

Ordinal	Arithmetic theories	Set theories	Inductive defs.	Type theories	Other...
	Q	$KP^-$			
$\omega^2$	RFA, $I\Delta_0$				
$\omega^3$	$RCA_0^*$ , $WKL_0^*$ , $I\Delta_0+exp$				
$\omega^n$	$I\Delta_0+\mathcal{E}_n$ is total				
$\omega^\omega$	$(RCA)_0$ , $(WKL)_0$ , PRA			CPRC	
$\varepsilon_0$	$(ACA)_0$ , $(\Delta_1^1-CA)_0$ , PA	$KP^{-\infty}$			$EM_0$
$\varepsilon_1$	$(ACA)_0+(KPHT)$				
$\varepsilon_\omega$	$(ACA)_0+(iRT)$				
$\varepsilon_{\varepsilon_0}$	(ACA)				
$\psi_{\Omega_1}(\Omega) = \zeta_0$	$(ACA)_0+(\forall X\exists Y(TJ(\omega, X, Y)))$				
$\psi_{\Omega_1}(\Omega^\omega)$	$(\Delta_1^1-CR)$		$ID_1^\#$		$EM_0+JR$
$\psi_{\Omega_1}(\Omega^{\varepsilon_0})$	$(\Delta_1^1-CA)$ , $(\Sigma_1^1-AC)$		$\widehat{ID}_1$	$ML_1$	$EM_0+J$
$\psi_{\Omega_1}(\Omega^\Omega) = \Gamma_0$	$(ATR)_0$ , $(\Delta_1^1-CA)+(BR)$	$KPi^-$	$\widehat{ID}_{<\omega}$	$ML_{<\omega}$ , MLU	U(PA)
$\psi_{\Omega_1}(\Omega^\Omega\varepsilon_0)$	(ATR)		$\widehat{ID}_\omega$		
$\psi_{\Omega_1}(\Omega^{\Omega+\omega})$	$(ATR)_0+(\Sigma_1^1-DC)$		$\widehat{ID}_{<\omega^\omega}$		
$\psi_{\Omega_1}(\Omega^{\Omega+\varepsilon_0})$	$(ATR)+(\Sigma_1^1-DC)$		$\widehat{ID}_{<\varepsilon_0}$		
$\psi_{\Omega_1}(\Omega^{\Omega+\Gamma_0})$			$\widehat{ID}_{<\Gamma_0}$	MLS	
$\psi_{\Omega_1}(\Omega^{\Omega^2})$		$KPh^-$	$Aut(\widehat{ID})$		
$\psi_{\Omega_1}(\Omega^{\Omega\omega})$		$KPM^-$			
$\psi_{\Omega_1}((^{(n+1)}\Omega)^{\Omega^\omega})$	$(ACA)_0+(\Pi_{n+2}^1-BI)$	$KP^-+\Pi_{n+2}^{set}\text{-Foundation}$			
$\psi_{\Omega_1}((^{(n+1)}\Omega)^{\Omega^{\varepsilon_0}})$	$(ACA)+(\Pi_{n+2}^1-BI)$	$KP^-+IND+\Pi_{n+2}^{set}\text{-Foundation}$			
$\psi_{\Omega_1}(\varepsilon_{\Omega+1})$	$(ACA)+(BI)$ , $(\Pi_1^1-CA)^-$	KP, $KP(\mathcal{P})$ , $KP+\Pi_2^{set}\text{-Reflection}$ , CZF	$ID_1$ , $ID_1^2$	$ML_1V$	
$\psi_{\Omega_1}(\Gamma_{\Omega+1})$					U( $ID_1$ )
$\psi_{\Omega_1}(\varepsilon_{\Omega_2+1})$		$KP+\Pi_1^{set}\text{-Coll}+V=L+\exists\omega_1$	$ID_2$ , $ID_2^2$		
$\psi_{\Omega_1}(\Omega_\omega)$	$(\Pi_1^1-CA)_0$ , $(\Delta_2^1-CA)_0$	$KPI^r$ , $KPi^r$ , $KP\beta^r$	$ID_{<\omega}$ , $(ID_{<\omega}^2)_0$		
$\psi_{\Omega_1}(\Omega_\omega\varepsilon_0)$	$(\Pi_1^1-CA)$	W-KPI	W- $ID_\omega$ , $ID_{<\omega}^2$		
$\psi_{\Omega_1}(\varepsilon_{\Omega_\omega+1})$	$(\Pi_1^1-CA)+(BI)$	KPI	$ID_\omega$ , $BID_\omega^2$		
$\psi_{\Omega_1}(\Omega_{\omega^\omega})$	$(\Delta_2^1-CR)$ , $(\Pi_1^1-CA_{<\omega^\omega})$	$KPI_{\omega^\omega}^r$	$ID_{<\omega^\omega}$		
