Introduction to Quantitative Geology
Lecture 9
Rock and ice as viscous materials

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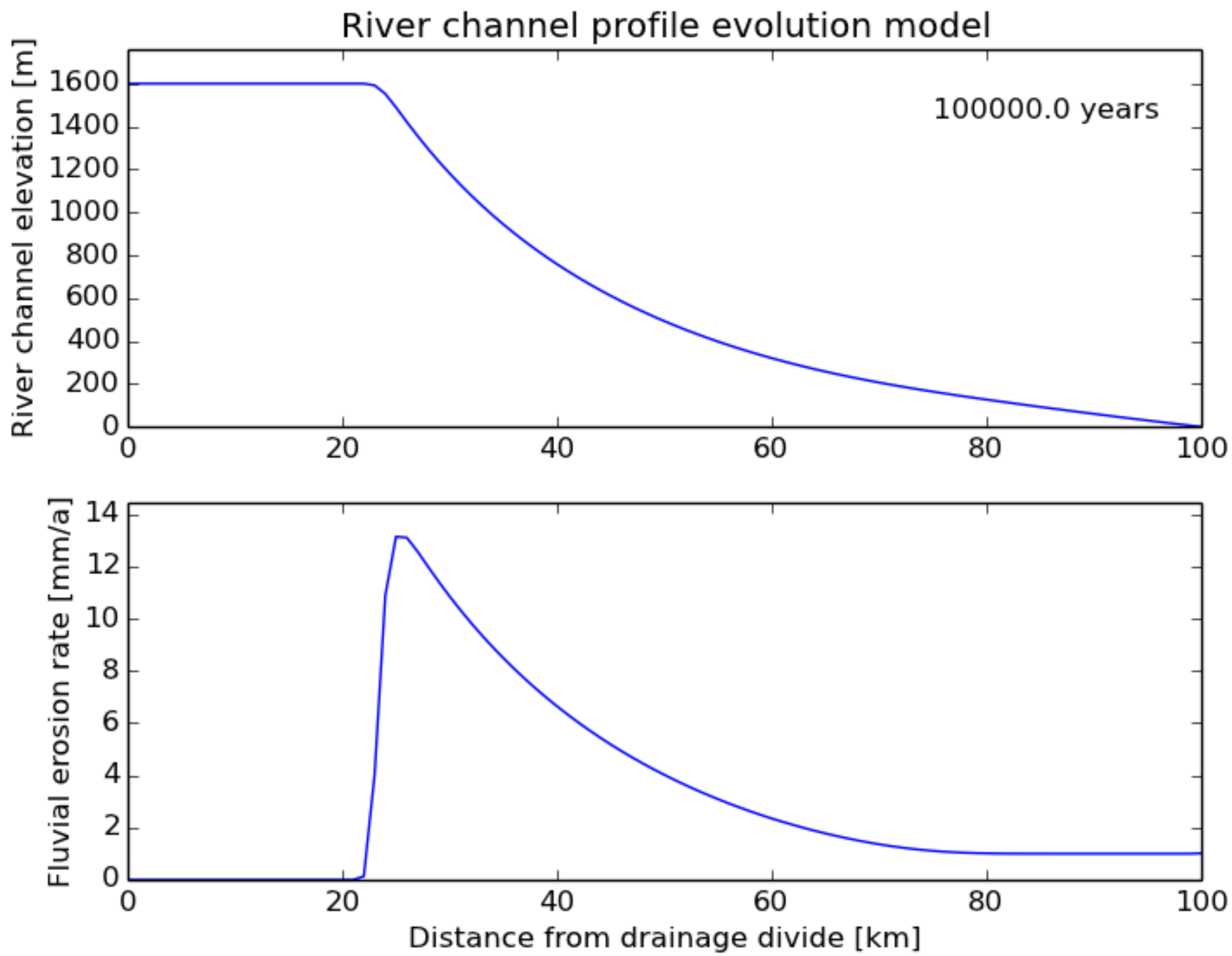
13.4.2015

Laboratory exercise 4



- Any questions or problems?

Mid-term feedback summary

- Lecture pace (too fast, too slow or about right?):
  - About right 😃
- Lecture content (too simple, too difficult or about right?):
  - Mostly about right 😀
  - Some things are too simple, some a bit difficult
  - Some material is only explained briefly making understanding difficult
  - Some of the mathematical explanations are a bit simple, but still useful

Mid-term feedback summary

- Exercise content (too simple, too difficult or about right?):
  - Mostly about right, parts are a bit difficult 😊
  - Exercises are heavily based on programming with less geology than the lectures
  - A bit difficult and time consuming to learn Python so quickly
- Hours per week on this course:
  - 2-3 to ~15 😊
  - 25 hours per credit point, 125 hrs expected for this course

Mid-term feedback summary

- Most helpful aspects of the course:
  - Combination of geology and programming exercises 😄
  - Learning to use Python and NumPy 😄
  - Visualisation of the advection and diffusion equations 😃
  - Clear exercises with no trick questions 😄
- Least helpful aspects of the course:
  - Can get stuck with programming issues and not be able to progress to understand the geological problems 😐
  - Lecture on geochronological ages unrelated to other course material 😐

Mid-term feedback summary

- Suggestions for improving the course: 😄
    - More time could be spent to qualitatively explain lecture material
    - Part of the exercises could be "brainless", where a set of instructions are followed to introduce concepts; other part could be the like recent exercises
    - Younger students could also be encouraged to take this course - Not just 3rd year and up
    - Questions can sometimes be hidden in the long exercise handouts, it would help to make the questions more clear

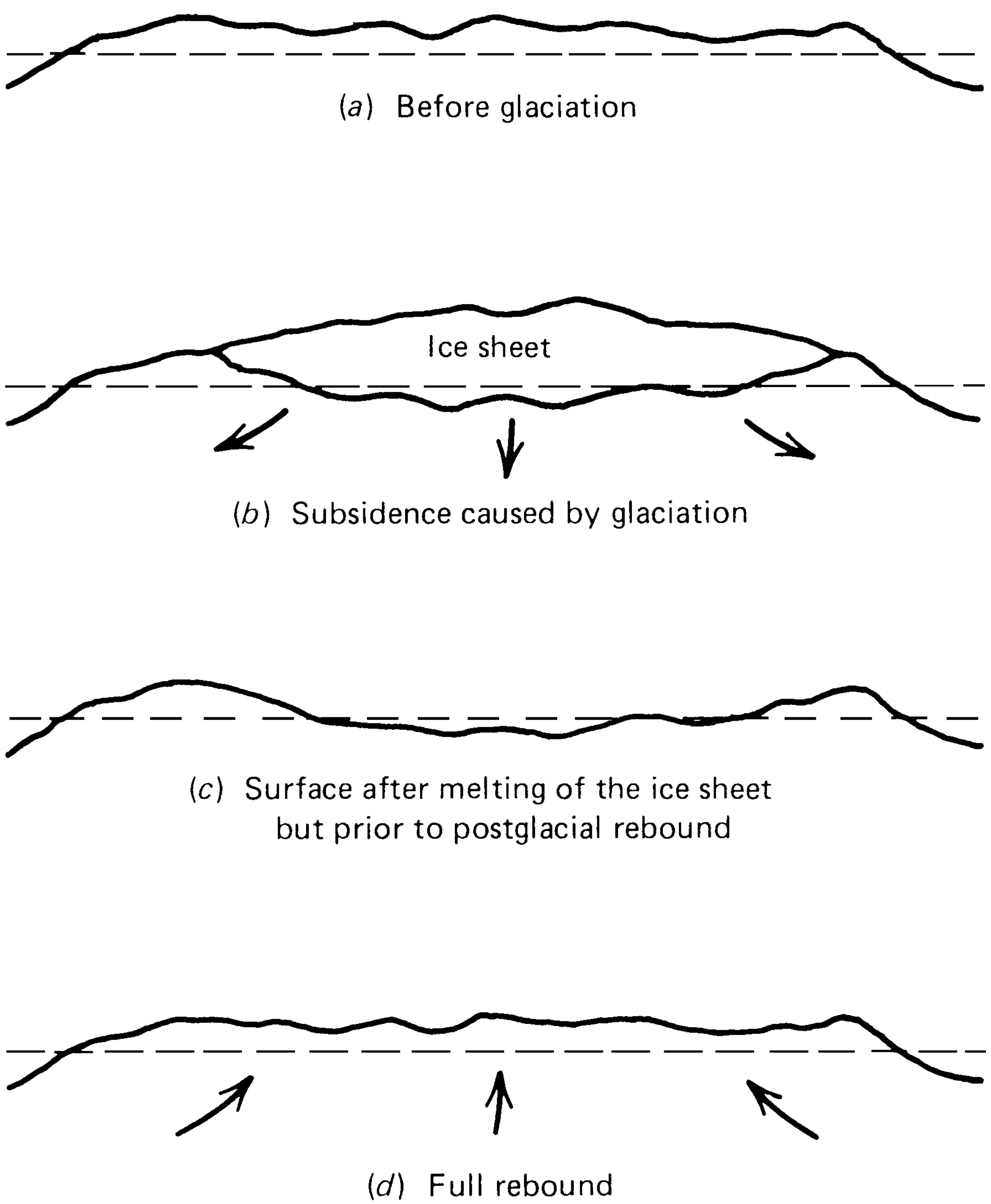
Goals of this lecture

- Introduce the basic relationship for viscous flow of rock and ice
- Explore two different end-member types of viscous flow in a channel
- Discuss the effects of temperature on viscosity and nonlinear viscosity

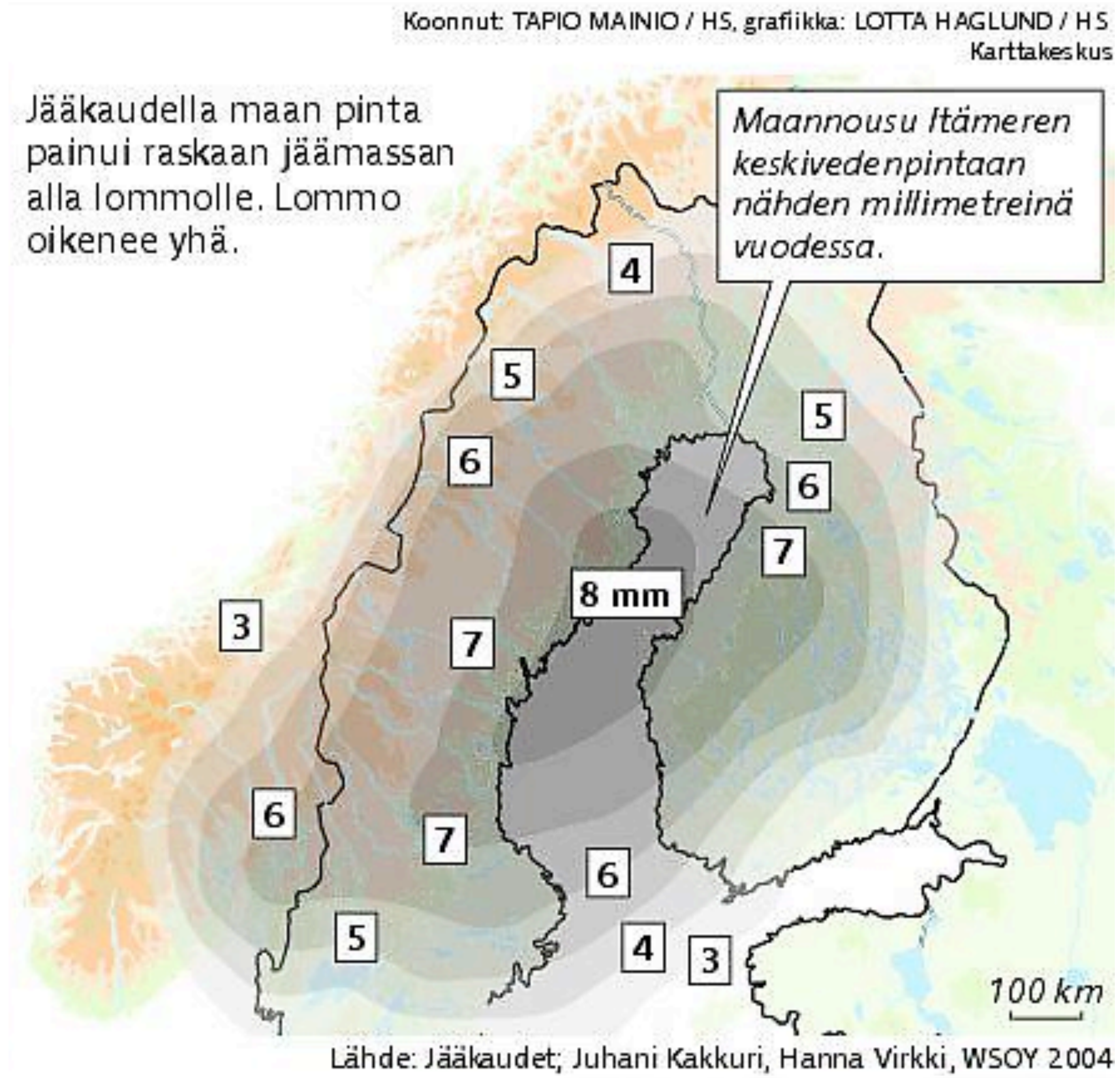
Examples of viscous flow: Alpine glaciers

