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AN ECONOMETRIC CHARACTERIZATION OF BUSINESS CYCLE DYNAMICS WITH FACTOR STRUCTURE AND REGIME SWITCHING*

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A dynamic factor model with regime switching is proposed as an empirical characterization of business cycles. The approach integrates the idea of comovements among macroeconomic variables and asymmetries of business cycle expansions and contractions. The first is captured with an unobservable dynamic factor and the second by allowing the factor to switch regimes. The model is estimated by maximizing its likelihood function and the empirical results indicate that the combination of these two features leads to a successful representation of the data relative to extant literature. This holds for within and out-of-sample and for both revised and real time data.

Running Head: Business Cycle Dynamics

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JEL Classification: C32, C50, E32

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1. INTRODUCTION

A great deal of attention has focused on measuring business cycles and identifying their turning points. The possibility of a set of indicators providing early signals of change in aggregate economic activity is important to any business or government affected by expansions and contractions. Based on the early work of Burns and Mitchell (1946), the U.S. Department of Commerce constructs economic indicators that are widely used to predict business cycle turning points. Burns and Mitchell provide a careful statistical description of the cyclical aspects of various time series, and classify macroeconomic variables as lagging, leading or coincident with economic activity. However, the analysis does not provide a formal mathematical treatment of business cycle measurement. In particular, an explicit probability model generating the aggregate time series is lacking. As a result, the Department of Commerce indicators do not contain all the information necessary to characterize business cycle dynamics at the time the events occur and are constantly revised *ex-post*. The NBER dating of business cycle turning points is also based on *a posteriori* inferences of coincident variables. Since revisions of the indicators can be substantial, real time assessment of economic conditions may be severely compromised.

This paper proposes a theoretical framework in which a formal underlying probability model is used to represent business cycles and to generate coincident indicators and inferred probabilities of expansions and recessions. The results are reproducible, and the method enables analysis of business cycles in real time. For example, the imminence of a recession starting in a certain month can be detected by the inferred probabilities or by the implied coincident indicator at the same time the macroeconomic variables are signaling the recession.

In the proposed model, business cycles are empirically characterized by a dynamic factor model with regime switching. The dynamic factor is an unobservable variable that summarizes the common cyclical movements of some coincident macroeconomic variables. This factor is subject to discrete shifts in order to capture the asymmetric nature of business cycle phases - expansions are gradual and display a high mean duration while recessions are shorter and steeper. Hence, the approach used in this paper models the idea of business cycles as the simultaneous movement of economic activity in various sectors by using an unobserved dynamic factor. In addition, the asymmetric nature of expansions and contractions is captured by assuming that the underlying factor switches regimes according to a Markov process.

Both of these ideas were fundamental elements of Burns and Mitchell's (1946) research. The two elements, however, have been studied separately. On the one hand, Stock and Watson (1989, 1991, 1993) develop a model where business cycles are measured by comovements in various components of economic activity in order to obtain an alternative index to the Department of Commerce indicators. Recessions (expansions) are generated by negative (positive) symmetric shocks to a linear and dynamically stable time series system. Unfortunately, Stock and Watson's model fails to account for the 1990 recession using a recession index extracted from their nonswitching dynamic factor representation. The linearity imposed by their model implies a built-in symmetry which forces expansions and contractions to have the same magnitude, duration, and amplitude. In addition, Stock and Watson's model does not take into account changes over time in the stochastic structure of the economy, such as shifts in policy, while an analysis of the long term trend of aggregate macroeconomic time series such as employment, sales, and output indicates several structural changes over this century.

On the other hand, Hamilton (1989) considers nonlinearities in business cycles by assuming that the growth rate of quarterly GNP follows a nonlinear stationary process and incorporates occasional discrete variations in the dynamic features of this time series with a Markov switching model. He finds evidence of asymmetries in cyclical expansions and contractions and ascertains the differences in the dynamics of business cycle phases. However, Hamilton's model, since it is univariate, can not capture the notion of economic fluctuations corresponding to comovements of many aggregate and sectoral variables. In addition, extensions of Hamilton's analysis to monthly growth rates fail to account for several of the historical recessions as determined by the NBER. It is possible that all underlying business cycle information can not be extracted from only one coincident variable. Also, individual coincident variables display movements that do not correspond to business cycle dynamics but instead to noise inherent in monthly data.

Diebold and Rudebusch (1996) suggest integrating the two ideas by fitting a univariate Markov model to the Department of Commerce coincident index and to its components. They show evidence of the suitability of a switching dynamic factor, although they do not fully estimate it.

The existing literature has not found a generally accepted framework that provides a unified explanation of business cycle asymmetries and the comovements of economic aggregates. In this paper, we construct an integrated representation of what Stock and Watson and Hamilton pursued in separate frameworks, as suggested in Diebold and Rudebusch (1996). The idea is that the integrated approach might capture in large part what Burns and Mitchell and the NBER have in mind in their description of business cycle comovements and asymmetries.

The novel aspect of this paper with respect to the existing literature is that we fully estimate a dynamic factor model with regime switches by maximizing its likelihood function. Methods for estimating this model were not available until recently. To estimate the model it is necessary to make inferences about both the unobserved nonlinear factor and the latent

Markov state. Hamilton's (1989) paper popularized the use of Markov regime switches, but the nonlinearity introduced by this precluded the estimation of multivariate unobservable dynamic models. In particular, Hamilton's nonlinear estimation algorithm can not handle models with a regime switching dynamic factor. The dynamic factor model proposed by Stock and Watson's (1989, 1991, 1993) is governed by a linear stochastic process, which implies that the estimation can be implemented by applying the Kalman filter. The estimation procedure undertaken in this paper consists of a combination of Hamilton's algorithm and a nonlinear discrete version of the Kalman filter, as proposed by Kim (1994).

The goals in building a dynamic factor model with regime switching are to obtain optimal inferences of business cycle turning points, and to construct alternative coincident indicators to the Department of Commerce coincident index. The empirical results indicate that the combination of a dynamic factor model with Markov switches leads to a successful representation of the sample data, relative to the existing literature, along several dimensions. This holds both within and out-of-sample, and for both revised and real time data sets. In particular, the results corroborate previous evidence about asymmetries of business cycle phases. The inferred probabilities are strongly correlated with the NBER business cycle dates and all recessions are well characterized for both quarterly and monthly data. In addition, the extracted coincident index is strikingly similar to the Department of Commerce coincident indicator. However, the advantages of the present framework in comparison to traditional approaches are that it allows a more rigorous and timely method for real time assessment of the economy, and results can be consistently reproduced. In contrast, the Department of Commerce and the NBER methodologies require ex-post revision in order to obtain results that we are able to attain in real time.

The paper is organized as follows: the second and third sections present the model and discuss the estimation procedure employed in our study of the switching dynamic factor framework. In the fourth section, the empirical results are presented and interpreted for both quarterly and monthly data. In the fifth section, the model is tested for out-of-sample performance and the sixth section concludes and suggests directions for future research.

2. THE MODEL

A vector of macroeconomic variables displaying comovements with aggregate economic conditions is modeled as composed of two stochastic autoregressive processes - a single unobserved component, which corresponds to the common factor among the observable variables, and an idiosyncratic component.[2] A stochastic trend is not included in the dynamic factor based on evidence that each of the series studied might be integrated but not cointegrated.[3] Therefore, the empirical analysis is undertaken using the log of the first difference of the observable variables.

The model is:

$$(1) \qquad \underset{n\times 1}{\Delta Y_{it}} = \underset{n\times 1}{\lambda_i(L)} \; \underset{1\times 1}{\Delta F_t} + \underset{n\times 1}{\Delta v_{it}} \qquad i = 1,\ldots,n$$

$$(2) \qquad \underset{1\times 1}{\Delta F_t} = \underset{1\times 1}{\alpha_1} \; S_t + \underset{1\times 1}{\alpha_2} + \underset{1\times 1}{\phi(L)} \; \underset{1\times 1}{\Delta F_{t-1}} + \underset{1\times 1}{\eta_{st}} \qquad S_t = 0,1$$

$$(3) \qquad \underset{n\times 1}{\Delta v_{it}} = \underset{n\times n}{D_i(L)} \; \underset{n\times 1}{\Delta v_{it-1}} + \underset{n\times 1}{\varepsilon_{it}} \qquad i = 1,\ldots,n$$

The assumptions of the model are:

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[2] The coincident variables considered are sales, personal income, employment and production.

[3] We performed a Dickey-Fuller test (1979) for the presence of unit roots in each of the coincident variables and it was not able to reject the null hypothesis of integration against the alternative of stationarity at the 10% level. We also tested whether the four coincident variables are not cointegrated against the alternative of cointegration using Stock and Watson's (1988) test, and it failed to reject the null.

$$\eta_{st} \sim N(0,\ s^{2}_{hst})$$

$$\varepsilon_{it} \sim \text{i.i.d. } N(0,\Sigma)$$

$$H_{st} \sim NI \begin{pmatrix} s^{2}_{hst} & 0 \\ 0 & \Sigma \end{pmatrix}$$

$$D_i(L) = \mathrm{diag}(d_1(L), \ldots, d_n(L))$$

$$p_{ij} = \mathrm{Prob}[S_{t=j} \mid S_{t-1}=i],\ \sum_{j=1}^{M} p_{ij} = 1 \quad \forall i \text{ for M states}$$

which implies that ΔF_t and Δv_{it}, for $\forall$ i = 1,..., n are mutually uncorrelated at all leads and lags. The vector ΔY_{it} is the log of the endogenous observable variables, the parameters λ_i are the factor loadings, which measure the sensitivity of the i^{th} series to the business cycle, and ΔF_t is the common factor. In addition, the variables Δv_{it} are the idiosyncratic terms, the ε_{it} are the measurement errors, and η_t is the transition or common shock. The functions $\lambda(L)$, $\phi(L)$ and D(L) are finite lag polynomials of orders l, f and b, respectively, where L is the lag operator and Δ=1-L.