$\psi_{\Omega_1}(\Omega_{\varepsilon_0})$	$(\Delta_2^1-CA)$ , $(\Sigma_2^1-AC)$ , $(\Pi_1^1-CA_{<\varepsilon_0})$	$KPI_{\varepsilon_0}^r$ , W-KPi, W-KP $\beta$	$ID_{<\varepsilon_0}$ , $ID_{<\varepsilon_0}^2$ , $ID_{<\varepsilon_0}^2$		
$\psi_{\Omega_1}(\Omega_\lambda)$	$(\Pi_1^1-CA_{<\lambda})$ , $(\Pi_1^1-CA_{<\lambda})+(BI)$	$KPI_\nu^r$	$ID_{<\lambda}$ , $BID_{<\lambda}^2$ , $ID_{<\lambda}^2+(BI)$		
$\psi_{\Omega_1}(\Omega_\mu)$	$(\Pi_1^1-CA_\mu)_0$	$KPI_\mu^r$	$(ID_\mu^2)_0$		
$\psi_{\Omega_1}(\Omega_\gamma)$	$(\Pi_1^1-CA_\gamma)$	W-KPI $_\gamma$	$ID_\gamma^2$		
$\psi_{\Omega_1}(\varepsilon_{\Omega_\nu+1})$			$ID_\nu$		
$\psi_{\Omega_1}(\varepsilon_{\Omega_\gamma+1})$	$(\Pi_1^1-CA_\gamma)+(BI)$	KPI $_\gamma$	$ID_\gamma$ , $BID_\gamma^2$ , $ID_\gamma^2+(BI)$		
$\psi_{\Omega_1}(\varepsilon_{\Omega_\Omega+1})$		$KPI^* = KPI_\Omega^r$	$ID_{<^*}$ , $BID^{2*}$ , $ID^{2*}+(BI)$		
$\psi_{\Omega_1}(\psi_I(0))$	$(\Pi_1^1-TR)_0$ , $(Aut-\Pi_1^1)_0$	Aut-KPI $^r$	$(Aut-ID)_0$		
$\psi_{\Omega_1}(\psi_I(0)\varepsilon_0)$	$(\Pi_1^1-TR)$ , $(Aut-\Pi_1^1)$	W-Aut-KPI	Aut-ID		
$\psi_{\Omega_1}(\varepsilon_{\psi_I(0)+1})$	$(Aut-\Pi_1^1)+(BI)$	Aut-KPI	Aut-BID		
$\psi_{\Omega_1}(\varepsilon_{I+1})$	$(\Delta_2^1-CA)+(BI)$ , $(\Sigma_2^1-AC' )+(BI)$	KPi, $KP\beta$ , CZF+REA			$T_0$
$\psi_{\Omega_1}(\Omega_{I+\omega})$		$KPi^+$		$ML_1W$ , $ML_1WV$ , $KP_1W$ , IARI	
$\psi_{\Omega_1}(\Omega_{L+1})$		KPh		$ML_{<\omega}W$	
$\psi_{\Omega_1}(\varepsilon_{M+1})$	$(\Delta_2^1-CA)+(BI)+(M)$	KPM, CZFM			
$\psi_{\Omega_1}(\Omega_{M+\omega})$		$KPM^+$		MLM	Agda
$\Psi_{\Omega_1}^0(\varepsilon_{K+1})$	$(ACA)+(BI)+(\Pi_4^1\text{-}\beta\text{-model Reflection})$	$KP+\Pi_3^{set}\text{-Reflection}$			
$\Psi_{\Omega_1}^0(\varepsilon_{\pi_n+1})$	$(ACA)+(BI)+(\Pi_{n+5}^1\text{-}\beta\text{-model Reflection})$	$KP+\Pi_{n+4}^{set}\text{-Reflection}$			
$\Psi_{\mathbb{X}}(\varepsilon_{\Xi+1})$	$(ACA)+(BI)+(\beta\text{-model Reflection})$	$KP+\Pi_\omega^{set}\text{-Reflection}$			
$\Psi_{\mathbb{H}}(\varepsilon_{\Upsilon+1})$		$KPi+\forall\alpha\exists\kappa\ L_\kappa\prec_1L_{\kappa+\alpha}$			
$\Psi_{\mathbb{K}}(\varepsilon_{\mathcal{I}+1})$	$(\Delta_2^1-CA)+(BI)+(parameter\text{-free}\ \Pi_2^1-CA)$	$KPi+\exists M(Trans(M)\wedge M\prec_1V)$			

$\nu$  is any nonzero ordinal,  $\gamma$  is a limit ordinal,  $\lambda$  is of the form  $\omega^\gamma$ ,  $\mu$  is additively indecomposable.  