- **Alpine glaciers flow downhill** under their own weight

Glacio isostatic adjustment



Turcotte and Schubert, 2002



Helsingin Sanomat, 19.3.2012

Surface uplift due to glacio isostatic adjustment is controlled by **flow of the underlying asthenosphere**

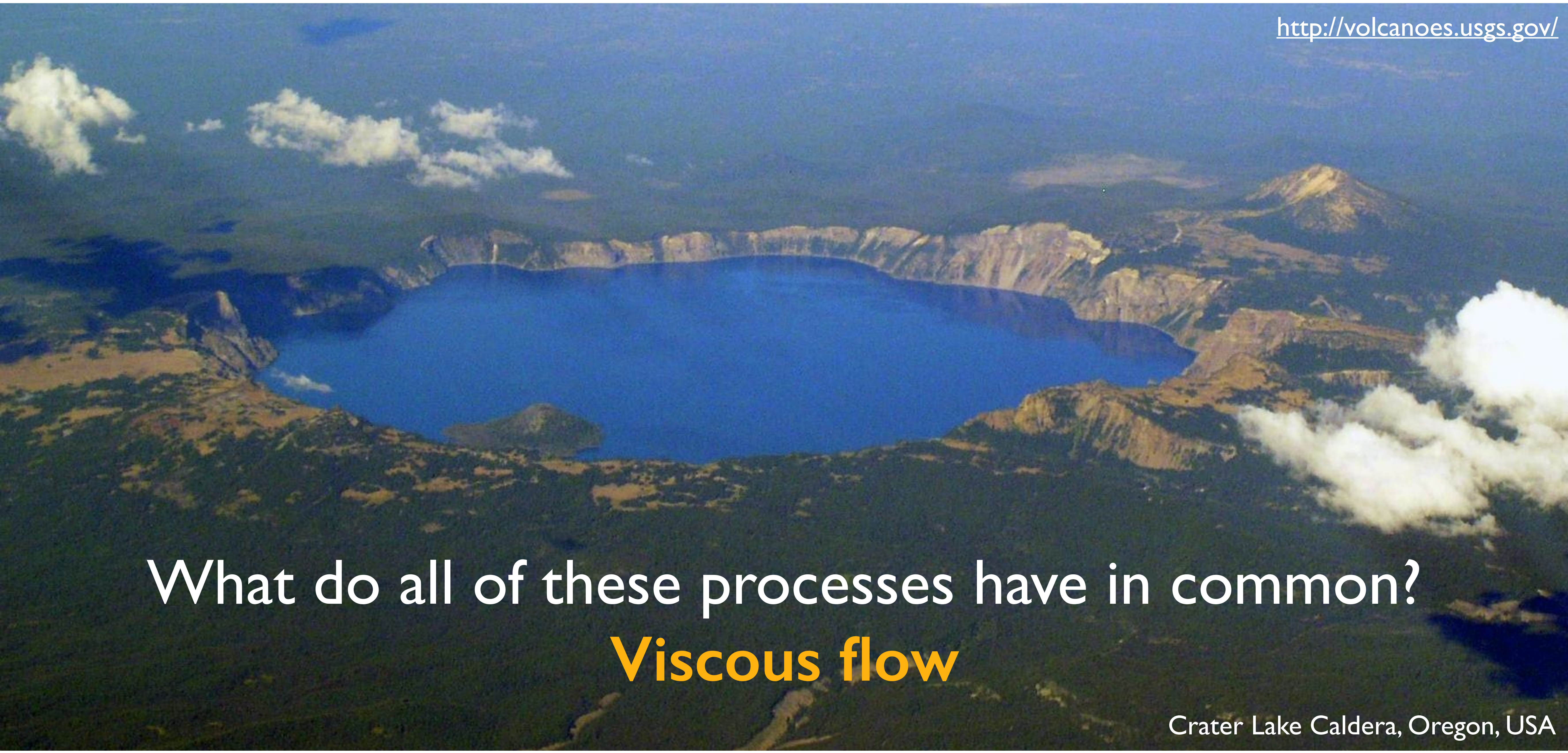
Lava flows…


A'a flowing during the 1984 eruption
of Mauna Loa, Hawai'i, USA
US Geological Survey

or resists flow (explodes)

Crater Lake Caldera, Oregon, USA

or resists flow (explodes)



What is a fluid?

- **Fluid**: Any material that flows in response to an applied stress
  - Deformation is continuous
  - Stress is proportional to strain rate

$$\tau \propto \frac{du}{dz}$$

where τ is the shear stress, du/dz is the velocity gradient (equivalent to strain rate) and u is the velocity in the x-direction

- **What does this suggest for deforming rock or ice?**

Viscosity, defined

http://en.wikipedia.org

- Constant of proportionality η is known as the **dynamic viscosity**, or often simply **viscosity**

1-D: $$\tau = \eta \frac{du}{dz}$$

- Viscosity has units of Pa s (Pascal seconds) or kg m^{-1} s^{-1}
- You can think of viscosity as a resistance to flow
  - Higher viscosity → more resistant to flow, and vice versa
- The terms kinematic viscosity and bulk viscosity (or compressibility) are not the same thing as the dynamic viscosity

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Approximate viscosities of common materials

Material	Viscosity [Pa s]
Air	10^{-5}
Water	10^{-3}
Honey	10^{1}
Basaltic lava	10^{3}
Ice	10^{10}
Rhyolite lava	10^{12}
Rock salt	10^{17}
Granite	10^{20}

A honey dipper works because of the viscosity of honey

- Viscosity of natural materials is hugely variable
  - Range of almost 20 orders of magnitude for rocks and lava

Newtonian (linear) viscosity

$$\tau = \eta \frac{du}{dz}$$

- A **Newtonian material** has a <u>linear relationship between shear stress and strain rate</u>
  - In other words, <u>η is a constant value</u> that does not depend on the stress state or flow velocity
- Air, water and thin motor oil are practically Newtonian fluids
- Rocks rarely deform as Newtonian fluids

Reynolds number: Laminar versus turbulent flow

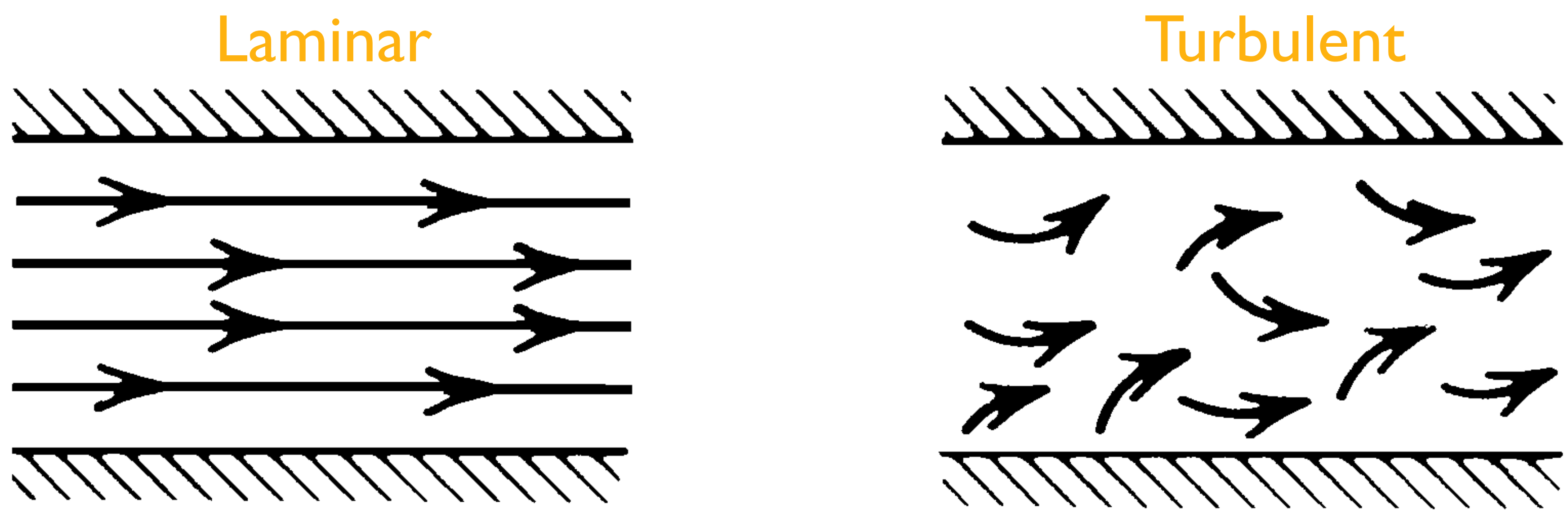


Fig. 6.8, Turcotte and Schubert, 2002

- We have assumed all of our flows are laminar thus far
- Flow behavior can be estimated using the **Reynolds number**