A nonlinear structure is introduced in the unobserved index in the form of a first order two-state Markov switching process. The mean growth rate of the dynamic factor is directly calculated from the nonlinear filter and it is subject to sporadic discrete regime shifts. That is, the economy can be either in a sluggish growth state, S_t=2, or in an accelerated expansion period, S_t=1, with the alternation between regimes ruled by the outcome of a Markov process. The factor mean or the factor variance switch between states, governed by the transition probabilities of the Markov process, p_{ij}. For example, $\mathrm{Prob}[S_t=1 \mid S_{t-1}=1] = p_{11}$ is the probability of an expansion given that the economy is expanding, and $\mathrm{Prob}[S_t=2 \mid S_{t-1}= 2] = p_{22}$ is the probability of recession given that the economy is in a recession.[4]

In the proposed model, cycles are generated from common shocks to the dynamic factor, η_{st}, and all idiosyncratic movements arise from ε_{it}. The only source of comovements among the observable variables comes from the dynamic factor, which can be interpreted as the business cycle.[5] The identification of the model is discussed in an appendix that is available from the author on request (see also Chauvet 1995).

A particular state space representation for the switching dynamic factor (1)-(3), with an AR(2) process for the factor and an AR(1) for the idiosyncratic term, is:

[4] We also consider another specification for the factor mean suggested by Hamilton (1994), in which the shifts depend on the dynamics of the autoregressive process, that is, $\phi(L)\,(\Delta F_t - \mu_{st}) = \eta_{st}$, for $S_t = 0,1$. This specification requires that the order of the Markov switching process be at least as long as the degree of the polynomial $\phi(L)$. Both models were estimated and compared using the same number of lags for the Markov process. However, anticipating the empirical results, this equation was slightly dominated by model (2), both by specification tests and in terms of performance in predicting business cycle turning points. Other variations on the basic models were also introduced, such as allowing the factor variance and mean to switch regimes, or holding the mean constant with a switching variance for the factor.

[5] The number of factors underlying the variables was verified by the usual procedure undertaken in exploratory factor analysis. That is, we check the eigenvalues of the correlation matrix containing the part of the total variance that is accounted for by the common factors. The magnitude of the eigenvalues for each factor, which conveys information about how much of the correlations among the observable variables a particular factor explains, indicated strong evidence for the single factor structure.

Measurement Equations:

$$
\begin{array}{cccc}
\Delta Y_t & & Z & & \xi_t \\
\begin{vmatrix} \Delta Y_{1t} \\ \Delta Y_{2t} \\ \Delta Y_{3t} \\ \Delta Y_{4t} \end{vmatrix} & = & \begin{vmatrix} l_1 & 0 & 1 & 0 & 0 & 0 & 0 \\ l_2 & 0 & 0 & 1 & 0 & 0 & 0 \\ l_3 & 0 & 0 & 0 & 1 & 0 & 0 \\ l_4 & 0 & 0 & 0 & 0 & 1 & 0 \end{vmatrix} & * & \begin{vmatrix} \Delta F_t \\ \Delta F_{t-1} \\ \Delta v_{1t} \\ \Delta v_{2t} \\ \Delta v_{3t} \\ \Delta v_{4t} \\ F_{t-1} \end{vmatrix}
\end{array} \qquad (4)
$$

Transition Equations:

$$
\begin{array}{ccccccccc}
\xi_t & & \alpha_{\xi st} & & T & & \xi_{t-1} & & u_t \\
\begin{vmatrix} \Delta F_t \\ \Delta F_{t-1} \\ \Delta v_{1t} \\ \Delta v_{2t} \\ \Delta v_{3t} \\ \Delta v_{4t} \\ F_{t-1} \end{vmatrix} & = & \begin{vmatrix} a_1 S_t + a_2 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{vmatrix} & + & \begin{vmatrix} f_1 & f_2 & 0 & 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & d_1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & d_2 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & d_3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & d_4 & 0 \\ 1 & 0 & 0 & 0 & 0 & 0 & 1 \end{vmatrix} & * & \begin{vmatrix} \Delta F_{t-1} \\ \Delta F_{t-2} \\ \Delta v_{1t-1} \\ \Delta v_{2t-1} \\ \Delta v_{3t-1} \\ \Delta v_{4t-1} \\ F_{t-2} \end{vmatrix} & + & \begin{vmatrix} h_{st} \\ 0 \\ e_{1t} \\ e_{2t} \\ e_{3t} \\ e_{4t} \\ 0 \end{vmatrix}
\end{array} \qquad (5)
$$

or

$$\Delta Y_t = Z \; \xi_t \qquad (4')$$

$$\xi_t = \alpha_{\xi st} + T \; \xi_{t-1} + u_t \qquad (5')$$

Notice that the term F_{t-1} is included in the state vector to allow estimation of the dynamic factor in levels from the identity $\Delta F_{t-1} = F_{t-1} - F_{t-2}$.

3. ESTIMATION PROCEDURE

We simultaneously estimate the dynamic factor model with regime switching by maximizing its likelihood function. In order to estimate the model it is necessary to make inferences about both the unobserved nonlinear factor and the latent Markov state. The nonlinearity imposed by the regime shifts precluded the estimation of a multivariate dynamic model until the estimation methods by Albert and Chib (1993), Shephard (1994) and Kim (1994) were developed.[6]

We use method developed by Kim (1994) to estimate our model. In that paper, Kim extended Hamilton's Markov-switching model to a general dynamic linear state-space framework where both the measurement and the transition equations are allowed to switch regimes. The parameters of the model are dependent upon a state variable S_t, which follows a stochastic process. A nonlinear discrete version of the Kalman filter is combined with Hamilton's nonlinear filter in one algorithm. This permits estimation of the unobserved state vector as well as the probabilities associated with the latent Markov state. An interesting aspect of Kim's filter for the present analysis is that his algorithm combines the estimation methods undertaken by Stock-Watson and Hamilton, while the proposed model integrates the frameworks underlying their models.

The objective of Kim's nonlinear filter is to form forecasts of the unobserved state vector and the associated mean squared error matrices. The forecasts are based on information available up to time t-1, $I_{t-1} \equiv [\Delta Y'_{t-1}, \Delta Y'_{t-2}, \ldots, \Delta Y_1']'$, on the Markov state S_t taking on the value j, and on S_{t-1} taking on the value i:[7]

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[6] Albert and Chib (1993) and Shephard (1994) proposed, independently, a modified version of the Gaussian filtering and smoothing procedures. They employ simulation and Gibbs sampling to obtain the exact likelihood function of a nonlinear time series. Basically, a Gaussian state-space is used to analyze non-Gaussian frameworks through simulation techniques, and the approach may be used to find maximum likelihood estimates of the Markov switching model. However, this method is still very costly in terms of computation time.

[7] In the empirical exercise, we chose to estimate the model including an additional state variable, s_{t-2}, to obtain more efficient estimates. In this case, equations (6) and (7) become, respectively: $x_{t|t-1}^{(h,i,j)} = E(\xi_t \mid I_{t-1}, S_t=j, S_{t-1}=i, S_{t-2}=h)$ and $P_{t|t-1}^{(h,i,j)} = E[(\xi_t - \xi_{t|t-1})(\xi_t - \xi_{t|t-1})' \mid I_{t-1}, S_t=j, S_{t-1}=i, S_{t-2}=h)]$.

$$x^{(i,j)}_{t|t-1} = E(\xi_t \mid I_{t-1}, S_t = j, S_{t-1} = i) \tag{6}$$

$$P^{(i,j)}_{t|t-1} = E[(\xi_t - \xi_{t\mid t-1})(\xi_t - \xi_{t\mid t-1})' \mid I_{t-1}, S_t = j, S_{t-1} = i)]. \tag{7}$$

For z lagged states conditioning the forecasts and M regimes, the algorithm calculates M^z forecasts for each date t, corresponding to every possible value of S_{t-z}. The filter uses as inputs the joint probability of the Markov-switching states at time t-2 and t-1 conditional on information up to t-1, $\{Prob(S_{t-2}=h, S_{t-1}=i \mid I_{t-1})\}$; an inference about the state vector using information up to t-1, given S_{t-2}=h and S_{t-1}=i, that is, $\{x^{i,j}_{t-1|t-1}\}$; and the mean squared error matrices, $\{P^{i,j}_{t-1|t-1}\}$. The outputs are their one-step updated values. For the probabilities we use as an initial condition the probabilities associated with the ergodic distribution of the Markov chain. For the state vector, its unconditional mean and unconditional covariance matrix are used as initial values.[8]

The nonlinear filter tracks the course of the state vector, which is calculated using only observations on ΔY_t, and computes recursively one-step-ahead predictions and updating equations of the state vector and the mean squared error matrices. The part of the filter that corresponds to a nonlinear discrete version of the Kalman filter, applied to the particular state-space (4)-(5) is:

(prediction equations)

$$x^{(i,j)}_{t|t-1} = a_j + T x^{i}_{t-1|t-1} \tag{8}$$

$$P^{(i,j)}_{t|t-1} = T P^{i}_{t-1|t-1} T' + H^{j} \tag{9}$$

(updating equations)

$$x^{(i,j)}_{t|t} = x^{(i,j)}_{t|t-1} + K^{(i,j)}_{t} N^{(i,j)}_{t|t-1} \tag{10}$$

$$P^{(i,j)}_{t|t} = (I - K^{(i,j)}_{t} Z) P^{(i,j)}_{t|t-1} \tag{11}$$

[8] The unconditional mean and variance-covariance matrix of the state vector are, respectively, $x^{i}_{0|0}=E(\xi_t)$ and $P^{i}_{0|0} = T P^{i}_{0|0} T' + \sigma^2_{\eta st}$. The unconditional mean of the probabilities are $\{Prob(S_{t-2}=h, S_{t-1}=i \mid I_{t-1}\} = Prob(S_0=i) = \pi_i$, i=1,2, where π_i is the ergodic probability.

