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The Dark Side of Loop Control Theory

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Course Agenda

- ❑ Introduction to Control Systems
- ❑ Shaping the Error Signal
- ❑ How to Implement the PID Block?
- ❑ The PID at Work with a Buck Converter
- ❑ Considering the Output Impedance
- ❑ Classical Poles/Zeros Placement
- ❑ Shaping the Output Impedance
- ❑ Quality Factor and Phase Margin
- ❑ What is Delay Margin?
- ❑ Gain Margin is not Enough

What is the Purpose of this Seminar?

- ❑ There have been numerous seminars on control loop theory
- ❑ Seminars are usually highly theoretical – link to the market?
- ❑ Control theory is a vast territory: you don't need to know everything!
- ❑ This 3-hour seminar will shed light on some of the less covered topics:
- ❖ PID compensators and classical poles/zeros compensation
- ❖ Output impedance considerations in a switching regulator
- ❖ Understanding delay and modulus margins
- ➢ In a 3-hour course, we are just scratching the surface…!

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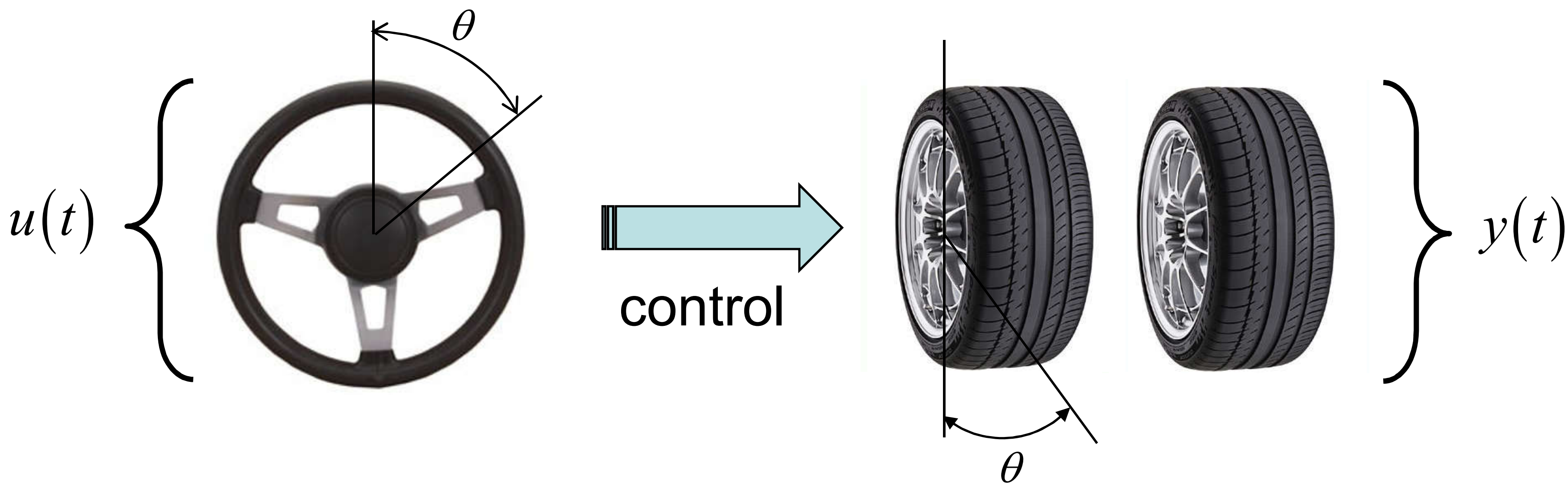
What is a Closed-Loop System?

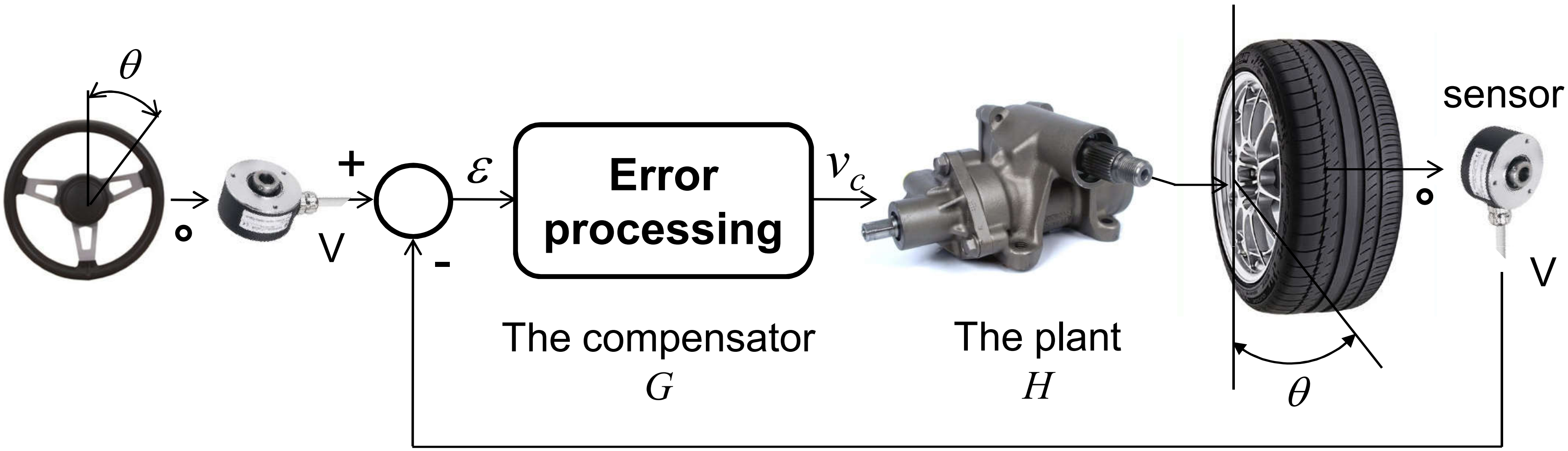
- A closed-loop system forces a variable to follow a setpoint
- The setpoint is the input, the controlled variable is the output
- French term is "enslavement": the output is slave to the input
- A car steering wheel is a possible example:



The input u The output y

Representing a Closed-Loop System

- ❑ A closed-loop system can be represented by blocks
- ❑ The output is monitored and compared to the input

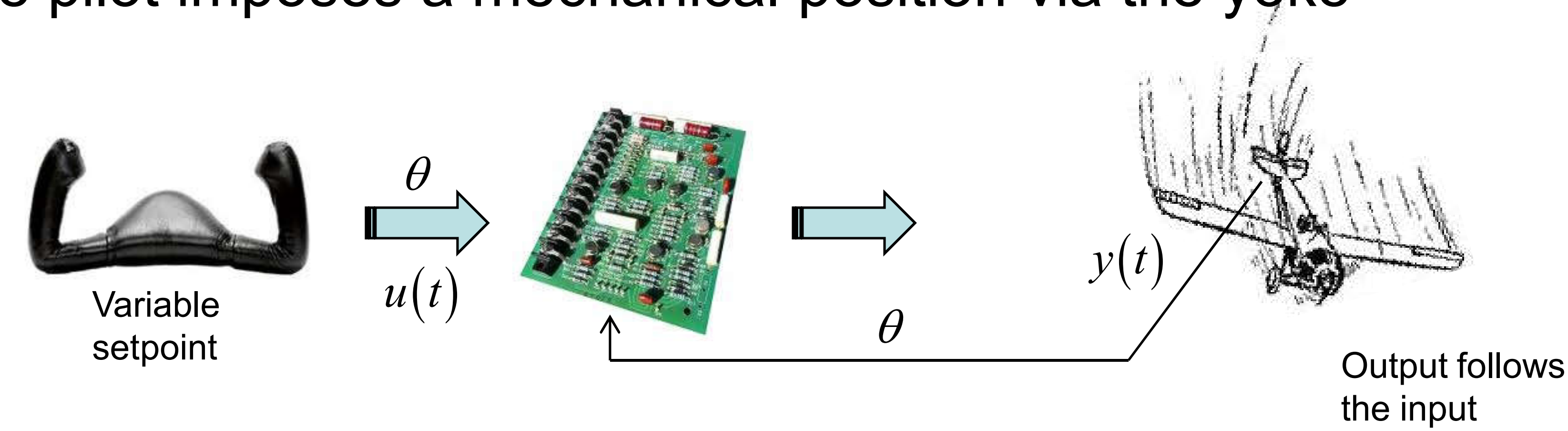


- ❑ Any deviation between the two gives birth to an error ε
- ❑ This error is amplified and drives a corrective action

A Servomechanism or a Regulator?

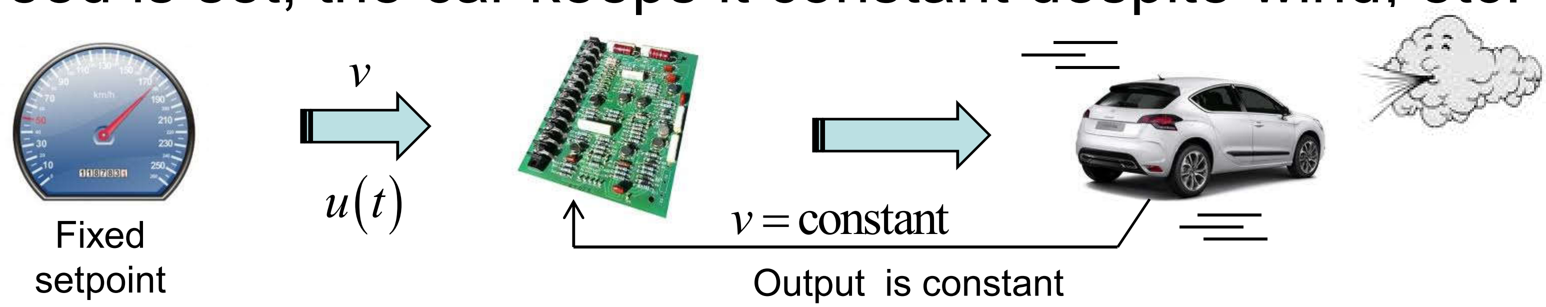
❑ Airplane elevator control is a servo-mechanism:

• The pilot imposes a mechanical position via the yoke



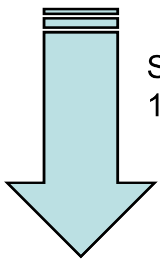
❑ Car cruise control is a regulator:

• The speed is set, the car keeps it constant despite wind, etc.

