Invited Paper

Performance Analysis of Precoded MIMO Systems with Adaptive Peak Power Suppression

Osamu Muta[†], Tomoya Kageyama^{††}

Abstract This paper presents two approaches for reducing the peak power in precoded multi-input multi-output (MIMO) systems with space division multiplexing (SDM). First, we present a peak power aware linear precoding scheme for MIMO-SDM single carrier systems, where the precoder is designed to restrict per-antenna transmit power below a permissible level while mitigating the performance degradation caused by per-antenna power restriction. Second, we present an analytical method to represent bit error rate (BER) performance of maximum ratio combining (MRC) precoded massive MIMO OFDM systems with adaptive peak cancellation, where peak amplitude is canceled below a given threshold to reduce peak-to-average power ratio (PAPR). With this method, we clarify the impact of adaptive peak cancellation on approximated BER of MRC-precoded massive MIMO OFDM with an arbitrary number of the transmit antennas and users. Numerical results prove that our proposed approaches are effective in restricting the peak power at each antenna while mitigating performance degradation in BER performance of the precoded MIMO-SDM systems.

Key words: MIMO, peak power, precoding, peak cancellation

1. Introduction

Multi-input multi-output (MIMO) technologies with space division multiplexing (SDM) is one of the key wireless transmission technologies for multimedia services since data rate can be increased without expanding the required bandwidths¹⁾⁻⁵⁾. MIMO technologies can be combined with multi-carrier modulation (i.e., orthogonal frequency division multiplexing: OFDM) as a signaling scheme for broadband wireless communications. However, OFDM signal exhibits high peak-toaverage power ratio (PAPR) which causes non-linear distortion and/or out-of-band (OoB) radiation due to nonlinear power amplifier $(PA)^{6}$. This problem may become more critical in massive MIMO (mMIMO) systems because per-antenna power amplification is needed for a large size antenna array. In addition, if spatial precoding (e.g., spatial filter) is adopted to MIMO systems, the maximum (peak) per-antenna transmission power is increased by the precoder even when total transmission power is fixed. Hence, it is requested to limit a perantenna power below a permissible level even in single carrier (SC) modulation systems. Thus, the reduction

(Fukuoka, Japan)

of per-antenna peak power is a challenging issue.

One of the solutions to the above problems is to apply a deliberate peak limiter such as clipping-and-filtering $(C\&F)^{6}$ to the transmitter. C&F is a simple technique but it will cause nonlinear distortion which destroys orthogonality between data streams in MIMO system with space division multiplexing (SDM). As an alternative approach, an adaptive bit loading and power allocation method has been investigated⁹⁾, where adaptive bit loading and power allocation are adopted to maximize the data rate under constraint of the maximum output power at each antenna. However, it is needed to reduce the total transmit power to keep perantenna transmit power below a permissible level. A weight optimization method for space division multiple access systems under per-antenna power constraint was proposed in¹⁰. However, the work in ¹⁰ is limited to single antenna user case.

In another viewpoint, in wireless communication systems, in-band distortion and non-linear distortion may be measured as error vector magnitude (EVM) and adjacent channel leakage power ratio (ACLR), respectively. In general, EVM and ACLR should be kept below a predefined threshold. To keep EVM and ACLR below an acceptable level, an adaptive peak cancellation method for MIMO OFDM using eigenbeam space division multiplexing (E-SDM) was proposed in¹¹⁾¹². In this method, peak amplitude that exceeds a given threshold is suppressed repeatedly by adding a peak

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 $[\]dagger$ Center for Japan-Egypt Cooperation in Science and Technology, Kyushu University

⁽Fukuoka, Japan)

^{††} Graduate School and Faculty of Information Science and Electrical Engineering

cancellation (PC) signal at a given time instance while keeping ACLR and EVM below pre-defined levels. Using this method, peak power is automatically reduced below a given threshold under constraints of the ACLR and EVM requirements. In order to design effective PAPR reduction schemes for mMIMO systems while keeping required quality of services, it is necessary to present an analytical framework to provide the relationship between BER performance and achievable PAPR characteristics of mMIMO systems with an arbitrary number of transmit antennas and the number of served users unlike the work in¹¹⁾¹².

