Parallax of the Moon in Terms of a World Geodetic System*

IRENE FISCHER
Army Map Service, Washington, D. C.

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The recent geodetic connection between Europe and South Africa along the 30th meridian east makes it possible to reinterpret the Greenwich and Cape observations of the lunar crater Mösting A made by Christie and Gill in the years 1905 to 1910. Crommelin's result for the correction of Hansen's value for mean parallax of the moon is expressed as a mean lunar distance in kilometers, which is compared with various results from dynamical theory, with O'Keefe and Anderson's determination from occultations, and with radar measurements. The Astrogeodetic World Datum serves as common reference system. As a by-product, the contradictions in the literature quoting Hansen's value for mean parallax are cleared up.

1. INTRODUCTION

IN the years 1905 through 1910 Christie at Greenwich and Gill at the Cape of Good Hope arranged for observations to be made of the moon, specifically of the crater Mösting A, at meridional transit on the same night whenever possible, in order to determine the mean equatorial horizontal parallax (Christie, Gill, Crommelin 1911). Crommelin made the computations and drew conclusions about the figure of the earth, regretting very much the uncertainty arising from the lack of geodetic connection between the observatories. That connection is available now.

The discrepancies between the geometric parallax as derived in this classical paper and the dynamical parallax have been analyzed by various authors. Crommelin put the entire discrepancy into the flattening of the earth and computed 1/294.4 as that flattening which would make the observed and calculated parallax agree. Lambert (1928) assumed 1/297 as certain and estimated the influence of the neglected deflections and geoidal heights. De Sitter (1927) also blamed the geoidal irregularities; he also pointed out the possibility of a systematic error due to the difference between the aspects of the crater as seen from the north and the south. Jeffreys (1948) analyzed the uncertainties, and seemed satisfied that each value was within the rms error of the other.

The present paper reinterprets Crommelin's paper by combining the old observations with new geodetic knowledge, eliminating the uncertainties due to geoidal heights and deflections. For comparison, the dynamical parallax is expressed in terms of the numerical constants considered best at present.

Two other geometric methods have been tried within recent years: by O'Keefe and Anderson (1952) using four occultations, and by Yaplee *et al.* (1958) using radar. The results from occultations are modified in the present paper by introducing recent geodetic knowledge. The results of all four methods are expressed as mean lunar distance in linear units.

2. CROMMELIN'S PROCEDURE AND RESULT

The north polar distances of Mösting A, observed when the crater was on the meridian of Greenwich and the Cape, respectively, were corrected for parallax, and the Cape observations were reduced to the time of the Greenwich observations by allowing for the change in declination during the time interval between the two transits. The difference between the geocentric values so derived gives a measure of the correction to be applied to Hansen's value of the mean parallax. The final correction $\Delta\pi$ was obtained as a mean of 100 measurements, with the alternatives of using a weighting system or applying equal weights. The calculations were carried out for two values of f: 1/293.5 and 1/300, taking the altitude of the instruments above sea level into account.

	$f=1/293.5$	$f=1/300$	
$\Delta\pi$	$+0''.50$	$+0''.12$	weighted
$\Delta\pi$	$+0''.52$	$+0''.14$	equal weights
d	1.349579	1.349728	

The chord distance d between the instruments, with the Cape instrument reduced to the Greenwich meridian, may easily be derived in units of the earth's equatorial radius from the assumption of the flattening. It is this distance that can now be computed from geodetic measurements in terms of kilometers.

3. GEODETIC CONNECTION BETWEEN THE OBSERVATORIES

In 1954 an Army Map Service field party under D. L. Mills completed the last link in triangulation along the 30th meridian east in Africa, so that we now have a continuous chain from Greenwich to the Cape. The position of the Greenwich Observatory is known on European Datum (Hayford ellipsoid) from the relative deflections published by the IAG in 1957 and the European geoid chart (Fischer 1959a). The position of the Cape on South African Arc Datum (Modified Clarke 1880 ellipsoid) is given in the 1894 Report of the South African Geodetic Survey. The conventional extension of the European Datum to South Africa was corrected for the separation between geoid and ellipsoid along the meridian (Fischer 1959b) and conver-

