

Reasoning: Odd one out



Learning objective

To use mathematical reasoning to identify the odd one out in a list of mathematical statements.

Links to year 6 problem solving and reasoning pupil target sheet

Reasoning
I can recognise the odd one out.
I can explain my reasoning using clear sentence structures, calculations and diagrams.
I can convince others by proving and justifying my answers, using further examples to back up my reasoning.

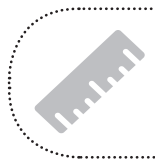
Teaching notes

- The purpose of odd one out questions is to provide pupils with the opportunity to justify their thinking using mathematical reasons. Often there is not an obvious 'right answer'. Try to devise questions that provide an element of ambiguity.
- It is vital to emphasise the use of correct mathematical vocabulary and these types of questions lend themselves to this.
- Give opportunities for the pupils to try to convince each other. It might be worth taking regular 'votes' to see who is 'winning the argument'.
- Questions and examples in this section do not have one 'right answer'. However, the pupils do need practice with questions that have a more obvious odd one out – for example, 24, 40, 64, 70, 88. (Here 70 is the odd one out because it is not a multiple of 8 although 64 could be the odd one out because it is the only square number, 24 could be the odd one out because it is the only number with a multiple of 6, etc.)

Example: Which is the odd one out in these multiplication and division number sentences?

$60 \times 80 = 4800$	$60 \times 8 = 80 \times 6$	$8 \times 0.6 = 4.8$
$4800 \div 60 = 80$	$48 \div 60 = 0.8$	$8 = 480 \div 60$

- Start by asking the pupils if all the number sentences are correct. Perhaps the odd one out is one that is wrong? (They are all correct but the discussion will identify misconceptions and provide opportunities for discussion about place value.) Having established that they are all correct, the pupils then have to think of other reasons why a certain calculation may be the odd one out.
- The pupils will come up with a range of reasons which they have to be prepared to justify. There may be a range of answers, all perfectly justified:
 - $8 = 480 \div 6$ (the only one where the answer is on the left)
 - $8 \times 0.6 = 4.8$ (the only one where you are multiplying or dividing by a decimal)
 - $60 \times 8 = 80 \times 6$ (the only one which does not calculate the answer)
- Emphasise that there is sometimes no correct answer, only a mathematically justifiable reason.



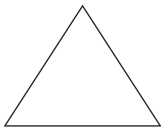
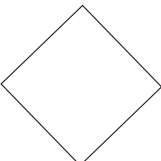
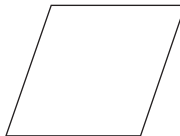
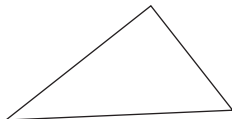
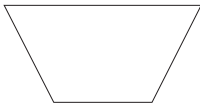
Reasoning: Odd one out

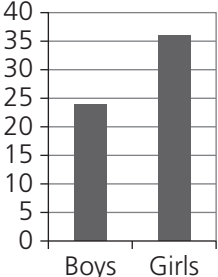
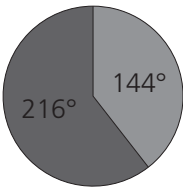
For pupils – Steps to success

1. Identify the odd one out from a set and explain why you have chosen it, with some mathematical justification.
2. Convince others of the reasons for your choice, perhaps changing their minds by your argument.
3. Justify your reasoning using correct mathematical language, both orally and using good written English.


Bank of 'Odd one out' questions

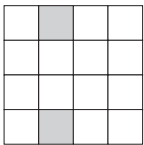
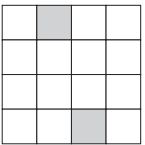
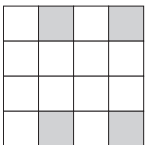
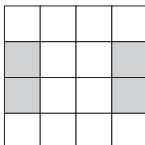
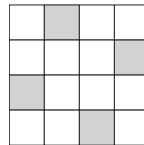
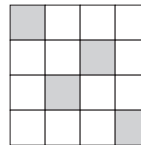
- 1 a) product b) multiple c) square number d) factor e) prime

- 2 a)  b)  c)  d)  e) 

- 3 a)  b)  c)

Gender	Number
Boys	24
Girls	36

 d) 

- 4 a)  b)  c)  d)  e)  f) 

- 5 a)  b)  c)  d)  e) 

- 6 a) $\frac{2}{3}$ b) $\frac{3}{5}$ c) $\frac{9}{16}$ d) $\frac{12}{15}$ e) $\frac{8}{12}$ f) $\frac{1}{4}$

See the pupil activities with the  icon on pages 34 to 68 for topic-based practice of this strategy.