 $\Omega_\nu = \nu$ th uncountable cardinal,  $I$  = least weakly inaccessible cardinal.  
 $M$  = least weakly Mahlo cardinal,  $K$  = least weakly compact cardinal.  
 $\pi_n$  = least  $\Pi_{n+2}^1$ -indescribable cardinal,  $\Xi$  = least  $\Pi_0^2$ -indescribable cardinal.  
 $\Upsilon$  = least cardinal such that:  $\forall\theta < \Upsilon\exists\kappa < \Upsilon(\kappa$  is  $\theta$ -indescribable) and  $\forall\theta < \Upsilon\forall\kappa < \Upsilon(\kappa$  is  $\theta$ -indescribable  $\implies \theta < \kappa)$   
 $\mathcal{I}$  = least inaccessible cardinal such that there is a cardinal  $\xi < \mathcal{I}$  that is  $\rho$ -reducible for all  $\xi \leq \rho < \mathcal{I}$ .  
The existence of all of the above cardinals follows from the existence of a subtle cardinal.

$S_0 = (\text{Ext})+(\text{Null})+(\text{Pair})+(\text{Union})+(\text{Diff})$	
$S_1 = S_0+(\text{Powerset})$	
$M_0 = S_1+(\Delta_0^{\text{set}}\text{-Separation})$	
$M_1 = M_0+(\text{Regularity})+(\text{Transitive Containmentment})$	
$KP^- = S_0+(\text{Infinity})+(\Delta_0^{\text{set}}\text{-Separation})+(\Delta_0^{\text{set}}\text{-Collection})$	
$KP^{-\infty} = S_0+(\text{Foundation})+(\Delta_0^{\text{set}}\text{-Separation})+(\Delta_0^{\text{set}}\text{-Collection})$	
$KP = KP^{-\infty}+(\text{Infinity}) = KP^-+(\text{Foundation})$	
$KPI = KP+(\text{universe limit of admissible sets})$	
$KPi = KP+(\text{recursively inaccessible universe})$	
$KPh = KP+(\text{recursively hyperinaccessible universe})$	
$KPM = KP+(\text{recursively Mahlo universe})$	
$ZBQC = M_0+(\text{Regularity})+(\text{Infinity})+(\text{Choice})$	$NFU+(\text{Infinity})+(\text{Choice})$
$MAC = M_1+(\text{Infinity})+(\text{Choice}) = ZBQC+(\text{Transitive Containmentment})$	
$MOST = MAC+(\Delta_0^{\text{set}}\text{-Collection}) = ZBQC+KP+(\Sigma_1^{\text{set}}\text{-Separation})$	
$Z = S_1+(\text{Regularity})+(\text{Infinity})+(\Sigma_\omega^{\text{set}}\text{-Separation})$	
$ZC = Z+(\text{Choice}) = ZBCQ+(\Sigma_\omega^{\text{set}}\text{-Separation})$	
$MAC+\forall m(\beth_m \text{ exists})$	$NFU+(\text{Counting})$
$Z+(\Pi_2^{\text{set}}\text{-Replacement})$	$NFU^* = NFU+(\text{Counting})+(\text{Strongly Cantorian Separation})$
$Z+(\Pi_m^{\text{set}}\text{-Replacement})$	
$ZF = Z+(\Pi_\omega^{\text{set}}\text{-Replacement})$	$AST, GB$