This paper presents two approaches for reducing the peak power in precoded MIMO systems^{*}. First, we present a peak-power aware linear precoding scheme for MIMO-SDM SC systems, where per-antenna transmission power is restricted below a given threshold while mitigating the degradation due to the power restriction. In this method, we investigate an alternative optimization algorithm that updates the precoding matrix and the virtual postcoding matrix alternatively under total transmission power and per-antenna power constraints. Second, we present an analytical method to approximately represent bit error rate (BER) of maximum ratio combining (MRC) precoded mMIMO OFDM systems with the adaptive peak cancellation, where inter-user interference (IUI) and in-band distortion due to peak cancellation are approximated as Gaussian random variables, respectively. With this analytical method, we investigate the impact of adaptive peak cancellation on approximated BER on mMIMO OFDM with arbitrary number of the transmit antennas and users.

2. A peak power aware precoding scheme for MIMO single carrier systems

The precoding operations may increase the peak power of the transmit signal at each antenna. In other words, in the precoded MIMO-SDM systems, there is a possibility that the transmission power is concentrated on a certain antenna even when the total transmission power is constant, because the transmission power at each antenna is changed according to precoding weights and current channel conditions. To solve this problem, we need to limit per-antenna transmission power within a certain threshold. This section presents a method to



Fig. 1 MIMO-SDM system.

restrict per-antenna power in MIMO-SDM systems as an extended work of¹⁰⁾.

2.1 System model

Figure 1 shows a MIMO system considered in this section, M, N and K denote the number of transmit antennas, receive antennas, and data streams, respectively. Here, $W_t \in \mathbb{C}^{K \times M}$ and $W_r \in \mathbb{C}^{K \times N}$ denote weighting matrices of the transmit and receive spatial filters, i.e., the precoding and postcoding matrices, respectively. The weight optimization part at the transmitter side is shown in Fig. 1(b). $\boldsymbol{x} = [x_1, ..., x_k, ..., x_K]^T$ denotes the transmit data stream vector, where x_k is the k-th stream. Let $\mathbf{H} \in \mathbb{C}^{N \times M}$ denote channel matrix. The received stream is de-multiplexed by multiplying the weighting matrix at the postcoder. Hence, the demultiplexed signal vector $\boldsymbol{y} = [y_1, ..., y_k, ..., y_K]^T$ is given as

$$\boldsymbol{y} = \boldsymbol{W}_{\boldsymbol{r}}^{H} \mathbf{H} \boldsymbol{W}_{\mathbf{t}}^{\mathbf{H}} \boldsymbol{x} + \boldsymbol{W}_{\mathbf{r}}^{\mathbf{H}} \boldsymbol{n}, \qquad (1)$$

where superscript H denotes transposed conjugate of a matrix and $\boldsymbol{n} = [n_1, ..., n_n, ..., n_N]^T$ is additive white Gaussian noise (AWGN) vector at each receive antenna.

2.2 Weight optimization method

In Fig. 1, we assume that the transmitter equips a virtual receiver that calculates the estimated received signal passing through an estimated channel and a virtual post coding matrix. Here, $\hat{H} \in \mathbb{C}^{N \times M}$ and $\hat{\boldsymbol{y}} = [\hat{y}_1, ..., \hat{y}_i, ..., \hat{y}_K]^T$ denote the estimated channel matrix and the virtual receive signal at the virtual receiver, respectively. \hat{W}_r denotes the virtual postcoding matrix. We assume that $\hat{\boldsymbol{n}} = [\hat{n}_1, ..., \hat{n}_n, ..., \hat{n}_N]^T$ denotes virtual noise vector whose element n_n is added to the virtual receive signal at the n-th receive antenna. $G = diag\{\alpha_1, ..., \alpha_k, ..., \alpha_K\}$ is a diagonal matrix whose diagonal element takes a real value used for purpose of automatic gain control of the k-th data stream. The kdimensional error vector between the transmit signal \boldsymbol{x} and the normalized virtual received signal $G\hat{y}$ is defined as

 $^{^{\}ast}$ The works in this paper are presented in part in international conferences. $^{(13)\,14)}$

$$e(W_t, G) = x - G\hat{y}$$

= $x - G\hat{W}_r\hat{H}W_t^H x - G\hat{W}_r\hat{n}$
= $x - G\hat{H}'W_t^H x - G\hat{W}_r\hat{n}$, (2)

where $\hat{H}' = \hat{W}_r \hat{H}$ and $e(W_t, G) = (e_1, \cdots e_k, \cdots, e_K)$.

