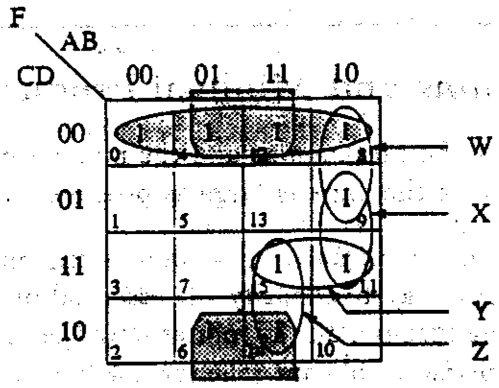
Finding Minimal Sums and Minimal Products

1. Loops on K-maps must be rectangular and the number of minterms included must be a power of 2. Form loops that are as large as possible.
2. Find all *essential prime implicants.* If a loop is the only loop covering a minterm, then it is essential. One way to identify the essential prime implicants is to begin by systematically checking through all the minterms in the map. When you find a minterm covered by only one prime implicant (the largest loop you can form that covers that minterm), that loop represents an essential prime implicant. Write down the product term corresponding to the essential prime implicant and check off the minterms it covers. Repeat the procedure for the remaining minterms.
3. For the minterms not covered by essential prime implicants, the appropriate choice is often obvious. If not, use either the branching method or Petrick's method (see following pages).
4. A loop in a K-map is read by identifying those variables that are always true or always false for all terms in the loop.
5. SOP expressions can be entered directly into K-Maps using a process that is the reverse of reading a looped K-map.
6. For "Don't Cares," include them when forming prime implicants, but do not worry about covering them.
7. Looping 1's in a K-map results in a minimal SOP expression for F.
8. Looping 0's in a K-map results in a minimal SOP expression for F'. Complement the F' expression to find a minimal POS expression for F.

Branching Method

First, find the essential prime implicants and write these terms down. Check off all the minterms covered by these essential prime implicants. For the remaining minterms, find all the prime implicants that cover them. Pick a a minterm that has not yet been checked off and arbitrarily choose one of the prime implicant loops that covers it. Write down the corresponding product term and mentally check off the minterms it covers. Now proceed with uncovered minterms until all minterms have been covered. Then, return to the original minterm and choose one of the other prime implicant loops and proceed as before. Repeat this procedure until all prime implicant loops covering the original minterm have been used. The process of covering uncovered terms may require several levels of nested branches. Examine the resulting expressions to find the minimal ones.
Consider the following example:



The essential loops and the minterms they cover have already been identified. Only minterms 9, 11, and 15 remain to be covered. Start with minterm 9 and notice that loop W or loop X will cover it. Pick loop W ($AB'C'$). Including the essential terms, we have

$$F = C'D' + BD' + \left\{ AB'C' + \right.$$

The parenthesis indicates that we are branching at this point. After we choose loop W, only minterms 11 and 15 need to be covered. The simplest way to do this is with loop Y. After adding the product term corresponding to loop Y, we have

$$F = C'D' + BD' + \left\{ AB'C' + ACD \right.$$

Now we return to minterm 9. and choose loop X.

$$F = C'D' + BD' + \begin{cases} AB'C' + ACD \\ AB'D + \end{cases}$$

Now only minterm 15 remains to be covered, and there are two ways to do this. Branching at minterm 15 gives

$$F = C'D' + BD' + \begin{cases} AB'C' + ACD \\ AB'D + \begin{cases} ACD \\ ABC \end{cases} \end{cases}$$

Examining the result, we see that there are three minimal SOP expressions for F.

$$F = C'D' + BD' + AB'C' + ACD$$
$$F = C'D' + BD' + AB'D + ACD$$
$$F = C'D' + BD' + AB'D + ABC$$

Petrick's Method

Petrick's method makes use of Boolean algebra. Consider again the example we used above. After the essential prime implicants have been identified, only minterms 9, 11, and 15 remain to be covered. Minterm 9 can be covered by loop W or loop X, minterm 11 can be covered by loop X or loop Y, and minterm 15 can be covered by loop Y or loop Z. All three of the minterms must be covered, so the resulting expression must use at least one of W or X and at least one of X or Y and at least one of Y or Z. We can write this requirement as a Boolean equation. That is,

$$1 = (W + X)(X + Y)(Y + Z),$$

where the Boolean variables W, X, Y, and Z are true if the corresponding loops are used. Expanding the equation results in

$$1 = WY + XY + XZ.$$

Thus, using loops W and Y, or loops X and Y, or loops X and Z will cover minterms 9, 11, and 15. The result is the same as that found using the branching method.

Minimizing Functions Using VEMs

A VEM (Variable Entered Map) is a K-map that has some variables from the function appearing in the cells of the map. Consider the following function:

$$F = A'B'C' + BC'D + A'BCD + AB'C'E' + d(A'B'C)$$

We can plot this function on a three-variable map that has A, B, and C as inputs and uses D and E as map-entered variables (MEVs). To enter the product terms for the above function proceed as follows. The term $A'B'C'$ does not depend on D or E, so place a 1 in cell 0. The term $BC'D$ depends on D, so place a D in cells 2 and 6. The term $A'BCD$ again depends on D, so place a D in cell 3. Similar reasoning results in placing E' in cell 4 and a don't care in cell 1.

F

C \ AB	00	01	11	10
0	1 (0)	D (2)	D (6)	E' (4)
1	Θ (1)	D (3)	0 (7)	0 (5)

The cells that contain D will only be true when the function of A, B, and C that correspond to the cells are true *and* when D is true. Similar reasoning is applied to the cell that contains E'.

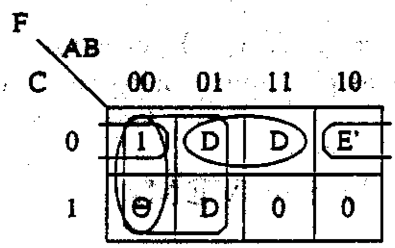
VEM Method

1. Pretend all MEVs are 0 and loop 1s. Then identify essential loops and read the map as usual.

2. Now, for each MEV, form loops containing only the MEV, 1's, and don't cares. Treat 1's as don't cares when looping MEVs. Also, treat an MEV and the complement of that MEV as if they were independent variables.

3. Finally, find the essential prime implicants for the MEVs and read the minimal expressions from the K-map as usual. All MEVs must be covered by some loop. A loop containing an MEV is read as usual and then the MEV is ANDed to the minterm.

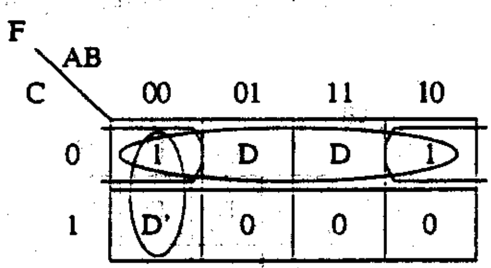
For the above example, we have



$$F = A'B' + A'D + BC'D + B'C'E'$$

Try to choose the variables to use as MEVs so as to minimize the number of MEVs. Consider the following example with MEV D.

$$F(A, B, C, D) = A'B'C' + A'B'CD' + A'BC'D + AB'C' + ABC'D$$



$$F = C'D + B'C' + A'B'D'$$