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sion formulas between the two geodetic systems were derived at Buffelsfontein in South Africa, the datum point of the South African Arc Datum. With both observatories referred to the same geodetic system, the chord distance between the instruments could be computed and modified in order to have the crater observations referred to the same meridian. The ratio between this distance of 8608.611 km and the distance d computed above in units of the equatorial radius leads to fictitious values for the equatorial radius consistent with Crommelin's assumptions:

$$a_{293.5}=6378.739 \text{ km}, \quad a_{300}=6378.033 \text{ km}.$$

From these we can compute the mean lunar distance $a/\sin(\pi_☾+\Delta\pi)$ in kilometers, if we know Hansen's value for mean parallax, to which Crommelin's corrections apply.

4. HANSEN'S VALUE OF MEAN PARALLAX

There are conflicting quotations of Hansen's value of the mean parallax in the literature. Crommelin quotes Newcomb (1895, p. 194) as having raised Hansen's value by 0''.45, and there Newcomb gives his own adopted value as 3422''.68 when referred to the Clarke 1880 ellipsoid ($a=6378.249$ km, $1/f=293.46$); thus by inference we know that Crommelin considered 3422''.23 as Hansen's value. This value is also quoted explicitly by Newcomb (1912), not with a verbatim reference from Hansen, but derived by sine to angle correction from Hansen's constant in the sine of the parallax, which Newcomb decided to raise by 0''.40, leading to $\pi_☾=3422''.63$ for Helmert's flattening 1/298.26. The authority for this value, Hansen's constant in the sine of the parallax, is given by Newcomb (1880, 1912) mostly as 3422''.06, but sometimes as ''.07 or ''.09.

On the other hand, Schrutka-Rechtenstamm (1955, 1956) repeatedly quotes Hansen's mean parallax as 3422''.28; and the *Berliner Jahrbuch* gives 57'2''.27 with reference to Peters (1895).

A discrepancy of 0''.05 changes the implications of Crommelin's results by more than 5.5 km in the mean distance of the moon.

$f=1/293\cdot5$	$f=1/300$	
3422''.23 (28)	3422''.23 (28)	Hansen's $\pi_☾$
3422''.73 (78)	3422''.35 (40)	$\pi_☾+\Delta\pi$ weighted
.75 (80)	.37 (42)	equal weights
6378.739 km	6378.033 km	equatorial radius
384 421.1 km (15.5)	384 4213 km (15.6)	mean distance weighted
384 418.9 km (13.3)	384 419.0 km (13.4)	equal weights

On the other hand, the change in flattening has practically no effect on the result when expressed in terms of kilometers (the variation being 100 to 200 m), while the resulting mean parallax is very different.

To remove the uncertainty due to Hansen's $\pi_☾$ we have searched Hansen's main publications (1857, 1864): there is no specific mention of a value of the mean parallax, but an earlier communication (Hansen 1840), to which Newcomb referred, gives the numerical derivation of the constant in the sine parallax 3422''.06. In Hansen's tables (1857), however, a slightly different value is used for the constant part of the inequalities in the logarithm of the radius vector of the moon, and this change produces one of 0''.06 in the mean parallax, bringing it up to the value quoted by Schrutka-Rechtenstamm, namely 3422''.28.

Incidentally, Newcomb's statement (1880) that Hansen contradicts himself by deriving 3422''.06 as constant in sine parallax and using 3422''.09 in the tables of the same paper (Hansen 1840), is a confusion of the constant in the *sine* (3422''.06) and the "*incomplete*" (unvollständige) constant in the *logarithm* of the sine($\log_{10}\sin 3422''.09$); the latter is made "*complete*" (vollständig) by subtracting the constant term in the logarithm of the radius vector of the auxiliary ellipse.