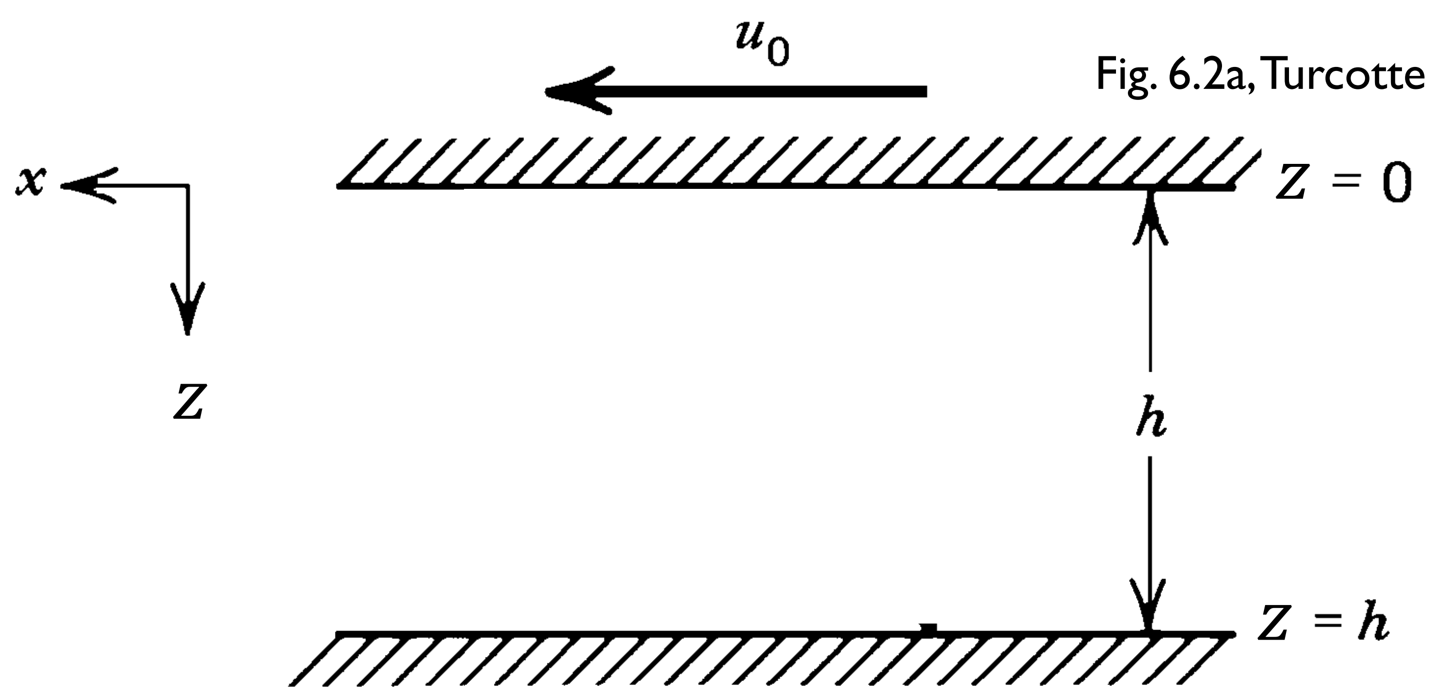
$$\mathrm{Re} = \frac{\rho \bar{u} D}{\eta}$$

where ρ is the fluid density, $\bar{u}$ is the mean velocity and D is the pipe diameter