First, we begin to derive an optimization algorithm by assuming that the postcoding matrix \hat{W}_r is identity matrix. Let ξ denote the maximum permissible transmit power at each antenna. In case where there is no correlation between data streams, relation between ξ and individual transmission power at the *m*-th antenna is expressed as

$$\sum_{k=1}^{K} E[|w_{km}x_k|^2] = \sum_{k=1}^{K} E[|x_k|^2]|w_{km}|^2 = \sum_{k=1}^{K} |w_{km}|^2 \leq \xi,$$
$$\forall m \in \{1, \cdots, M\}, \quad (3)$$

where M and K denote the numbers of transmit antennas and data streams, respectively. The above relation regarding the *m*-th antenna can be formulated as a function of the precoding weight matrix W_t ;

$$f_m(\mathbf{W}_t) = \sum_{k=1}^K |w_{kj}|^2 - \xi \le 0, \ \forall m \in \{1, \cdots, M\}.(4)$$

Similarly, the relation between the total transmission power Γ and the precoding matrix W_t is expressed as

$$f(\boldsymbol{W}_t) = \sum_{k=1}^{K} \sum_{m=1}^{M} |w_{km}|^2 - \Gamma \le 0,$$
 (5)

where w_{km} denotes (k, m) element of the matrix W_t . Hence, we formulate an optimization problem that minimize the mean square error under the total transmission power constraint and per-antenna power constraint¹⁰;

Minimize
$$E[\|\boldsymbol{e}(\boldsymbol{W}_{t})\|_{2}^{2}]$$

Subject to $f_{m}(\boldsymbol{W}_{t}) \leq 0 \qquad \forall m \in 1, ..., M$
 $f(\boldsymbol{W}_{t}) \leq 0$ (6)

where $\|\cdot\|_2$ denotes the Euclidean norm of a vector.

The constrained optimization problem in (6) can be relaxed into the following non-constrained problem:

Minimize
$$\epsilon(\boldsymbol{W}_t, \boldsymbol{G}) = E[\|\boldsymbol{e}(\boldsymbol{W}_t, \boldsymbol{G})\|_2^2] + r \cdot \eta(\boldsymbol{W}_t)$$
 (7)

where $r \ (> 0)$ is a small real constant. $\eta(\mathbf{W}_t)$ denotes a penalty function that is defined as $\eta(\mathbf{W}_t) = \{Q(\mathbf{W}_t) + V(\mathbf{W}_t)\}$ that takes a large value at boundary¹⁴⁾¹⁵⁾. Here,

$$Q(\boldsymbol{W}_t) = \begin{cases} -\sum_{\substack{m=1 \ M}}^{M} (f_m(\boldsymbol{W}_t))^{-1}, & \text{if } f_m(\boldsymbol{W}_t) \leq \zeta \\ -\sum_{\substack{m=1 \ M}}^{M} \zeta^{-2} \left(2\zeta - f_m(\boldsymbol{W}_t) \right), & \text{if } f_m(\boldsymbol{W}_t) > \zeta \end{cases}$$
(8)

$$V(\boldsymbol{W}_t) = \begin{cases} -\left(f(\boldsymbol{W}_t)\right)^{-1} & \text{if } f(\boldsymbol{W}_t) \leq \zeta \\ -\zeta^{-2} \left(2\zeta - f(\boldsymbol{W}_t)\right) & \text{if } f(\boldsymbol{W}_t) > \zeta, \end{cases}$$
(9)

where ζ is a pre-determined small real constant value. The following recursive equations with respect to W_t and G are used;

$$W_t(m+1) = W_t(m) - \mu \nabla_{W_t}[\epsilon(W_t, G)]$$

$$G(m+1) = G(m) - \mu \nabla_G[\epsilon(W_t, G)], \quad (10)$$

where $\mu(>0)$ is a step size.