Here, $K_t^{(i,j)} = P_{t|t-1}^{(i,j)} Z'[Q_t^{(i,j)}]^{-1}$ is the Kalman Gain, $N_{t|t-1}^{(i,j)} = \Delta Y_t - Z\, x_{t|t-1}^{(i,j)}$ is the conditional forecast error of ΔY_t, and $Q_t^{(i,j)} = ZP_{t|t-1}^{(i,j)} Z'$ is its conditional variance. The nonlinear filter allows recursive calculation of the predicted equations, given the parameters in T, Z and H and initial conditions for $x_{t|t}^{j}$ and $P_{t|t}^{j}$.

In the second part of Kim's filter, the probability terms are computed using Hamilton's nonlinear filter. This provides the conditional probability of the latent Markov state at time t. The conditional likelihood of the observable variable is evaluated as a by-product of the algorithm at each t, which allows estimation of the unknown model parameters. The log likelihood function is:

$$(12)\ \text{LogL} = \log f(\Delta Y_T, \Delta Y_{T-1}, \ldots \mid I_0)) = \sum_{t=1}^{T} \log \sum_{j=1}^{M} \sum_{i=1}^{M} \{(2p^{-n/2} |Q_t^{(i,j)}|^{-1/2} \exp(-\frac{1}{2} N_{t|t-1}^{(i,j)'} Q_t^{(i,j)\,-1} N_{t|t-1}^{(i,j)})\}$$

The filter evaluates this likelihood function, which can be maximized with respect to the model parameters using a nonlinear optimization algorithm. The parameters estimated and the sample data are then used in a last application of the filter to draw inferences about the dynamic factor and probabilities based on information available at time t.[9]

For each date t the nonlinear filter computes M^z forecasts, which implies that at each iteration the number of cases is multiplied by M, where M is the number of regimes and z is the number of states conditioning the forecasts. Thus, if the filter does not reduce the number of terms at each time t, it becomes computationally unfeasible even in the simplest two-state case. Kim proposed an approximation introduced through $x_{t|t}^{j}$ and $P_{t|t}^{j}$ for t>1, based on the work of Harrison and Stevens (1976). The approximation consists of a weighted average of the

[9] We also obtain full sample inferences about the state vector and unobserved regimes using Kim's (1994) smoothing algorithm.

updating procedures by the probabilities of the Markov state, in which the mixture of M^z Gaussian densities is collapsed, after each observation, into a mixture of M^{z-1} densities.[10] That is:

$$x_{t|t}^{j} = \frac{\sum_{i=1}^{M} \mathrm{Prob}[S_{t-1} = i, S_t = j | I_t] x_{t|t}^{(i,j)}}{\mathrm{Prob}[S_t = j | I_t]},$$

$$P_{t|t}^{j} = \frac{\sum_{i=1}^{M} \mathrm{Prob}[S_{t-1} = i, S_t = j | I_t] \{P_{t|t}^{(i,j)} + (x_{t|t}^{j} - x_{t|t}^{(i,j)})(x_{t|t}^{j} - x_{t|t}^{(i,j)})'\}}{\mathrm{Prob}[S_t = j | I_t]}.$$

4. EMPIRICAL RESULTS

Data

The empirical analysis focuses on both quarterly and monthly data. For the monthly study, the data sample is from 1952.04 to 1993.03, and for the quarterly, from 1952.2 to 1993.1.[11] The goals are to obtain optimal inferences of business cycle turning points and to construct alternative coincident indicators to the Department of Commerce coincident index. Thus, we focus on the same four sectoral variables utilized by the NBER and the Department of Commerce, which were selected based on the work of Burns and Mitchell (1946). The series

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10 This filter is an optimal estimator in the sense that no other estimator based on a *linear* function of the information set yields a smaller mean squared error. Smith and Makov (1980) examine the nature of this approximation through simulations to verify its performance in terms of jump estimation and detection as well as its fit to the optimal solution. Using an extensive range of initial conditions for the inputs and starting parameter values, Smith and Makov conclude that the approximation performs well in terms of minimizing the sum of the squared errors compared with other nonlinear approximation methods. Also it is the method that tracks most closely the true observations and it is the best in estimating the jumps.

11 The sample range was chosen to be close to the one used by Hamilton, in order to facilitate comparisons. Estimations were also performed for data prior to 1952 and the results are similar to the ones obtained here. One difference is that the probability of staying in a recession is smaller, which may be explained by the magnitude of sudden changes in the growth of the variables studied during the 40s.

used include manufacturing and trade sales in 1982 dollars (MTS), total personal income less transfer payments in 1987 dollars (PILTP), employees on nonagricultural payrolls (ENAP), and industrial production (IP). As alternatives, we also examine gross domestic product (GDP), hours of employees on nonagricultural payrolls (HENAP), total civilian employment (TCE), and non-agricultural civilian employment (NACE).[12] In both quarterly and monthly studies, the data are transformed by computing one hundred times the first difference of the logarithm of each series.[13]

The switching factor coincident index (SFC) estimated in this paper is compared to the Composite Coincident Index of the Department of Commerce (CCI 1982=100) and the coincident index proposed by Stock and Watson (SW). The series SW uses the same variables as CCI with the exception of employees on non-agricultural payrolls (ENAP), which is substituted with hours of employees on non-agricultural payrolls (HENAP).

Model Selection and Specification Tests

Several different specifications of the various models were estimated, including AR(1), AR(2) and AR(3) processes for the factor in the transition and measurement equations, AR(1) and AR(2) for the idiosyncratic terms, and combinations of these using different coincident variables. More highly parameterized models were also estimated, but the coefficients of

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[12] The data, kindly supplied by Frank Diebold and Glenn Rudebusch, were obtained from the Federal Reserve Board’s databank, released in June 1993.

[13] We also estimated the model using linear detrended series as well as by applying the H-P filter detrending technique. Under the linear detrending method, if a structural break in the 70s is not taken into account, the sample data identify the switching as permanent changes rather than cycling back and forth. On the other side, the H-P filter is designed to remove aspects of the data, such as low frequency cycles. Thus, since our goal is to uncover dynamics underlying the data without imposing any *a priori* information on it, these detrending methods turn out to be inappropriate for this paper.

higher dynamic orders were not significant at the 5% statistical level.[14] Akaike Information Criterion, Schwarz Criterion and the likelihood ratio test were used to choose among alternative specifications of the model.

In order to check the adequacy of the model specification, we analyze the disturbances in the observable variables. If the model is correctly specified, the estimated residuals for each observable variable are serially uncorrelated and nearly uncorrelated with each other. Thus, the residuals' sample autocorrelation should be close to zero for observations more than one period apart and ε_t should be a white noise. We also use Brock, Dechert, and Scheinkman's (1987) BDS test for nonlinear models to check the i.i.d. assumption for the disturbances.[15] The diagnostic tests for both quarterly and monthly data indicate that the specifications selected are adequate for all equations. The BDS test fails to reject the i.i.d. hypothesis for the residuals. In addition, the autocorrelation functions for the disturbances are within the limit of two times their asymptotic standard deviation, and the pairwise covariance between the disturbances is nearly zero.

We also test for the number of states, as was done in Diebold and Rudebusch (1996). In particular, we employ the approach described in Garcia (1992). Garcia shows that if the transition probabilities are treated as nuisance parameters, results from Hansen (1993, 1996)

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[14] The extracted switching factors are almost identical for all specifications when using the same variables.

[15] For a vector $\varepsilon_t^m = \varepsilon_t, \varepsilon_{t+1}, \varepsilon_{t+2}, ..., \varepsilon_{t+m-1}$, we use m=2,..., 5 and λ=standard deviation of ε_t, where λ is the distance between any two vectors, ε_t^m and ε_s^m. The test amounts to estimating the probability that these vectors are within the distance λ.

can be applied to test regime switching models.[16] We construct Garcia's test statistic and use critical values that are reported in his paper. Although these critical values are designed for an AR(1) regime switching model and the test is parameter dependent, the highest value in Garcia's table for the 1% significance level is about 2.5 times smaller than the likelihood ratio test for the dynamic factor with regime switching.[17] Hence, this test provides some evidence rejecting the one state null hypothesis.

