, the units are in thousands. The graph shows that ABC sold 1,200 thousand sport utility vehicles and XYZ sold 1,005 thousand sport utility vehicles; 1,200 + 1,005 = 2,205 thousand total sport utility vehicles, which equals 2,205,000 vehicles, because $2{,}205 \times 1{,}000 = 2{,}205{,}000$.

Use the following bar graph to answer the next two examples:

Examples:
The bar graph below shows the population of Maincity, USA.



What is the largest population shown?

The tallest bar is in the year 1997. This bar represents 205 hundred thousand people. The largest population is 20,500,000, since $205 \times 100{,}000 = 20{,}500{,}000$.

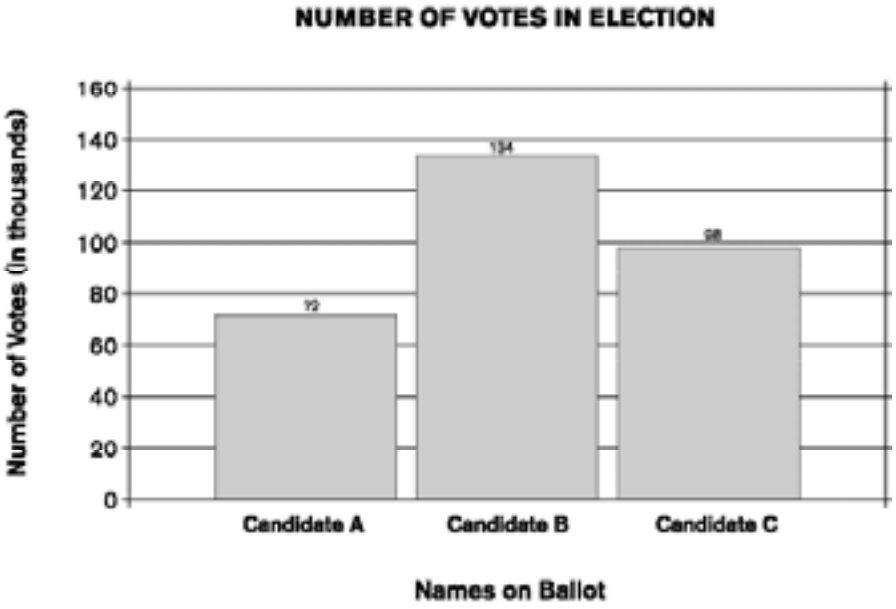
Between which years was there the biggest change in population?

To find the biggest change, observe the largest gap from one bar to another. This occurs between 1998 and 1999, where the population decreased by six million people.

HISTOGRAMS

When data is represented by frequency, and shown pictorially, it is called a *histogram*. A histogram is just a special type of bar graph that shows frequency on the vertical axis. For histograms, there are no spaces between the bars.

Use the histogram shown below with the following two examples:



Examples:
How many people voted in total?

Add up the frequencies for each candidate; 72 + 134 + 98 = 304. The frequency is shown to be in thousands; 304,000 voters voted in the election.

What percentage of the people voted for candidate B, to the nearest percent?

From the previous example, there were 304,000 total voters. Of these, 134,000 voted for candidate B. The percentage is a ratio: $\frac{\text{part}}{\text{whole}} = \frac{\%}{100}$. Substitute in the given numbers; $\frac{134{,}000}{304{,}000} = \frac{p}{100}$. Cross-multiply to get $304{,}000 \times p = 134{,}000 \times 100$, or $304{,}000 \times p = 13{,}400{,}000$. Divide 13,400,000 by 304,000 to get approximately 44%.

PROBABILITY

Probability is the likelihood that an event will occur. This event is called a "favorable outcome," whether in fact it is favorable to the situation or not. We find out the probability of rain in the forecast. If the probability of rain is 70%, then 70 out of 100 times it is expected to rain. The rain is considered a favorable outcome in this instance, even if rain is not desired.

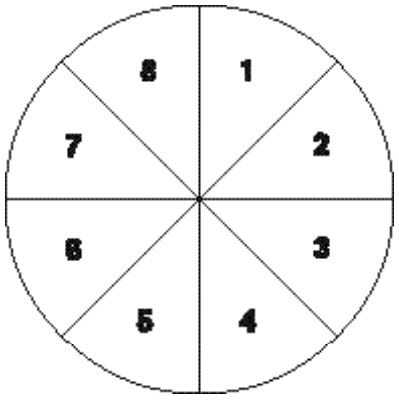
GLOSSARY

PROBABILITY of an event is a ratio, expressed as a fraction, decimal, or percent that defines $\frac{\text{number of favorable outcomes}}{\text{number of total possible outcomes}}$. The notation for the probability of an event is P(event).

You can assume, in probability problems, that all outcomes occur at random, unless otherwise noted. If the events described concern dice, assume that the dice always lands "flat" on a number. If the events concern a spinner, assume that the spinner never lands on a dividing line.

Example:
What is the probability of rolling a five on a die?
There are six possible outcomes, one of which is a five, so $P(5) = \frac{1}{6}$.

Example:
Given the spinner shown below, what is the probability of spinning an even number?



There are eight possible outcomes, four of which are even numbers. The probability of spinning an even number, P(even) = $\frac{4}{8}$ or $\frac{1}{2}$, in lowest terms.

Example:
In a standard deck of playing cards, what is the probability of randomly picking a diamond suit?

There are thirteen diamond cards in a standard deck of 52 cards; P(diamond) = $\frac{13}{52}$ or $\frac{1}{4}$.

RULE BOOK

The probability of an impossible event is zero; P(impossible) = 0.

The probability of an event that is certain is one; P(event that is certain) = 1.

All probabilities are a number between zero and one; $0 \leq$ P(event) ≤ 1.

Because an event, E, will either occur or it will not occur, P(E) + P(not E) = 1

Example:
When a die is rolled, what is the probability of NOT rolling a four?

Since P(4) + P(not 4) = 1, then P(not 4) = 1 – P(4). There are six possible outcomes, one of which results in a roll of a four; P(not 4) = $1 - \frac{1}{6} = \frac{5}{6}$.

Example:
When a die is rolled, what is the probability of rolling a seven?

It is impossible to roll a seven when rolling one die; P(7) = 0.

MUTUALLY EXCLUSIVE EVENTS

GLOSSARY

MUTUALLY EXCLUSIVE EVENTS events that cannot occur simultaneously. For example, when a die is rolled, a five can be rolled, or a six can be rolled, but both a five and a six cannot be rolled simultaneously. So rolling a five and rolling a six are mutually exclusive. However, in a deck of standard playing cards, when a card is chosen, the card can be a spade, or the card can be a queen. A spade and a queen can be chosen, so choosing a spade and choosing a queen are NOT mutually exclusive.

RULE BOOK

For two events, E_1 and E_2, that are mutually exclusive, the probability of E_1 OR the probability of E_2 is equal to the sum of the probability of the events; $P(E_1 \text{ or } E_2) = P(E_1) + P(E_2)$.

Examples:

When rolling a die, what is the probability of rolling a four or a two?

The P(4) or the P(2) is equal to P(4) + P(2). There are six possible outcomes on a die, one of which is a four, and one of which is a two; P(4 or 2) = $\frac{1}{6} + \frac{1}{6} = \frac{2}{6}$ or $\frac{1}{3}$.

When a coin is tossed, what is the probability of tossing a head or a tail?

There are two possible outcomes, one of which is a head, one of which is a tail. P(head or tail) = $\frac{1}{2} + \frac{1}{2} = 1$. Alternately, remember that it is certain that the coin will either land on a head or a tail, and the P(event that is certain) = 1.

There were 125 total raffle tickets sold, and there will be one winner. Sara purchased one ticket, Joan purchased five tickets, and Pete purchased ten tickets. What is the probability that either Sara or Joan or Pete will win?

The events of Sara winning, Joan winning, and Pete winning are all mutually exclusive. Out of the 125 possible outcomes, one favors Sara, five favor Joan, and ten favor Pete; P(Sara or Joan or Pete) = $\frac{1}{125} + \frac{5}{125} + \frac{10}{125} = \frac{16}{125}$.

If two events are NOT mutually exclusive, you must add the probabilities of the events and then subtract out the duplicate events.

Example:

In a standard deck of playing cards, what is the probability of randomly choosing a king or a diamond?

The events of choosing a king and choosing a diamond are not mutually exclusive; the king of diamonds is both a king and a diamond. To find this probability, add the probability of a king to the probability of a diamond and subtract out the probability of choosing the king of diamonds. Out of the total possible outcomes of 52 cards, four are kings, thirteen are diamonds, and one is the king of diamonds. By substitution, P(king) + P(diamond) – P(king of diamonds) becomes $\frac{4}{52} + \frac{13}{52} - \frac{1}{52} = \frac{16}{52}$ or $\frac{4}{13}$.

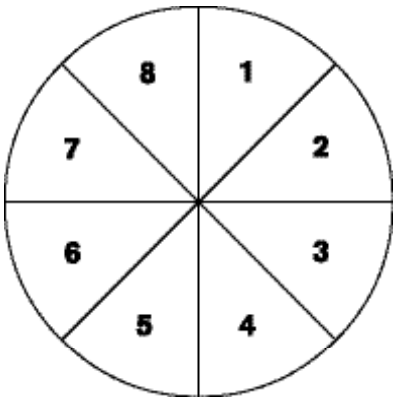
RULE BOOK

The probability of one event, E_1, AND then another event, E_2, is $P(E_1$ and $E_2) = P(E_1) \times P(E_2)$. This is sometimes written as $P(E_1, E_2)$.

Example:
What is the probability, when rolling a die, of rolling a four and then another four?

There are six possible outcomes, one of which is a four;
P(4 and 4) = $\frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$.

Example:
On the spinner shown below, what is the probability of spinning a three and then spinning an even number?



There are eight possible outcomes, one of which is a three, and four of which are even numbers; P(3 and even) = $\frac{1}{8} \times \frac{4}{8} = \frac{4}{64}$ or $\frac{1}{16}$.

Probability problems concerning two dice rolled are common examples on tests. It is helpful to make a table of all possible outcomes as shown below:

	1	2	3	4	5	6
1	1,1	1,2	1,3	1,4	1,5	1,6
2	2,1	2,2	2,3	2,4	2,5	2,6
3	3,1	3,2	3,3	3,4	3,5	3,6
4	4,1	4,2	4,3	4,4	4,5	4,6
5	5,1	5,2	5,3	5,4	5,5	5,6
6	6,1	6,2	6,3	6,4	6,5	6,6

In the table it is obvious that there are 36 possible outcomes, and each possibility for the two dice are shown. If the problem asks for the sum of two dice rolled, make a table of sums:

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Examples:

What is the probability of rolling doubles when rolling two dice?