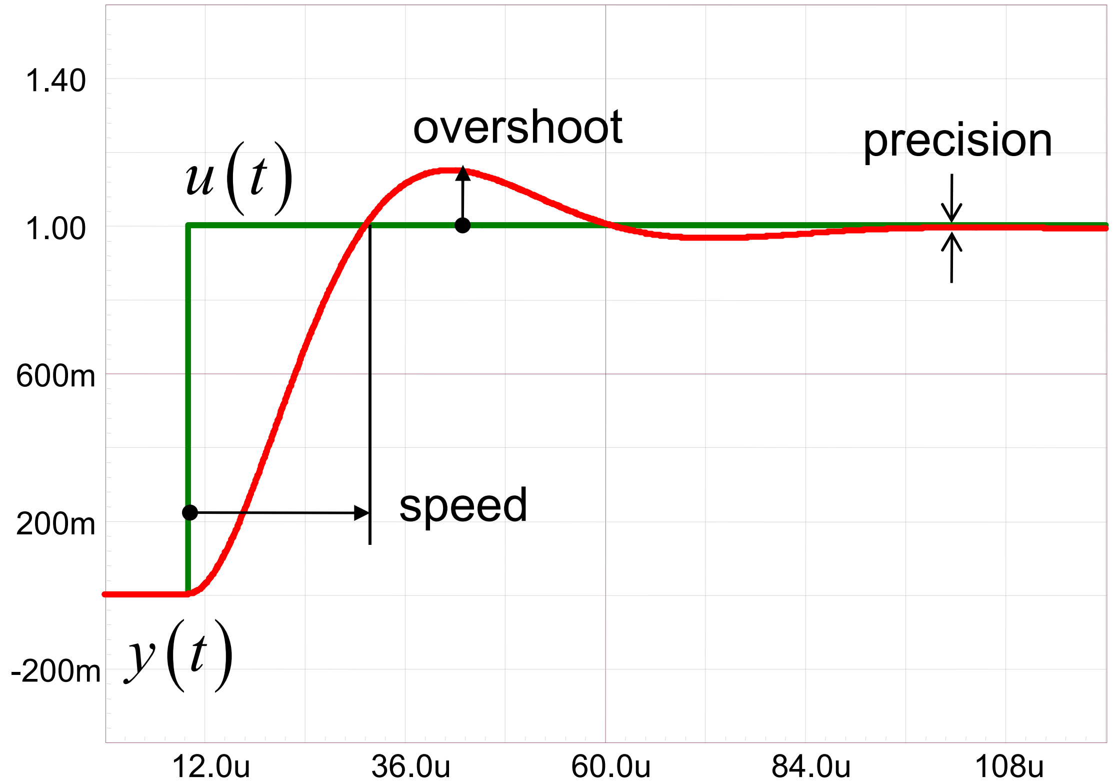


Processing the Error Signal

- ❑ The error signal is processed through the compensator G
- ❑ We want the following operating characteristics:
- ✓ Speed
- ✓ Precision
- ✓ Robustness



PID



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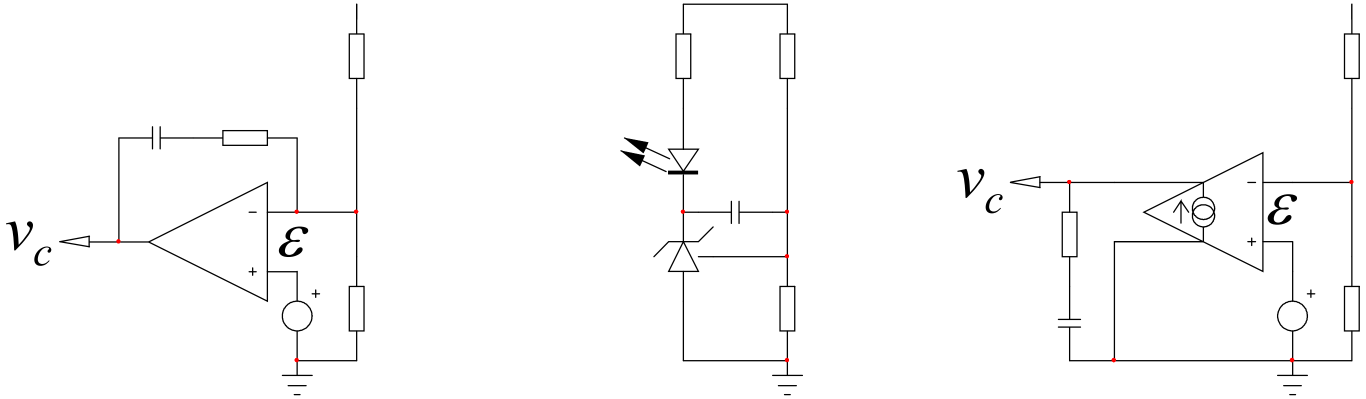
Where do You Shape the Signal?

- The compensator is the place where you apply corrections

The compensator: G

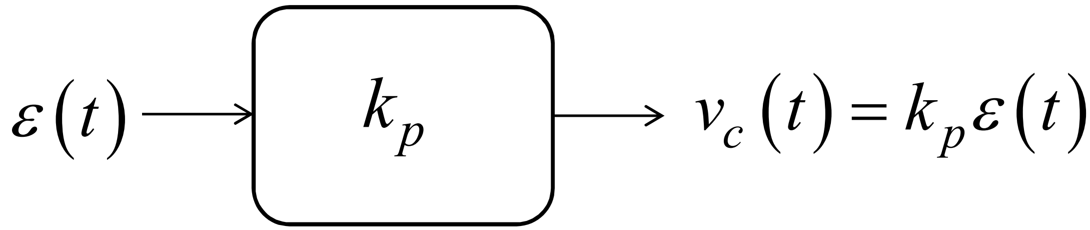
- The compensator is built with an error amplifier:

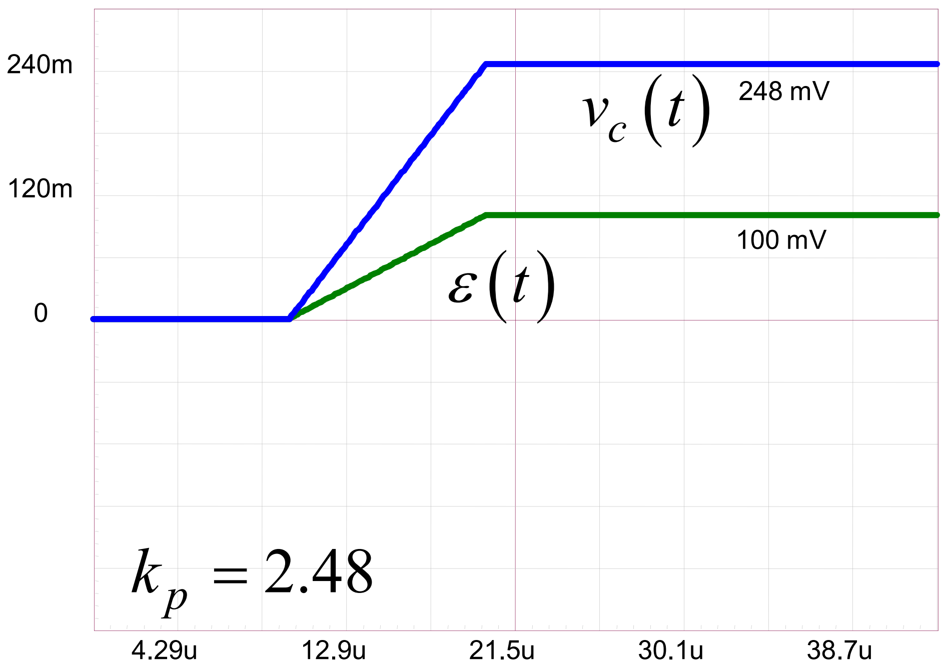


Op amp TL431 OTA

The PID Compensator

- A **P**ID welcomes a Proportional block





k_p is high:

✓ reaction speed

❖ risks of overshoot

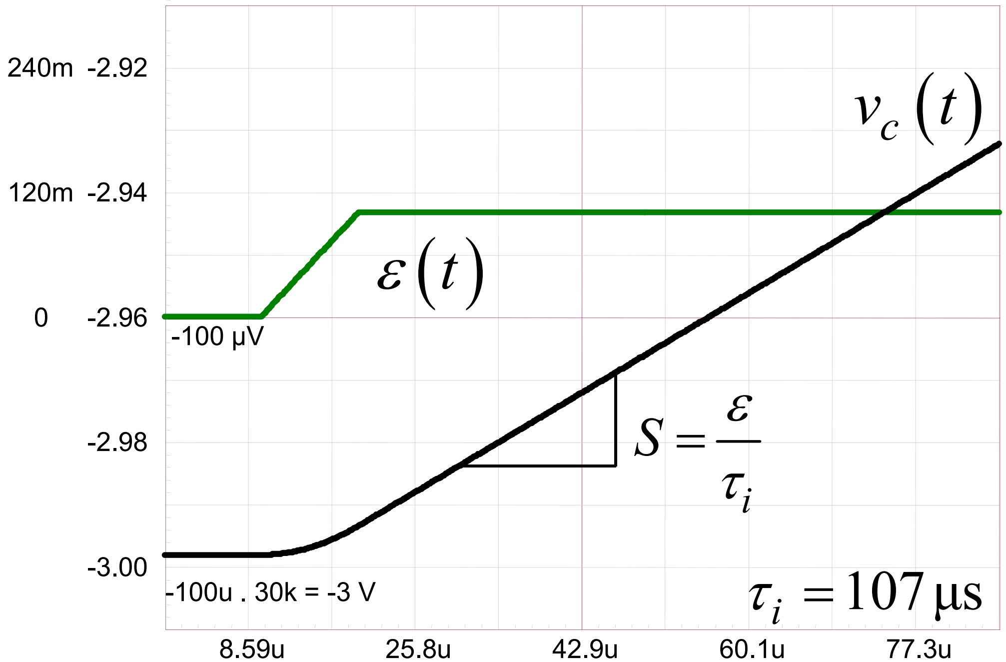
k_p is low:

❖ sluggish response

The PID Compensator

- A PID includes an Integrating block

$$\varepsilon(t) \rightarrow \boxed{\frac{k_p}{\tau_i}\int} \rightarrow v_c(t) = \frac{k_p}{\tau_i}\int_0^t \varepsilon(\tau)\,d\tau$$



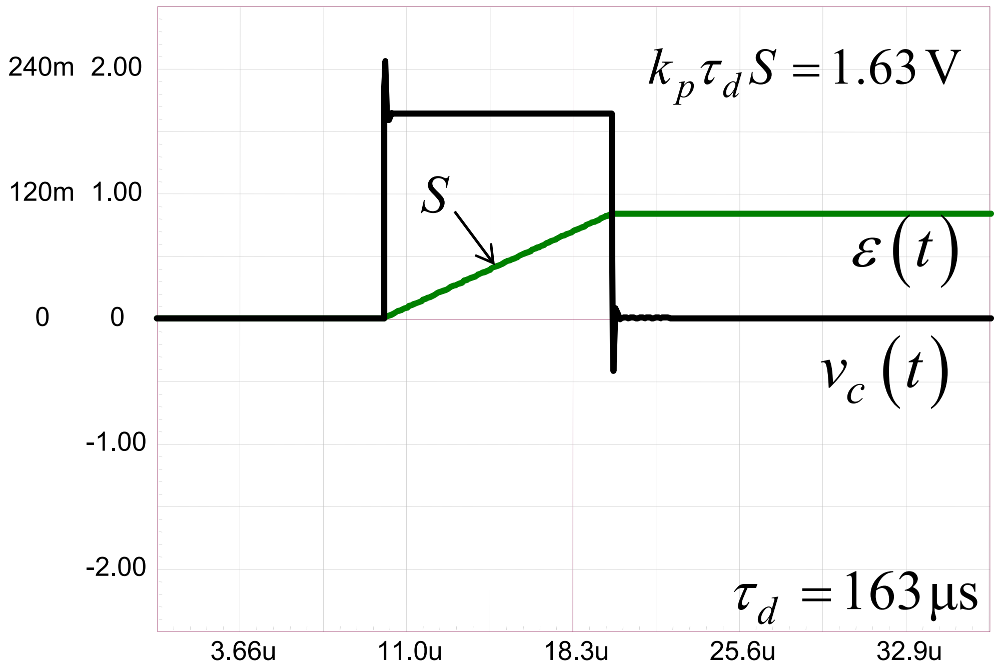
Integral action:

- ✓ no static error
- ❖ slow response
- ❖ decreases stability
- ❖ large overshoots

The PID Compensator

- A PID offers a Derivative block

$$\varepsilon(t) \rightarrow \boxed{k_p \tau_d \frac{d}{dt}} \rightarrow v_c(t) = k_p \tau_d \frac{d\varepsilon(t)}{dt}$$