Reasoning: Always, sometimes, never true



Learning objective

To decide if a statement is always, sometimes or never true by considering it from a number of standpoints.

Links to year 6 problem solving and reasoning pupil target sheet

Reasoning
I can identify if a statement is always, sometimes or never true.
I can explain my reasoning using clear sentence structures, calculations and diagrams.
I can convince others by proving and justifying my answers, using further examples to back up my reasoning.
I am beginning to use simple algebraic expressions to help explain my reasoning.

Teaching notes

- Try to choose a statement that addresses a misconception, a common error or something that is intuitively always true or always false – for example, doubling a whole number is always less than squaring it ($1 + 1 > 1^2$ and $2 + 2 = 2^2$ shows this statement is sometimes true).
- Guide the pupils a little more than in some of the other reasoning sections, possibly through the use of ‘What if...?’ type questions. If, for example, the pupils believe that multiplication always makes numbers bigger, ask them to consider $6 \times \frac{1}{2}$.
- Point out that you only have to find one thing that does not fit the statement for it to not be always or never true – for example, prime numbers are odd numbers is sometimes true because 2 is a prime number and it is even.
- As with all these reasoning skills, expect clear explanations using correct mathematical language.

Example: *The sum of two fractions is greater than their product.*

- First ask if the sum of two positive whole numbers is greater than their product. Give the pupils opportunities to try some.
- Intuitively it might seem that this is never true – for example, $3 + 4 < 3 \times 4$. Hopefully, someone will identify a calculation where it is greater – for example, $1 + 2 > 1 \times 2$. In fact, if one of the numbers is 1, then it is always true ($1 + a > 1 \times a$ when $a > 0$). Therefore, elicit from the pupils that the statement is sometimes true.
- Now pose the question about fractions. Let the pupils try some – some pupils could work on fractions with the same denominator, some with numbers where one denominator is a multiple of the other and some where they need to find a common denominator. Display the results in a table like the one below. From this, the statement would appear to be always true.

Fraction A	Fraction B	Sum	Product	Sum is greater than/less than/equal to the product
$\frac{1}{5}$	$\frac{3}{5}$	$\frac{4}{5}$	$\frac{3}{25}$	greater
$\frac{3}{7}$	$\frac{2}{7}$	$\frac{5}{7}$	$\frac{6}{49}$	greater
$\frac{2}{3}$	$\frac{4}{9}$	$\frac{10}{9}$	$\frac{8}{27}$	greater
$\frac{1}{5}$	$\frac{3}{10}$	$\frac{5}{10}$	$\frac{3}{50}$	greater
$\frac{2}{3}$	$\frac{3}{5}$	$\frac{19}{15}$	$\frac{6}{15}$	greater



Reasoning: Always, sometimes, never true

- Ask the pupils if $\frac{7}{3}$ and $\frac{5}{2}$ are fractions? Establish that they are. So what about $\frac{4}{2}$ and $\frac{12}{3}$? Some pupils might link the question to the first bullet about whole numbers.
 - Ask them to try with $\frac{7}{3}$ and $\frac{5}{2}$ ($\frac{14}{6} + \frac{15}{6} < \frac{35}{6}$ so the sum is less).
 - What happens if they choose a fraction smaller than 1 and a fraction bigger than 1?
 - Could the pupils conjecture a hypothesis that is always true?
 - Establish that the statement is sometimes true because, when each of the fractions is greater than 1, the sum is less than the product.

For pupils – Steps to success

1. Understand that if one fact does not fit a statement, it means the statement cannot be 'always true'.
2. Test a conjecture with a range of different examples and form a reasoned view about whether it is always, sometimes or never true.
3. Begin to prove why something is always or never true.