- Flows typically become turbulent for $\mathrm{Re} > 2200$

Styles of viscous flow: Couette flow

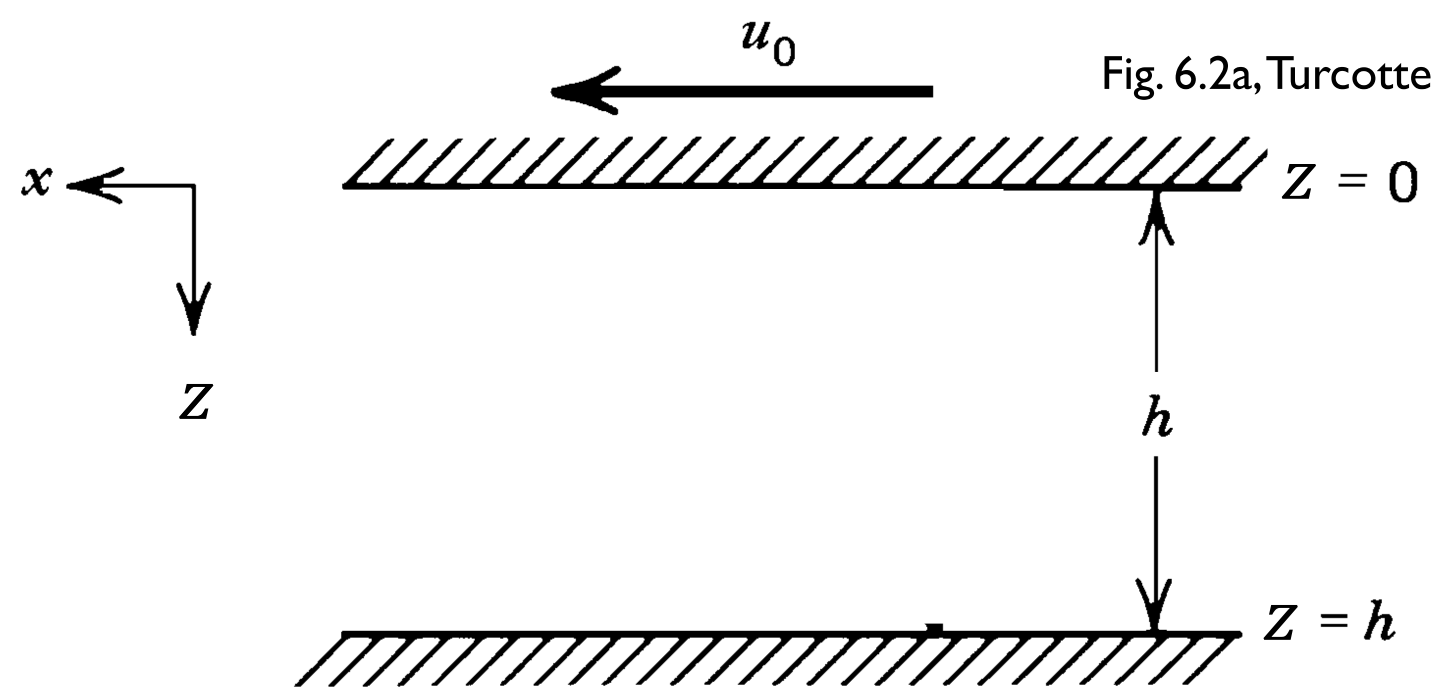
Fig. 6.2a, Turcotte and Schubert, 2002



(a) Couette flow $\frac{dp}{dx} = 0, u_0 \neq 0$

Styles of viscous flow: Couette flow

Fig. 6.2a, Turcotte and Schubert, 2002

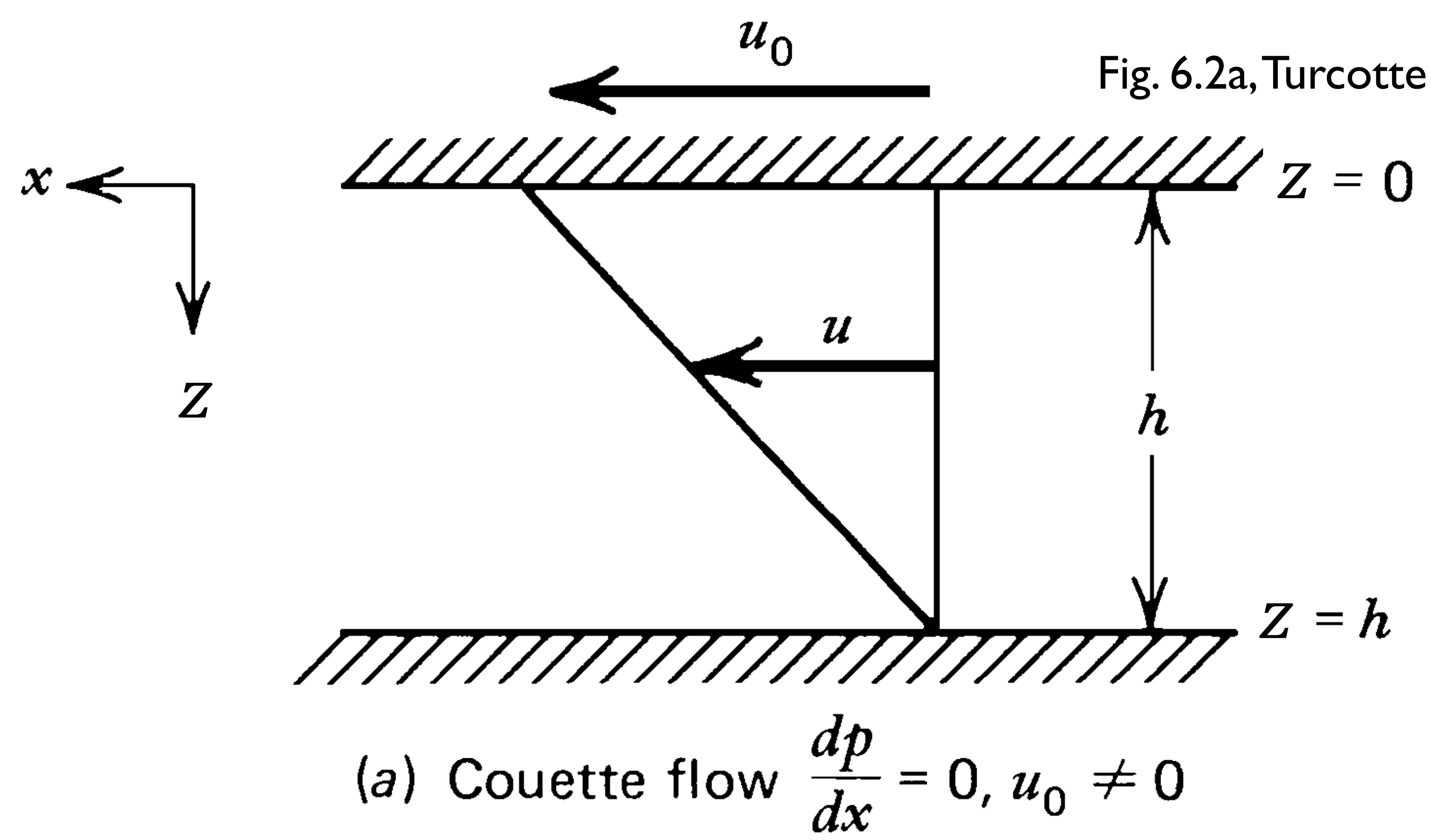


(a) Couette flow $\frac{dp}{dx} = 0,\ u_0 \neq 0$

- **What is the velocity distribution across this channel?**

Styles of viscous flow: Couette flow

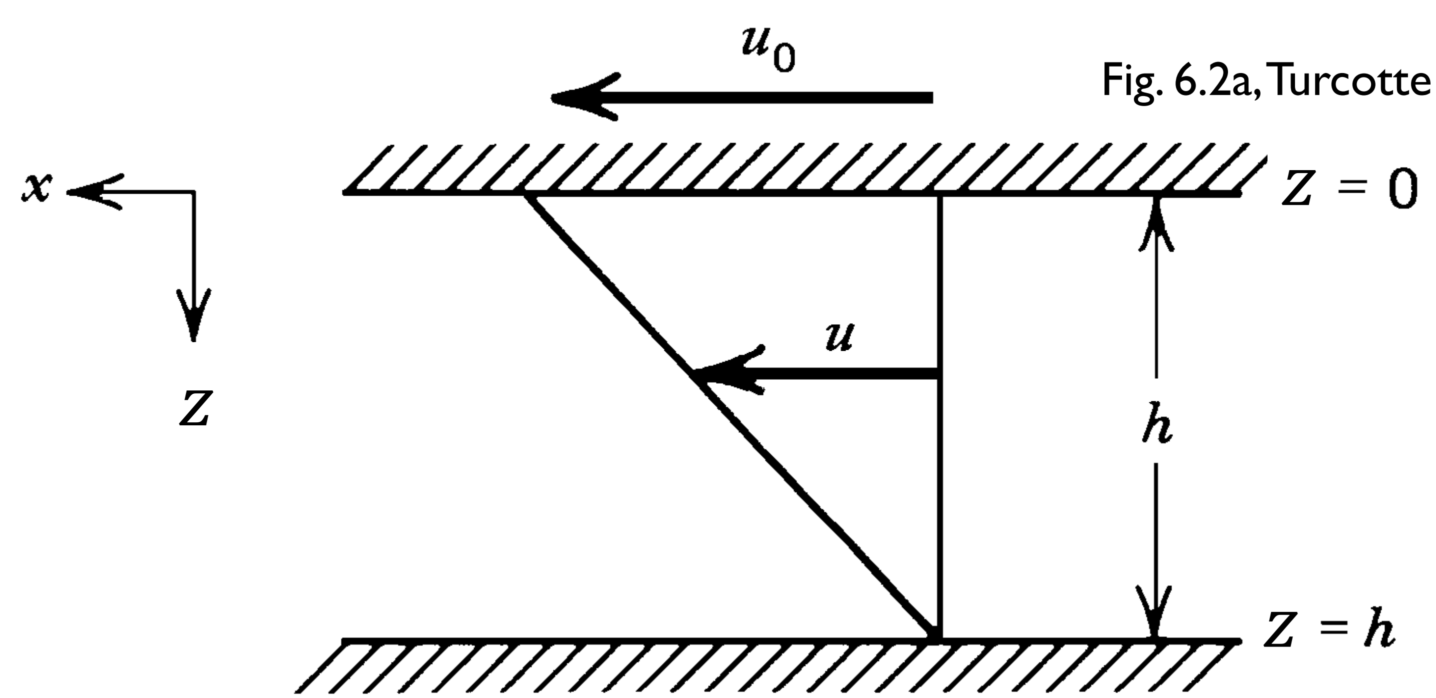
Fig. 6.2a, Turcotte and Schubert, 2002



- **What is the velocity distribution across this channel?**
- Couette flow occurs when there is (1) a difference in velocity between the channel boundaries and (2) effectively no pressure gradient

Couette flow solution

Fig. 6.2a, Turcotte and Schubert, 2002



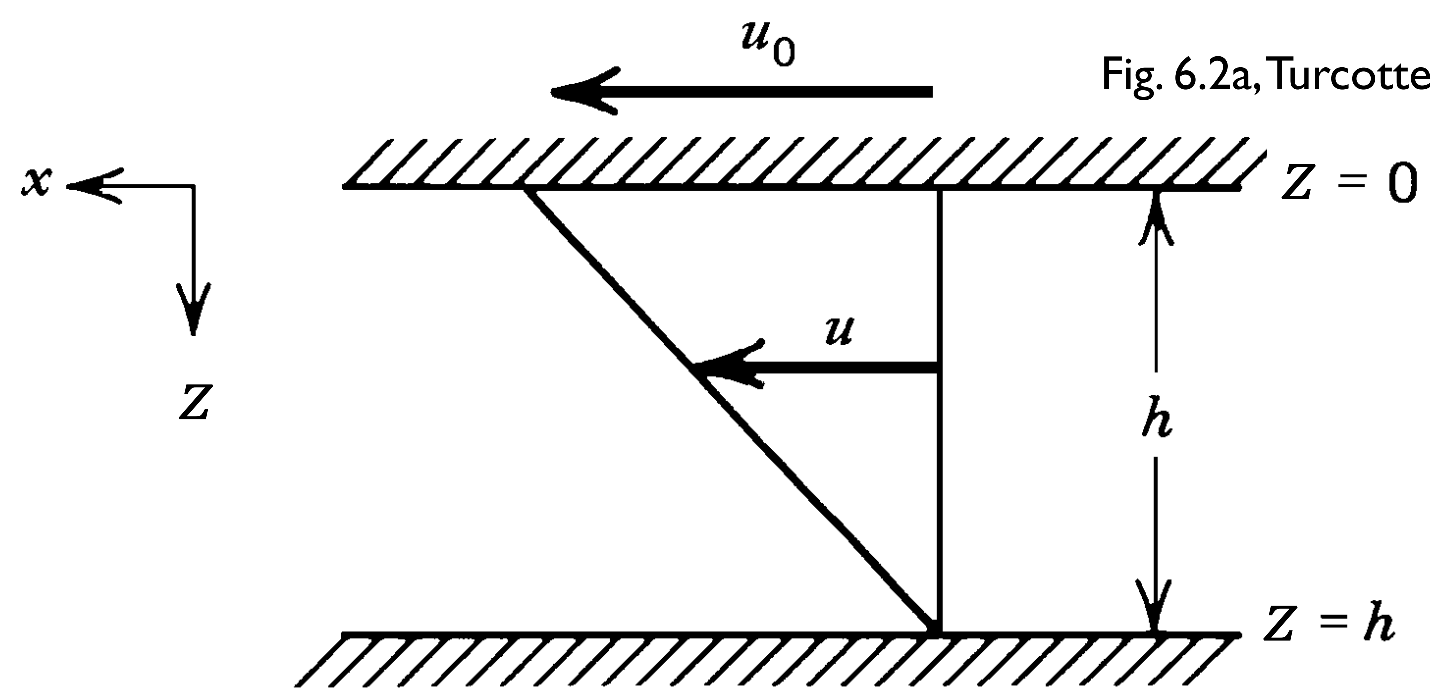
(a) Couette flow $\frac{dp}{dx} = 0,\ u_0 \neq 0$

- The general solution for the 1-D velocity of a fluid across a channel with boundary conditions (1) $u = 0$ at $z = h$ and (2) $u = u_0$ at $z = 0$ is

$$u = \frac{1}{2\eta}\frac{dp}{dx}\left(z^2 - hz\right) - \frac{u_0 z}{h} + u_0$$

where dp/dx is the applied pressure gradient

Couette flow solution



(a) Couette flow $\frac{dp}{dx} = 0,\ u_0 \neq 0$

Fig. 6.2a, Turcotte and Schubert, 2002

- If we assume dp/dx = 0,

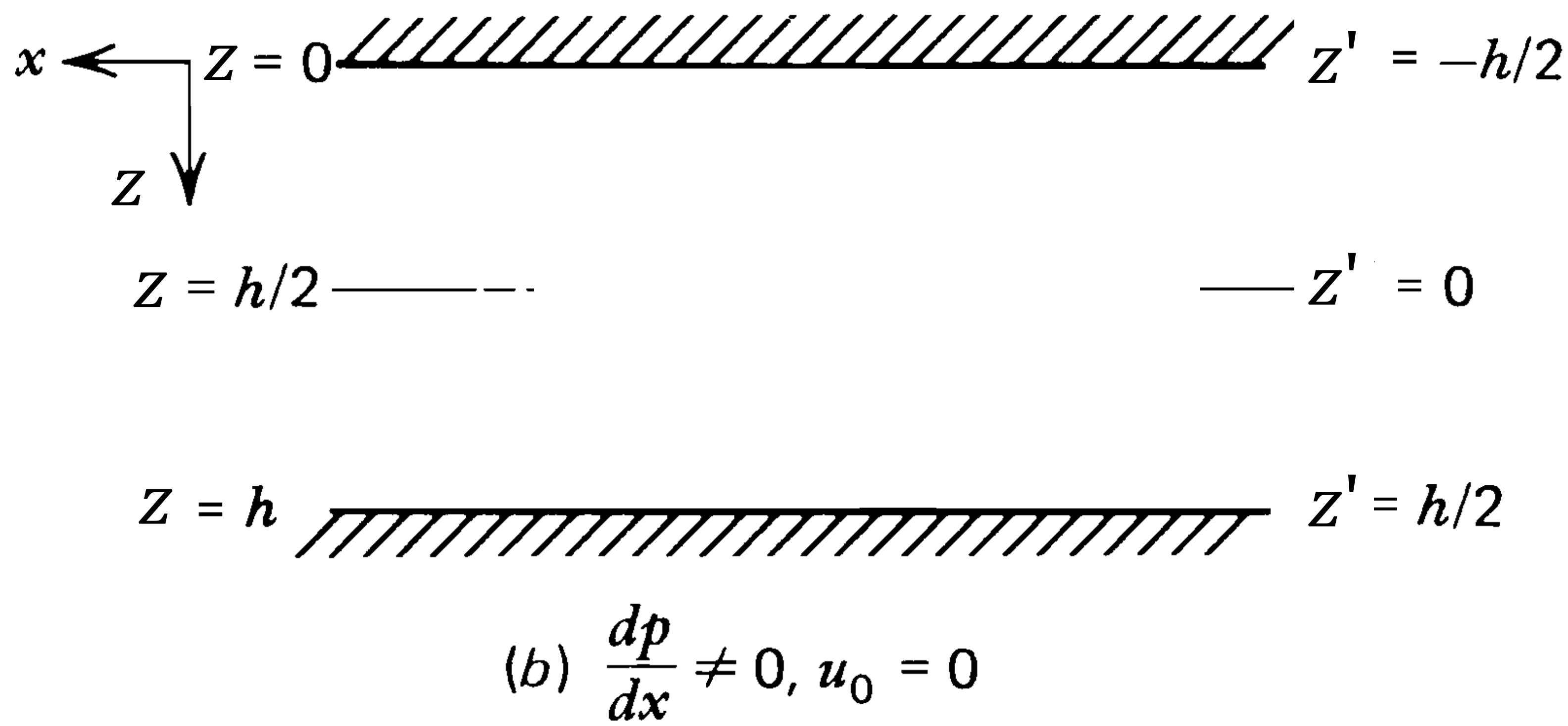
$$u = \frac{1}{2\eta}\frac{dp}{dx}\left(z^2 - hz\right) - \frac{u_0 z}{h} + u_0$$