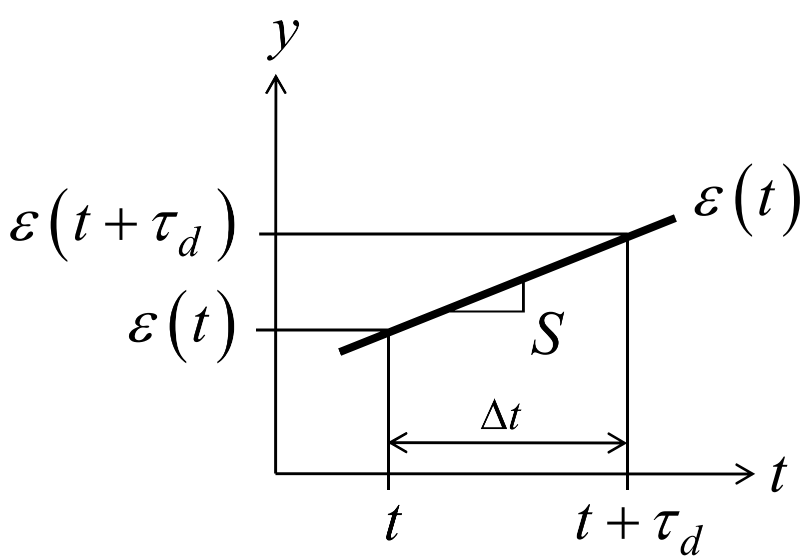


Derivative action:

✓ perturbation variation is known: anticipation
✓ stabilizes the response
○ no static effect

The PID Compensator

- The Derivative block anticipates the signal evolution and speed



$$\frac{\Delta\varepsilon(t)}{\Delta t} = \frac{\varepsilon(t+\tau_d) - \varepsilon(t)}{\tau_d}$$

if τ_d small

$$\varepsilon(t+\tau_d) \approx \varepsilon(t) + \tau_d \frac{d\varepsilon(t)}{dt}$$

later now

Combining the Blocks

- You can formulate the PID transfer function in different ways:
  - differentiation: $v_c(t) = \frac{d\varepsilon(t)}{dt} \rightarrow V_c(s) = \varepsilon(s)s$
  - integration: $v_c(t) = \int \varepsilon(t)dt \rightarrow V_c(s) = \frac{\varepsilon(s)}{s}$
- The standard form:

$$G(s) = k_p\left(1 + \frac{1}{s\tau_i} + s\tau_d\right)$$

- The parallel form:

$$G(s) = k_p + \frac{k_i}{s} + sk_d$$

The derivative term cannot be physically implemented:

$$\lim_{s\to\infty} s\tau_d = \infty$$

Need a pole

$$s\tau_d \rightarrow \frac{s\tau_d}{1 + \frac{s\tau_d}{N}}$$

Combining the Blocks

- The transfer function becomes a filtered PID:

$$G(s) = k_p \left(1 + \frac{1}{s\tau_i} + \frac{s\tau_d}{1 + \frac{s\tau_d}{N}} \right)$$

- If we develop, we obtain a more familiar expression:

$$G(s) = \frac{1 + s\left(\frac{\tau_d}{N} + \tau_i\right) + s^2\left(\frac{\tau_d \tau_i}{N} + \tau_d \tau_i\right)}{s\frac{\tau_i}{k_p}\left(1 + \frac{\tau_d}{N}s\right)}$$

A double zero
An origin pole
A high-frequency pole

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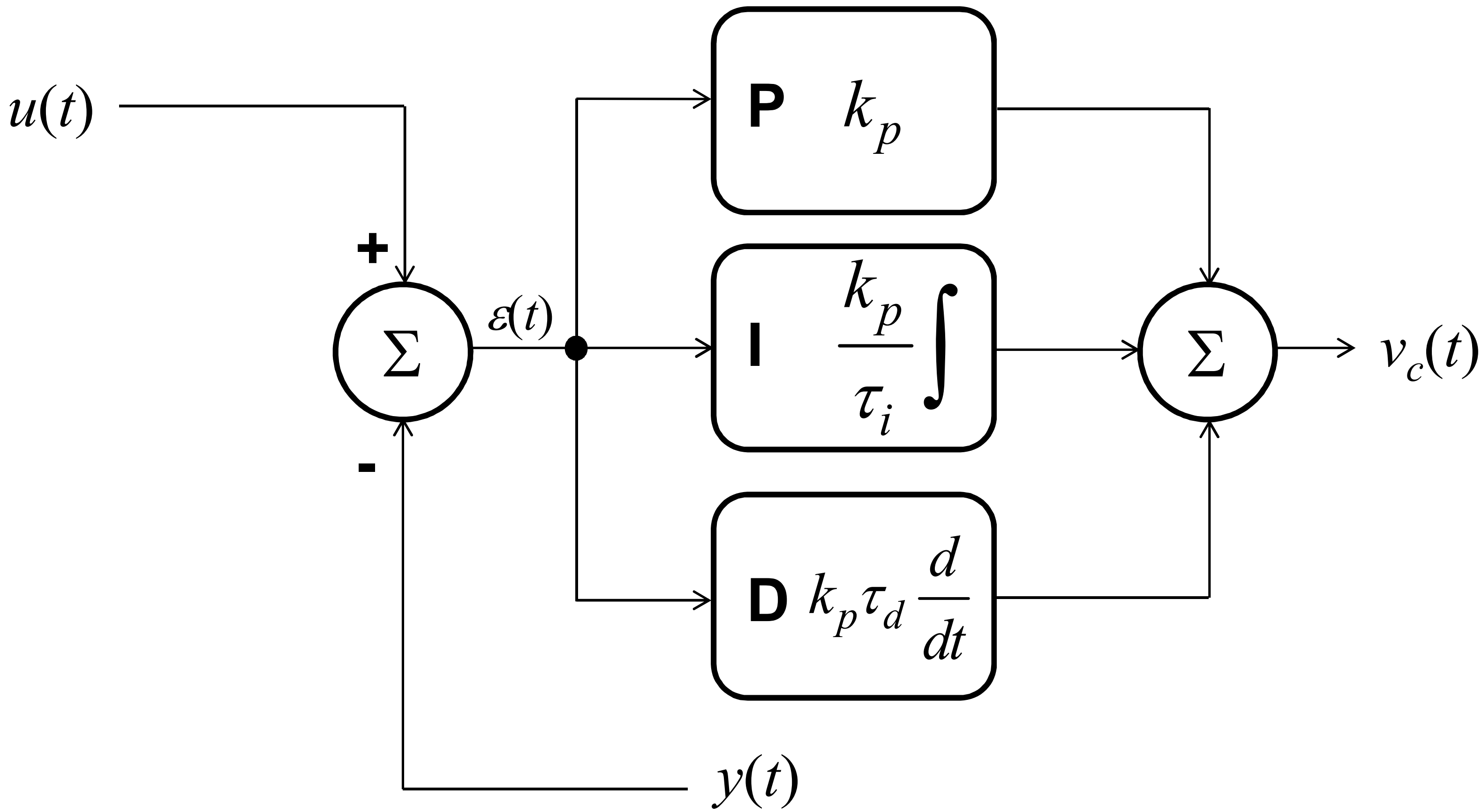
Practical Implementation