Refer to the table showing dice results. Out of 36 possible outcomes, there are six possibilities that are doubles; P(doubles) = $\frac{6}{36}$ or $\frac{1}{6}$.

When rolling two dice, what is the probability of rolling a sum of seven?

Refer to the table showing dice sum results. Out of 36 possible outcomes, there are six that have a sum of seven; P(sum of 7) = $\frac{6}{36}$ or $\frac{1}{6}$.

EXTRA HELP

If you feel you need additional help on the topics of probability and statistics, Practical *Math Success in 20 Minutes a Day,* published by LearningExpress, has two chapters devoted to these topics: Lesson 13 and Lesson 14.

There are several useful websites that deal with probability and statistics. Use the Internet to gain further practice if needed:

1. **Visit *www.aaamath.com* and, under *Math Topics* on the right-hand side of the screen, scroll down to click on *Statistics*. There are brief descriptions, followed by interactive practice problems dealing with mean, median, mode, and range.**
2. **The website *www.mathgoodies.com/lessons* is another good resource for practice with both statistics and probability. Once on this page, scroll down to *Introduction to Statistics*, or alternately, *Probability*. From this topic, click on the *X* under Description, then scroll down to *Challenge Exercises*. Apart from the exercises, there are selected lessons you can view.**
3. **A third Internet resource is *www.mathleague.com/help/help.htm.* From this page, click on either *using data and statistics*, or *percent and probability*.**

TIPS AND STRATEGIES

- Know the common statistical measures and how to calculate them:
  - mean
  - median
  - mode
  - range
- Be able to recognize the three common statistical graphs:
  - circle graphs
  - bar graphs
  - histograms
- Carefully read graphs for units and keys.
- Probability is a ratio: $\frac{\text{number of favorable outcomes}}{\text{number of total possible outcomes}}$.
- Mutually exclusive events cannot happen simultaneously.

- The probability of event E_1 OR event E_2 is $P(E_1) + P(E_2)$, when E_1 and E_2 are mutually exclusive.
- For probability, *or* means *add*.
- The probability of event E_1 *and* event E_2 (or E_1 *and* then E_2) is $P(E_1) \times P(E_2)$.
- For probability, *and* means *multiply*.

PRACTICE

Now that you have studied these lessons, try this set of practice problems to gauge your success. Carefully read over the answer explanations.

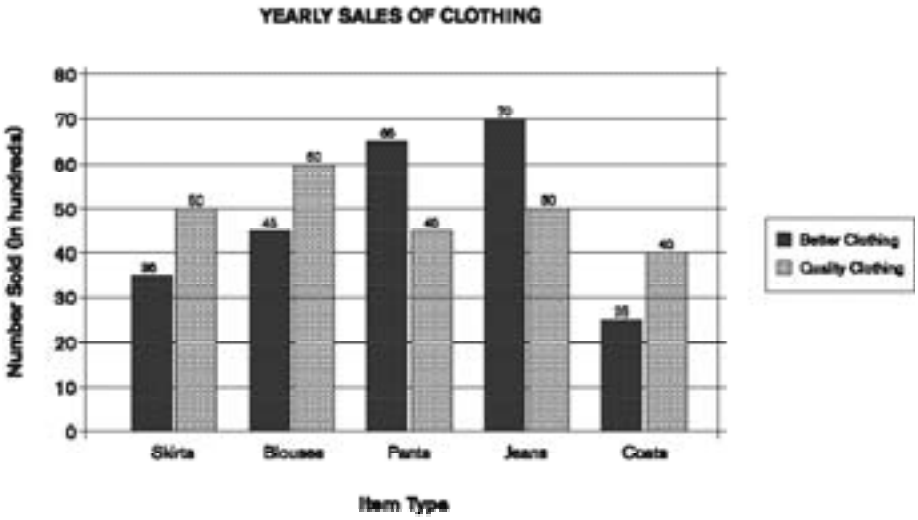
1. Find the mean of the following set of data:
{32, 34, 34, 35, 37, 38, 34, 42}.
a. 34
b. 35.75
c. 10
d. 36
e. 34.5

2. What is the mode of {71, 68, 71, 77, 65, 68, 72}?
a. 71
b. 68
c. none
d. 9
e. 71 and 68

3. The ages at a day camp were as follows:
9, 12, 9, 10, 9, 13, 11, 8, 17, 10. What is the median age?
a. 10
b. 9
c. 10.8
d. 11
e. 10.4

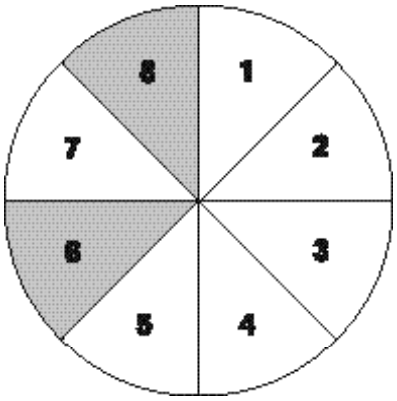
4. What is the range of the temperatures listed:
43°, 47°, 43°, 52°, 42°, 78°, 84°, 80°?
a. 43°
b. 42°
c. 37°
d. 47°
e. 58.625°

5. In the bar chart, what is the total number of jeans sold?



a. 70
b. 120
c. 12,000
d. 5000
e. 1,200

Use this spinner for questions 6 and 7.



6. Using the spinner shown, what is the probability of spinning an odd number or a gray space?

a. $\frac{2}{8}$

b. $\frac{8}{64}$

c. $\frac{1}{2}$

d. $\frac{3}{4}$

e. 1

7. Using the spinner shown, what is the probability of spinning an even number or a gray space?

a. $\frac{1}{2}$

b. $\frac{8}{64}$

c. $\frac{2}{8}$

d. $\frac{3}{4}$

e. 1

8. What is the probability of randomly choosing an ace from a standard deck of playing cards?

a. $\frac{4}{13}$

b. $\frac{4}{26}$

c. $\frac{1}{13}$

d. $\frac{1}{52}$

e. $\frac{4}{13}$

9. What is the probability of rolling a single die, and first rolling a five and then rolling a five again?

a. $\frac{2}{6}$

b. $\frac{2}{12}$

c. $\frac{5}{36}$

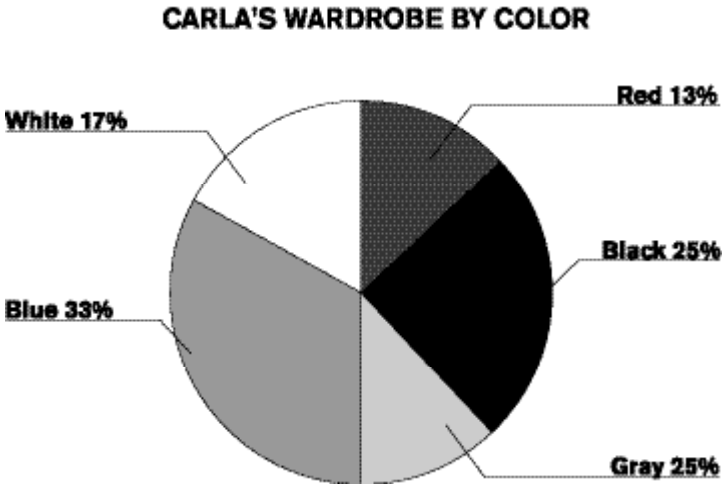
d. $\frac{1}{5}$

e. $\frac{1}{36}$

10. When rolling two dice, what is the probability of getting a sum of seven?

a. $\frac{7}{36}$
b. $\frac{1}{6}$
c. $\frac{1}{7}$
d. $\frac{2}{7}$
e. $\frac{7}{12}$

11. In the circle graph shown below, if Carla has a total of 60 items in her wardrobe, how many of them are black?



a. 25
b. 20
c. 15
d. 22
e. 10

12. When rolling two dice, what is the probability of NOT rolling doubles?

a. $\frac{5}{6}$
b. $\frac{1}{6}$
c. $\frac{6}{36}$
d. $\frac{4}{6}$
e. $\frac{4}{12}$