reduces to

$$u = u_0\left(1 - \frac{z}{h}\right)$$

Poiseuille flow

Fig. 6.2b, Turcotte and Schubert, 2002



Poiseuille flow

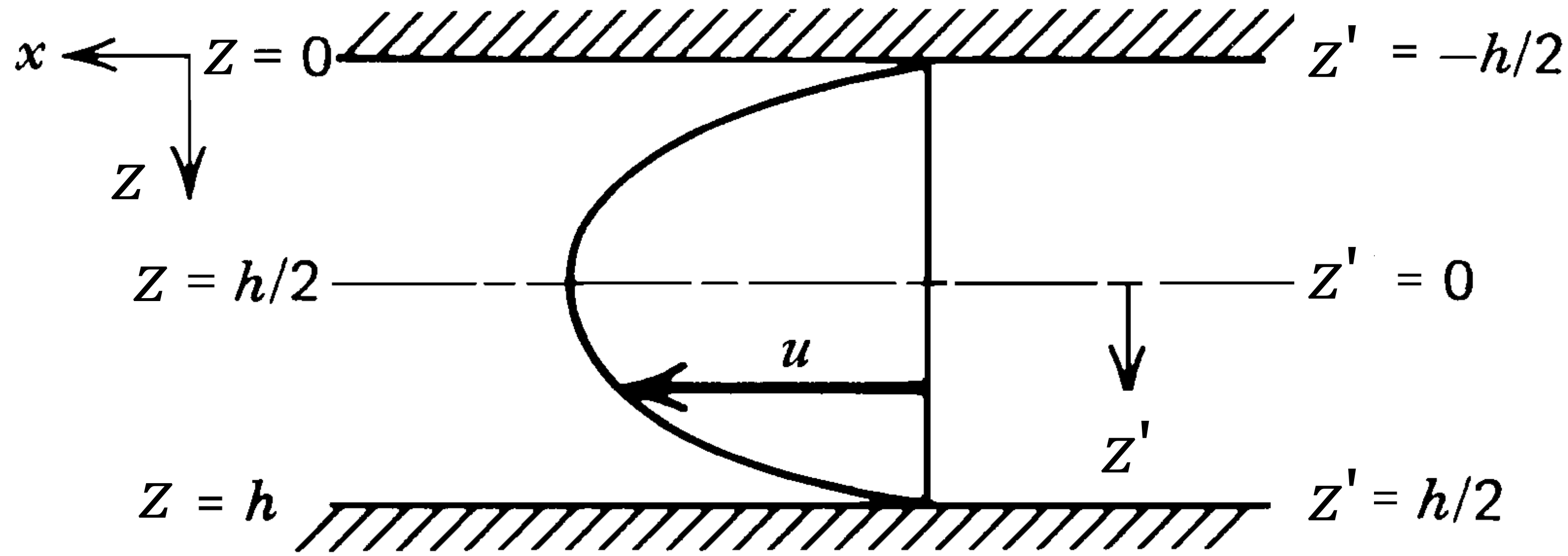
Fig. 6.2b, Turcotte and Schubert, 2002

(b) $\frac{dp}{dx} \neq 0,\ u_0 = 0$

- **What is the velocity distribution across this channel?**

Poiseuille flow

Fig. 6.2b, Turcotte and Schubert, 2002

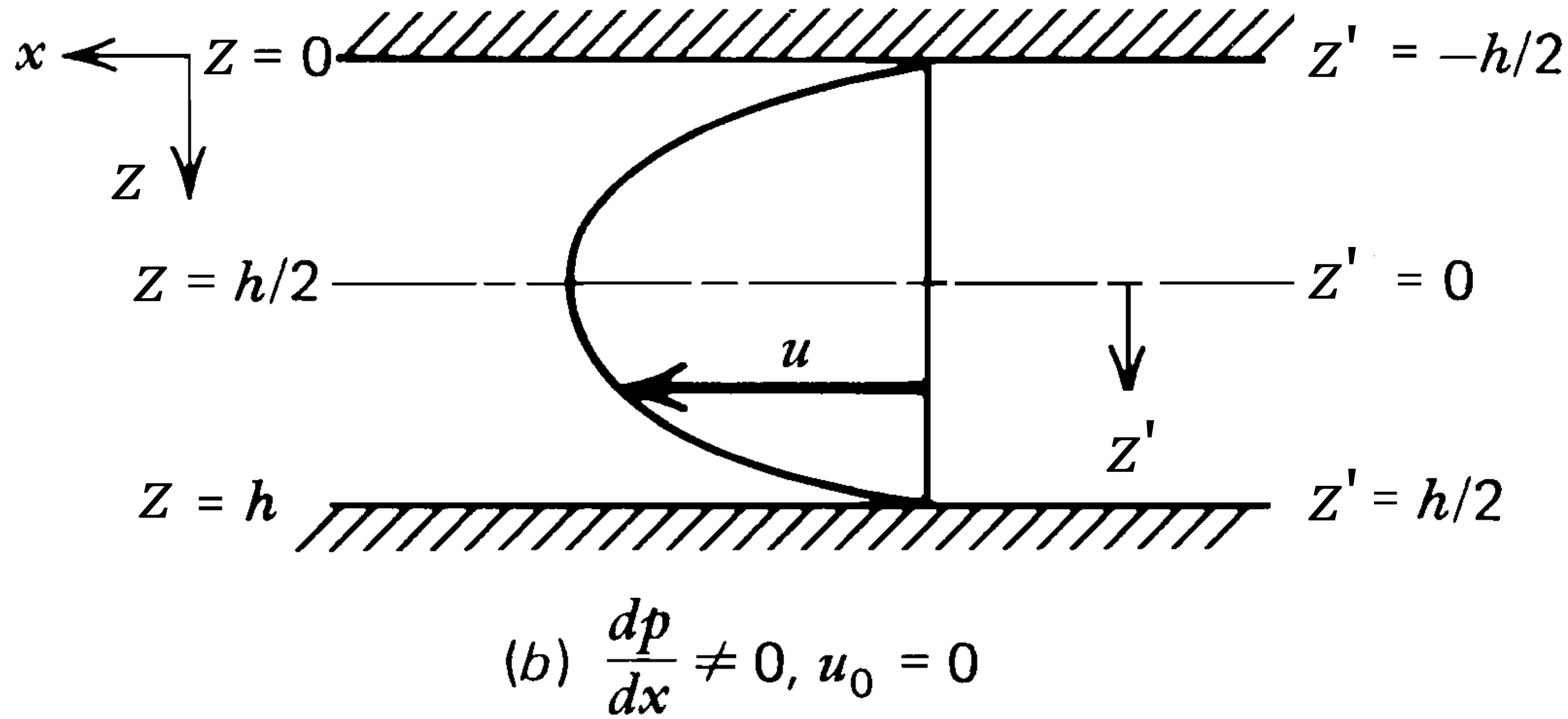


$(b)\ \frac{dp}{dx} \neq 0,\ u_0 = 0$

- **What is the velocity distribution across this channel?**
- **Poiseuille flow** occurs when (1) there is no velocity difference between the walls of the channel and (2) a pressure gradient is applied

Poiseuille flow solution

Fig. 6.2b, Turcotte and Schubert, 2002



- Using the same equation as we have previously, we can start with the general solution

$$u = \frac{1}{2\eta}\frac{dp}{dx}\left(z^2 - hz\right) - \frac{u_0 z}{h} + u_0$$

- If we set $u_0 = 0$, the velocity solution becomes

$$u = \frac{1}{2\eta}\frac{dp}{dx}\left(z^2 - hz\right)$$

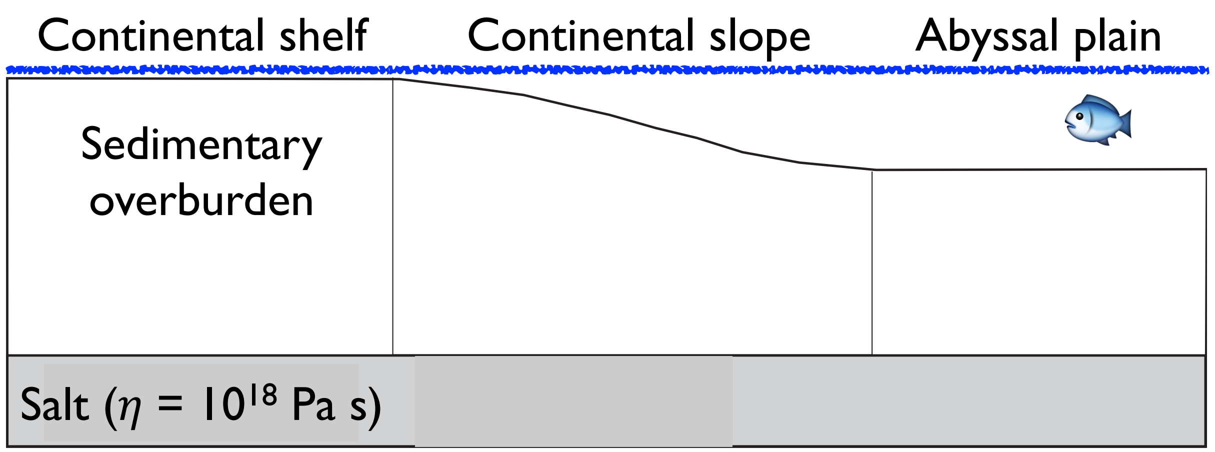
Salt tectonics

- Let's look at an example geological system that can exhibit both **Couette** and **Poiseuille** flow behavior:
  Sediment atop rock salt

Salt tectonics

- Let's look at an example geological system that can exhibit both **Couette** and **Poiseuille** flow behavior: Sediment atop rock salt

A generic salt tectonic system model

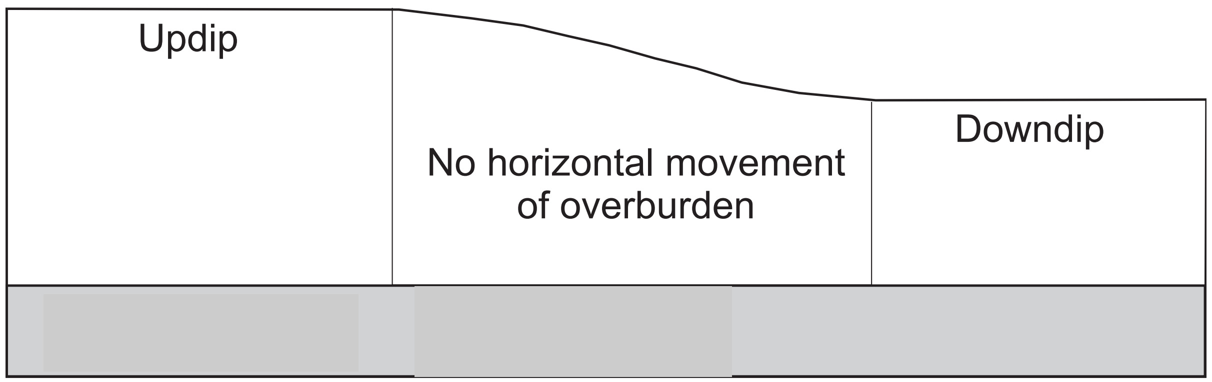


Gemmer et al., 2004

- **Salt tectonics** refers to the deformation of rock as a result of the presence of significant salt layers or bodies

- In this example, we have a simple system of an ocean adjacent to a continent and underlain by a uniform salt layer

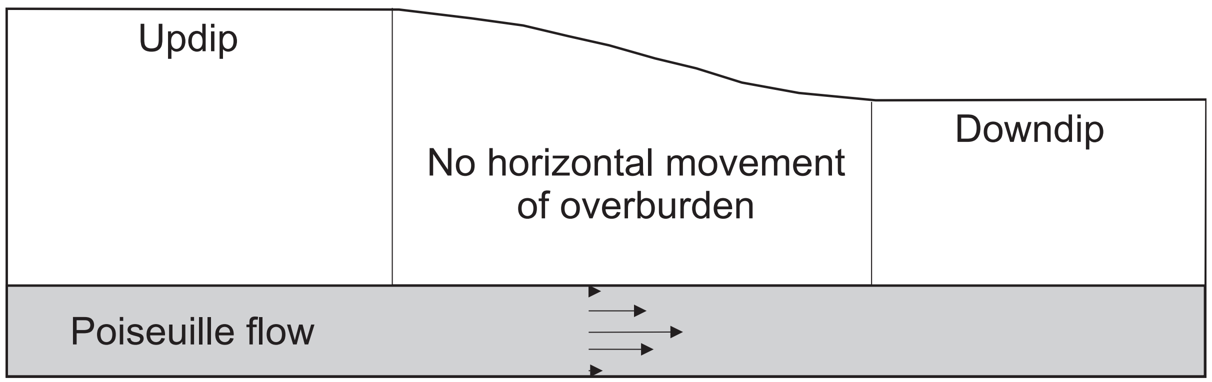
A generic salt tectonic system model



- Let's assume the overburden is stable, **which way would the salt flow and what is the distribution of velocities?**