We selected the best performing model for each data frequency conditional on the results of the diagnostic tests. The quarterly specification, henceforth Model 1, is composed of the coincident variables MTS, PILTP, ENAP and GDP. Both the disturbances, Δv_t, and the factor, ΔF_t, follow a second order autoregressive process in the transition equations (b=2,f=2). For monthly ENAP, it is necessary to introduce a high order autoregressive process to eliminate the misspecification in its equation. Since this would amount to studying a lagging indicator, we also examined other alternative measures of employment such as NACE, TCE or HENAP. Parsimonious versions of the switching dynamic factor model pass specification tests when we use these coincident variables. Thus, for the monthly analysis (Model 2), the series used are

[16] Markov switching models require non-standard testing methods since several of the classical assumptions of asymptotic distribution theory do not hold. For example, the transition probabilities are not identified under the null hypothesis, which implies that the likelihood function with respect to them is flat at the optimum point. Hansen (1993, 1996) proposes simulation methods to approximate the asymptotic null distribution of a standardized likelihood test under non-standard conditions. If the transition probabilities are treated as nuisance parameters, the asymptotic one state null distribution is the supremum over all admissible values in the space of transition probabilities.

[17] We specify a grid in the space of the Markov parameters p_{11} and p_{22}, where the range is from 0.2 to 0.95 in steps of 0.5. In order to overcome the problem of local maxima, we also estimated the likelihood function under the alternative using many different sets of starting values.

MTS, PILTP, NACE and IP. A first order autoregressive process was selected for both the disturbances and for the factor (b=1,f=1).

Estimation Results

The quarterly data are obtained as simple averages of the monthly data, which tends to smooth the series somewhat. For monthly analysis, empirical models have not been as successful in extracting information about fluctuations in economic activity, possibly because of noise inherent to the data. For example, a univariate Markov switching model, such as the one studied in Hamilton (1989), fails to account for several of the historical recessions when applied to some monthly coincident variables. In addition, according to Stock and Watson (1993), their monthly experimental index gives only mild and delayed signals of the last recession and their out-of-sample estimated probabilities fail to forecast it.

In contrast with the existing literature, the inferred probabilities for monthly data estimated from the switching dynamic factor model are highly correlated with NBER business cycle dating. In particular, in an out-of-sample exercise, the inferred probabilities predict the last recession. The estimates obtained through numerical maximization of the conditional log likelihood function (12) are presented in Tables 1 and 2, for quarterly and monthly data, respectively.[18] The empirical results provide support to the Markov switching framework. Both monthly and quarterly samples are well characterized by the two-state specification. There is a significantly positive growth rate in state 1 and a significantly negative growth rate in state 2. The asymmetries in the phases of the business cycle are also well characterized by the switching dynamic factor. The estimated transition probabilities are substantially

[18] We also summarize the results of a more highly parameterized specification for the monthly frequency, although the specification tests favor more parsimonious versions.

significant and the probability of staying in expansion, p_{11}, is higher than the probability of staying in a contraction, p_{22}. This confirms previous findings that the average duration of recessions is smaller than the duration of expansions. The expected duration for recession and expansion implied by the quarterly switching models is, respectively, 5 and 13.7 quarters, compared to 4.4 and 14.3 quarters implied by the NBER dating technique.[19]

INSERT TABLES 1 AND 2

With respect to the factor loadings, sales (MTS) and industrial production (IP) have the highest coefficients and variances in both models, supporting the observation that they are the most sensitive coincident variables to business cycles. In fact, a historical examination of U.S. business cycles indicates that sales and production respond immediately and more intensively to changes in economic conditions close to turning points than the other variables analyzed.

We also estimated the model allowing the factor variance to be state dependent. When the variance and the mean follow a Markov process, the asymmetry in the data is mostly absorbed by the variances. There is a low variance state with a high positive mean, corresponding to the long and gradual expansions, and a high variance state with a positive low mean, associated with steep and short contractions.

Probabilities of Turning Points

The NBER's business cycle dating is generally recognized as the official chronology of turning points. The determination of peaks and troughs is the result of a consensus among the Dating Committee members, who each use different procedures to examine business cycle phases. Various coincident macroeconomic variables are examined and turning points are chosen based on quasi-simultaneous accumulation of inflection points. The subjective aspect of

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[19] The expected duration of recession is determined by the formula: $\sum_{k=1}^{\infty} k p_{22}^{k-1} (1 - p_{22}) = 1 / (1 - p_{22})$.

the NBER decision process may involve the use of different criteria over time, which suggests that conclusions about some features of expansions and contractions should be confronted with alternative dating methodologies.[20] Another drawback of the NBER dating is that decisions about turning points are released with a long delay.

In our case, inferences of the filtered and smoothed probabilities can be used to identify peaks and troughs. Hence, our method allows for real time assessment of the economy and results can be consistently reproduced. Figures 1 and 3 graph the estimated probability that the economy is in the recession state at time t, based on information up to time t, $\text{Prob}(S_t=2 \mid I_t)$, for Models 1 and 2, respectively. Figures 2 and 4 report the corresponding full sample smoother, $\text{Prob}(S_t=2 \mid I_T)$, for Models 1 and 2.[21] The plots show that the probabilities of the recession state are remarkably similar to the NBER dating of business cycles. All recessions are well characterized by both filtered and smoothed probabilities, including the last recession of 1990. For comparison, we also plot the smoothed probabilities of a recession obtained by fitting Hamilton's univariate model to monthly growth rates in industrial production, using an AR(8) process (Figure 5).[22] The inferred probabilities are not strongly correlated with the NBER

[20] Boldin (1994) presents an extensive review and analysis of existing methods for dating business cycle turning points.

[21] The bars in the figures represent historical recessions as determined by the NBER. This information was not used in estimating the model and is shown only for comparison with the inferred probabilities. The NBER business cycle dating is obtained from *Business Conditions Digest*, June 1993.

[22] Although it may be appropriate to use 12 lags, to correspond to the four quarters in Hamilton's specification, the algorithm becomes computationally prohibitive for higher autoregressive processes.

dating of business cycle turning points and they fail to account for the 1970, 1982 and 1990 recessions.[23]

INSERT FIGURES 1 TO 5

Table 3 reports recession turning points derived from our smoothed probabilities, using Hamilton's (1989) criterion to characterize peaks and troughs.[24] Changes in the probabilities track very closely historical turning points, and discrepancies with the NBER dating are very small (no more than 2 periods), with the exception of the 1957 and 1990 recessions. For the 1957 recession, the estimated smoothed probabilities for both models indicate a peak before the official NBER date. The probabilities suggest, as also observed by Hamilton (1989), that the 1957-58 recession came as an immediate response to the oil price shock in the first quarter of 1957. In general, dating differences between the proposed method and the NBER dating are concentrated in the determination of peaks more than in troughs. In particular, the troughs from Models 1 and 2 almost all coincide with the NBER's, with the exception of the last recession.

INSERT TABLE 3

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[23] Hamilton's univariate Markov switching model does not succeed in yielding results that are strongly correlated with the NBER recession dating when applied to some coincident macroeconomic variables, including monthly industrial production (IP), sales (MTS), personal income (PILTP), DOC coincident indicator (CCI) and employment (ENAP). It is successful when applied to employment (HENAP, NACE, TCE). The order of the assumed autoregressive process also affects the results.

[24] Under this method, the economy is in a recession if the full-sample smoothed probability of recession is greater than 0.5, $Prob(S_t=2 \mid I_T)>0.5$. This metric is not necessarily optimal and flexibility should be considered in dubious cases, such as when the recession probabilities are clustered in the interval (0.3, 0.7). We follow the NBER rule in considering the minimum duration of a recession to be six months.

The economic recession in 1990 exhibited some unusual features not observed in previous ones, in particular during the final stage as the economy began to recover. Although, as in prior contractions, production and sales had a steep upturn near the trough, real personal income recovered very slowly, and employment remained low for a long time after the NBER declared the recession to be over in March 1991.[25] Generally, a faster rise in employment and income are observed during the recovery phase. This feature is captured by the switching dynamic factor structure and is especially accentuated in the model using quarterly data, in which the probabilities of recession decrease very slowly in the end of the 1990 recession. In fact, for Model 1, the probabilities indicate that the recession did not end until the first quarter of 1992.[26] According to the Bureau of Economic Analysis' (BEA) official dating, which is based on the DOC coincident indicator, the trough occurred in January 1992.[27] This is in close agreement with the findings from our extracted coincident indicator using quarterly data.

The inferred probabilities are not only useful in identifying the beginning and end of recessions as they occur, but they also reveal moments of great uncertainty in the economy. Almost all recessions were preceded by an increase in the recession probabilities immediately before it. Also, a mild rise in the probabilities 5 to 12 months before a recession forewarns a subsequent downturn, with the exception of the 1970 and 1975 recessions. While it is the case that not all recessions had these prior indicative spikes in the probabilities, every time there

[25] The ensuing uncertainty in the economy during 1991-1992 caused the NBER to delay its decision about the trough of the recession for over a year. The trough in March 1991 was chosen by the NBER based primarily on the recovery of industrial production, although a general rebound happened almost two years later.

[26] Boldin (1994), using a Markov switching model for unemployment to date business cycle turning points, finds that an expansion did not begin until mid 1992.