If Crommelin had used the correct value for Hansen's mean parallax and, incidentally, also quoted himself correctly (on p. 539 he uses +''.49 in his observational equation, while on p. 536 he gives his result as +''.50 weighted and +''.52 for equal weights), then his conclusions about the flattening of the earth would have been 1/295.5 (weighted) or 1/296.1 (equal weights) instead of the much quoted 1/294.4. Brown's (1914, 1915) choice of 1/294.0 for his lunar tables, although not based on Crommelin's result, was very much influenced by the apparent agreement, which gave his result from lunar theory a middle value between Newcomb's (1/293.5) and Crommelin's (1/294.4) values.

Even if Crommelin had used the correct values, however, his procedure of deriving the flattening of the earth from equating his geometrical with Newcomb's gravitational parallax is objectionable because of two tacit assumptions: (1) that all of Newcomb's factors in the gravitational parallax other than the flattening are the true values so that a change in the latter will not affect the others; (2) that his rigid relation between assumed flattening and resulting observed mean parallax, as shown in his table on p. 540, is a real one. The first assumption is probably justifiable on grounds of expediency, speculating about one constant with a large uncertainty, while assuming for the others the best (i.e., most recently observed) values as true (or with small uncertainties) even though later values often are outside the quoted uncertainties. The second assumption is more serious, because it confuses a reasonable technique of guessing the distance between the observatories in the absence of a geodetic connection, with a statement about the earth as a whole. The elements in the geometric problem are essentially three points of a huge plane triangle in space, or four points if the center of the earth is included. This configuration is unaffected by the shape of the earth at the poles, in Africa, or elsewhere.

5. GEOMETRIC PARALLAX AS REFERRED TO THE ASTROGEODETIC WORLD DATUM

The Astrogeodetic World Datum (Fischer 1960a) is the result of fitting an ellipsoid of revolution with unknown parameters and position to the bulk of all geodetic observations available at present, which include astrogeodetic as well as gravimetric data and satellite observations. The resulting parameters are $f=1/298.3$ and $a=6378.166$ km.

Although the chord distance between the observatories is an invariant under coordinate transformation, the introduction of the World Datum into the problem of the parallax is important because it represents our present idea of the position of the earth's center in relation to these observatories.

The change to the World Datum produces changes in Crommelin's listed results for $f=1/293.5$ that vary from $-0''.301$ to $-0''.307$, the mean being $-0''.304$ for both of Crommelin's weighting systems:

Weighted	Equal weights	
$+''.196$ $(+.241)$	$+''.213$ $(+.258)$	WD corr. to Hansen
$3422''.28$	$3422''.28$	Hansen's $\pi_☾$
$3422''.476$ $(.521)$	$3422''.493$ $(.538)$	WD $\pi_☾$
6378.166 (.250) km	6378.166 (.250) km	equatorial radius
384 415.1 (15.1) km	384 413.2 (13.2) km	corresp. mean distance

The figures in parentheses refer to an alternative reference system. Since the geodetic observations available at present cover only a relatively small part of the earth's surface it is conceivable that the World Datum derived from the present material will undergo significant changes when the geodetic coverage will be significantly enlarged. O'Keefe *et al.* (1959) derived 6378.255 km as equatorial radius of the earth by forcing agreement between the dynamical and geometric parallax, the latter given in the form of Yaplee's lunar radar measurements (see Brouwer *et al.* 1960). In order to show that even such a big difference of 90 m in equatorial radius has no essential influence on the purely geometric problem between two observatories and the moon, an ellipsoid of revolution with parameters $f=1/298.3$ and $a=6378.250$ km was fitted to the same geodetic material as before. It can be seen that the result is quite different if expressed as mean parallax, but unchanged if expressed as mean distance in terms of kilometers.

6. DYNAMICAL PARALLAX

The dynamical mean horizontal equatorial parallax $\pi_☾$ is defined by

$$\sin\pi_☾=\frac{a}{a'/(1+\nu_4)}$$

or

$$\pi_☾'^3=\frac{a^3n'^2(1+\nu_4)^3}{GM(1+\mu)(86\,400)^2\sin 1''},$$

where a is the equatorial radius of the earth, M the mass of the earth, G the gravitation constant, and a', ν_4, $\pi_☾'$, n', μ are taken from De Sitter and Brouwer's (1938) notation.