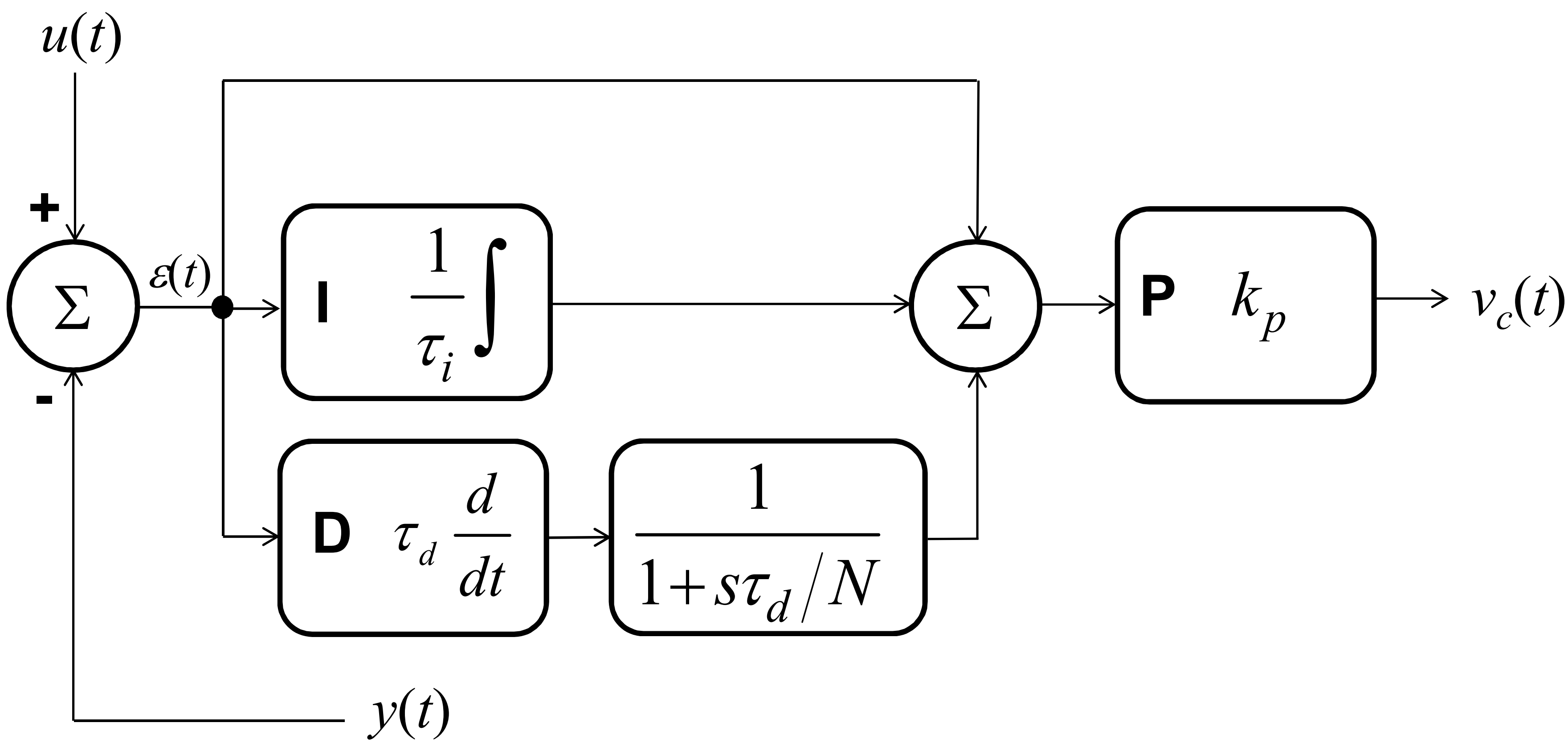
- Sum up the output of each individual block:



- This is the parallel form of the PID

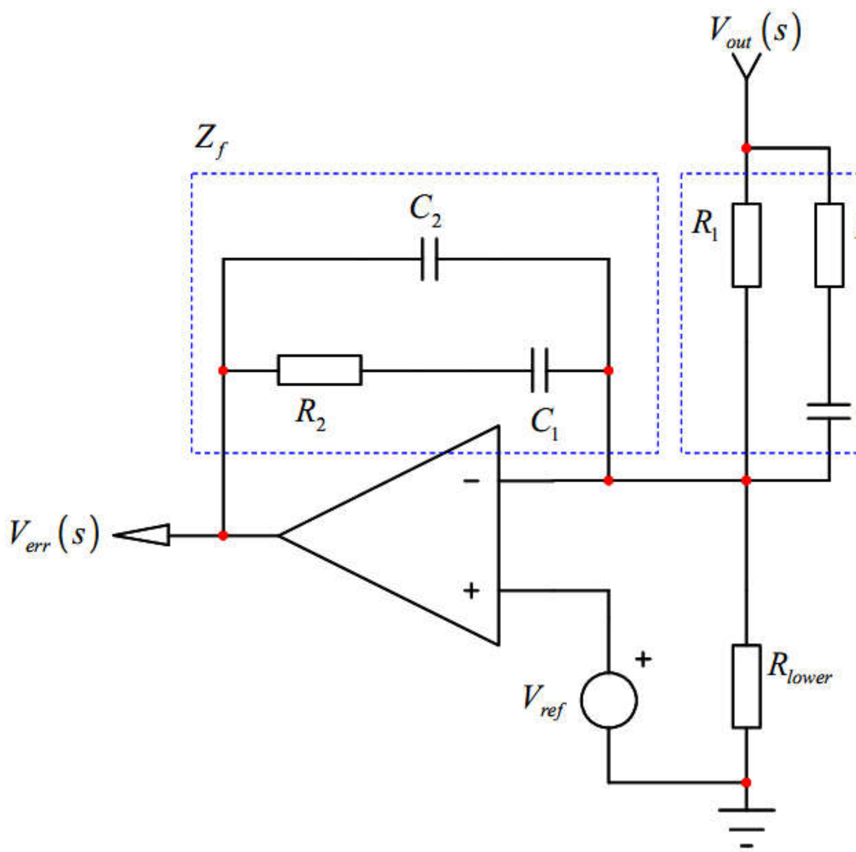
Practical Implementation

- This is the filtered standard form of the PID



Bridging a PID to a Type 3

- A type 3 is implemented around an op amp



$$G(s)=\frac{\left(1+s/\omega_{z_1}\right)\left(1+s/\omega_{z_2}\right)}{\frac{s}{\omega_{po}}\left(1+\frac{s}{\omega_{p_1}}\right)\left(1+\frac{s}{\omega_{p_2}}\right)}$$

$$G(s)=\frac{1+s\left(\frac{1}{\omega_{z_1}}+\frac{1}{\omega_{z_2}}\right)+s^2\left(\frac{1}{\omega_{z1}\omega_{z_2}}\right)}{\frac{s}{\omega_{po}}\left(1+\frac{s}{\omega_{p_1}}\right)\left(1+\frac{s}{\omega_{p_2}}\right)}$$

Added pole

Bridging a PID to a Type 3

- Identify the terms and write the equations:

$$G(s)=\frac{1+s\left(\frac{\tau_d}{N}+\tau_i\right)+s^2\left(\frac{\tau_d\tau_i}{N_1}+\tau_d\tau_i\right)}{s\frac{\tau_i}{k_p}\left(1+\frac{\tau_d}{N}s\right)}$$

$$G(s)=\frac{1+s\left(\frac{1}{\omega_{z_1}}+\frac{1}{\omega_{z_2}}\right)+s^2\left(\frac{1}{\omega_{z1}\omega_{z_2}}\right)}{\frac{s}{\omega_{po}}\left(1+\frac{s}{\omega_{p_1}}\right)}$$

- Four unknowns, four equations:

$$\frac{\tau_d}{N}+\tau_i=\frac{1}{\omega_{z_1}}+\frac{1}{\omega_{z2}}$$

$$\frac{\tau_d\tau_i}{N}+\tau_d\tau_i=\frac{1}{\omega_{z_1}\omega_{z_2}}$$

$$\frac{\tau_i}{k_p}=\frac{1}{\omega_{po}}$$

$$\frac{\tau_d}{N}=\frac{1}{\omega_{p_1}}$$

Bridging a PID to a Type 3

- From Type 3 to PID:

$$\tau_d = \frac{\left(\omega_{p_1} - \omega_{z_1}\right)\left(\omega_{p_1} - \omega_{z_2}\right)}{\left(\omega_{p_1}\omega_{z_1} + \omega_{p_1}\omega_{z_2} - \omega_{z_1}\omega_{z_2}\right)\omega_{p_1}} \qquad N = \frac{\omega_{p_1}{}^2}{\omega_{p_1}\omega_{z_1} + \omega_{p_1}\omega_{z_2} - \omega_{z_1}\omega_{z_2}} - 1$$

$$\tau_i = \frac{\omega_{z_1} + \omega_{z_2}}{\omega_{z_1}\omega_{z_2}} - \frac{1}{\omega_{p_1}} \qquad k_p = \frac{\omega_{po}}{\omega_{z_1}} - \frac{\omega_{po}}{\omega_{p_1}} + \frac{\omega_{po}}{\omega_{z_2}}$$

- From PID to Type 3:

$$f_{z_1} = \frac{\tau_d - \sqrt{-4N^2\tau_d\tau_i + N^2\tau_i{}^2 - 2N\tau_d\tau_i + \tau_d{}^2} + N\tau_i}{2\tau_d\tau_i\left(1+N\right)2\pi} \qquad f_{p_1} = \frac{N}{2\pi\tau_d}$$

$$f_{z_2} = \frac{\tau_d + \sqrt{-4N^2\tau_d\tau_i + N^2\tau_i{}^2 - 2N\tau_d\tau_i + \tau_d{}^2} + N\tau_i}{2\tau_d\tau_i\left(1+N\right)2\pi} \qquad f_{po} = \frac{k_p}{2\pi\tau_i}$$

Testing the Conversion

- Assume a type 3 compensator calculated for:

$G_{f_c} = 1$ at a crossover of $f_c = 2740$ Hz

$f_{z_1} = 200$ Hz $\quad f_{z_2} = 600$ Hz $\quad f_{p_1} = 21400$ Hz $\quad f_{p_2} = 21400$ Hz

High-frequency pole

- The "0-dB crossover" pole is placed at:

$$f_{po} = \frac{\sqrt{1+\left(\frac{f_c}{f_{p_1}}\right)^2}\sqrt{1+\left(\frac{f_c}{f_{p_2}}\right)^2}}{\sqrt{1+\left(\frac{f_{z_1}}{f_c}\right)^2}\sqrt{1+\left(\frac{f_c}{f_{z_2}}\right)^2}} G_{f_c} f_{z_1} = 43.4\text{ Hz} = 272\text{ rd/s}$$

$\tau_d = 194u \quad \tau_i = 1.05m \quad N = 25.6 \quad k_p = 0.287$

What is the "0-dB Crossover" Pole?