[27] Since this paper was completed, the BEA, through extensive revisions of the series, changed the dating of the trough so that it coincides with the NBER dating.

was an abnormal change in the average expansion probabilities, a recession followed between half a year and a year later. The most noticeable spikes occurred in mid 1956 and mid 1959. These events are not considered recessions due to their very short duration of at most a quarter. Thus, the switching dynamic factor model might be useful in capturing signals of an imminent recession that are implicit in coincident macroeconomic variables.[28]

Comparison with the Department of Commerce Indicator

The similarities between the growth rates of the extracted factor and the Department of Commerce coincident indexes are striking. As seen in Table 4, the growth rates of the extracted coincident factors and the DOC indexes are highly correlated for both models. In particular, the extracted factor and CCI growth rates for Model 1 exhibit a correlation of 0.96. The standard deviations of these indexes are very close, differing by only 11 percent. For Model 2, the correlation of SFC with CCI is 0.94. The SFC growth rates for both models are plotted on Figures 6 and 7.

INSERT TABLE 4

INSERT FIGURES 6 AND 7

[28] The purpose of the coincident indicator is not to forecast business cycles, but to obtain real time prediction of the state of the economy and to date turning points. A leading indicator would be a more appropriate tool for forecasting business conditions.

For graphical analysis it is easier to examine the indicators in levels.[29] The level of the extracted index (SFC) for Models 1 and 2, as well as the Composite Coincident Index from the Department of Commerce are plotted in Figures 8 and 9, respectively. For quarterly data, the estimated coincident indicator tracks very closely the CCI index and, for almost the whole sample studied, the two series show the same pattern regarding amplitude, timing and duration of fluctuations. The time series for the monthly SFC and CCI in levels are also very similar with respect to the timing and duration of the cycles, although the estimated coincident index exhibits more accentuated oscillations.

INSERT FIGURES 8 AND 9

For the 1990 recession, the extracted SFC indicators for both models show a deeper decline than the CCI. A possible reason for this is related to the weight of each coincident variable in the index construction. The Department of Commerce calculates its coincident index as a weighted average of individual components, where the weights are inversely related to the series' estimated volatility. This implies, for example, that employment and personal income receive the highest weight in the construction of CCI. In our model, we do not place any *a priori* restrictions on the weights assigned to each variable that enters our coincident indicator. As it turns out, the most volatile variables, sales and production, are the ones more highly correlated to the extracted coincident index, although employment and personal income also

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[29] We use the identity $F_{t-1}=\Delta F_{t-1}+F_{t-2}$ in the nonlinear filter to obtain the factor level, FL, where $F_{t-1}=100*\log(FL_{t-1})$. From the estimated results, the extracted factor in levels is obtained by exponentiating $0.01*F_{t-1}$. The Department of Commerce calculates the coincident index as a weighted average of its individual components, ΔCCI_t , that is: $\Delta CCI_t = \sum_{i=1}^{n} b_i \Delta Y_{i,t} + t$, where, b_i are the weights, ΔY_{it} are the growth rate of the coincident variables and t (=-3% a month) is an adjustment so that the index has the same trend as GDP. For graphical comparison, we also account for the trend adjustment imposed on the calculation of the DOC coincident index.

play an important role. The more volatile series are the ones that displayed a steeper decline during the last recession, which might account for the difference between our coincident indicator and the DOC index.

The slow recovery following the 1990 recession is captured by our SFC index and the Department of Commerce indicator, both of which indicate an economic contraction lasting until the beginning of 1992. The monthly SFC index shows a shorter recession in 1990 than the quarterly extracted index. This might be caused by the faster recovery of the monthly IP in the first quarter of 1991. This is the variable with the highest estimated weight in our monthly coincident indicator.

Figure 10 shows the CCI and the monthly index obtained by Stock and Watson (1991). Stock and Watson's index has a standard deviation 80% smaller than the CCI index. Thus, any comparison of these two measures involves an extra correction.[30] The SW index is more jagged, has a smaller mean than the CCI, and underpredicts it for the whole sample period. With respect to the 1990 recession, the SW index provides weak and late signals of a recession exhibiting a fast revival at a time when the economy was still in a slow recovery as indicated by the CCI index. Our extracted switching factor series, as seen in Figures 8 and 9, captures in a timely manner the last recession as well as the slow recovery.

INSERT FIGURE 10

5. OUT-OF-SAMPLE PERFORMANCE - THE 1990 RECESSION

We examine the performance of inferred probabilities in predicting turning points in an out-of-sample exercise. The probability forecasts are evaluated with respect to their accuracy

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[30] Stock and Watson (1991) use a modified factor growth rate to extract the level, whose standard deviation is set to the same value as the DOC coincident index and its mean is kept the same. The series is then scaled to equal 100 in July 1982.

and calibration in predicting observed realizations by using the quadratic probability score (QPS), the raw correlation, and the global square bias (GSB), for both the NBER and the BEA dating.[31] Two sets of post-sample data are examined: historically revised data as well as partially revised or real time data. The idea is to compare and evaluate not only the model performance of ex-post forecasts, but also real time ex-ante forecasts using only data available at the time of forecasting. As discussed in Diebold and Rudebusch (1991), since macroeconomic series undergo several revisions and definitional changes over time, the use of real time data in an out-of-sample exercise provides a more rigorous test of model performance for ex-ante forecasts. The parameters were estimated using data up to 1989.4, for quarterly data, and up to 1989.12, for monthly data. The in-sample estimates were then used to generate out-of-sample forecasts of the filtered probabilities. For quarterly analysis, out-of-sample performance is analyzed from 1990.1 through 1993.1 and for monthly, from 1990.01 through 1993.03. Given the sample selection, the out-of-sample exercise amounts to testing the model for a very unusual period corresponding to the economic downturn in 1990 and the sluggish recovery of the economy in 1991-92.

The dynamic factor models with regime switching successfully characterize the last recession using both filtered and smoothed probabilities and for both quarterly and monthly data. The monthly model yields dating of turning points closer to the NBER while the quarterly model is more in accord with the BEA dating.

Ex-Post Performance - Revised Data

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[31] The quadratic probability score and the global square bias are, respectively: $QPS = \frac{2}{T}\sum_{t=1}^{T}\{Prob[S_t = 2|I_t] - N_t\}^2$,

and $GSB = 2\{\frac{1}{T}\sum_{t=1}^{T}\{Prob[S_t = 2|I_t] - \frac{1}{T}\sum_{t=1}^{T}N_t\}^2$, where N_t is the 0/1 dummy for the NBER or BEA recessions and $Prob[S_t=2 \mid I_t]$ are the filtered probabilities of recession.

Table 5 reports the out-of-sample performance of the probabilities in forecasting turning points. The dynamic factor model displays a slightly better post sample performance in terms of the QPS for regime forecasts than the in-sample results. Also, the proposed model shows better out-of-sample performance in terms of the QPS for the NBER regime forecasts when compared to alternative models. The QPS obtained for Models 1 and 2 are, respectively, 0.29 and 0.11. These values are smaller, for example, than the QPS=0.34 obtained by Hamilton and Perez-Quiros (1996) for a bivariate Markov Switching VAR of the DOC leading indicator and GNP. In fact, the inferred probabilities from their model miss the 1990 recession. In the middle of the recession, their inferred probabilities of a recession were no greater than 25%.

INSERT TABLE 5

The uncertainty in the path of the economy during 1991-92 is also captured by the out-of-sample filtered probabilities obtained from both Models 1 and 2, but it is particularly accentuated for the quarterly frequency (Figure 11).[32] A graphical comparison of the out-of-sample filtered probabilities for Model 1 and the DOC Coincident Indicator (Figure 11) shows that the probabilities of recession characterize the economy in a very similar way as the CCI. The trough determined by the NBER to be in the first quarter of 1991 corresponds to a period of flatness in the CCI. It also coincides with the moment in which the probabilities of recession start decreasing. However, these probabilities decrease very slowly during the first and second quarter of 1991, and increase again in the third quarter when the CCI also shows a decline. Only in the first quarter of 1992 did the probabilities fall below 50%, indicating the end of the recession. This is in closer agreement with the trough decided by the BEA. The QPS

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[32] This difference might be explained by the fact that the monthly model uses the variable IP, which showed a steep upturn in March 1991. The variable GDP, used in the quarterly model, shows a mild increase in the first three quarters of 1991 and a steeper rebound only in the last quarter.

measuring the closeness of the filtered probabilities to the BEA dating is only 0.078, and the GSB=0.039.

INSERT FIGURE 11

Real Time Analysis

Out-of-sample performance is examined with real time monthly data available at the date of each forecast, obtained from the Survey of Current Business.[33] Table 5 reports the out-of-sample performance for monthly data, using our real time data set. Model 2 performs well in terms of forecasting the NBER regimes, achieving a QPS=0.16, a GSB=0.003 and a raw correlation between the filtered probabilities and the NBER business cycle dates of 0.6.

Figures 12 and 13 show the filtered probabilities of recession for revised and real time monthly data. The probabilities using real time data are more volatile, reflecting the uncertainty of the economy during the period. In the beginning of 1990 the probability of a recession had a short and pronounced increase to 0.8, indicating the subsequent economic contraction. The probabilities were above 50% again in May 1990. That is, the switching factor model indicates an economic downturn at the same time the economy was signaling a recession, using only data available then. The model also forecasts the beginning of the recession before it occurred. The one-step ahead probability of a recession, based on real time data up to May, signals a 51.4% chance of a recession in June (Figure 14). The zero-step ahead probability forecasts identify the end of the recession in March 1991. These probabilities also indicate uncertainty in the economy at the end of 1991, increasing close to the 50% level at the

[33] Real time date t corresponds to data released at t+2. For example, data for January 1990 was obtained from the March 1990 issue of the Survey of Current Business. This data selection is based on Diebold and Rudebusch's (1991) evidence that using preliminary and incomplete data, as released at t+1, leads to a poor forecasting performance.

time sales and industrial production showed a modest decline.