Gemmer et al., 2004

A generic salt tectonic system model



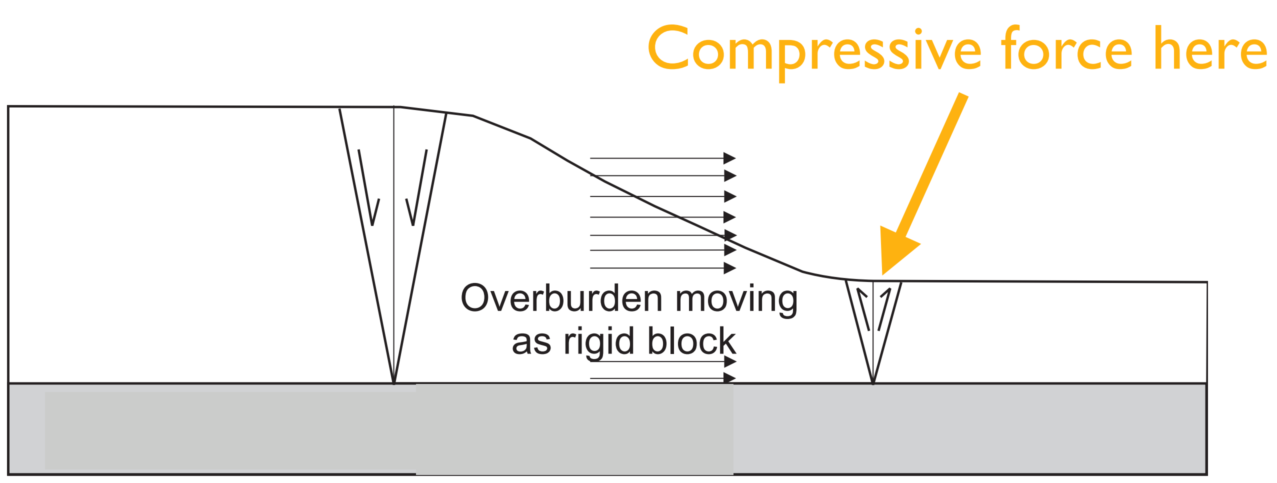
Gemmer et al., 2004

- Let's assume the overburden is stable, **which way would the salt flow and what is the distribution of velocities?**

- If the overburden is stable, a pressure gradient resulting from the different sediment thicknesses drives **Poiseuille flow in the salt**

A generic salt tectonic system model

- A thicker sedimentary sequence on the continental shelf produces a force on the sediments in the model abyssal plain
    - If this force gets large enough, those sediments can fail and the continental slope sediments can become mobile

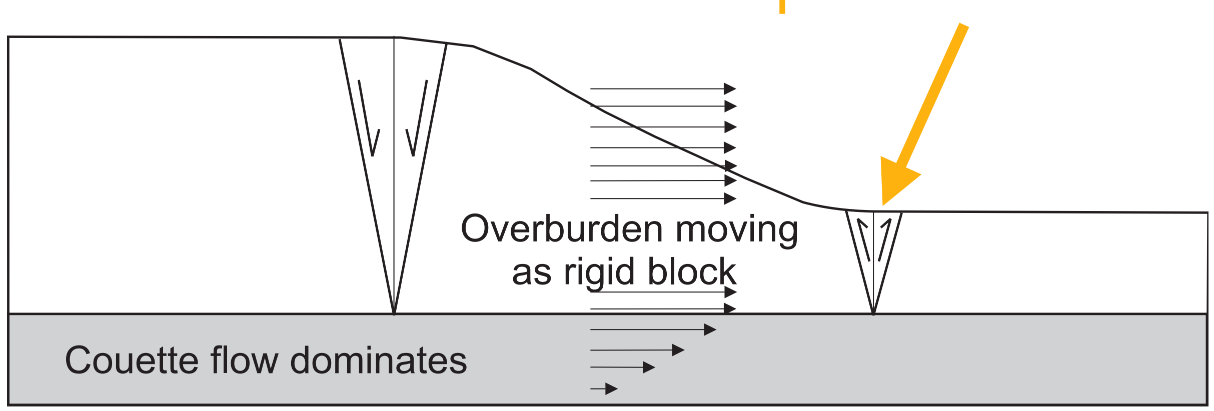


Gemmer et al., 2004

A generic salt tectonic system model

- A thicker sedimentary sequence on the continental shelf produces a force on the sediments in the model abyssal plain
  - If this force gets large enough, those sediments can fail and the continental slope sediments can become mobile
- Mobility of the slope sediments drives dominantly **Couette flow in the salt layer**

Compressive force here



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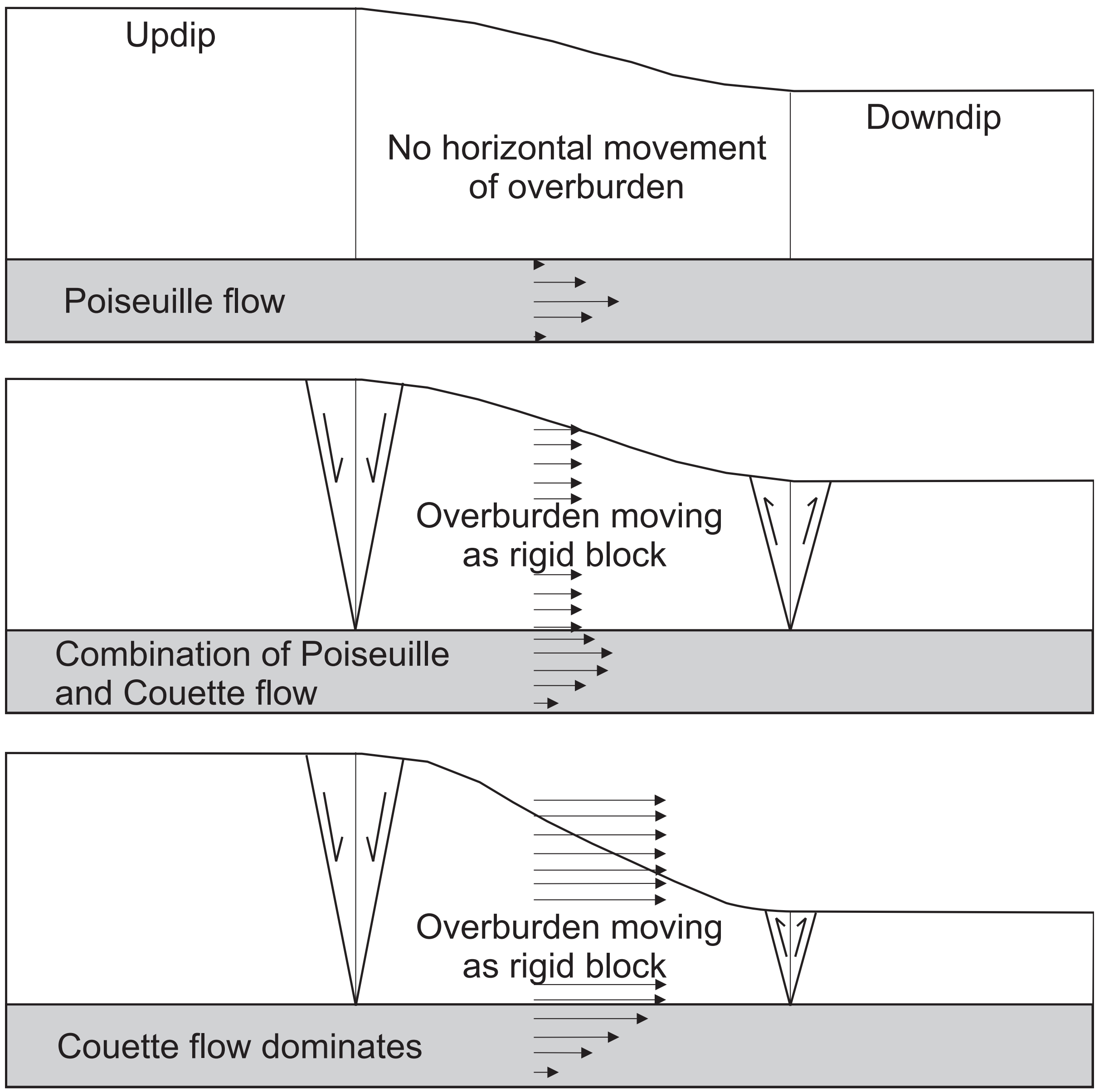