Next, we consider case where the virtual postcoding matrix \hat{W}_r is not identity matrix. Let \hat{y} denote the virtual received stream vector given as

$$\hat{\boldsymbol{y}} = \hat{\boldsymbol{W}}_r \hat{\boldsymbol{H}} \boldsymbol{W}_t^H \boldsymbol{x} + \hat{\boldsymbol{W}}_r \hat{\boldsymbol{n}} = \hat{\boldsymbol{W}}_r \hat{\boldsymbol{x}} + \tilde{\boldsymbol{n}}, \qquad (11)$$

where $\hat{x} = \hat{\mathbf{H}} W_t^H x$ denotes the virtual received signal. Let $r_{\hat{x}x}$ denote cross-correlation vector between transmit signal vector x and receive signal vector \hat{x} . The alternative error function is given as

$$\boldsymbol{e}(\hat{\boldsymbol{W}}_r) = \boldsymbol{x} - \hat{\boldsymbol{y}} = \boldsymbol{x} - \hat{\boldsymbol{W}}_r \boldsymbol{x}' - \tilde{\boldsymbol{n}}$$
(12)

The virtual postcoding matrix is given as an minimum mean square error (MMSE) filter which minimizes the means square error $E[\|\boldsymbol{e}(\hat{\boldsymbol{W}}_r)\|_2^2]$.

In this method, an alternative iteration algorithm is used where the precoding matrix $\mathbf{W}_{\mathbf{t}}$ and the virtual postcoding matrix $\hat{\mathbf{W}}_r$ are updated alternatively. The algorithm is summarized as follows: First, $\hat{\mathbf{W}}_r$ is set to identity matrix. Then, transmit weight matrix $\mathbf{W}_{\mathbf{t}}$ is updated. After that, $\hat{\mathbf{W}}_r$ is updated to minimize square error of the error signal in Eq.(12). Similar to the above, $\mathbf{W}_{\mathbf{t}}$ and $\hat{\mathbf{W}}_r$ are updated alternatively until number of alternative iterations come to a given number N_r .

On the receiver side, similarly to the virtual receive weight on the transmitter side, the MMSE based spatial filtering is carried out to detect the received signal.

2.3 Performance evaluation

Performance of a MIMO SDM system using the proposed method is evaluated by computer simulation. Simulation block diagram is shown in Fig.2. We consider MIMO flat Rayliegh fading channels, where there are no correlation between paths of any pairs of transmit and receive antennas. Offset QPSK (OQPSK) with coherent demodulation is adopted. The number of transmit and receive antennas are set to M = 4 and N = 2, respectively. The number of steams is set to K = 2. $F(\omega)$ is frequency transfer function of raised cosine roll-off filter. For comparison purpose, we also evaluate an eigen-mode SDM (E-SDM) system using



a linear scaling algorithm that reduce the transmission power so that the maximum per-antenna power is equal to or less than a given threshold. This paper defines the normalized maximum permissible average transmit power per antenna element: $\gamma = \xi/\Gamma$, $1/N \leq \gamma \leq 1.0$, where ξ and Γ denotes the individual-antenna maximum permissible transmission power and the maximum permissible total transmission power, respectively. $\gamma = 1.0$ corresponds to case where peak power of the transmit signal is not limited. To evaluate the influence on per-antenna power restriction, $E_b/N_0|_{\gamma=1}$ is defined as E_b/N_0 in case of $\gamma = 1$.

Figure 3(a) shows the complementary cumulative distribution functions (CCDFs) of per-antenna transmission power for different channel realizations. CCDF is defined as the probability that random variable Z exceeds a certain value z, i. e., $CCDF(z) = Prob(Z \ge z)$. From this figure, we can see that the proposed scheme works effectively to restrict the maximum per-antenna power below a given threshold. Figure 3(b) shows in-



Fig. 4 BER performance of MIMO SDM system with the proposed precoding, where M = 4 and N = K = 2.

stantaneous power of the transmit signal normalized by the maximum permissible average power per antenna element, where the number of antennas are set to M = 4and N = 2. From this figure, it can be seen that the normalized instantaneous power at CCDF= 10^{-3} can be reduced by about 2.5dB by using the proposed algorithm. In both the proposed and linear scaling methods, total transmission power is reduced by limiting perantenna transmission power compared to case without antenna power limitation.

Figure 4 shows BER performance of the MIMO SDM system using the proposed scheme as a function of $E_b/N_0|_{\gamma=1}$, where the number of antennas is set to M = 4 and N = 2 and the number of alternative iterations is set to $N_{it} = 3$. For comparison, E-SDM with the linear-scaling technique is used to limit perantenna power, where the optimum power allocation on BER minimization criterion is adopted to each data streams. It can be seen that the proposed system improves BER performance by about 2dB compared with case using the linear scaling method.