INSERT FIGURES 12 AND 13

According to Stock and Watson (1993), their experimental index fails to characterize and forecast the last recession. Their out-of-sample estimated probabilities signal a recession only in November of 1990. In October, a quarter after the beginning of the recession, the zero-step ahead recession probability was only 0.28. The three-month ahead probability forecast was 0.23 in November, based on information up to August, and the one-step ahead probability of a recession was 0.05 in both May and in June, failing to forecast the last recession.

6. SUMMARY AND CONCLUDING REMARKS

This paper proposes a model in which business cycles are empirically characterized by a dynamic factor with regime switches. The approach captures both the idea of business cycles as comovements in several macroeconomic variables and the asymmetric nature of business cycle phases. The optimally inferred dates of business cycle turning points display a strong correlation with the NBER dating of business cycles and the extracted dynamic factor is remarkably similar to the Department of Commerce coincident indicator. In particular, the results highlight the importance of nonlinearities in business cycles.

The model provides a more rigorous and timely approach for dating business cycle turning points than traditional methods. In particular, our approach is based on a probabilistic framework that can be used in real time to assess the state of the economy and that can be replicated consistently at any time.

The results suggest that a very satisfactory representation of the sample data is obtained by modeling business cycles as the common element underlying a set of coincident variables subject to sporadic regime shifts. The proposed framework is also a useful tool for ex-ante prediction of business cycle turning points. Investigation using both revised data and information available to agents in real time indicates that the extracted coincident index and estimated probabilities perform very well in their ability to characterize these turning points.

In the future, it might be worthwhile to investigate whether extending the approach in this paper to include leading macroeconomic variables might yield a leading indicator that could be successfully used to forecast turning points. Also, including additional states in the Markov stochastic process might improve the performance of the model. For example, Burns and Mitchell (1946) conceive business cycles as composed of four distinct periods: prosperity, crisis, depression and revivals. It might be interesting to investigate this notion using the framework studied in this paper.

In closing, let us address the relationship of this paper to the contemporaneous and independent work of Kim and Yoo (1995). Although the Kim-Yoo approach is similar to ours, there are important differences, related to choice of sample period, model specification, and the variables used. As regards sample period, the Kim-Yoo sample starts in 1960, which excludes two recessions compared to our sample data. Thus, our estimation uses information obtained from 8 recessions, while theirs uses information from 6 recessions. As regards model specification and the variables used, Kim and Yoo use the ENAP employment variable, which according to Stock and Watson (1991) requires extra lags in its equation to avoid model misspecification. This results in a mixed coincident/lagging index specification, which is unfortunate given that the objective is to construct a coincident index model. As was discussed in Section 4, we obtain a better-specified version of the switching dynamic factor model by using the employment series NACE instead of ENAP.

The upshot is simply that, because of differences in sample period, model specification, and the variables used, the Kim-Yoo coincident indicator behaves very differently from ours. The Kim-Yoo coincident indicator is highly correlated with Stock and Watson's (the correlation between the two is greater than 0.99). Hence, both the Stock-Watson and Kim-Yoo coincident indexes fail to indicate the slow recovery following the last recession. This feature follows from the fact that their indexes are more correlated with changes in Industrial Production than with other variables, in particular for the subperiod around the 1990 recession. As discussed in the empirical section of our paper, Industrial Production shows a steep upturn near the trough of the 1990 recession. Although our index is highly correlated with Industrial Production as well, it captures the sluggish recovery because it is also highly correlated with variables that grew very slowly following the last recession, such as real personal income. Our conclusions about the last recession and the period that followed it are confirmed by an out-of-sample exercise using unrevised data, which Kim and Yoo did not perform.

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Figure 1 - Probability of Recession at t Using Information up to t, Prob(S_t=2|I_t) from Model 1, and Ex-Post NBER-Dated Recessions.

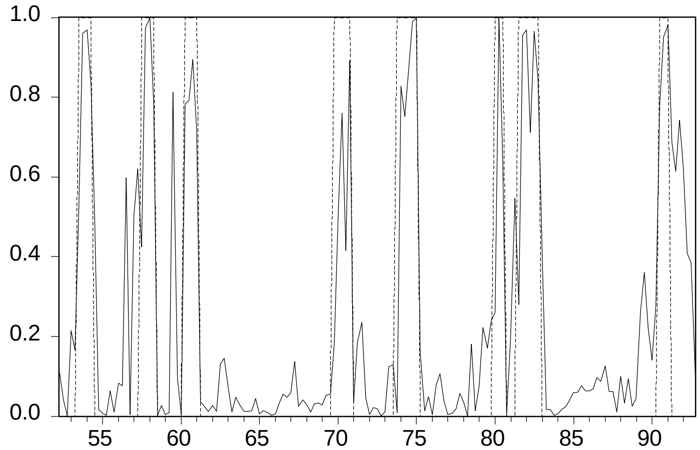

Figure 2 - Probability of Recession at t Using Full Sample Information: Prob(S_t=2|I_T) from Model 1, and Ex-Post NBER-Dated Recessions.

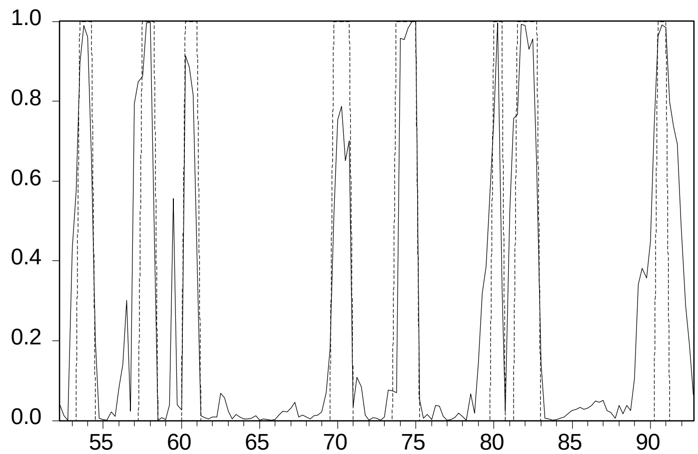

Figure 3 - Probability of Recession at t Using Information up to t, Prob(S_t=2|I_t) from Model 2, and Ex-Post NBER-Dated Recessions.

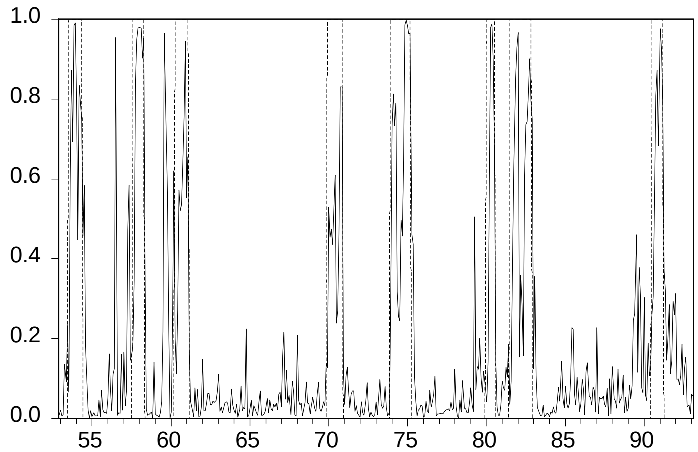

Figure 4 - Probability of Recession at t Using Full Sample Information: Prob(S_t=2|I_T) from Model 2, and Ex-Post NBER-Dated Recessions.

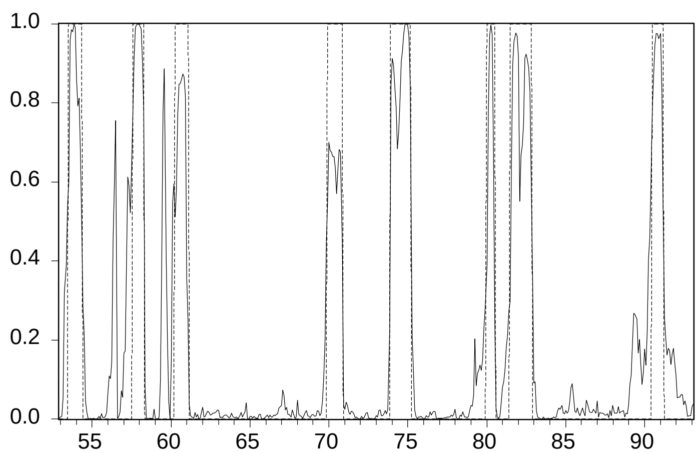

Figure 5 - Probability of Recession at t Using Full Sample Information: Prob(S_t=2|I_T) from Hamilton's Univariate Model Fitted to Monthly IP Growth Rates, and Ex-Post NBER-Dated Recessions.