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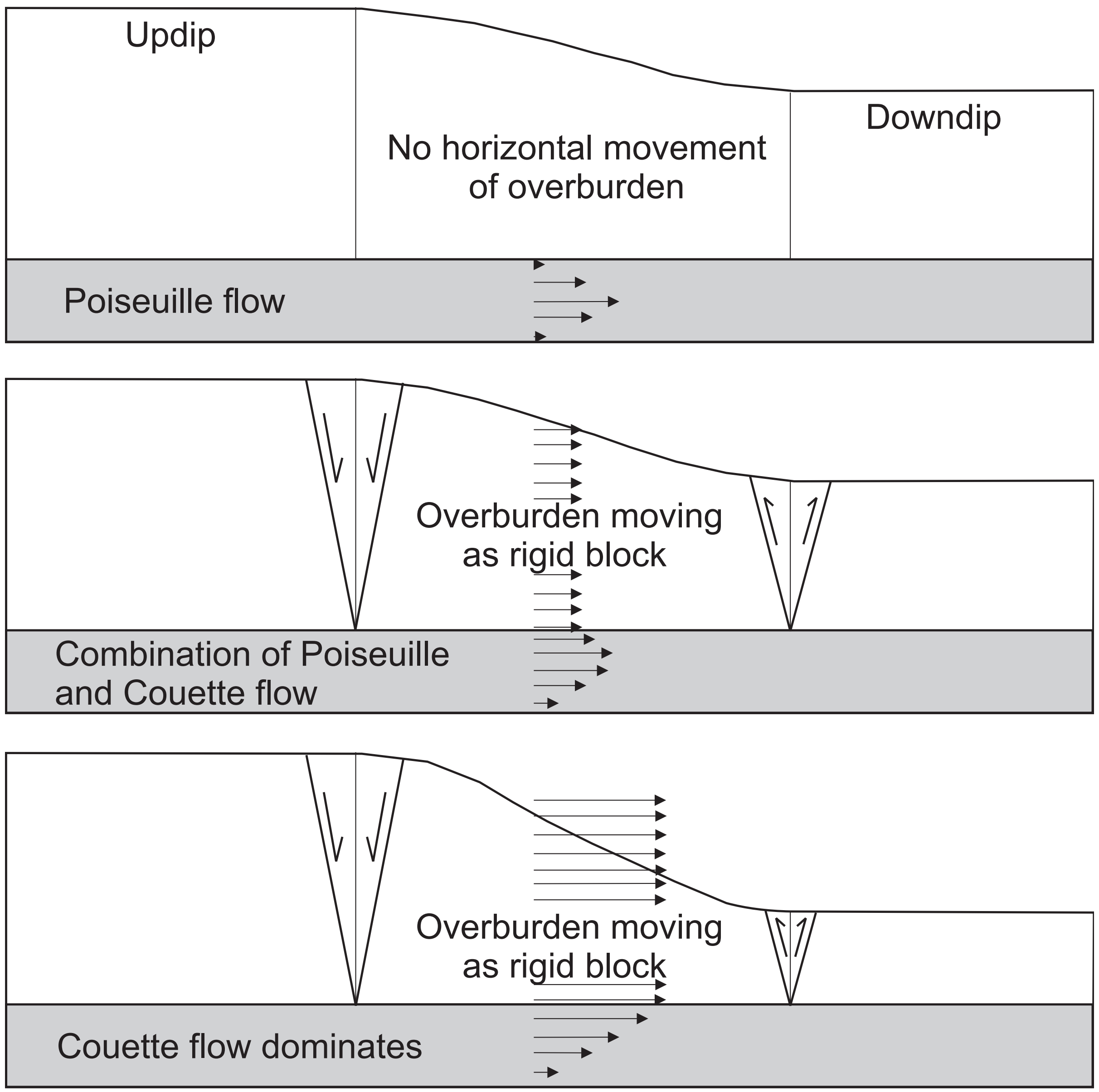
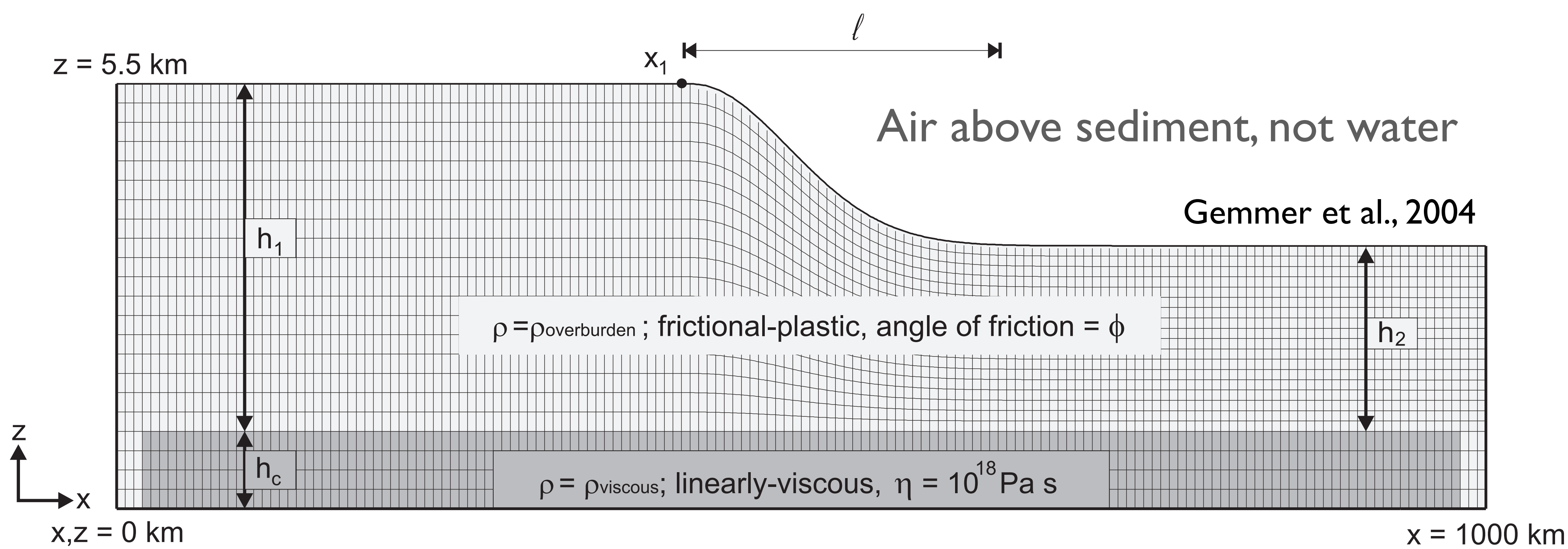


Gemmer et al., 2004

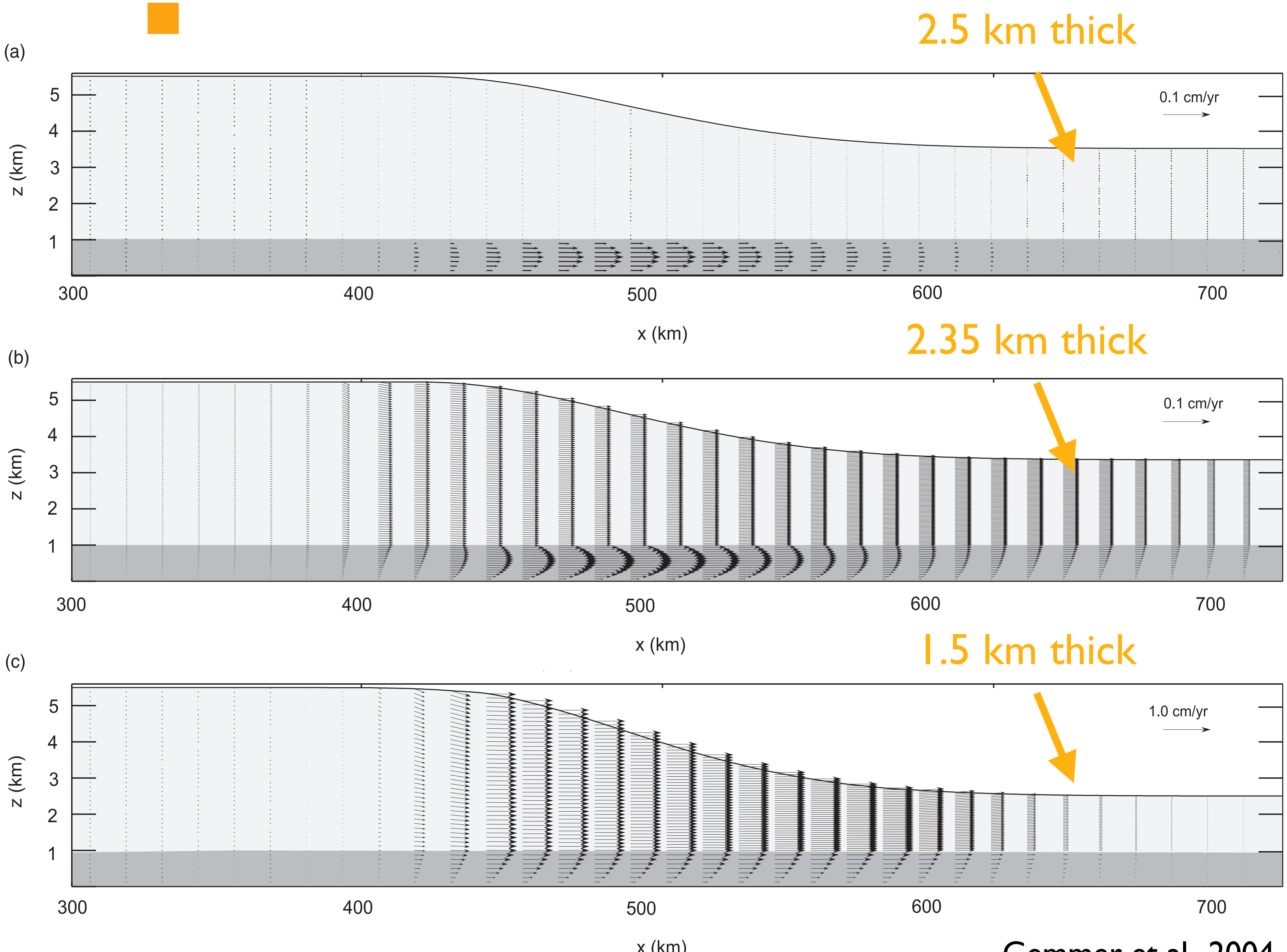
- Often, the deforming salt layer will exhibit both **Couette** and **Poiseuille** flow behaviors

Salt tectonic numerical model



- Salt/sediment deformation behavior was explored using 2D viscous-plastic numerical models
- 1 km of linear viscous salt overlain by 1.5-4.5 km of sediment
- No imposed velocity at margins, purely gravity driven

Quick summary of model results



Gemmer et al., 2004

The Gemmer et al. model shows the salt tectonic behavior and flow style in the salt depends on the thickness of the downdip sedimentary layer

Temperature dependence

- In general, rock viscosity depends strongly temperature

$$\eta = A_0 \mathrm{e}^{Q/RT_\mathrm{K}}$$

where A_0 and Q are material properties known as the **pre-exponent constant** and **activation energy**, R is the universal gas constant and T_K is temperature in Kelvins

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- **What happens to rock viscosity at T_K approaches absolute zero?**

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- **What happens to rock viscosity at T_K approaches absolute zero?**
- **What happens as T_K approaches infinity?**

Temperature-dependent viscosity

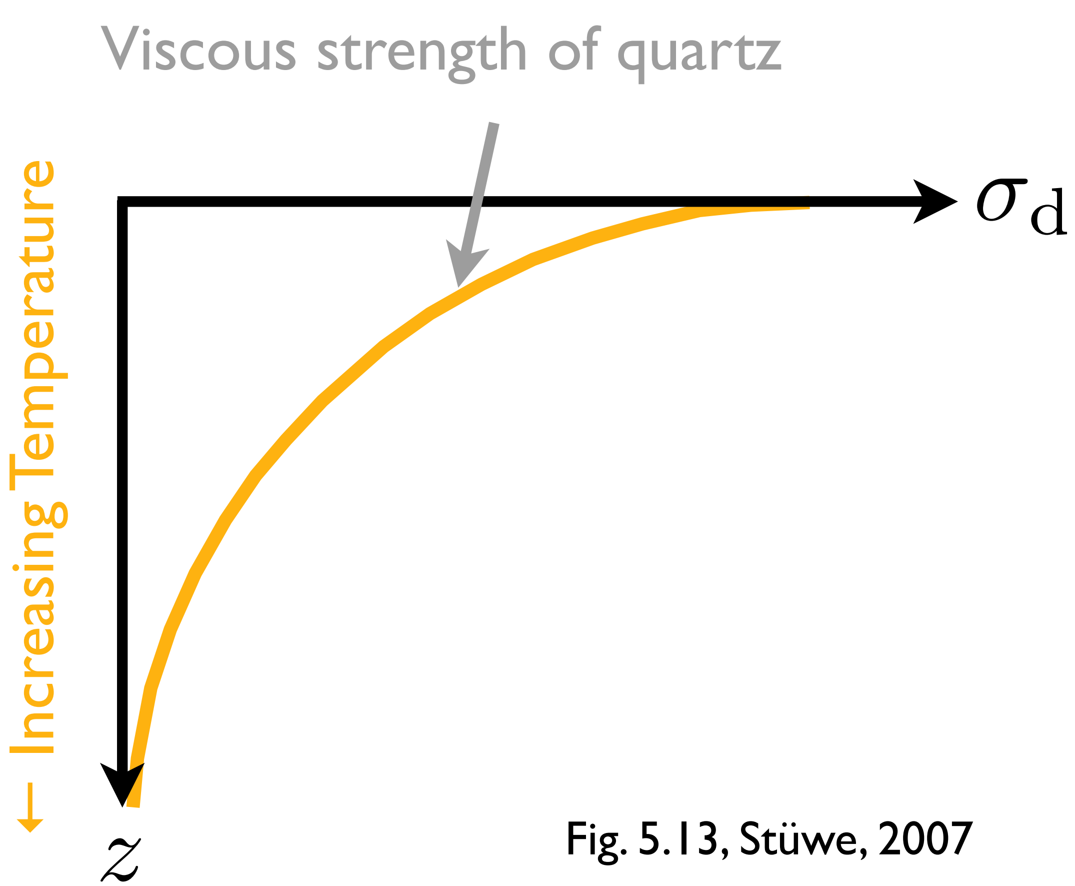


Fig. 5.13, Stüwe, 2007

- The viscous strength of quartz, for example, rapidly decreases with increasing temperature
- Note that the viscous strength is simply the viscosity η multiplied by a nominal strain rate
- **How might temperature-dependent viscosity be important in the Earth?**