The choice of the equatorial radius as terrestrial unit of length in this definition was considered by Newcomb (1912) "a defect in astronomical practice" since the mean radius is the one which is "best determined by geodetic measurements, and for which gravity is best ascertained from pendulum observations." De Sitter, Jeffreys, and others concur.

Accordingly, Newcomb uses gravity at mean latitude, assumed to be the same as if the mass of the earth were concentrated in its center, corrects it for centrifugal force to obtain the mean attraction g_m. The mass of the earth is set equal to the mass of a mean sphere with radius r_m and attraction g_m, so that $GM=g_m r_m^2$. The flattening becomes now part of the parallax expression because of the (arbitrary) presence of the equatorial radius.

Thus the numerical value of the dynamical parallax depends on estimates of gravity, radius, flattening of the earth, and mass of the moon, all of which have changed very much with the times. The efforts of Newcomb (1895) and De Sitter and Brouwer (1938) for at least theoretical consistency between these and the other astronomical constants are being renewed at present.

In Table I the numerical constants considered best by Hansen, Newcomb, Brown, Lambert, and at present, are listed together with the resulting dynamical parallax and the corresponding lunar distance. The values marked a are inferred from information given in other form:

Hansen (1840, 1857) gives length of the seconds pendulum and centrifugal correction instead of g_m. The Bessel (1837) ellipsoid is referred to in the theoretical derivations, but the *Tables de la Lune* are based on the rounded value of 1/300, which produces the slight difference of $0''.009$. Here is the explanation why Peters quoted $3422''.27$ as Hansen's mean parallax (derived with $1/f=300.7047$). Hansen's value for $(1+\nu_4)$ can be deduced from a comparison of his procedure with Newcomb's, if allowance is made for Hansen's using the anomalistic instead of the sidereal period of the moon, and an auxiliary instantaneous orbital ellipse with constant parameters: Hansen gives numerically the constant in the logarithm of sine parallax as

$$\log\frac{a[\![.0028398]\!]}{\bar{a}[\![.0003275]\!]},$$

where double bracket means antilogarithm, and $\bar{a}$=orbital semimajor axis, defined by $\bar{a}^3 4\pi^2/T_a^2=GM\times(1+\mu)$. He gives the correction from the antilogarithm of this constant to the constant in the sine parallax as

TABLE I. Constants for dynamical parallax.

	Ellipsoid	$1/f$	a 6370 km+	r_m 6370 km+	g_m gal	GM megameter³/kilosec²	$1/\mu$	$1+\nu_4$	π 57′+	$a/\sin\pi_{☾}$[a] km
Hansen 1857	Bessel 1837	300.7047	7.1569	0.063	981.980[a]		80	1.000919[a]	2″.27[a]	384 377.4
		300							2″.28[a]	384 376.3
Newcomb 1895	Clarke 1880	293.465	8.249	1.004	982.030	398.603 Hill	81.45	1.000907	2″.68	384 397.2
Newcomb 1912	Helmert 1903	298.26	7.980	0.843	981.996	398.569 Helmert	81.45	1.000907	2″.63	384 386.6
Brown 1915	modified from Newcomb 1912	294.0	8.074[a]	0.843[a]	981.996[a]	398.569[a]	81.53	1.0009076	2″.70[a]	384 384.7
Brown and Hayford	Hayford	297.0	8.388	1.229[a]	982.038[a]	398.634[a]			2″.678[a]	384 405.8
					982.022[a]	398.628[a]			2″.70[a]	384 403.7
Lambert 1928	Hayford	297.0	8.388	1.229[a]	982.041[a]	398.636[a]	81.53	1.0009078[a]	2″.68(2)	384 405.4
World Datum		298.3	8.166	1.0388	982.0222	398.604	81.375	1.0009076	2″.624	384 398.5
							81.219		2″.597	384 401.5

[a] By inference.