Bank of 'Always, sometimes, never true?' questions

- 1 If the denominator of a fraction is 6 more than its numerator, the fraction is smaller than $\frac{1}{2}$.
- 2 A pyramid has five faces.
- 3 A 4-digit number between 1000 and 9999 multiplied by a 2-digit number between 10 and 99 has six digits in the answer.
- 4 A number between 1000 and 9999 divided by a 2-digit whole number has three digits in the answer.
- 5 If $3x + y = 20$ and x and y are positive whole numbers, then x and y are both odd numbers.
- 6 Dividing a fraction between 0 and 1 by a whole number gives a smaller answer.
- 7 Multiplying a prime number by 8 gives an even number.
- 8 The difference between a 3-digit number and its reverse is a multiple of 9.
- 9 The diagonal of a quadrilateral divides it into two triangles with the same area.
- 10 Subtracting 10% from an amount and then subtracting 10% of what is left is the same as finding 20%.
- 11 Rounding a number to the nearest 10 000 and rounding to the nearest 1000 will give different answers.
- 12 If you double the area of a rectangle, the perimeter doubles in length.
- 13 A pentagon has more right angles than a rectangle.
- 14 If x and y are whole numbers, $2x + 4y$ is an even number.
- 15 The perimeter of the larger of two similar shapes is the perimeter of the smaller shape multiplied by a scale factor.
- 16 If two numbers are 200 when rounded to the nearest 100, the largest possible difference between the numbers is 99.

See the pupil activities with the  icon on pages 34 to 68 for topic-based practice of this strategy.

Division

Remember the division fact associated with each multiplication fact in your times tables. Always estimate your answer.

Example: What is the closest multiple of 37 to 5000?

We need to do a long division.

We know there are about three 37s in 100, so there would be 30 in 1000 and 150 in 5000. This means that the answer is about 150, or probably a bit less because $3 \times 37 = 111$

37×3	→	1 1 1	A reasonable answer as we estimated 150
37×5	→	1 8 5	The remainder is 5 so 5000 is not an exact multiple of 37 However, $135 \times 37 = 4995$ and this is only 5 away from 5000 so it is the closest multiple of 37 to 5000

3 7	5 0 0 0	1 3 5
	3 7	↓
	1 3 0	↓
	1 1 1	↓
	1 9 0	↓
	1 8 5	↓
	5	

Getting started



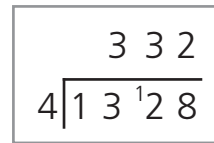
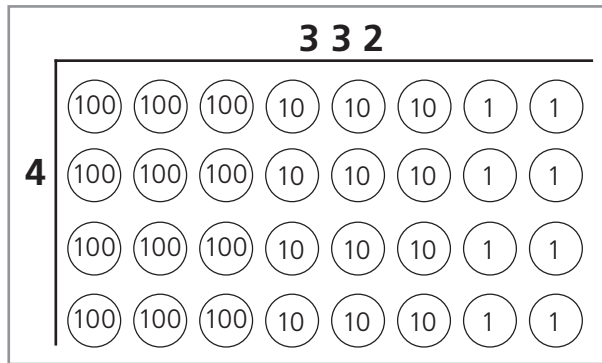
- 1 Tom has £10 to buy some first class stamps. A first class stamp costs 63p. How many stamps can he buy? How much more money would he need to buy 25 stamps?



- 2 Are these statements always, sometimes or never true?
- If a number ends in 5, it is divisible by 15
 - If the remainder is 6, then the remainder given as a fraction is bigger than $\frac{1}{2}$
 - If the remainder is 4, then the decimal is 0.4
 - If you divide one square number by another square number you get a square number.
 - If a number is even and the digits add up to a multiple of 9, then the number is divisible by 18



3 What is the same and what is different about these two ways of showing $1328 \div 4$?



Challenge 1



4 Look at this sequence of calculations.

$$2700 \div 4 = 675 \qquad 2500 \div 4 = 625 \qquad 2300 \div 4 = 575$$

What is the 20th term?

Explain the pattern and your answer.



5 Emma knows that $5695 \div 67 = 85$

She subtracts 67 from 5695

She says: 'That is the answer to 84×67 '

Is she correct? Explain how you know.