13. What is the probability of choosing a two or a heart in a standard deck of playing cards?

a. $\frac{17}{52}$

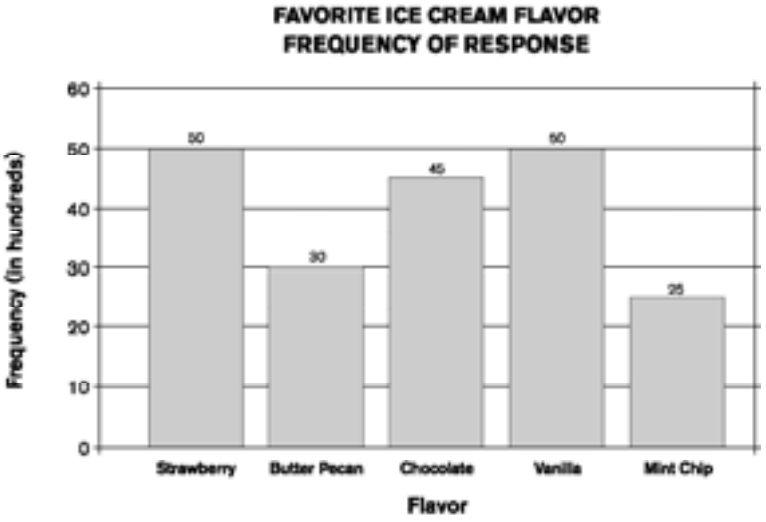
b. $\frac{16}{52}$

c. $\frac{1}{52}$

d. $\frac{1}{13}$

e. $\frac{4}{13}$

14. In the histogram, how many people prefer chocolate or vanilla ice cream?



a. 95

b. 950

c. 200

d. 20,000

e. 9,500

15. In the following set of data, what is the mode?
{15, 12, 13, 11, 14, 10, 8}

a. 0

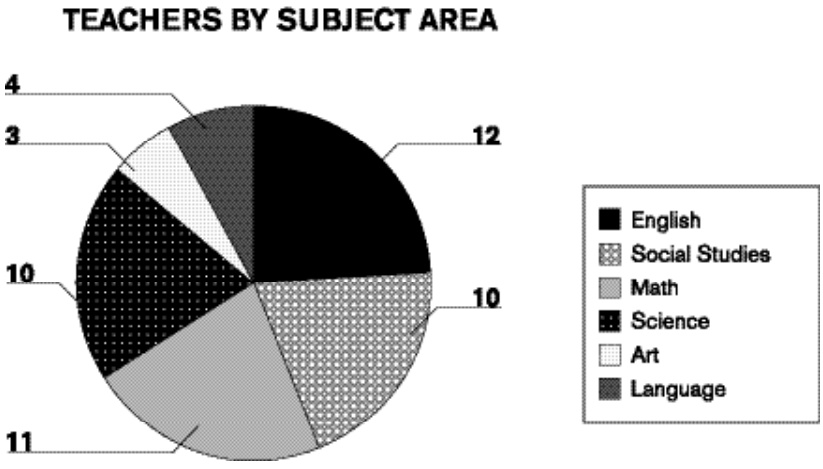
b. none

c. 11

d. 7

e. 12

16. In the circle graph below, if there are 50 total teachers, what percent of them are Social Studies teachers?



a. 10%
b. 5%
c. 20%
d. 2%
e. 25%

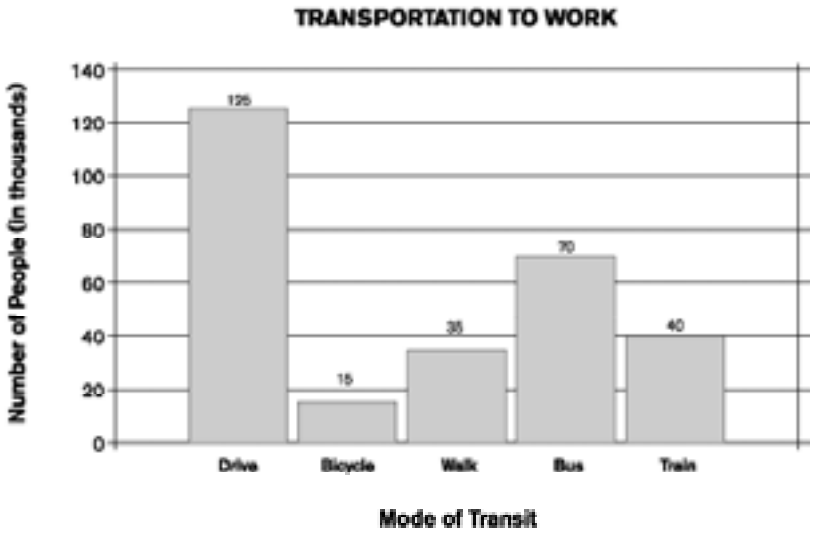
17. There were 225 raffle tickets sold, and there will be one winner. What is the probability of you being the winner if you bought five tickets?
a. $\frac{1}{5}$
b. $\frac{1}{45}$
c. $\frac{1}{225}$
d. $\frac{1}{20}$
e. $\frac{1}{55}$

18. In the following set of data, which statement below is NOT true?
{43, 48, 47, 46, 45, 43, 43}
a. The mean is 45.
b. The mode is 43.
c. The range is 5.
d. The median is 43.
e. The mean is equal to the median.

19. What is the mean of this data set? {56, 58, 60, 62, 64}

a. 8
b. 61
c. 60.5
d. none
e. 60

20. In the histogram shown below, how many people walk to work?



a. 35
b. 35,000
c. 3,500
d. 4,000
e. 350

21. If two dice are rolled, what is the probability of rolling a sum of ten?

a. $\frac{1}{12}$
b. $\frac{1}{10}$
c. $\frac{10}{36}$
d. $\frac{1}{18}$
e. $\frac{2}{10}$

22. There are three red marbles, five green marbles, and seven blue marbles in a bag. When a marble is chosen, it is placed back in the bag. What is the probability of choosing a green marble, and then choosing a red marble?

a. $\frac{8}{15}$

b. $\frac{8}{30}$

c. $\frac{35}{225}$

d. $\frac{1}{15}$

e. $\frac{21}{15}$

23. There are four red marbles, three green marbles, and nine blue marbles in a bag. What is the probability of choosing a red or a blue marble?

a. $\frac{13}{32}$

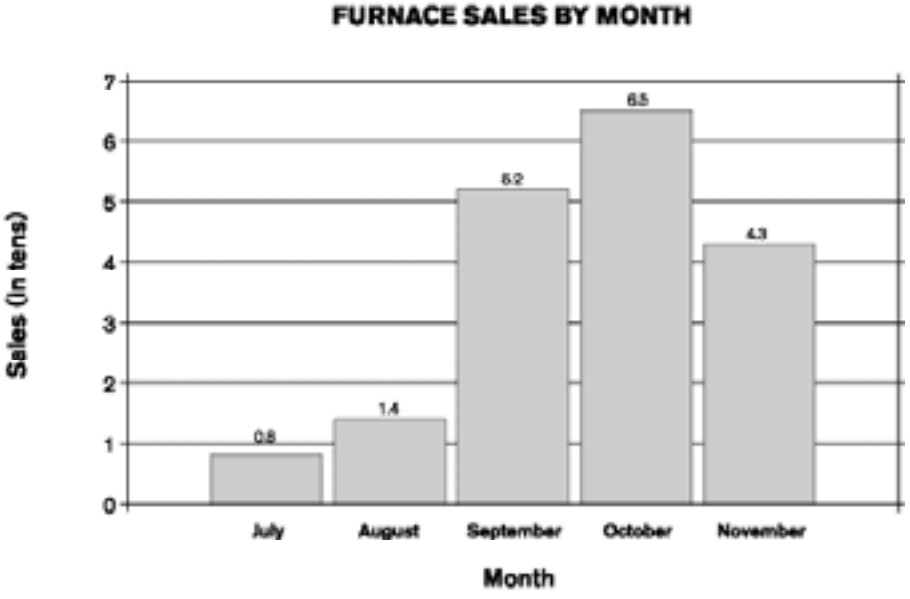
b. $\frac{13}{16}$

c. $\frac{36}{256}$

d. $\frac{12}{16}$

e. $\frac{3}{4}$

24. In the bar chart, between which two consecutive months was there the biggest difference in sales, and what is the difference?



a. between July and October, 57 furnaces
b. between July and October, 5.7 furnaces
c. between August and September, 380 furnaces
d. between October and November, 22 furnaces
e. between August and September, 38 furnaces

25. In the game of blackjack, the highest points are awarded for an ace and a black jack. In a set of shuffled playing cards, what is the probability of turning over an ace and then turning over a black jack?

a. $\frac{8}{2{,}652}$
b. $\frac{8}{2{,}704}$
c. $\frac{6}{52}$
d. $\frac{8}{52}$
e. $\frac{4}{2{,}704}$

ANSWERS

1. b. To find the mean, add up all of the data values, and divide by the number of items, which is eight; 32 + 34 + 34 + 35 + 37 + 38 + 34 + 42 = 286; 286 ÷ 8 = 35.75.

2. e. There are two modes for this data set. Both 71 and 68 appear in the set twice.

3. a. First, arrange the data into increasing order: 8, 9, 9, 9, 10, 10, 11, 12, 13, 17. There are an even number of data values, so the median is the mean of the two middle values. The middle values are 10 + 10 = 20, and 20 divided by 2 is 10.

4. b. The range is the difference between the highest and lowest values in the set of data. The highest temperature is 84° and the lowest temperature is 42°; 84° – 42° = 42°.

5. c. The chart shows that both stores sold jeans. The total number of jeans sold is 70 + 50 = 120 hundred jeans; 120 × 100 = 12,000 total jeans sold.

6. d. Probability is a ratio of $\frac{\text{number of favorable outcomes}}{\text{number of total outcomes}}$.

Odd numbered sectors and gray sectors are mutually exclusive, so P(odd) or P(gray) = P(odd) + P(gray). There are 8 total outcomes,

four that are odd, and two that are gray; P(odd or gray) = $\frac{4}{8} + \frac{2}{8} = \frac{6}{8}$ or $\frac{3}{4}$ in lowest terms.

7. a. Probability is a ratio of $\frac{\text{number of favorable outcomes}}{\text{number of total outcomes}}$. Even-numbered sectors and gray sectors are NOT mutually exclusive, because two of the sectors are both gray and even. You must add the two probabilities, and then subtract the two repeats. There are eight total outcomes, four that are even, and two that are gray; P(even or gray) = $\frac{4}{8} + \frac{2}{8} = \frac{6}{8}$, and $\frac{6}{8} - \frac{2}{8} = \frac{4}{8}$, or $\frac{1}{2}$ in lowest terms.

8. c. There are 52 total outcomes, and there are 4 aces, which is the favorable outcome. Probability is a ratio of $\frac{\text{number of favorable outcomes}}{\text{number of total outcomes}}$, so P(ace) = $\frac{4}{52}$ which is $\frac{1}{13}$ in lowest terms.

9. e. The probability of rolling a five, P(5), and then P(5) again is P(5) × P(5). Probability is a ratio of $\frac{\text{number of favorable outcomes}}{\text{number of total outcomes}}$, and there are six total outcomes, one of which is a five; $\frac{1}{6} \times \frac{1}{6} = \frac{1}{36}$.

10. b. Refer to the table of possible outcomes for dice sums:

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

There are 36 total outcomes, six of which have a sum of seven. The $\frac{\text{number of favorable outcomes}}{\text{number of total outcomes}}$ is $\frac{6}{36}$, or $\frac{1}{6}$ in lowest terms.

11. c. From the circle graph, 25% of her wardrobe is black. Twenty-five percent of 60 is 0.25 × 60 = 15 items.

12. a. Refer to the table of possible outcomes for two dice:

	1	2	3	4	5	6
1	1,1	1,2	1,3	1,4	1,5	1,6
2	2,1	2,2	2,3	2,4	2,5	2,6
3	3,1	3,2	3,3	3,4	3,5	3,6
4	4,1	4,2	4,3	4,4	4,5	4,6
5	5,1	5,2	5,3	5,4	5,5	5,6
6	6,1	6,2	6,3	6,4	6,5	6,6

Of the 36 total outcomes, six are doubles. So the probability of rolling doubles is $\frac{6}{36}$ or $\frac{1}{6}$. The probability of NOT rolling doubles is $1 - \frac{1}{6} = \frac{5}{6}$.

13. b. Probability is a ratio of $\frac{\text{number of favorable outcomes}}{\text{number of total outcomes}}$, and *or* means to add. There are 52 total outcomes, 13 of which are hearts, 4 of which are twos, and one of which is the two of hearts. You must subtract the repeat; P(two or heart) = $\frac{13}{52} + \frac{4}{52} - \frac{1}{52} = \frac{16}{52}$.