$ZFC = ZF+(\text{Choice})$	$NBG = GBC = GB+(\text{Global Choice})$
$NBG+(\text{Class Forcing Theorem})$	$NBG+(\text{Clopen Class Game Determinacy})$
$MK = NBG+(\Pi_\infty^{\text{class}}\text{-CA})$	
$ZFC+(\text{there is an inaccessible cardinal})$	$ZFC+(\Pi_1^1 \text{ Perfect Set Property}), ZFC+(\Sigma_3^1 \text{ Lebesgue measurability})$
$ZFC+(\text{there are } \omega \text{ inaccessible cardinals})$	$ZFC+(\forall \alpha(\omega \leq \alpha \leq \aleph_\omega \implies  V_\alpha \cap L  =  \alpha ))$
$ZFC+(\text{there is a proper class of inaccessible cardinals})$	$ZFC+(\text{Grothendieck Universe Axiom})$
$ZFC+(\text{there is a } \Sigma_n^{\text{set}}\text{-reflecting cardinal})$	
$ZFC+(\text{there is a } \Sigma_\omega^{\text{set}}\text{-reflecting cardinal}), ZFC+(\text{Ord is Mahlo})$	
$ZFC+(\text{there is an uplifting cardinal})$	$ZFC+(\text{Resurrection Axioms})$
$ZFC+(\text{there is a Mahlo cardinal})$	
$SMAH = ZFC+(\text{there is a } n\text{-Mahlo cardinal})_{n \in \mathbb{N}}$	$NFUA = NFU+(\text{Infinity})+(\text{Cantorian Sets})$
$SMAH^+ = ZFC+\forall n(\text{there is a } n\text{-Mahlo cardinal})$	
$MK+(\text{Ord is weakly compact})$	$GPK_\infty^+ = GPK^+(\text{Infinity}), NFUB = NFU+(\text{Infinity})+(\text{Cantorian Sets})+(\text{Small Ordinals})$
$ZFC+(\text{there is a weakly compact cardinal})$	$ZFC+(\omega_2 \text{ has the tree property})$
$ZFC+(\text{there is a totally indescribable cardinal})$	
$ZFC+\forall n(\text{there is a } n\text{-subtle cardinal})$	
$ZFC+(\text{there is a totally ineffable cardinal})$	
$ZFC+\forall \alpha(\alpha < \omega_1 \implies \text{there is a } \alpha\text{-Erdős cardinal})$	
$ZFC+(0^\# \text{ exists}), ZFC+(L \models \aleph_\omega \text{ is regular})$	$ZFC+\forall \alpha(\alpha \geq \omega \implies  V_\alpha \cap L  =  \alpha ), ZFC+(\text{parameter-free } \Sigma_1^1\text{-determinacy})$
$ZFC+\forall x(x \in \mathbb{R} \implies x^\# \text{ exists})$	$ZFC+(\Sigma_1^1\text{-determinacy})$
$ZFC+\forall x(x^\# \text{ exists})$	$ZFC+(\Sigma_2^1 \text{ universal Baireness})$
$ZFC+(\text{there is a } \omega_1\text{-Erdős cardinal})$	$ZFC+(\text{Chang's Conjecture})$
$SRP = ZFC+(\text{there is cardinal with the } n\text{-stationary Ramsey property})_{n \in \mathbb{N}}$	
$SRP^+ = ZFC+\forall n(\text{there is a cardinal with the } n\text{-stationary Ramsey property})$	
$MK+(\text{Ord is measurable})$	$NFUM = NFU+(\text{Infinity})+(\text{Large Ordinals})+(\text{Small Ordinals})$