3. BER analysis of MRC-precoded massive MIMO OFDM systems with peak cancellation

Adaptive peak cancellation was proposed for reducing PAPR of OFDM signals¹²⁾. This technique is used to reduce PAPR of OFDM signal while keeping EVM and ACLR below their permissible levels. In this section, we propose an analytical framework to mathematically express the BER performance of MRC-precoded massive MIMO OFDM systems with peak cancellation.

3.1 System model

Figure 5 illustrates the mMIMO-OFDM downlink system block diagram considered in this section, where M and N denote the number of transmit antennas and the number of single antenna users. Let $\mathbf{s}^l \in \mathbb{C}^{N \times 1}$ denotes the complex transmit QAM symbol vector at l-th subcarrier, where l denotes subcarrier index. We assume MRC precoding² whose precoding matrix $\mathbf{W}_t^l \in \mathbb{C}^{M \times N}$ is given as

$$\mathbf{W}_{t}^{l} = \begin{bmatrix} \frac{h_{11}^{l*}}{\sum_{m=1}^{M} |h_{1m}^{l*}h_{1m}^{l}|} & \cdots & \frac{h_{N1}^{l*}}{\sum_{m=1}^{M} |h_{Nm}^{l*}h_{Nm}^{l}|} \\ \vdots & \vdots & \vdots \\ \frac{h_{1M}^{l*}}{\sum_{m=1}^{M} |h_{1m}^{l*}h_{1m}^{l}|} & \cdots & \frac{h_{NM}^{l*}}{\sum_{m=1}^{M} |h_{Nm}^{l*}h_{Nm}^{l}|} \end{bmatrix}, (13)$$

where h_{mn}^l denotes the frequency response between *m*th transmit antenna and *n*-th received antenna at *l*-th subcarrier. (·)* denotes conjugate operation. The precoded transmit signal $\mathbf{x}^l \in \mathbb{C}^{M \times 1}$ is expressed as

$$\mathbf{x}^{l} = \frac{1}{P_{t}} \mathbf{W}_{t}^{l} \mathbf{s}^{l}, \tag{14}$$

where P_t is the normalized factor of the transmit power. Let $\mathbf{H}^l \in \mathbb{C}^{N \times M}$ denotes the channel matrix between the base station and user equipments. The received signal vector $\mathbf{y}^l \in \mathbb{C}^{N \times 1}$ at *l*-th subcarrier is given as

$$\mathbf{y}^{l} = \frac{1}{P_{t}} \mathbf{H}^{l} \mathbf{x}^{l} + \mathbf{n}^{l}, \tag{15}$$

where $\mathbf{n}^l \in \mathbb{C}^{N \times 1}$ denotes the noise vector. The received signal of the *n*-th is given as

$$y_n^l = \frac{1}{P_t} \underbrace{\boldsymbol{h}_n^l \boldsymbol{w}_t^{(n,l)} \boldsymbol{s}_n}_{\text{desired}} + \frac{1}{P_t} \underbrace{\sum_{k \neq n}^N \boldsymbol{h}_k^l \boldsymbol{w}_t^{(n,l)} \boldsymbol{s}_k}_{\text{IUI}} + n_n^l, \quad (16)$$

where \boldsymbol{h}_{n}^{l} and $\boldsymbol{w}_{t}^{(n,l)}$ are the *n*-th row vector of \mathbf{H}^{l} and *n*-th column vector of \mathbf{W}_{t}^{l} , respectively. In Eq.(16), the first and second terms in the bracket represent desired signal and inter-user interference (IUI), respectively.

3.2 Adaptive peak cancellation

In the adaptive peak cancellation method¹¹⁾¹²⁾, the maximum signal amplitude exceeding a given threshold level is suppressed by adding a PC signal illustrated in Fig. 6. The PC signal is a scaled OFDM symbol whose subcarriers are added up to be in-phase at a given time instance of the symbol duration; the PC signal is generated by scaling the following basic function g(t);

$$g(t) = \frac{1}{N} \sum_{k=1}^{N} h(t) e^{j\omega_k t},$$
(17)

where N is the number of subcarriers, ω_k is the angular





(a) Time-domain signal waveform. (b) Frequency-domain spectrum.

Fig. 6 The illustration of the basic PC signal.