14. e. From the histogram, add up the frequencies for chocolate and vanilla; 50 + 45 = 95. Note that the frequency is in hundreds, so 95 hundred, or 9,500 people prefer chocolate or vanilla.

15. b. There is no number value repeated, so the mode is *none*.

16. c. Percent is a ratio of $\frac{\text{part}}{\text{whole}}$. There are 50 teachers, 10 of which are Social Studies; $\frac{10}{50} = 0.20$, and 0.20 written as a percent is 20%.

17. b. The probability of winning is $\frac{\text{number of favorable outcomes}}{\text{number of total outcomes}}$, which is $\frac{5}{225}$ or $\frac{1}{45}$ in lowest terms.

18. d. Calculate the statistical measures. First, arrange the data in increasing order: {43, 43, 43, 45, 46, 47, 48}. The mode is 43, the value that occurs the most. The median is the middle number, that is 45. The mean is 43 + 43 + 43 + 45 + 46 + 47 + 48 = 315, and 315 divided by 7, the number of data values, is 45. Statement **d** is NOT true.

19. e. The mean is 60. Because all the data points increase in order by twos, the mean is the middle value in the set.

20. b. From the graph, the number of people who walk is 35 thousand, or 35,000.

21. a. Refer to the chart that shows sum outcomes for two dice:

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

There are 36 possible outcomes, three of which have a sum of ten. Probability, the $\frac{\text{number of favorable outcomes}}{\text{number of total outcomes}}$, is $\frac{3}{36}$ or $\frac{1}{12}$ in lowest terms.

22. d. The P(green) and then P(red) is P(green) × P(red). There are 15 total marbles, five that are green and three that are red; P(green, red) = $\frac{5}{15} \times \frac{3}{15} = \frac{15}{225}$ or $\frac{1}{15}$.

23. b. The P(red) or P(blue) is P(red) + P(blue), since choosing a red marble and choosing a blue marble are mutually exclusive events. There are 16 total marbles, four that are red and nine that are blue; P(red or blue) = $\frac{4}{16} + \frac{9}{16} = \frac{13}{16}$.

24. e. The problem asks for the biggest difference in consecutive months, not the largest range. The biggest difference is between August and September, and the difference is 5.2 – 1.4 = 3.8. The number sold is expressed in tens, so the difference is 3.8 times 10, or 38 furnaces.

25. a. The P(ace) and then P(black jack) is P(ace) × P(black jack). The probability, which is $\frac{\text{number of favorable outcomes}}{\text{number of total outcomes}}$, of turning over an ace on the first card is $\frac{4}{52}$; there are two black jacks, but now there are only 51 cards left in the deck. The probability of now turning over a black jack is $\frac{2}{51}$; $\frac{4}{52} \times \frac{2}{51} = \frac{8}{2{,}652}$.

10

Word Problems

The final chapter to review is math word problems. Word problems cover any of the topics in previous chapters. Word problems are daunting for many people. Follow the suggestions in this chapter to meet this math challenge and score high on your test. Before you begin, spend a few minutes to take this ten-question *Benchmark Quiz*. These questions are similar to the type of questions that you will find on important tests. When you are finished, check the answer key carefully to assess your results. Your Benchmark Quiz analysis will help you determine how much time you need to spend on word problems and the specific areas in which you need the most careful review and practice.

BENCHMARK QUIZ

1. Train A leaves the station heading due east at 12:00 P.M. at a constant rate of speed of 40 miles per hour. Train B leaves the same station, also heading due east at 1:00 P.M., at a constant rate of 50 miles per hour on a parallel track. When will train B meet train A?
a. 6:00 P.M.
b. 4:00 P.M.
c. They will never meet.
d. 5:00 P.M.
e. 10:00 P.M.

2. If the sides of a square are increased by 12 inches, the new perimeter is 56 inches. What is the length of a side of the original square?
a. 14 inches
b. 8 inches
c. 26 inches
d. 16 inches
e. 2 inches

3. The sum of four consecutive odd integers is 8. What is the value of the largest integer?
a. 8
b. 3
c. 5
d. 9
e. 7

4. The Best Corporation has a phone tree to pass on important information to its employees. The initiator of the tree makes three calls to start the tree on round one. On round two, nine calls are made. On round three, 27 calls are made, and so on in this fashion. How many calls are made on round ten?
a. 59,049
b. 130
c. 51
d. 300
e. 19,685

5. An eight-inch by ten-inch photo is surrounded by a mat that is one inch wide on each side. What is the area of the mat?
a. 120 in^2
b. 40 in^2
c. 80 in^2
d. 44 in^2
e. 36 in^2

6. The temperature rose six degrees, dropped three degrees, and then rose again four degrees, to arrive at a final temperature of 78 degrees. What was the original temperature?
a. 85 degrees
b. 65 degrees
c. 91 degrees
d. 71 degrees
e. 79 degrees

7. The coin bank has $2.30 in total value, consisting of exactly 29 coins, all of which are nickels and dimes. How many nickels are there?
a. 29
b. 17
c. 12
d. 20
e. 18

8. In the parking lot, every sixth car receives a 50% discount coupon on the windshield. Every fourteenth car receives a gift certificate on the windshield. What is the number of the first car to receive BOTH a coupon and a gift certificate?
a. 14
b. 20
c. 40
d. 12
e. 42

9. Brittany has $45.00 saved for a ski trip, which costs $135.00. If she earns $4.50 an hour working, how many hours must she work to add to her savings to have enough money for the trip?
a. 20
b. 2
c. 30
d. 23
e. 40

10. On the farm, there are a total of 20 animals, all of which are either cows or chickens. There are 52 legs total. How many chickens are on the farm, assuming each animal has all of its legs?
a. 26
b. 6
c. 10
d. 15
e. 14

BENCHMARK QUIZ SOLUTIONS

How did you do on the word problems Benchmark Quiz? Check your answers here, and then analyze your results to figure out your plan of attack to master this topic. Keep in mind that there are many ways to solve word

problems, and your approach may have been different than the one used for a specific problem, but you still may arrive at the correct answer.

1. d. Make a table to organize the data, using the formula described in Chapter 6, which is distance equals rate times time, or $D = R \times T$. 12:00 P.M. will be considered time zero. Train B's time is one hour less (one hour later).

TIME	DISTANCE OF TRAIN A	DISTANCE OF TRAIN B
0 (12:00 P.M.)	$D = 40 \times 0 = 0$	
1 (1:00 P.M.)	$D = 40 \times 1 = 40$	$D = 50 \times 0 = 0$
2 (2:00 P.M.)	$D = 40 \times 2 = 80$	$D = 50 \times 1 = 50$
3 (3:00 P.M.)	$D = 40 \times 3 = 120$	$D = 50 \times 2 = 100$
4 (4:00 P.M.)	$D = 40 \times 4 = 160$	$D = 50 \times 3 = 150$
5 (5:00 P.M.)	$D = 40 \times 5 = 200$	$D = 50 \times 4 = 200$

When the distance for train A equals the distance for train B, the trains meet. This occurs at 5:00 P.M.

2. e. Use the strategy of working backward. If you knew the original length, you would:

Step 1: add 12.

Step 2: multiply this length by 4 to get the perimeter of 56.

Work backward, by reversing the steps:

Reverse step 2: divide 56 by 4 to get the new length of 14 inches.

Reverse step 1: subtract 12 to get 14 – 12 = 2 inches, the original length of the square.

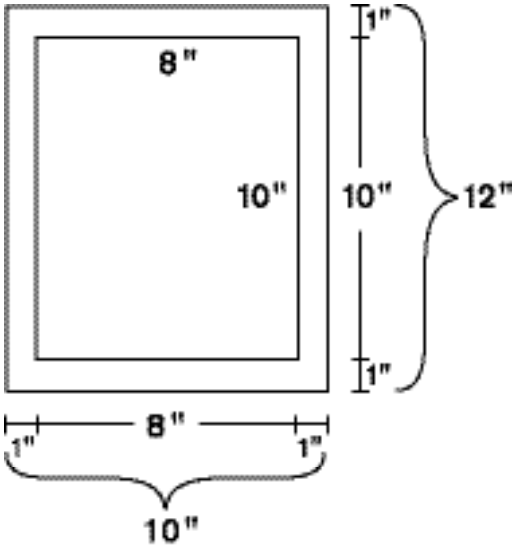
3. c. Use the answer choices and the guess and check strategy. Start with the middle value of the answer choices as your first guess, which is 7. If the largest was 7, then the sum of the four consecutive odd integers would be 1 + 3 + 5 + 7 = 16, which is too big. Choose the next smallest answer choice, which is 5. This sum would be –1 + 1 + 3 + 5 = 8. The largest integer is therefore 5.

4. a. Use the strategy of making a table and then looking for a pattern. The table with the given information is:

ROUND	NUMBER OF CALLS	RULE
1	3	3^1
2	9	3^2
3	27	3^3

The first column increases by one each time, and the second column is multiplied by three each time. Repeated multiplication by three is the powers of three. This is how the third column, the rule, was generated. On round 3, the number of calls is $3^3 = 27$, so on round 10, the number of calls is $3^{10} = 59{,}049$.

5. b. Make a picture of the situation:



To find the area of the mat, you must find the total area of the large rectangle and subtract out the area of the smaller rectangle. This was reviewed in Chapter 8. The area of a rectangle is $A = b \times h$. So $\text{Area}_{\text{large}} - \text{Area}_{\text{small}}$ is $10 \times 12 - 8 \times 10$. Using order of operations, first multiply left to right, and then subtract; $120 - 80 = 40$ square inches.

6. d. Use the strategy of working backward. If you knew the original temperature, you would:

Step 1: add 6.
Step 2: subtract 3.
Step 3: add 4 to get the final temperature of 78 degrees.

Work backward by reversing the steps:

Reverse step 3: subtract 4 to get $78 - 4 = 74$.
Reverse step 2: add 3 to get $74 + 3 = 77$.
Reverse step 1: subtract 6 to get $77 - 6 = 71$ degrees.