$ZFM = ZFC+(\text{there is a measurable cardinal})$	$ZFC+(\text{NS}_{\omega_1} \text{ is precipitous}), ZF+(\omega_1 \text{ is measurable})$
$ZFC+(\text{there is a measurable cardinal } \kappa \text{ such that } o(\kappa) = 2)$	$ZFC+(\text{NS}_{\omega_2} \text{ is precipitous})$
$ZFC+(\text{there is a measurable cardinal } \kappa \text{ such that } o(\kappa) = \kappa^{++})$	$ZFC+\neg \text{SCH}, ZFC+(2^{\aleph_\omega} = \aleph_{\omega+2})$
$MK+(\text{Ord is Woodin})$	$Z_3+(\text{parameter-free } \Delta_2^1\text{-determinacy})$
$ZFC+(\text{there is a Woodin cardinal})$	$ZFC+(\Delta_2^1\text{-determinacy}), ZFC+(\text{OD} \models \text{AD}), ZFC+(\text{NS}_{\omega_1} \text{ is } \omega_2\text{-saturated})$
$ZFC+(\text{there are } n \text{ Woodin cardinals})_{n \in \mathbb{N}}$	$ZFC+(\text{PD}), Z_2+(\text{AD})$
$ZFC+(\text{there are } \omega \text{ Woodin cardinals}),$	$ZF+(\text{AD}), ZFC+(L(\mathbb{R}) \models \text{AD}), ZFC+(\text{OD}(\mathbb{R}) \models \text{AD}),$
$ZF+\text{DC}+(\omega_1 \text{ is } \mathcal{P}(\omega_1)\text{-strongly compact})$	$ZF+(\text{AD}), ZFC+(\text{NS}_{\omega_1} \text{ is } \omega_1\text{-dense})$
$ZF+\text{DC}+(\omega_1 \text{ is } \mathcal{P}(\mathbb{R})\text{-strongly compact})$	$ZF+\text{DC}+(\text{AD}_{\mathbb{R}})$
$ZFC+(\text{there is a superstrong cardinal})$	
$ZFC+(\text{there is a subcompact cardinal})$	$ZFC+(V = L[\vec{E}])+\exists \kappa(\neg \square_\kappa)$
$ZFC+(\text{there is a strongly compact cardinal})$	$ZFC+(\text{Proper Forcing Axiom})$
$ZFC+(\text{there is a supercompact cardinal})$	$ZFC+(\text{Martin's Maximum})$
$ZFC+\forall n(\text{there is a proper class of } C^{(n)}\text{-extendible cardinals})$	$ZFC+(\text{Vopěnka's Principle})$
$ZFC+(\text{there is a high-jump cardinal})$	
$HUGE = ZFC+(\text{there is a } n\text{-huge cardinal})_{n \in \mathbb{N}}$	
$HUGE^+ = ZFC+\forall n(\text{there is a } n\text{-huge cardinal})$	
$ZFC+(\text{Wholeness Axiom } \text{WA}_n)$	
$ZFC+\text{I3} = ZFC+\exists \lambda(E_0(\lambda))$	
$ZFC+\text{I2} = ZFC+\exists \lambda(E_1(\lambda))$	
$ZFC+\text{I1} = ZFC+\exists \lambda(E_\omega(\lambda))$	
$ZFC+\text{I0}$	
$ZF+\text{DC}+\exists \lambda \exists j : V_{\lambda+2} \prec_{\Sigma_\omega^{\text{set}}} V_{\lambda+2}$	
$ZF+\text{DC}+(\text{there is a Reinhardt cardinal})$	
$ZF+\text{DC}+(\text{there is a Berkeley cardinal})$	