2″.71/206265, so that

$$\sin\pi_{☾} = \left[\frac{a^3 4\pi^2}{GM(1+\mu)}\right]^{\frac{1}{3}}\left[\frac{1.0058016}{T_a^{\frac{2}{3}}} + (.4471)(10^{-7})\right]$$

for his adopted constants (Table I). The second factor should be equivalent to $(1+\nu_4)/T_s^{\frac{2}{3}}$; thus

$$1+\nu_4 = (T_s/T_a)^{\frac{2}{3}}(1.0058016) + T_s^{\frac{2}{3}}(0.4471)(10^{-7}) = 1.000919.$$

The derivation given in Jordan-Eggert (1941) is erroneous.

Brown (1914, 1915, 1919) gives the sine mean parallax 3422″.54 as the basic value, and his adopted changes from the constants used by Newcomb in 1912, which do not include r_m and g_m; the different flattening implies a small modification in a.—Brown's statement (1914, 1918) that his adopted 2″.54 "agrees exactly with the value which Crommelin found from observation (for the same flattening)" obviously refers to Crommelin's listing 0″.47 for $f=1/294$; then $\Delta\pi_{☾}' =$ ″.31, which together with Hansen's 2″.23 would give exactly 2″.54. Hansen's value, however, was *not* 2″.23 but 2″.28, which makes Brown's statement erroneous. On the other hand, Brown had started computing his Tables with 2″.70 as sine mean parallax, which was an observational value (Brown 1896, p. 124, 131) of the same type as Crommelin's, derived by Breen (1863) and later by Stone (1865) from grouping and averaging various pairs of observations at the Cape of Good Hope and at Greenwich, Edinburgh, and Cambridge, referred to the Bessel ellipsoid and $f=1/300$. Brown, however, decided later to reject this value in favor of a dynamically derived one.—In the *American Ephemeris* this dynamical value is used in connection with the Hayford (International) ellipsoid and the International gravity formula, which produce a value of 2″.678. This inconsistency could be resolved by changing g_m to 982.022 gals, close to the value used for the World Datum.

Lambert (1928) gives equatorial gravity 978.052 gals and $\log(1+\nu_4)^3 = 0.0011822$. The third decimal in 2″.682 is uncertain.

The World Datum constants are those of the Astro-geodetic World Datum (Fischer, 1960a) as employed earlier, combined with Brown's $(1+\nu_4)$ and an alternative of Rabe's (81.375±.026) and Delano's (81.219 ±.030) values (Makemson *et al.* 1961) for the mass of the moon. These alternatives were introduced to show today's uncertainty, each value quoted with an rms error excluding the other; they make a difference of 3 km in the lunar distance. From Table 7 of the Makemson paper it appears that this so-called Delano value was derived from Delano's constant of the lunar equation $L=6.4430$ using an adopted solar parallax of 8″.790. With 8″.795 for the solar parallax, as indicated by recent radar measurements of the distance to Venus, (Jet Propulsion Laboratory, Lincoln Laboratory, Jodrell Bank), Delano's L gives $1/\mu = 81.268$ (almost Spencer Jones' value as modified by Jeffreys), leading to $\pi_{☾} = 57'2''.605$ and $a/\sin\pi_{☾} = 384\,400.6$ km for the World Datum.—The mean attraction g_m includes the decrease by 12.8 mgals proposed by the Special Study Group No. 5 of the IUGG (Morelli 1960) as well as a decrease of 2 mgals derived by Kaula (1959).

The distances of the moon, computed from dynamical theory with these sets of constants very by 30 km; none of these distances matches Crommelin's result. Since there was no other geometric determination until recently, the often-stated disagreement between dynamical and geometrical parallax actually referred to various repeatedly modernized dynamical values in comparison to the one geometrical value. The comparison emphasizes the need to have other geometric determinations.

7. LUNAR DISTANCE FROM OCCULTATIONS

One such is that by O'Keefe and Anderson (1952), who determined the moon's distance from observations of four occultations at nine stations in the United States. This is comparable to Crommelin's determination in the sense that one and the same point on the moon is observed from two or more stations.

The published numerical results of this paper are between those implied by the Crommelin paper and those derived from dynamical theory. A recomputation inserting recent geodetic information, however, produces a significant change.