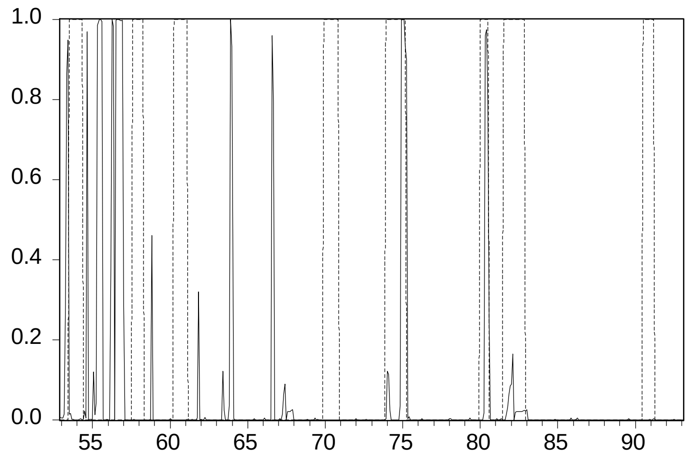

Figure 6 - Growth Rates of the Switching Factor Index and NBER-Dated Recessions, Model 1.

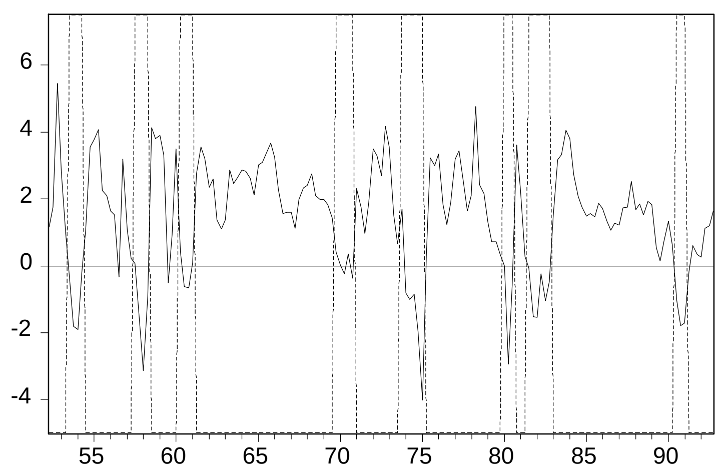

Figure 7 - Growth Rates of the Switching Factor Index and NBER-Dated Recessions, Model 2.

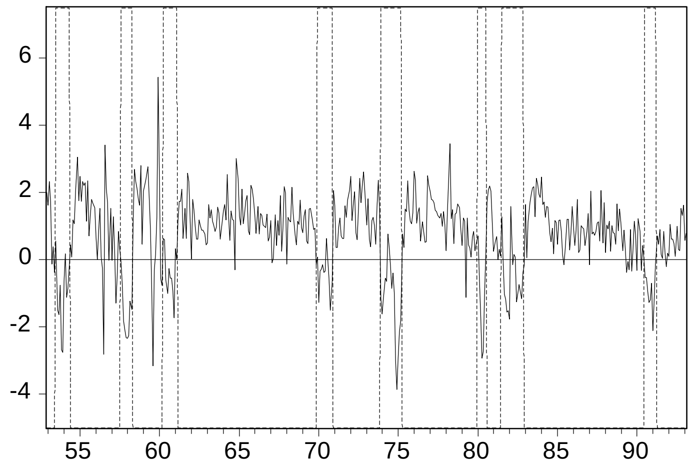

Figure 8 - Switching Factor Index (—) and Department of Commerce Coincident Indicator (---), Model 1.

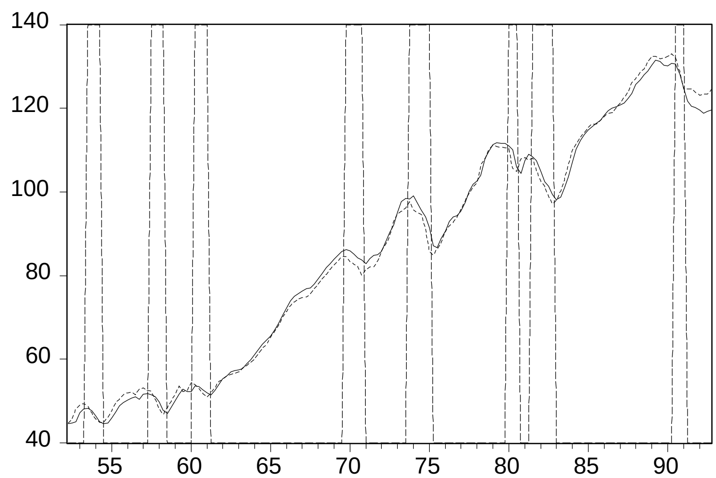

Figure 9 - Switching Factor Index (—) and Department of Commerce Coincident Indicator (---), Model 2.

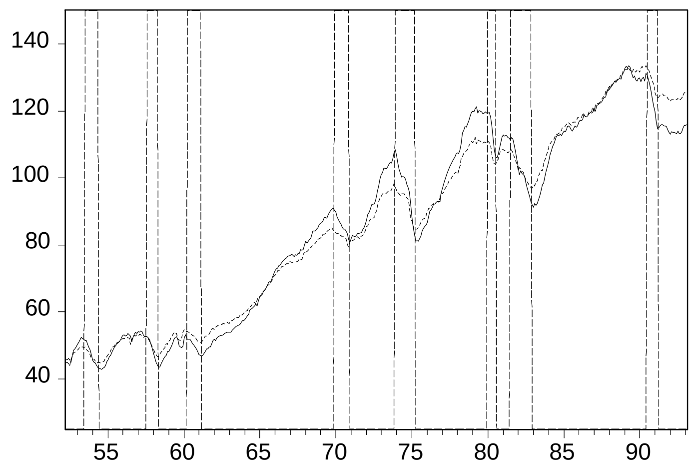

Figure 10 - Stock-Watson Coincident Index (—) and Department of Commerce Coincident Indicator (---), Monthly Data.

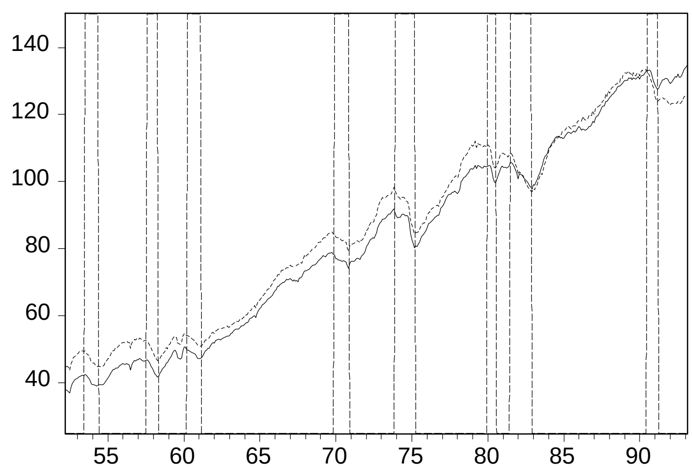

Figure 11 - Out-of-Sample Filtered Probability of Recession, Prob(S_t=2|I_t), from Model 1; Revised Data (lower), and Department of Commerce Coincident Index (upper): 1990.01 to 1993.01.

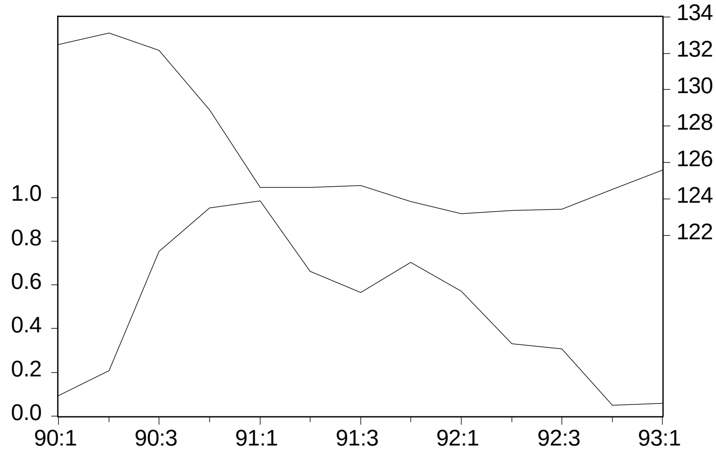

Figure 12 - Out-of-Sample Filtered Probability of Recession, Prob(S_t=2|I_t), from Model 2; Revised Data: 1990.01 to 1993.03.

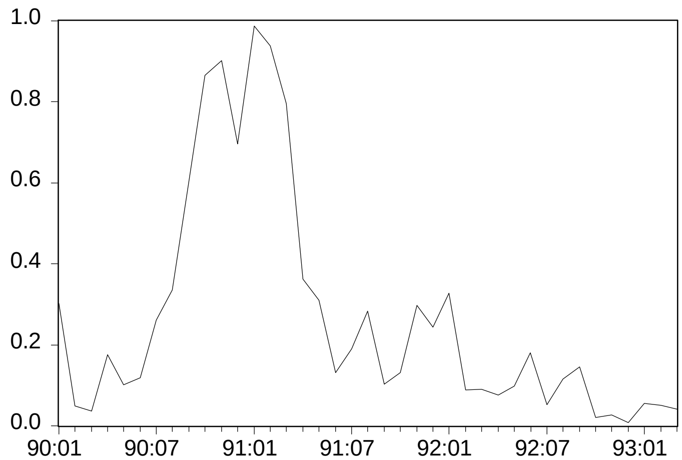

Figure 13 - Out-of-Sample Filtered Probability of Recession, Prob(S_t=2|I_t), from Model 2; Real Time Data: 1990.01 to 1993.03.