Fig. 7 The illustration of the distortion due to the adaptive PC.

frequency in k-th subcarrier signal, and h(t) is the impulse response of the transmit band-pass filter, and we assume that square wave expressed by h(t) = 1 when $0 \leq t \leq T$, where T is OFDM symbol duration. Let x(t) denotes complex multicarrier signal. The maximum peak amplitude $|x(t_0)|$ is detected at time $t = t_0$. If $|x(t_0)|$ exceeds a given threshold value A_{th} , peak amplitude is reduced to the A_{th} by adding the PC signal to the transmit signal. The added PC signal is expressed as

$$p_r(t - t_0) = -A_p e^{j\theta} g'(t - t_0), \qquad (18)$$

where g'(t) = w(t)g(t), and w(t) is windowing function to truncate the time-domain waveform of g(t). Here, $A_p = |x(t_0)| - A_{th}$ and θ denotes the phase of the maximum peak amplitude $|x(t_0)|$. In MIMO OFDM system, PC signal is individually added to transmit signals at each antenna. The transmit signal in *m*-th antenna after adding the *i*-th PC signal is given as

$$x_m^{(i)}(t) = x_m^{(i-1)}(t) + p_r^{(i)}(t-t_0),$$
(19)

where $x_m^{(i)}(t)$ is the transmit signal after adding the *i*-th PC signal $p_r^{(i)}(t - t_0^{(i)})t_0^{(i)}$ denotes time instance when the *i*-th maximum peak amplitude is detected. Then, we can estimate distortion power added on the transmit signal by calculating the increased ACLR and EVM due to peak cancellation¹².

Figure 7 illustrates the effect of in-band distortion and out of band (OoB) radiation due to adding the PC signal to cancel the peak amplitude. The above operations are continued until the maximum amplitude value $|x(t_0)|$ in each frame is below the threshold A_{th} or the number of iterations N_{it} comes to a given iteration number. If either the estimated ACLR value or EVM value exceeds the acceptable value in *i*-th iteration, the PAPR reduction procedure stops before adding the *i*-th PC signal. Finally, the precoded transmit signal vector after PC $\tilde{\mathbf{x}}^l \in \mathbb{C}^{M \times 1}$ can be expressed as

$$\tilde{\mathbf{x}}^l = \mathbf{x}^l + \mathbf{d}^l,\tag{20}$$

where $\tilde{\mathbf{d}}^l \in \mathbb{C}^{M \times 1}$ denotes the in-band distortion vector due to adaptive PC. EVM is defined below as the mean square error of transmission signals before and after PC normalized by the average power follows.

$$EVM = \frac{\|\tilde{\mathbf{x}}^{1} - \mathbf{x}^{1}\|_{2}^{2}}{\|\tilde{\mathbf{x}}^{1}\|_{2}^{2}} = \frac{\|\mathbf{d}^{1}\|_{2}^{2}}{\|\tilde{\mathbf{x}}^{1}\|_{2}^{2}},$$
(21)

where $\|\mathbf{a}\|_2$ denotes the Euclidean norm of a vector.

3.3 BER analysis

In this section, we present the approximated BER expression of MRC precoded mMIMO OFDM with the peak cancellation, where the IUI and the in-band distortion due to PC are approximated as Gaussian random variables. The received signal of the *n*-th user in case with adaptive PC \tilde{y}_n^l is given as



Fig. 8 BER analysis model on the receiver (user) side.

$$\tilde{y}_{n}^{l} = \frac{1}{P_{t}} \left(\boldsymbol{h}_{n}^{l}(\boldsymbol{w}_{t}^{(n,l)}\boldsymbol{s}_{n} + \mathbf{d}^{l}) + \sum_{k \neq n}^{N} \boldsymbol{h}_{k}^{l}\boldsymbol{w}_{t}^{(k,l)}\boldsymbol{s}_{k} \right) + n_{n}^{l} \\
= \underbrace{\frac{1}{P_{t}}\boldsymbol{h}_{n}^{l}\boldsymbol{w}_{t}^{(n,l)}\boldsymbol{s}_{n}}_{\text{desired}} + \underbrace{\frac{1}{P_{t}}\sum_{k \neq n}^{N} \boldsymbol{h}_{k}^{l}\boldsymbol{w}_{t}^{(k,l)}\boldsymbol{s}_{k}}_{\text{IUI}} \\
+ \underbrace{\frac{1}{P_{t}}\boldsymbol{h}_{n}^{l}\mathbf{d}^{l}}_{\text{in-band distortion}} + n_{n}^{l}, \quad (22)$$

where the third term corresponds to in-band distortion due to peak cancellation.