Challenge 2



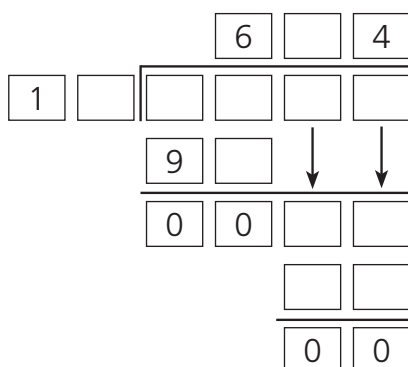
6 Sahid thinks that 647 is a prime number.

Is he correct? Explain how you know.

Explain why you do not have to check for any factor bigger than 26



7 What are the missing digits in this long division? There are two possible answers.



How did you do?   

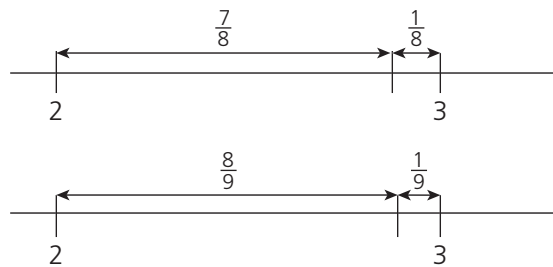
Fractions I

Remember that a fraction expresses a ratio or a proportion. A fraction is also a number in its own right – every fraction has a position on a number line. A fraction can be a calculation, for example $\frac{3}{4} = 3 \div 4$

Example: Which is the larger fraction: $2\frac{8}{9}$ or $2\frac{7}{8}$? Explain how you know.

$2\frac{8}{9}$ is $\frac{1}{9}$ less than 3 and $2\frac{7}{8}$ is $\frac{1}{8}$ less than 3

$\frac{1}{8}$ is larger than $\frac{1}{9}$ because the denominator is smaller and the numerators are the same.



Therefore, $2\frac{7}{8} < 2\frac{8}{9}$

Note: You could also write $\frac{7}{8}$ and $\frac{8}{9}$ as equivalent fractions to compare the numerators, e.g. $\frac{7}{8} = \frac{63}{72}$ and $\frac{8}{9} = \frac{64}{72}$ so $2\frac{8}{9}$ is bigger.

Getting started

1 Which calculation is the odd one out in each row?



a) $18 - 4\frac{3}{5}$ $6 \times 2\frac{1}{5}$ $8\frac{2}{5} + 4\frac{4}{5}$



b) $9 - \frac{1}{3} \times \frac{1}{2}$ $\frac{1}{2} \times 17 + \frac{1}{3}$ $2 \times 4\frac{1}{3} + \frac{1}{6}$



c) $\frac{1}{5} \div 3$ $\frac{2}{15} \times \frac{1}{2}$ $\frac{2}{20} - \frac{1}{5}$



2 Are the following statements always, sometimes or never true? Explain why.

- a) $\frac{1}{2}$ of a pizza is more than $\frac{3}{4}$ of a pizza.
- b) The product of two fractions is greater than $\frac{1}{2}$
- c) $4\frac{a}{b}$ is bigger than $\frac{b}{a}$
- d) If the numerator of a fraction is larger than the denominator, then the fraction is bigger than 1 but smaller than 2
- e) If the highest common factor of the numerator and the denominator is equal to 1, then the fraction is in its simplest form.

Challenge 1



3 Find all the possible fractions between 0 and 1 where the numerator and the denominator are prime numbers less than 10 and arrange them in order of size.



4 The men's long jump world record is 29 feet $4\frac{2}{5}$ inches.

Zoe says: 'That's over 9 metres.'

Is she correct? Explain how you know.

Challenge 2



5 a) Explain the rule which gets you the next calculation in the sequence and find the next three terms.

$$\frac{1}{2} \times \frac{3}{2}, \quad \frac{1}{3} \times \frac{4}{3}, \quad \frac{1}{4} \times \frac{5}{4}, \quad \frac{1}{5} \times \frac{6}{5}$$

b) What will be the first calculation smaller than $\frac{1}{10}$? Explain why.



6 Are the following statements always, sometimes or never true?

Explain why. Give reasons to explain your answers.

- a) Between any two fractions there is another fraction.
- b) A unit fraction can be expressed as the sum of two **different** unit fractions,
e.g. $\frac{1}{4} = \frac{1}{5} + \frac{1}{20}$
- c) $2\frac{1}{2}a > 1$

How did you do?