7. c. Use guess-and-check, and the first guess can be the middle value in the answer choices, which is 18 nickels. Make a table to organize the data:

NUMBER OF NICKELS	NUMBER OF DIMES	TOTAL MONEY VALUE
N	$29 - N$	$0.05 \times N + 0.10 \times (29 - N)$
18	11	$0.05 \times 18 + 0.10 \times 11 = 2.00$

This money value, $2.00, is too small, so there must be fewer nickels, and more dimes. Try again, with a smaller nickel answer choice, such as 17:

NUMBER OF NICKELS	NUMBER OF DIMES	TOTAL MONEY VALUE
N	$29 - N$	$0.05 \times N + 0.10 \times (29 - N)$
18	11	$0.05 \times 18 + 0.10 \times 11 = 2.00$
17	12	$0.05 \times 17 + 0.10 \times 12 = 2.05$

This is still too small. The answer must be 12 nickels, the only other answer choice that is smaller. To check,

NUMBER OF NICKELS	NUMBER OF DIMES	TOTAL MONEY VALUE
N	$29 - N$	$0.05 \times N + 0.10 \times (29 - N)$
18	11	$0.05 \times 18 + 0.10 \times 11 = 2.00$
17	12	$0.05 \times 17 + 0.10 \times 12 = 2.05$
12	17	$0.05 \times 12 + 0.10 \times 17 = 2.30$

8. e. Make a chart, showing which cars get which flyers. The first number that appears in both columns is the first car to get both.

COUPON	GIFT CERTIFICATE
6	14
12	28
18	42
24	
30	
36	
42	

Car number 42 is the first car to get both coupon and gift certificate. This is the least common multiple of 6 and 14.

9. a. Every hour, Brittany will earn $4.50. Think of a pattern for her earnings: if she works n hours, she will earn $4.50 \times n$ dollars. She needs to earn $135.00 – $45.00 = $90.00 dollars. Set up $90.00 = $4.50 \times n$. Divide 90 by 4.5 to get 20 hours.

10. e. Use guess-and-check, knowing that each cow has four legs and each chicken has two legs. Place the data in an organized table. Start your guess with the middle value in the answer choices: 14 chickens.

NUMBER OF CHICKENS	NUMBER OF COWS	NUMBER OF LEGS
c	$20 - c$	$2 \times c + 4 \times (20 - c)$
14	6	$2 \times 14 + 4 \times 6 = 52$

This is the correct number of legs. The first guess was successful, so no other guesses are needed. There are 14 chickens.

BENCHMARK QUIZ RESULTS

If you answered 8–10 questions correctly, you are a good problem solver. Read through the lesson to discover alternate methods of solution. Chances are, there will be new techniques explained that will be of benefit to you. Try all the examples and practice questions at the end of the chapter; there may be different types of problems than the ten given thus far.

If you answered 4–7 questions correctly, you need a more focused and careful review. If the questions you answered wrong deal with topics covered earlier in this book, you may need to go back and review these topics again. Read through the lesson presented in this chapter and work through all examples. Pay attention to the various tips, rules, and shortcuts.

If you answered 1–3 questions correctly, you need to make a concentrated effort to practice word problems. Perhaps you become intimidated as soon as you encounter words in a math problem. Read this chapter carefully. There are many suggestions and approaches described to make solving word problems simpler. Practice is the key to success. Pay attention to all tips, rules, and shortcuts and visit the suggested websites for further practice. You may want to refer to *Practical Math Success in 20 Minutes a Day*, published by LearningExpress, which has two Lessons, 15 and 16, devoted to word problems.

JUST IN TIME LESSON—WORD PROBLEMS

Word problems are the most challenging type of math problems because they involve both computation proficiency and an understanding of how the language translates into math. Word problems encompass all topics in math. Some of the examples in previous chapters were word problems. There are two main challenges when dealing with word problems:

- difficulty translating words into mathematical operations
- lack of "prescription," or set procedure, to use for solution

This chapter will focus on methods to use when faced with a word problem. Topics in this chapter include:

- translating words to mathematical operations
- the problem-solving process
  - read carefully
  - estimate the answer
  - plan and then solve
  - check
- smart strategies
  - guess and check
  - work backward
  - look for patterns
  - make a table or tree diagram
  - make a picture or drawing
- consecutive integers

TRANSLATING WORDS TO MATHEMATICAL OPERATIONS

Study the following chart that shows English words translated into the given mathematical operations:

ADD	SUBTRACT	MULTIPLY	DIVIDE	EQUALS
sum	difference	of	per	is
more than	fewer than	product	quotient	are
greater than	less than	times		has
older than	younger than			
further than	remained			
increased by	decreased by			
total				
altogether				
exceeded				

Also, there are key words that indicate a formula or type of problem:

Distance Formula $D = R \times T$	Ratios
plane	colon symbol (:)
train	percent symbol (%)
boat	per
travel	for each
verbs (walk, run, climb, swim)	for every

THE PROBLEM-SOLVING PROCESS

You can think of the problem-solving process as having four steps: read carefully, estimate the answer, plan and then solve, and check.

◗ *Read Carefully*

When approaching a word problem, you must be willing to read the problem several times. Read once to get the general idea and topics. Read again, and attempt to separate the problem into chunks, making notes as you read. Cross out any unnecessary information. Be clear on the goal—what is being asked for in the problem. Do not lose focus on the end result.

◗ *Estimate the Answer*

Before proceeding, make an estimate of the answer. This will help to balance your tendency to avoid the "check" at the end of the solution process. If your solution is not close to your estimation, reconsider your solution. In addition, read over the answer choices if it is a multiple-choice test. Often, you can use the answer choices and a strategy such as guess and check to

solve the problem. Sometimes, it is very difficult to make an estimate and you may not be confident of your guess. Problem-solving strategies will be reviewed to help in these situations.

Plan and then Solve

In this phase of problem solving, decide on which mathematical operations and topics are appropriate to the question. Ask yourself if you may have solved a similar problem in the past. Make a plan for solution and then execute the plan. This phase may have to be tried again. Keep in mind that solving word problems often involves the philosophy of "try and try again." The key to mastery is accepting this fact and sticking with your attempts. Planning often involves choosing a strategy, many of which are described in later sections of this chapter.

Check

After arriving at a solution, it is important to go back and double-check the answer. Be aware that double-checking may mean that you have to solve the problem three times: if the check does not agree with the original estimate, a third attempt at solution is necessary to "break the tie." Go back to your original estimate and compare it to your newfound solution. When checking word problems, you must go back to the words in the problem to check. If time permits, the most effective method of checking a problem is to resolve the problem using a different method or strategy.

SMART STRATEGIES

This section will review many strategies for solving problems, with one or more examples for each one. Keep in mind that you may use several strategies to solve a given problem.

Guess-and-Check

Guess-and-check is a common strategy used in elementary school that somehow came into disfavor for older students. Guess-and-check, especially on multiple-choice tests, is a very effective strategy for success in problem solving. Guess-and-check means that you make the best educated guess that you can, check this solution, and then revise with a better guess based on the results of the last guess. You may need to do this process several times until the solution is found. This strategy is effective when you are not sure

how to set up the problem, and other methods of solution would require the use of algebra.

Example:
Two trains are 440 miles apart, traveling toward each other from opposite directions on parallel tracks that are next to each other. Train A is traveling north at 60 miles per hour and Train B is traveling south at 50 miles per hour. How long will it take before they pass each other?

In addition to guess and check, this solution will create a table to organize the guesses. The key word "train" indicates the use of the distance formula, $D = R \times T$. The answer will be the time when the total miles equal 440, since the trains are that many miles apart. If the first guess is six hours, the table will be:

TIME	TRAIN A	TRAIN B	TOTAL MILES
	$D = R \times T$	$D = R \times T$	(Train A + Train B)
6 hours	$60 \times 6 = 360$	$50 \times 6 = 300$	$360 + 300 = 660$

Six hundred sixty miles is too big; therefore, six hours was too long of a guess. Revise the guess to be perhaps three hours:

TIME	TRAIN A	TRAIN B	TOTAL MILES
	$D = R \times T$	$D = R \times T$	(Train A + Train B)
6 hours	$60 \times 6 = 360$	$50 \times 6 = 300$	$360 + 300 = 660$
3 hours	$60 \times 3 = 180$	$50 \times 3 = 150$	$180 + 150 = 330$

Three hundred thirty is too small; yet it is closer to 440 miles than the first guess. The next guess is therefore four hours:

TIME	TRAIN A	TRAIN B	TOTAL MILES
	$D = R \times T$	$D = R \times T$	(Train A + Train B)
6 hours	$60 \times 6 = 360$	$50 \times 6 = 300$	$360 + 300 = 660$
3 hours	$60 \times 3 = 180$	$50 \times 3 = 150$	$180 + 150 = 330$
4 hours	$60 \times 4 = 240$	$50 \times 4 = 200$	$240 + 200 = 440$

Success after three tries! It will be four hours before they pass each other.

SHORTCUT

For a problem in which guess-and-check is a good strategy, use the answer choices as the guesses. Start the first guess as the middle value of the answer choices. Then, adjust the answer tries from there. For a five-answer multiple-choice question, you would have at most three tries to arrive at the answer.

Example:
Katy has a pocketful of 19 coins, consisting of quarters and dimes only. The coins have a value of $2.95. How many quarters and dimes does she have?

a. 15 quarters, 1 dime
b. 13 quarters, 3 dimes
c. 3 quarters, 22 dimes
d. 11 quarters, 4 dimes
e. 7 quarters, 12 dimes

For the guess-and-check, choose the middle value for quarters, that is choice **d**, 11 quarters and 4 dimes. The money value of this answer is $0.25 \times 11 + 0.10 \times 4$.

Using order of operations, $2.75 + 0.40 = 3.15$. This money value is too high; the answer must therefore be either choice **c** or **e**. The money value on the first guess was fairly close to the real value, so the next guess should be choice **e**, 7 quarters and 12 dimes. This money value is $0.25 \times 7 + 0.10 \times 12 = 1.75 + 1.20$. This is the correct value of $2.95.

◗ *Work Backward*

Use this strategy when a problem is presented with a result, and you are asked to find an initial value. You begin with the final result and examine the steps, in reverse, that lead to the result.

Example:
Jeans are on sale at the ABC department store, marked down $7.00. Elisabeth bought three pairs of jeans. A sales tax of $4.80 was added to the bill, bringing the total price to $67.80. How much was the original price of one pair of jeans?

If you knew the original price, you would:

Step 1: subtract $7.00 from the price.
Step 2: multiply this result by 3.
Step 3: and then add $4.80 for the sales tax.

Work backward, as follows:

Reverse step 3 above, by subtracting $4.80 to get $67.80 – $4.80 = $63.00.
Reverse step 2 above, by dividing by 3 to get $63.00 ÷ 3 = 21.00$.
Reverse step 1 above, by adding 7.00 to get $21.00 + $7.00 = $28.00.