In this analysis, we approximate the IUI and the inband distortion as Gaussian random variables with different variances, where g_{IUI} and g_{PC} are added to the receiver side as illustrated in Fig. 8, respectively. Here, we assume that g_{IUI} and g_{PC} follow the same statistical characteristics for all users and all subcarriers and we omit the user index and subcarrier index. The received signal is approximately expressed as

$$\tilde{y}_{n}^{l} = \frac{1}{P_{t}} \boldsymbol{h}_{n}^{l} \boldsymbol{w}_{t}^{(n,l)} s_{n} + g_{IUI} + g_{PC} + n_{n}^{l}$$
$$= \frac{1}{P_{t}} \boldsymbol{h}_{n}^{l} \boldsymbol{w}_{t}^{(n,l)} s_{n} + g + n_{n}^{l}.$$
(23)

Let g denote the sum of g_{IUI} and g_{PC} , g is also Gaussian random variables, and its average and variance is the sum of those averages and variances, respectively. Let $\sigma_g^2(M, N)$ and $\mu_g(M, N)$ denote the variance and the average of g, where these parameters depend on the number of transmit antennas and the number of users, respectively. We assume that the average received power of each user are the same. In that case, the power of the IUI is proportional to the number of the interference users (N-1) and thus the variance of g is expressed as follows.

$$\sigma_g^2(M,N) = \begin{cases} (N-1)\sigma_g^2(M,2) & N \ge 2\\ \sigma_{PC}^2 & N = 1, \end{cases}$$
(24)

 $\sigma_g^2(M, N)$ with arbitrary number of M and N can be given by using the variance when M=2, $\sigma_g^2(M, 2)$.

Since we assume that the average transmit power of each antenna is also the same, the average power of the IUI is given as 1/M of the desired signal power. The variance of g with M transmit antennas is given as

$$g(M,2) = \frac{\sigma^2(2,2)}{M}.$$
 (25)

From Eq.(24) and Eq.(25), $\sigma_g^2(M, N)$ for arbitrary number of M and N is given as

$$\sigma_g^2(M,N) = \begin{cases} \frac{(N-1)}{M} \sigma_g^2(2,2) & N \ge 2\\ \sigma_{PC}^2 & N = 1, \end{cases}$$
(26)

where $\sigma_g^2(M, N)$ is given by an off-line simulation. When N = 1, only in-band distortion due to PC occurs because there is no IUI. σ_{PC} depends on the required in-band distortion power and it can be theoretically derived¹².

The average BER of the QPSK modulated signal of the MRC precoded mMIMO OFDM system for M and N under the presence of in-band distortion is approximated as

$$P_b(\gamma, \mu_g, \sigma_g^2(M, N))$$

$$= \int_0^\infty \frac{1}{\sqrt{2\pi(\sigma_n^2 + \sigma_g^2(M, N))}}$$

$$\cdot \exp\left(-\frac{(x - (\mu_g + A))^2}{2(\sigma_n^2 + \sigma_g^2(M, N))}\right) dx, \qquad (27)$$

Hence, average BER of the MRC precoded mMIMO OFDM system under i.i.d. Rayleigh fading is approximated as

$$\bar{P}_b(\bar{\gamma}) = \int_0^\infty f(\gamma, \bar{\gamma}) \left(P_b(\gamma, \mu_g, \sigma_g^2(M, N)) \right) d\gamma, (28)$$

where $f(\gamma, \bar{\gamma})$ denotes the PDF of the received signalto-noise ratio (SNR) of the received MRC given as¹⁶.

$$f(\gamma,\bar{\gamma}) = \frac{1}{(M-1)!\bar{\gamma}^M} \gamma^{(M-1)} e^{-\frac{\gamma}{\bar{\gamma}}},\tag{29}$$

where $\bar{\gamma}$ is the average received SNR.

3.4 Performance evaluation

In this section, we evaluate the PAPR characteristics and BER performance of MRC precoded mMIMO OFDM system with adaptive peak cancellation. The number of subcarriers is set to 64 and subcarrier modulation is QPSK.