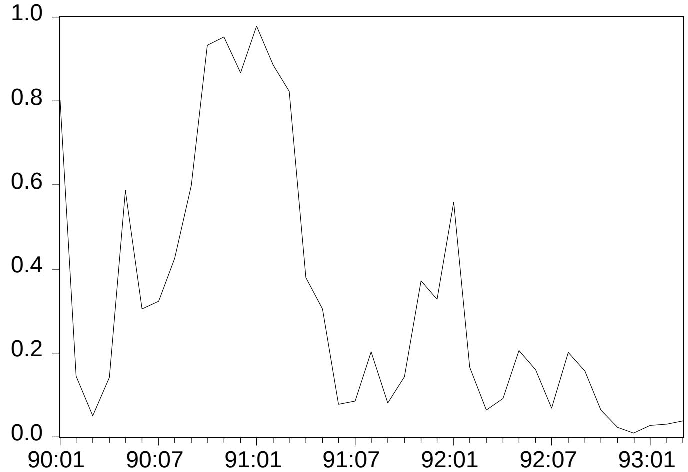

Figure 14 - Out-of-Sample One-Step Ahead Probability of Recession, Prob(S_{t+1}=2|I_t), from Model 2; Real Time Data: 1990.01 to 1993.03.

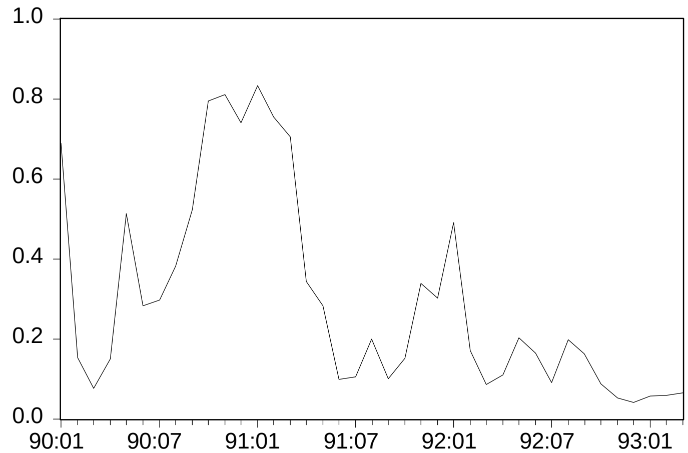

Table 1
Maximum Likelihood Estimates - Model 1
Quarterly Data: 1952.2-1993.1

$$\Delta Y_{it} = \lambda_i \Delta F_t + \Delta v_{it}$$
$$\Delta F_t = \alpha_2 + \alpha_1 S_t + \phi_1 \Delta F_{t-1} + \phi_2 \Delta F_{t-2} + \eta_t \qquad S_t=0,1$$
$$\Delta v_{it} = d_i \Delta v_{it-1} + d_i^* v_{it-2} + \varepsilon_{it}$$

(i = Sales, PIncome, Employm, GDP)

Parameters		Parameters	
α_1	1.975	λ_{gdp}	0.461
	(.369)		(.049)
α_2	-0.490	d_{sales}	0.101
	(.292)		(.089)
ϕ_1	0.583	$d_{pincome}$	-0.007
	(.096)		(.067)
ϕ_2	-0.230	$d_{employm}$	0.920
	(.008)		(.135)
$\sigma^2\varepsilon_{sales}$	1.290	d_{gdp}	-0.217
	(.163)		(.112)
$\sigma^2\varepsilon_{pincome}$	0.249	d^*_{sales}	-0.021
	(.042)		(.114)
$\sigma^2\varepsilon_{employ}$	0.048	$d^*_{pincome}$	-0.290
	(.015)		(.097)
$\sigma^2\varepsilon_{gdp}$	0.247	$d^*_{employm}$	-0.303
	(.044)		(.120)
λ_{sales}	0.739	d^*_{gdp}	0.093
	(.089)		(.101)
$\lambda_{pincome}$	0.456	p_{11}	0.913
	(.047)		(.035)
$\lambda_{employm}$	0.312	p_{22}	0.754
	(.028)		(.093)
LogL(θ)	-670.835		
LR	28.92		

Asymptotic standard errors in parentheses correspond to the diagonal elements of the inverse hessian obtained through numerical calculation. LR is the likelihood ratio test for the number of states. The variables used are: MTS, PLITP, ENAP and GDP, and the recession and expansion means are respectively $\mu_2 = \alpha_2 / (1- \phi_1 - \phi_2) = -0.76$ and $\mu_1 = (\alpha_1+\alpha_2) / (1- \phi_1 - \phi_2) = 2.29$.

Table 2
Maximum Likelihood Estimates - Model 2
Monthly Data: 1952.04-1993.03

$$\Delta Y_{it} = \lambda_i \Delta F_t + \Delta v_{it}$$
$$\Delta F_t = \alpha_2 + \alpha_1 S_t + \phi \Delta F_{t-1} + \eta_t \qquad S_t=0,1$$
$$\Delta v_{it} = d_i \Delta v_{it-1} + \varepsilon_{it}$$
(i = Sales, PIncome, Employm, IP)

Parameters		Parameters	
α_1	1.591	λ_{employ}	0.150
	(.332)		(.012)
α_2	-0.746	λ_{ip}	0.569
	(.309)		(.045)
ϕ	0.291	d_{sales}	-0.240
	(.085)		(.052)
$\sigma^2\varepsilon_{sales}$	0.827	$d_{pincome}$	-0.087
	(.066)		(.057)
$\sigma^2\varepsilon_{pincome}$	0.156	$d_{employm}$	-0.171
	(.013)		(.052)
$\sigma^2\varepsilon_{employm}$	0.084	d_{ip}	0.199
	(.006)		(.069)
$\sigma^2_{\varepsilon ip}$	0.448	p_{11}	0.964
	(.049)		(.013)
λ_{sales}	0.478	p_{22}	0.855
	(.040)		(.070)
$\lambda_{pincome}$	0.259		
	(.018)		
LogL(θ)	-1776.91		
LR	25.12		

Asymptotic standard errors in parentheses correspond to the diagonal elements of the inverse hessian obtained through numerical calculation. LR is the likelihood ratio test for the number of states. The variables used are: MTS, PLITP, NACE and IP. The recession and expansion means are $\mu_2=\alpha_2/(1-\phi_1) = -1.05$ and $\mu_1=(\alpha_1+\alpha_2)/(1-\phi_1) = 1.19$. The log likelihood for a second-order autoregressive specification for both the factor and the disturbances is LogL(θ) = -1769. 59.

Table 3
Dating of U.S. Business Cycle Turning Points:
NBER, BEA and Smoothed Probabilities of Recession
Models 1 and 2

Dates NBER		Dates BEA Official		Dates Factor Model 1(*)		Dates Factor Model 2	
Peak	Trough	Peak	Trough	Peak	Trough	Peak	Trough
1953:07	1954:05	1953:06	1954:08	1953:II	1954:II	1953:06	1954:05
1957:08	1958:04	1957:02	1958:04	1957:I	1958:II	1957:04	1958:04
1960:04	1961:02	1960:01	1961:02	1960:II	1960:IV	1960:02	1960:12
1969:12	1970:11	1969:10	1970:11	1969:IV	1970:IV	1969:12	1970:10
1973:11	1975:03	1973:11	1975:03	1974:I	1975:I	1973:12	1975:03
1980:01	1980:07	1980:01	1980:07	1979:IV	1980:II	1980:02	1980:06
1981:07	1982:11	1981:07	1982:12	1981:II	1982:IV	1981:08	1982:11
1990:07	1991:03	1990:06	1992:01	1990:II	1992:I	1990:06	1991:03
				QPS_{NBER}=0.1230		QPS_{NBER}=0.0645	
				QPS_{BEA}=0.0976		QPS_{BEA}=0.0927	

(*) Denotes quarterly dating.
The economy is assumed to be in a recession if $P(S_t=2 \mid I_T) > 0.5$.

Table 4
Statistics for the Extracted Coincident Factor Index (SFC) and the Department of Commerce Coincident Index (CCI,): Quarterly and Monthly Data - Models 1 and 2

	Model 1				Model 2			
	Growth Rates		Level		Growth Rates		Level	
Statistics	CCI	SFC	CCI	SFC	CCI	SFC	CCI	SFC
Mean	0.631	1.449	86.187	86.195	0.210	0.741	86.347	86.180
Stand Dev.	1.863	1.661	27.854	27.760	0.875	1.203	27.858	27.827
Corr.(SFC,CCI)	--	0.963	--	0.998	--	0.941	--	0.984

Table 5
Evaluation of Turning Point Forecasts of the Filtered Probabilities of Recession: Revised and Real Time Data

		Revised Data		Real Time Data
		Model 1	Model 2	Model 2
In-Sample	QPS_{NBER}	0.119	0.122	-
	QPS_{BEA}	0.101	0.192	-
Out-of-Sample	QPS_{NBER}	0.289	0.113	0.164
	QPS_{BEA}	0.078	0.464	0.438
	Corr. with NBER	0.815	0.775	0.588
	Corr. with BEA	0.897	0.595	0.523

The criterion adopted to determine if the economy is in a recession is whether the filtered probability of recession is greater than 0.5, $P(S_t=2 | I_t) > 0.5$. The Quadratic Probability Score is: QPS $= \frac{2}{T}\sum_{t=1}^{T}\{p[S_t = 2|I_t] - N_t\}^2$, where N_t is a 0/1 dummy corresponding to the BEA or NBER dating