TABLE II. Modification of O'Keefe and Anderson's (1952) computation, to be compared with Table III on p. 120 of their publication.[a]

Station	Star	d m	$\delta(d)$ m	$\delta\xi$ m	$\delta\eta$ m	σ m	Tangent Hayford $\Delta\sigma=\sigma-k$ m	$\langle\Delta\sigma\rangle=\Delta\sigma$ −mean	World Datum $\Delta\sigma'$ m	$\langle\Delta\sigma'\rangle=\Delta\sigma'$ −mean
1	2540	+10.5	−2.30	−.4	−2.1	1 740 128.9	2141.3	−7.05	2030.7	−1.1
2	2540	−1.5	−5.67	−1.9	−5.1	1 740 143.0	2155.4	+7.05	2032.9	+1.1
3	501	−2.0	−28.0	−21.1	−9.3	1 740 930.7	2943.0	+.45	2781.9	+2.1
4	501	+2.5	−19.5	−16.2	−7.0	1 740 929.7	2942.1	−.45	2777.7	−2.1
5	348	+1.5	−16.5	−13.5	−7.8	1 740 054.0	2066.4	−12.7	1862.7	−12.6
6	348	+1.0	−19.0	−15.9	−9.1	1 740 079.4	2091.8	+12.7	1887.9	+12.6
7	996	+3.5	−19.5	−13.7	−7.1	1 739 851.8	1864.2	+16.1	1664.7	+16.8
8	996	−18.5	−47.5	−37.0	−17.1	1 739 820.0	1832.3	−15.8	1631.9	−16.0
9	996	−12.5	−42.5	−34.9	−15.2	1 739 835.4	1847.8	−.3	1647.0	−.9

[a] d = geoid above ellipsoid. $\delta(d)$ = discrepancy between new and old geoidal heights.

The new geodetic information available now, but not at the time of the occultations refers to the geoidal map of North America (Fischer 1960b). For reading the geoidal heights of the nine stations, a detailed, large-scale map was used, however, of which the map in the reference is only a simplified version. The geoidal heights and their differences from the estimates used by O'Keefe are listed in Table II. The rest of the table contains the computational consequences of this change in geoidal heights and will be understood in connection with Table III on p. 120 of the O'Keefe and Anderson publication. We made two solutions: one for the Hayford ellipsoid tangent to the Clarke 1866 ellipsoid at Meades Ranch, differing from O'Keefe's Solution III only through the change in geoidal heights; the other for the World Geodetic System. In O'Keefe's Eq. (20) on p. 110,

$$\Delta\sigma=b_1\Delta u+b_2\Delta v+b_6\Delta p+U,$$

$\Delta u=\Delta v=0$ for the first solution, and $\Delta u=+16.9$ m, $\Delta v=+205.6$ m for the second solution. Least-squares solutions give

$$\Delta p_1=+.4 \text{ km (Tangent Hayford)},$$
$$\Delta p_2=-2.8 \text{ km (World Datum)}.$$

The basic value p to be corrected by Δp is the one consistent with Brown's lunar tables in conjunction with the Hayford ellipsoid. O'Keefe used $p\sim 384\ 403$ km; more closely

$$p=\frac{6378.388 \text{ km}}{3422.540 \sin 1''}=384\ 403.7 \text{ km},$$

leading to the following results:

Tangent Hayford: $p_1=384\ 404.1$ km and $\pi_{☾.1}=3422''.693$,

World Datum: $p_2=384\ 400.9$ km and $\pi_{☾.2}=3422''.603$.

Since it is now generally recognized that the Hayford ellipsoid is not a good fit as a World ellipsoid, $p_2=384400.9$ km may be regarded as the updated result of the occultation method, agreeing very well with the dynamical result of about 384 400±2 km, referred to the same World Datum.