Figure 9(a) denotes the CCDFs of the normalized instantaneous power of the transmit signals per antenna, where the horizontal axis is normalized by the average power of the transmit signal before peak cancellation. In Fig. 9 (a), the black line shows the original OFDM signal. From this figure, normalized instantaneous power at CCDF= 10^{-4} is reduced by 4.4 dB by using adaptive PC, where EVM is kept below -20dB. Figure 9(b) denotes the relationship between EVM and



Fig. 9 PAPR reduction performance of mMIMO OFDM systems using adaptive PC.

normalized instantaneous power at $CCDF = 10^{-4}$. In Fig. 9 (b), the black line shows the original OFDM signal that any PAPR reduction is not adopted. From this figure, in-band distortion (EVM) increases as PAPR (i.e., the peak detection threshold) decreases.

To clarify the validity of BER expression in Sect. 3.3, we compare Eq.(28) with simulation results under the same conditions. Figures 10 (a) and (b) depict BER expression in (28) and its simulation result for MRCprecoded mMIMO-OFDM systems with the peak cancellation, where the horizontal axis represents average SNR $(\bar{\gamma})$ per transmit antenna^{*}. In Figs. 10 (a) and 10(b), the number of users are set to $N_u=2$ and $N_u=4$, respectively. Pre-determined EVM and ACLR upper limits are set to -20 dB and -50dB, respectively. In this figure, it is clear that irreducible errors occur at large SNR. This is mainly due to residual inter-user interference in MRC precoding case. The error floor (irreducible error) can be mitigated by increasing the number of transmit antennas M and consequently improving achieved signal-to-interference and noise ratio

^{*} In this section, we assume the transmit signal power is the same as the receive signal power, i.e., the channel gain is assumed to be unity.



(c) Relation between the number of transmit antennas and BER expression in Eq.(28).

Fig. 10 BER performance of MRC-Precoded QPSKmMIMO OFDM systems using adaptive PC with EVM=-20dB.

(SINR). These results prove that analytical results show good agreement with their simulation results.

Figure 10 (c) shows the impact of the number of transmit antennas on the achieved BER in Eq.(28), where $\bar{\gamma}$ is set to -15dB and N_u =8. The black line corresponds to the case without any degradation due to peak cancellation (i.e., this corresponds to BER lower bound). The green, blue and red lines shows the BER using adaptive PC and EVMs are kept below -10, -15

and -20 dB, respectively. From Fig. 10 (c), it can be seen that BER degradation due to peak cancellation is controlled by using a proper EVM constraint for a given number of the transmit antennas.

4. Conclusion

In this paper, we have presented two approaches for reducing the peak power in MIMO-SC and mMIMO OFDM systems, respectively. First, we have presented a peak power aware precoding scheme that restricts perantenna transmission power and consequently reduces peak power of the transmit signal. Then, we have presented an analytical method to evaluate the achievable BER of MRC precoded mMIMO OFDM systems with adaptive peak cancellation. We derived an approximated BER expression to clarify the impact of the peak cancellation on BER performance of mMIMO OFDM for an arbitrary number of the transmit antennas and users. Numerical results show that the proposed approaches are effective in reducing per-antenna peak power while mitigating performance degradation due to peak power suppression. In broadcasting systems, field pick-up units (FPU) employ precoded MIMO transmission, and thus it is expected that the proposed techniques are also useful for future FPUs.

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Osamu Muta received B.E. degree from Ehime University, in 1996, M.E. degree from Kyushu Institute of Technology, Japan, in 1998, and Ph.D. degree from Kyushu University in 2001. In 2001, he joined the Graduate School of Information Science and Electrical Engineering, Kyushu University as an assistant professor. Since 2010, he has been an associate professor in Center for Japan-Egypt Cooperation in Science and Technology, Kyushu University. His current research interests include signal processing techniques for wireless communications and powerline communications, MIMO, and nonlinear distortion compensation techniques for high-power amplifiers. Dr. Muta is a senior member of IEICE and a member of IEEE.



Tomoya Kageyama received B.E. degree and M.E. degree from Kyushu University, Japan, in 2015 and 2017, respectively. Presently, he is a doctoral student at Kyushu University, Japan. His research interests include digital signal processing techniques for MIMO wierless communications.