Look for Patterns

This strategy is based on the knowledge that a lot of mathematics is based on patterns that follow a set rule. Examining patterns enables you to solve problems that can seem "out of reach" by using other methods.

Example:
With living cells, cell division works as follows. In round one, one cell divides into two cells, then in round two the two cells divide into four cells; in round three the four cells divide into eight cells. How many cells will there be after round 25?

This is not a problem for guess and check, or working backwards. An organized table could be used, but it would need 25 entries, which would take up time. The strategy will be to start the table, and then look for a pattern:

ROUND	NUMBER OF CELLS	RULE FOR PATTERN
1	2	
2	4	
3	8	

The rule for column one is just to add one each time. The rule for column two is to multiply by two each time. Repeated multiplication by two is also the powers of two (from Chapter 7 of this book). So the table would now be:

ROUND	NUMBER OF CELLS	RULE FOR PATTERN
1	2	2^1
2	4	2^2
3	8	2^3
4	16	2^4

By examination, it is not necessary to generate the table through 25 entries; the twenty-fifth round will be 2^{25}, or 33,554,432 cells.

Make a Table or Tree Diagram

These strategies involve taking the information and organizing it in a meaningful way to solve a problem. A table can help with the solution to a problem that involves a set of numbers that are related. This strategy is often used in conjunction with other strategies; we have seen two previous problems that used a table to help with the process.

Example:
Lara decides to save $150.00 per month, and starts her savings in April. In June, Emily decides to start saving $225.00 per month. In what month will their savings be equal?
Make a table, calling April month #1 of savings:

MONTH #	LARA'S SAVINGS	EMILY'S SAVINGS
1 (April)	$150.00	0
2 (May)	$300.00	0
3 (June)	$450.00	$225.00
4 (July)	$600.00	$450.00
5 (August)	$750.00	$675.00
6 (September)	$900.00	$900.00

In September, their savings will be equal. This problem could also have been solved with algebraic equations.

Tables are very helpful for speed and distance problems, as was shown earlier under the guess and check strategy.

Example:
At the candy factory, every twelfth chocolate is checked for color, and every fifteenth chocolate is checked for taste. What is the first chocolate that will be checked for color and taste?

This problem is essentially asking for the least common multiple of 12 and 15, but if this was not obvious, a table will work fine to solve the problem:

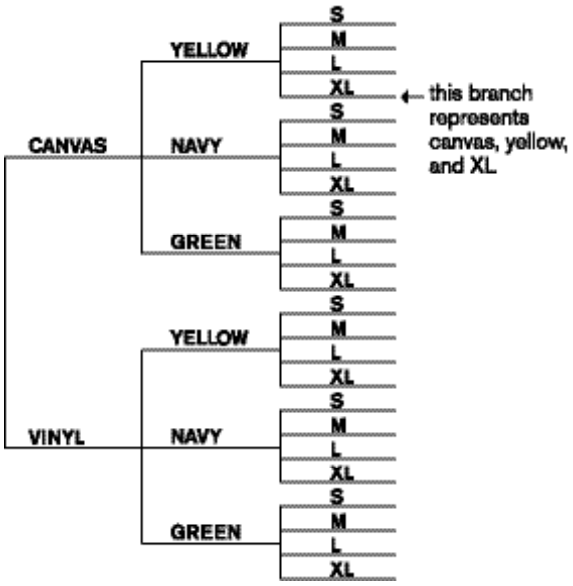
COLOR TEST	TASTE TEST
12	15
24	30
36	45
48	60
60	

Chocolate number 60 will be the first one tested for both color and taste.

Tree diagrams are helpful when a problem asks you to find the number of ways something can be done.

Example:
A specific style of raincoat is sold in either canvas or vinyl, in three colors (yellow, navy, or green) and four sizes (S, M, L, XL). How many different raincoats are available?

Make a tree diagram, showing all the possibilities:



The rightmost branches represent the number of possibilities, in this case 24 different raincoats.

Sometimes, reverse trees are needed.

Example:
Eight teams are playing in a doubles tennis tournament. How many games must be played to arrive at one final winner? Two teams compete, and then only the winning teams move on to play another winning team.

Set up a tree diagram as shown:

4 GAMES	2 GAMES	1 GAME
#1 *vs* #2		
#3 *vs* #4		
#5 *vs* #6		
#7 *vs* #8		

Reading the diagram, there are seven games that need to be played to arrive at one winner.

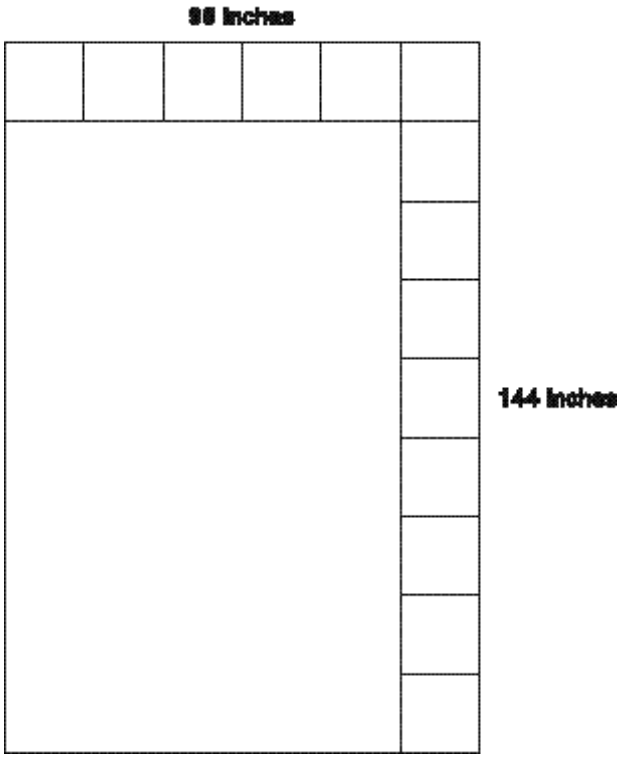
◗ *Make a Picture or Drawing*

Remember the old adage "A picture is worth a thousand words." Pictures help to solve a problem by visualizing the situation. Pictures are especially helpful when solving geometric problems.

Example:
A ceramic tile floor is made from 16-inch squares. The final dimensions are 8 feet by 12 feet. How many of the 16-inch squares are needed to make this floor?

After reading carefully, notice that the units are not consistent. Change the feet units to inch units, by multiplying by 12, before

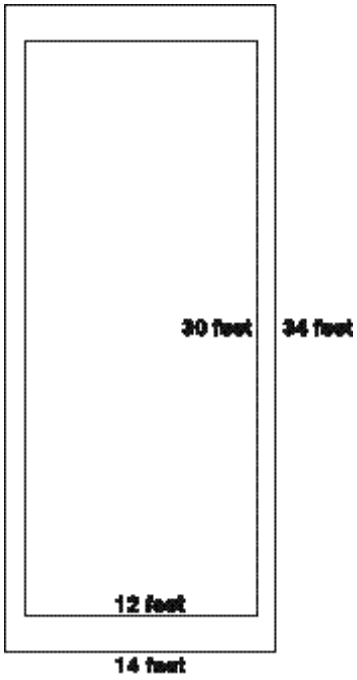
proceeding. Eight feet becomes 96 inches, and 12 feet becomes 144 inches. Now, make a drawing:



By working with the picture, there are 6 squares to make up the width and 9 squares to make up the length. The amount of squares needed is $9 \times 6 = 54$ squares.

Example:
A swimming pool measures 30 feet by 12 feet. There is a two-foot border surrounding the pool. How much fencing is needed to enclose this pool and border?

Draw a sketch of the pool and border. Recognize that fencing, which surrounds the area, is a perimeter problem:



Adding up all of the sides yields 34 + 14 + 34 + 14 = 96 feet of fencing.

CONSECUTIVE INTEGERS

Throughout the course of this book, we have looked at many word problems. Several problems involving distance and speed, percents, simple interest, and ratio and proportions have been reviewed. One other type of word problem not reviewed previously is consecutive integer problems. These problems are relatively easy to solve on multiple-choice tests.

GLOSSARY

CONSECUTIVE INTEGERS integers that follow one after the other in order. They differ by one. Example: 4, 5, 6, 7, 8, 9 . . . .

CONSECUTIVE EVEN INTEGERS even integers that follow one another in order. They differ by two. Example: −4, −2, 0, 2, 4, 6 . . .

CONSECUTIVE ODD INTEGERS odd integers that follow one another in order. They also differ by two. Example: 11, 13, 15, 17, 19 . . .

The guess and check strategy is an effective way to solve consecutive integer problems.

Example:
The four sons in the Johnson family have ages that are consecutive even integers. If the sum of their ages is 84, how old is the youngest?

a. 16
b. 18
c. 22
d. 20
e. 24

Start with the middle age in the answer choice, which is 20. If the youngest is 20, then the sum of their ages would be 20 + 22 + 24 + 26 = 92. This is too big. Next, try choice **b**, which is 18 years. This sum would be 18 + 20 + 22 + 24 = 84. Therefore, the correct answer choice is **b**, 18 years old.

Example:
Carlos's and James's ages are consecutive integers. James is older. Four years ago, Carlos was half the age that James is now. How old is James now?

a. 6
b. 7
c. 8
d. 9
e. 10

Find the middle age among the answer choices. The middle age is choice **c**, 8 years. If James is 8 now, Carlos is 7 now. Four years ago, Carlos was 7 – 4 = 3 years old. This is not one half of 7. Try choice **d**, 9 years. If James is 9 now, Carlos is 8 years old now. Four years ago, Carlos was 8 – 4 = 4 years old. This is not half of 9. Try choice **e**, where James is 10 years old now. Since James is 10, Carlos is 9; four years ago Carlos was 9 – 4 = 5 years old. Five is half of 10, so the correct answer is choice **e**. James is 10 years old.

Two strategies have been discussed for using multiple-choice answer choices to your advantage. One advantage is with guess and check type problems. Another is with consecutive integer problems. Keep in mind that for word problems, it is helpful to examine the answer choices in the planning phase. Through examination you can get hints as to what form of the answer may be required. For example, if all of the answer choices are in decimal form, take a decimal approach to your plan. Or, for geometric problems, if all answer choices are in centimeters, first convert all units to be centimeters.