The new geoidal heights were inserted also into O'Keefe's Solutions I and II. These solutions had been designed to extract a value for the earth's equatorial radius from the observed lunar distance by utilizing the dynamical relationship between them. The same approach was used by Crommelin to extract a value for the earth's flattening from the observed parallax. The procedure requires assumptions about the other dynamical constants involved. The uncertainty in our knowledge of the mass of the moon alone causes a large scatter in the derived value for equatorial radius as seen below. Inserting three current estimates:

Rabe	$1/\mu=81.375$,	recommended by Makemson *et al.* (1961),
Spencer Jones	$1/\mu=81.27$,	adopted in O'Keefe's paper,
Delano	$1/\mu=81.219$,	an alternative used in the World Datum

(In the following discussion the values resulting from $1/\mu=81.268$ are almost the same as those given for Spencer Jones.) O'Keefe's Eq. (35) on p. 112 reads

$$\Delta p=\frac{2}{3\sin\pi}\Delta a+\frac{\cot\pi}{\sin\pi}\begin{cases}.7731 \text{ for Rabe,}\\ 1.3297 \text{ for Spencer Jones,}\\ 1.7626 \text{ for Delano,}\end{cases}$$

leading to the following results, referred to the fixed flattening 1/297:

I. With Rice's 1952 estimate of gravity deflections at Meades Ranch

	a	$\pi_☾$	$a/\sin\pi_☾$
Rabe	6 378 270 m	57′2″.651	384 401.7 km
Spencer Jones	6 378 199	2″.620	384 400.9
Delano	6 378 143	2″.596	384 400.2

II. Without gravity estimate

	a	$\pi_☾$	$a/\sin\pi_☾$
Rabe	6 378 249	2″.647	384 400.9
Spencer Jones	6 378 178	2″.616	384 400.1
Delano	6 378 122	2″.592	384 399.4

It can be seen that the scatter in the resulting values for equatorial radius is too large to make the derivation significant; but it also can be seen that these values are balanced by a compensatory variation in $\pi_☾$ to give a fairly stable result for the lunar distance.

Thus the significant result of the O'Keefe and Anderson paper is the determination of the distance of the moon, not that of the earth's equatorial radius.

8. LUNAR DISTANCE FROM RADAR MEASUREMENT

The source for lunar distance carrying the most conviction at the present time is, of course, Yaplee's current determination by radar. So far only preliminary results and tentative interpretations of a one day (Yaplee *et al.* 1958; O'Keefe; Eckels *et al.* 1959) and of a one month series of observations (O'Keefe; Roman *et al.* 1959; Brouwer *et al.* 1960) have been published. The preliminary result is 384 402±1.2 km.

Combining this geometric result with the dynamical parallax O'Keefe derived an equatorial radius of 6 378 255 m, adopting as flattening of the earth 1/298.25, equatorial gravity 978.030 gals, and Rabe's value 1/81.375 for the mass of the moon (Brouwer *et al.* 1960). For Delano's value $\mu = 1/81.219$, however, the resulting equatorial radius would decrease to about 6 378 182 m.

Yaplee's result for the distance of the moon combines the actual radar measurements to the surface of the moon with an assumption of 1740 km for the radius of the moon. The reason for this choice is not mentioned. In the occultation method the radius of 1738 km had been used in conformity with the American Ephemeris. It would seem that the radar result, if computed with 1738 km for the sake of a fair comparison, should decrease to about 384 400 km. The corresponding derivation of the equatorial radius would lead to about

6 378 205 m (Rabe),
6 378 155 m (Spencer Jones),
6 378 132 m (Delano).

The statement that the equatorial radius of the earth is 6 378 255 m on authority of the radar measurements should be taken *cum grano salis*.

9. CONCLUSION

Referred to the World Geodetic System ($f = 1/298.3$, $a = 6378.166$ km) the dynamical mean parallax is

$$3422''.610 \pm .013,$$

and the corresponding distance of the moon is

$$384400 \pm 2 \text{ km},$$

the uncertainty largely due to a realistic estimate of the uncertainty in the mass of the moon. The dynamical result agrees very well with the observational results

by occultations 384 400.9 km and 3422″.603 (O'Keefe, recomputed),
and by radar 384 402 km (Yaplee, for $r_☾ = 1740$ km),
384 400 km (for $r_☾ = 1738$ km).

It does not agree with Crommelin's result: 384 415 km (weighted) or 384 413 km (equal weights).

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