Nonlinear viscosity

- In general, rocks will deform about 8 times as quickly when the applied force is doubled
  - Relationship between shear stress and strain rate is thus **NOT linear**
- Mathematically, we can say

$$\tau^n = A_{\text{eff}} \frac{du}{dz}$$

where n is the **power law exponent** and A_{eff} is a material constant

- The power law exponent for many rocks is 2-4
- A_{eff} is similar to η, but has units of Pa^n s

Flow of glaciers

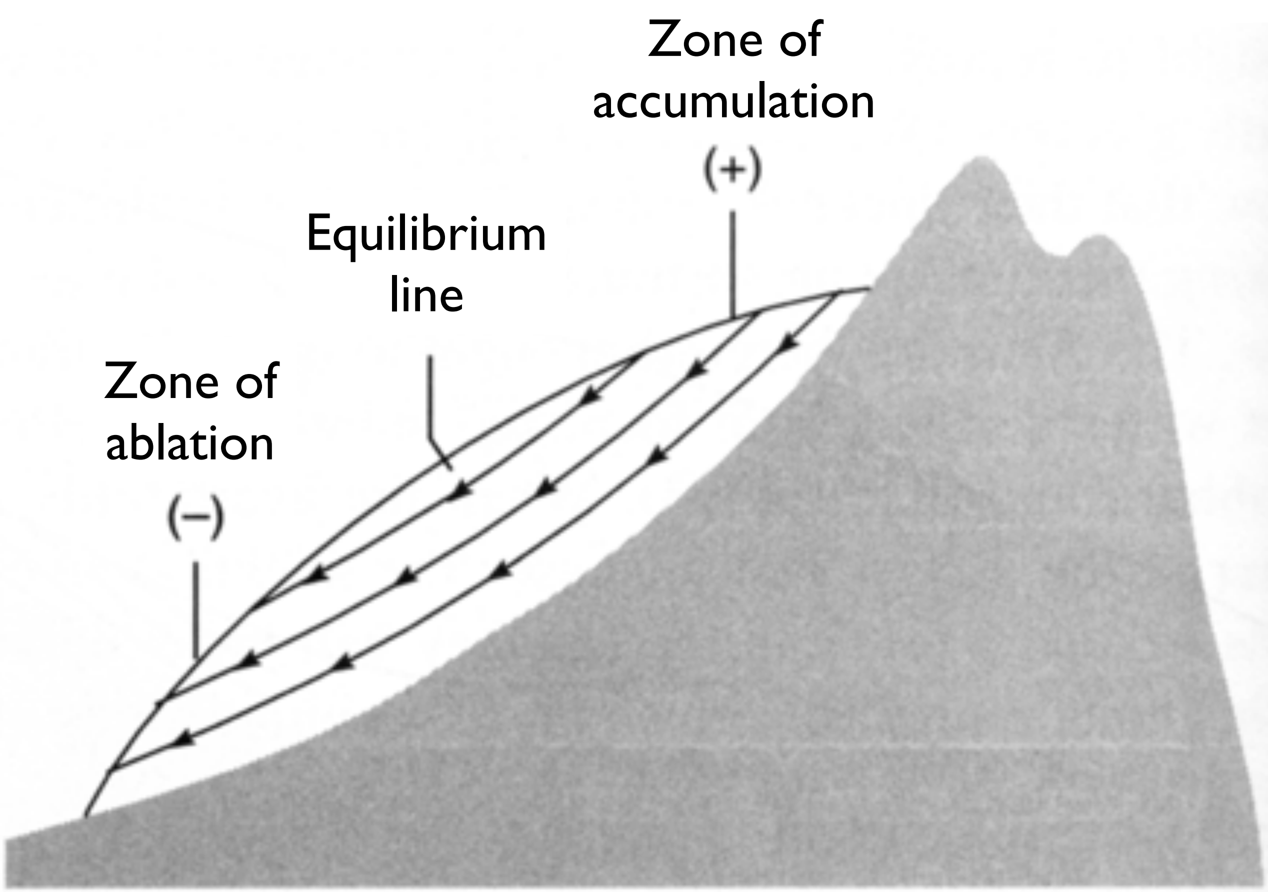


Fig. 9.14, Ritter et al., 2002

- Gravity drives the flow of alpine glaciers from higher elevation zones of **accumulation** to lower elevation zones of **ablation**

- Depending on the temperature of the region and the ice itself, the glacier may either be frozen to the bedrock (**cold-based**) or sliding along the bedrock (**warm-based**)

How do glaciers move?

- **Basal sliding**
  - Bottom of the glacier sliding along the substrate
  - Can occur as a result of slip atop a thin water layer, melting/re-freezing or slip atop water-saturated sediment
- **Internal deformation**
  - Ice flow is <u>nonlinear viscous</u> and <u>sensitive to temperature</u>
  - Deformation is <u>concentrated near the bed</u>

Flow of glaciers

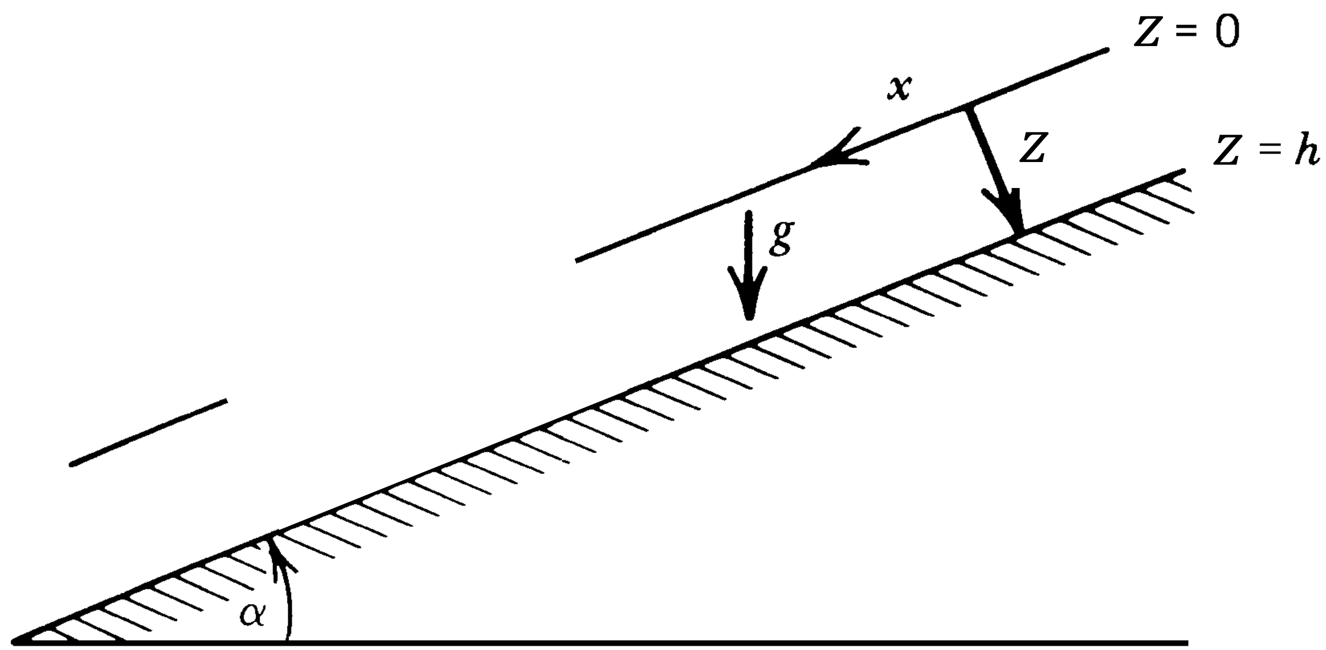
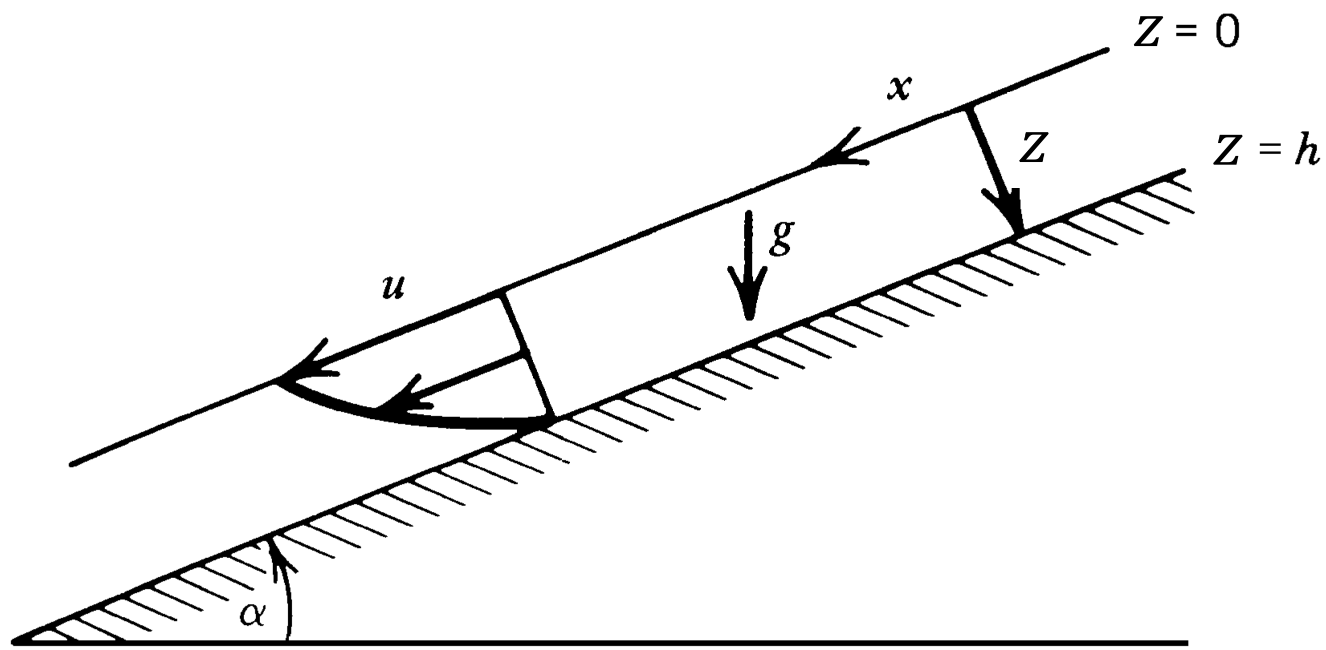


Fig. 6.3, Turcotte and Schubert, 2002

- In the next lecture and in the laboratory exercise this week, we will look more closely at glacial flow
  - Flow down an inclined slope
  - Flow velocity across a glacial valley

Flow of glaciers



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Recap

- **Viscous flow** is a common deformation behavior for rock and ice, where the deformation rate is proportional to the applied shear stress

- **Couette** and **Poiseuille** flows refer to end-member behaviors of linear viscous channel flows, and depend on the channel boundary velocities and pressure changes along the channel

- Most rocks do not exhibit a linear relationship between stress and strain rate (nonlinear viscosity), and their viscosity is strongly temperature-dependent

References

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