EXTRA HELP

If extra lessons or practice are in order, refer to *Practical Math Success in 20 Minutes a Day*, Lessons 15 and 16, published by LearningExpress. There are three good websites that review the problem-solving strategies and give several practice problems for each strategy:

1. **Visit the website *www.mathcounts.org/problems/strategies.html* for a description of the problem-solving process and a list of strategies with example problems and solutions.**
2. **The site *http://huisam.virtualave.net* also has many strategies described. Scroll down and click on *Problem-solving strategies*.**
3. **Go to *www.fcps.k12.va.us/DeerParkES/kids/diane/Math/math.htm*, and click on *Problem-Solving Strategies*. Scroll down, and click on the pictures to the left of each strategy for a sample problem with solution.**

TIPS AND STRATEGIES

- Memorize the table "Translating Words to Mathematical Operations" on page 241.
- Follow the four recommended steps in the problem-solving process:
  - read carefully, several times
  - estimate your answer when possible
  - make a plan and execute the plan
  - check your solution
- Be prepared to try and try again.
- Use multiple-choice answers to your advantage.
- Be aware of the many strategies available to help solve problems, including:
  - guess-and-check
  - work backward
  - look for patterns

- make a table or tree diagram
- make a drawing or picture

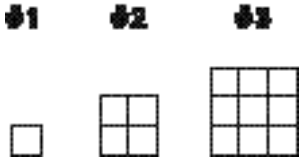
PRACTICE

Now that you have learned some important problem-solving strategies, try this set of practice problems to gauge your success. Carefully read over the answer explanations.

1. Miguel has 43 coins, consisting of quarters and dimes. If the total value of his coins is $8.05, how many dimes does he have?
a. 25
b. 23
c. 21
d. 20
e. 18

2. To reach her destination on time, Sarah must drive an average of 50 miles per hour. She has traveled at the following speeds in miles per hour: 43, 60, 50, 46, 57, 50. What should be her final speed, to the nearest mile per hour, to have the average of 50 miles per hour? Assume the given speeds were measured every hour.
a. 50 miles per hour
b. 44 miles per hour
c. 51 miles per hour
d. 53 miles per hour
e. 58 miles per hour

3. Given the pattern shown, pattern two has three more squares than pattern one. Pattern three has the four squares of pattern two, plus five additional squares. How many squares must be added to pattern six to get pattern seven?



a. 13
b. 7
c. 11
d. 36
e. 49

4. A rectangular swimming pool measures 9 feet by 12 feet. How many feet are there across the diagonal of the pool?
a. 21 feet
b. 15 feet
c. 10.5 feet
d. 13.5 feet
e. 112.5 feet

5. The product of three consecutive even integers is –192. What is the largest of the three integers?
a. –8
b. –81
c. –4
d. –18
e. –10

6. When rolling two dice, what is the probability of rolling a sum of four?
a. $\frac{3}{36}$
b. $\frac{4}{12}$
c. $\frac{4}{36}$
d. $\frac{1}{4}$
e. $\frac{2}{36}$

7. At the ice cream store, there are two choices of size (small and large), three choices of flavors (vanilla, chocolate, strawberry) and a choice of two toppings (nuts or whipped cream) to make a sundae. How many sundaes, consisting of one size, one flavor, and one topping are possible?
a. 6 sundaes
b. 4 sundaes
c. 24 sundaes
d. 12 sundaes
e. 3 sundaes

8. On the skating trail, there are a total of 29 skateboards and scooters. Skateboards have four wheels and scooters have two wheels. There are 100 wheels in total. How many scooters are there?
a. 17
b. 5
c. 8
d. 15
e. 16

9. The computer manufacturer offered a total of $145.00 in rebates. The cost of the computer after rebates and $51.10 in tax came to $781.10. What was the original price of the computer?
a. $832.20
b. $875.00
c. $845.00
d. $926.10
e. $730.00

10. On the first day, Tom's mother gave him two cents. Each day, she promised to give him an amount that is double the amount of the day before. How much money will she give him on the fifteenth day?
a. $0.30
b. $327.68
c. $32.77
d. $30.00
e. $3,276.80

11. There were five people at the business meeting. How many handshakes will occur at introduction, if every person shakes every other person's hand?
a. 25
b. 20
c. 10
d. 24
e. 7

12. The sides of a rectangle are consecutive even integers. What is the longer side, if the area is 168 square centimeters?
a. 14 cm
b. 12 cm
c. 10 cm
d. 21 cm
e. 8 cm

13. Two cars start on the interstate at the same location and the same time, heading in the same direction. Car A is traveling at 65 miles per hour. Car B is driving at 58 miles per hour. How long will it take for the cars to be 21 miles apart?
a. 20 minutes
b. 2 hours
c. $1\frac{1}{2}$ hours
d. 3 hours
e. 4 hours

14. A six-foot square garden is surrounded by a fence. There are fence posts at each corner, and posts every 18 inches along the perimeter. How many total fence posts are there?
a. 20 posts
b. 24 posts
c. 8 posts
d. 12 posts
e. 16 posts

15. Two trains are 780 miles apart and are traveling toward each other on parallel tracks. One is heading east at 75 miles per hour; the other is heading west at 55 miles per hour. How long until the trains meet?
a. 8 hours
b. 2.75 hours
c. 6 hours
d. 3.25 hours
e. 5 hours

16. The perimeter of a square is tripled to get an area of 225 square meters. What is the original length of a side of the square?
a. 5 meters
b. 18.75 meters
c. 15 meters
d. 7.5 meters
e. 37.5 meters

17. The ratio of foxes to rabbits is 2 : 7. If there are 360 total foxes and rabbits, how many rabbits are there?
a. 28
b. 280
c. 80
d. 210
e. 40

18. Eight friends went to a restaurant and decided to split the bill evenly. They included a tip of $10.00, making the total money spent $72.40. How much did each spend before the tip was included?
a. $9.05
b. $8.05
c. $19.05
d. $7.80
e. $6.55

19. Joe has 39 ceiling tiles that he wants to paint. On day one, he paints 15 tiles. Each day thereafter, he paints 8 tiles. How many days will it take to paint the ceiling?
a. 4.875 days
b. 5 days
c. 3 days
d. 6 days
e. 4 days

20. The product of three consecutive odd integers is 3. What is the largest of the integers?
a. 3
b. –1
c. 1
d. 5
e. 0

21. Andy had to wait in two lines to board his airplane, one to buy the ticket and the other to go through security. He waited 15 minutes longer in one line than the other. If the total wait was 79 minutes, how long was the wait in the shorter line?
a. 32 minutes
b. 47 minutes
c. 30 minutes
d. 15 minutes
e. 64 minutes

22. A cereal manufacturer is placing a toy in every sixth cereal box. In every fifth box there is a coupon for "buy one, get one free." What numbered box will be the first to get both surprises?
a. box #15
b. box #18
c. box #60
d. box #30
e. box #12

23. Emmanuel has three suits, two belts, and three ties for dress clothes. How many different outfits can he wear consisting of a suit, belt, and tie?
a. 3
b. 6
c. 18
d. 8
e. 81

24. The post office charges 37 cents for the first ounce and 23 cents for each additional ounce. What is the maximum possible weight for an item that costs $1.06 to mail?
a. 3 ounces
b. 5 ounces
c. 2 ounces
d. 6 ounces
e. 4 ounces

25. A circle is inscribed in a square, which means that the edges of the circle touch the sides of the square, with the circle inside. What is the radius of the circle, if the area of the square is 289 square inches?
a. 17 inches
b. 8.5 inches
c. 144.5 inches
d. 72.25 inches
e. 7 inches

ANSWERS

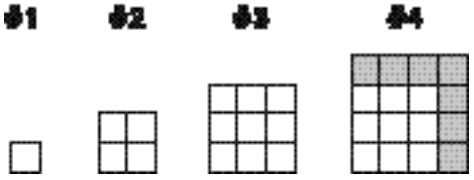
1. e. Use guess and check, starting with the middle value in the answer choices, which is 21 dimes. If there are 21 dimes, then there are 22 quarters. Check the monetary value; $21 \times 0.10 + 22 \times 0.25$. Use order of operations; multiply left to right, and then add to get the value of $7.60. This value is too low; try a choice that uses fewer dimes. The answer is not very close; try the answer choice of 18 dimes, and therefore 25 quarters; $18 \times 0.10 + 25 \times 0.25 = 8.05$, which is correct.

2. b. Use the working backward strategy. There are six intervals of speed, plus one more to make seven. Seven intervals with an average of 50 miles per hour would result in 350 miles. The seventh interval must be the number you would add to the sum of the other six to get 350 miles; $43 + 60 + 50 + 46 + 57 + 50 = 306$; $350 - 306 = 44$.

3. a. Use the strategies of looking for a pattern and making a table. Starting with pattern two, show the table for pattern number and additional squares:

PATTERN NUMBER	ADDITIONAL SQUARES
2	3
3	5

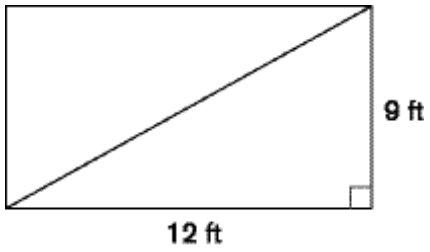
The first column increases by one, and the second column seems to increase by two. Double-check this pattern theory by creating the next pattern in the sequence:



The pattern is correct; make the table up to the sixth and seventh pattern:

PATTERN NUMBER	ADDITIONAL SQUARES
2	3
3	5
4	7
5	9
6	11
7	13

4. b. Draw a picture.



The diagonal divides the pool into two right triangles. Use the Pythagorean theorem (covered in Chapter 8) where the diagonal is the hypotenuse. The theorem, $a^2 + b^2 = c^2$, with substitution is $9^2 + 12^2 = c^2$, or $81 + 144 = c^2$, or $225 = c^2$. The diagonal is therefore the square root of 225, which is 15 feet.

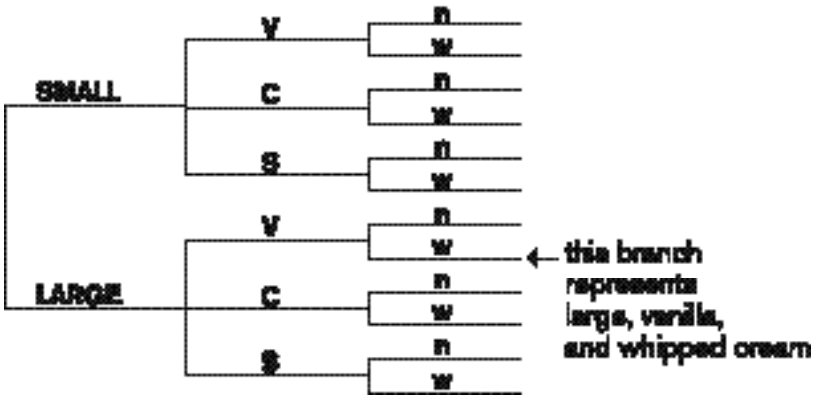
5. c. Use guess and check, remembering that the largest negative integer will have the smallest absolute value. The middle value of the answer choices, –10, can be eliminated because the product does not end in a zero. Try –8 as the first guess. The integers would be –12, –10, –8. This is not possible because –10 is a factor. Try –4; $-8 \times -6 \times -4 = -192$.

6. a. Make an organized table, showing the sums possible for two dice.

	1	2	3	4	5	6
1	2	3	4	5	6	7
2	3	4	5	6	7	8
3	4	5	6	7	8	9
4	5	6	7	8	9	10
5	6	7	8	9	10	11
6	7	8	9	10	11	12

Probability is a ratio, the $\frac{\text{number of favorable outcomes}}{\text{number of total outcomes}}$. From the table, there are 36 total outcomes and three of these have a sum of four. So the probability is $\frac{3}{36}$.

7. d. Make a tree diagram.



The branches on the far right show how many sundaes are possible. There are 12 different sundaes available.

8. c. Use guess and check; start with the middle value of the answer choices, which is 15 scooters. If there are 15 scooters, then there are 14 skateboards, to have a total number of wheels of $(15 \times 2) + (14 \times 4) = 86$ wheels. This number is too low, so there must be a smaller number of scooters. Try the next value, 8 scooters, and therefore 21 skateboards; $(8 \times 2) + (21 \times 4) = 100$ wheels. Success after two attempts.

9. b. Work backward to solve. If the original price was known, you would:

Step 1: subtract \$145.00 for the rebates.
Step 2: add \$51.10 for the tax to get \$781.10.

Working backward:

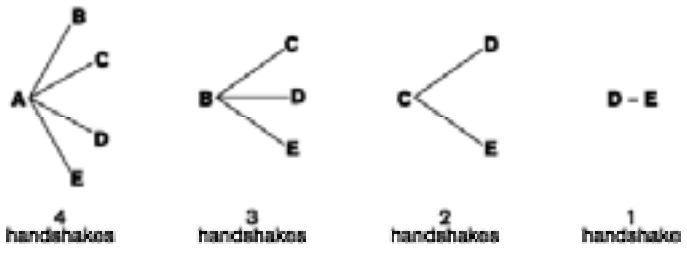
Reverse step 2: subtract \$51.10 from \$781.10 to get \$730.00.
Reverse step 1: add \$145.00 to \$730.00 to get \$875.00.

10. b. Look for a pattern by making a table. Since each day the amount doubles, generate the next entry in column two by doubling the previous entry. Successive doubling is the powers of two, to generate column three, the rule.

DAY #	AMOUNT IN CENTS	RULE
1	2	2^1
2	4	2^2
3	8	2^3

It is not necessary to take the table to 15 days. On day 15, Tom will receive $2^{15} = 32{,}768$ cents. Divide this amount by 100 to get the monetary value in dollars.

11. c. You can make a sketch of the situation, naming the people A, B, C, D, and E.



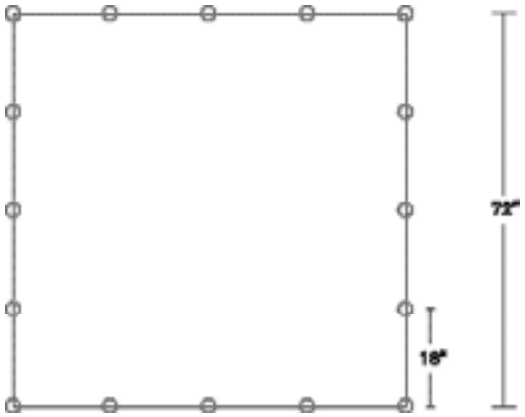
In the second set, B has already shook A's hand, so B only has to shake C, D, and E's hand. The sketch continues in this manner. Use the sketch to count the number of handshakes, 10.

12. a. Use guess-and-check. Use 12 cm as the first guess, which makes the shorter side 10 cm. The formula for the area of a rectangle is $A = b \times h$, or 12 times 10, which is 120 cm^2. This is too small. Try the next largest value, 14, to get $14 \times 12 = 168$ cm^2.

13. d. Make a table:

TIME (HOUR)	CAR A DISTANCE	CAR B DISTANCE	DISTANCE APART (COLUMN 1 – COLUMN 2)
1	65	58	7
2	130	116	14
3	195	174	21

14. e. First, change 6 feet to inches by multiplying by 12. The garden is 72 inches on each side. Seventy-two divided by 18 is 4 lengths, or 3 posts on each side. Make a drawing and count the 16 posts.



15. c. This problem uses the distance formula, where distance equals rate times time. Find the distance for each train and add them. Try guessing and checking; starting with the middle value of the answer choices, 5 hours; $(75 \times 5) + (55 \times 5) = 650$ miles. This is smaller than the wanted distance; try the next highest time, 6 hours; $(75 \times 6) + (55 \times 6) = 780$ miles.

16. a. Work backward. If you knew the original side of the square, you would:

Step 1: multiply the side by 3.

Step 2: square the side to get the area of 225 m^2.

Working backward:

Reverse step 2: take the square root of 225 to get 15 meters.

Reverse step 1: divide by 3 to get the original length of 5 meters.

17. b. Use guess and check. You can eliminate any answer choices that are not multiples of 7; which is the ratio for rabbits. This leaves the possibilities as 28, 210, or 280 rabbits. Try the middle value, 210 rabbits. Divide 210 by 7 to get 30; 30 times 2 is 60, to get the number of foxes; 210 + 60 = 270 total. This is too small. There must be 280 rabbits.

18. d. Work backward. If you knew how much each spent, you would:
Step 1: multiply this by 8.
Step 2: add $10.00 for the tip to get $72.40.
Work backward:
Reverse step 2: subtract $10.00 from $72.40 to get $62.40.
Reverse step 1: divide by 8 to get $7.80 each.

19. e. On day one, Joe paints 15 tiles; thereafter it is 8 tiles per day. Subtract 15 from 39 to get 24 tiles left. At 8 tiles per day, it will be 3 additional days to finish the job.

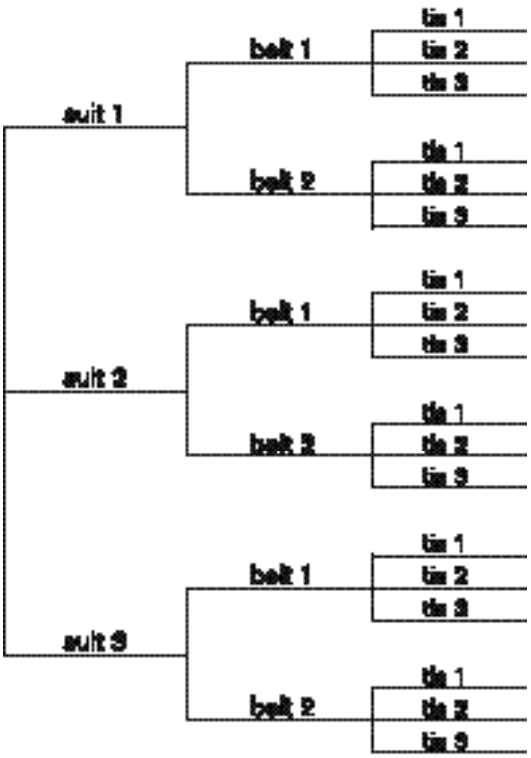
20. c. Use guess and check. The answer value of 5, choice **d**, is not possible. The value of 0, choice **e**, is not possible either. Take the middle value of the remaining choices, which is 1. If one is the largest of the three, then the other two are –3 and –1. Check: $-3 \times -1 \times 1 = 3$. Success!

21. a. Use guess and check. Try the middle value of 47 minutes. That would make the other time be 47 + 15 = 62 minutes. Total waiting time in this case is 62 + 47 = 109 minutes, too large. Try the next smallest time, 32 minutes. The long line time would be 32 + 15 = 47. Total minutes is 47 + 32 = 79 minutes.

22. d. Make a table, and the first number in common is 30:

TOY	COUPON
6	5
12	10
18	15
24	20
30	25
	30

23. c. Make a tree diagram.

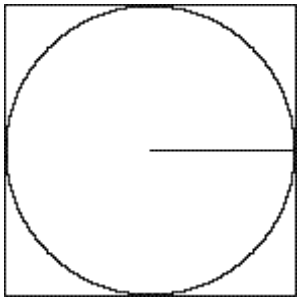


The number of outfits is represented on the far right branches, of which there are 18.

24. e. Make a table:

WEIGHT	PRICE
1 oz	$0.37
2 oz	$0.60
3 oz	$0.83
4 oz	$1.06

25. b. Draw a picture.



If the area of the square is 289 square inches, the side of the square is $\sqrt{289}$ = 17 inches. From the picture it is seen that the diameter of the circle is also 17 inches. The radius is one half of the diameter.

ADDITIONAL RESOURCES

This book has provided you with focused practice and an essential review of your math skills. If you need more practice, these resources offer good places to find what you need to pass your test.

BOOKS

Bobrow, Jerry. *Cliffs Notes Quick Review: Basic Math and Pre-Algebra* (New York: Wiley, 2001).

LearningExpress. *501 Math Word Problems* (New York: LearningExpress, 2003).

Rabinovitz, Judith. *Practical Math Success in 20 Minutes a Day, 2nd Edition* (New York: LearningExpress, 1998).

Slavin, Steve. *All the Math You'll Ever Need: A Self-Teaching Guide* (New York: Wiley, 1999).

Sobanski, Jessika. *Visual Math: See How Math Makes Sense* (New York: LearningExpress, 2002).

ONLINE RESOURCES

http://library.thinkquest.org/20991/home.html—"Math for Morons Like Us" offers a user-friendly series of tutorials from pre-algebra up through calculus.

www.math.com—This comprehensive math site offers helpful tutorials, practice exercises, and a "worksheet generator" for extra practice.

www.mathcounts.org/problems/strategies.html—This site provides students with a wide variety of problem-solving strategies for different skill levels and learning styles.

www.mathgoodies.com/lessons—"Mrs. Glosser's Math Goodies" includes practice problems that are instantly scored, as well as printable puzzles and worksheets for additional practice.