- s appears as an isolated term in $N(s)$, it is an origin pole

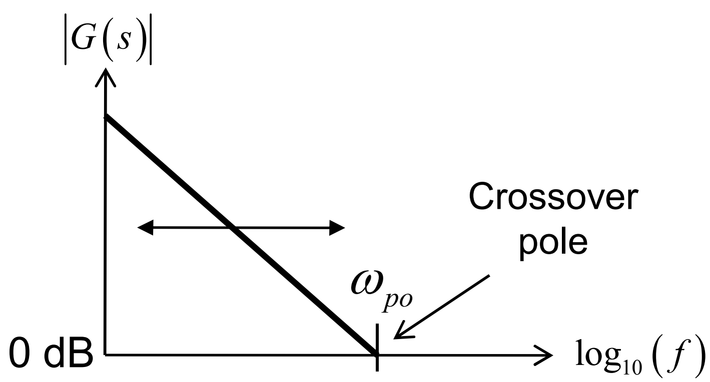
$$G(s) = \frac{1}{\boxed{s}\left(1 + s/\omega_{p_1}\right)}$$

s = 0 is the origin pole

- If s is affected by a coefficient it is the "0-dB crossover pole"

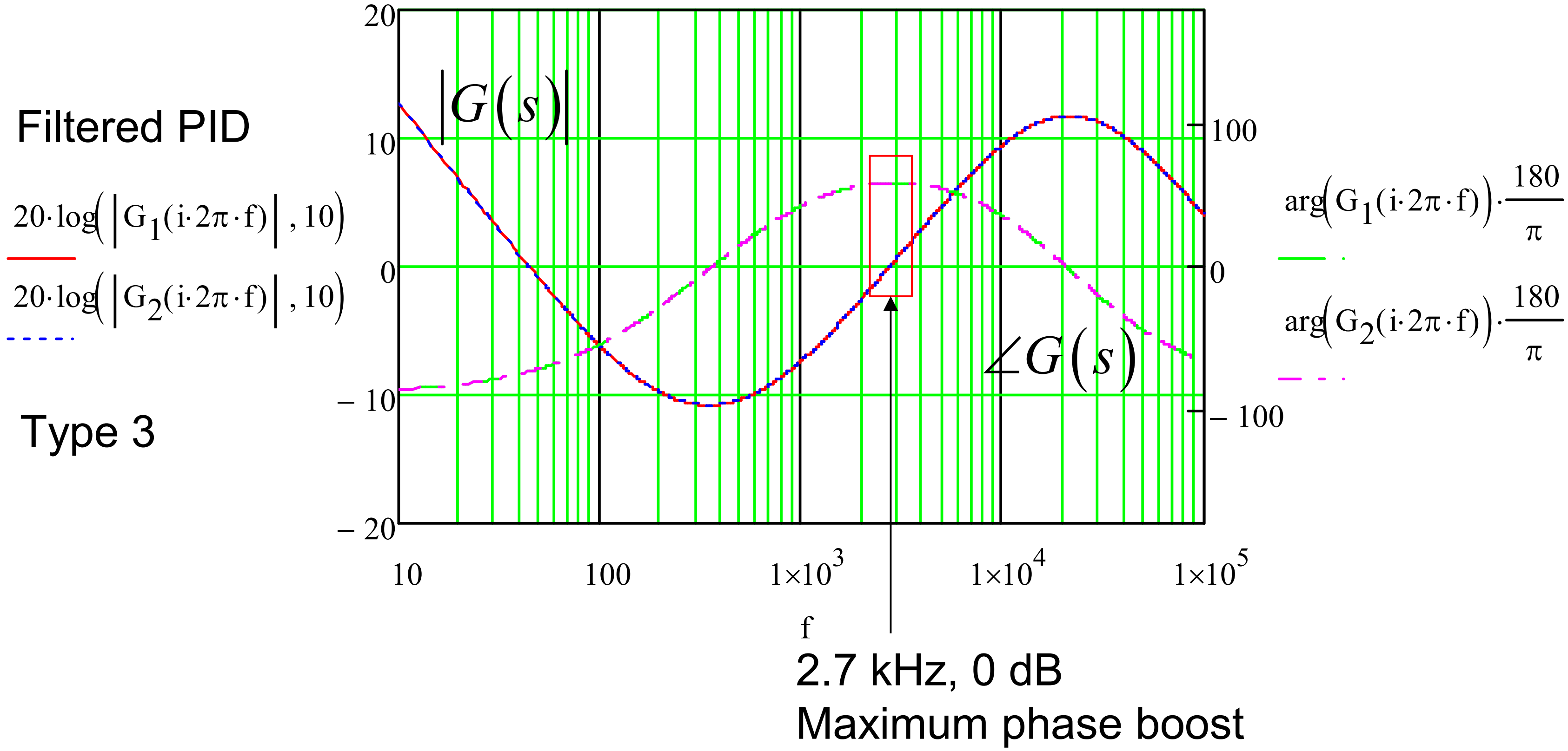
$$G(s) = \frac{1 + s/s_{z_1}}{As\left(1 + s/\omega_{p_1}\right)} = \frac{\frac{s}{s_{z_1}}\left(s_{z_1}/s + 1\right)}{\frac{s}{\boxed{\omega_{po}}}\left(1 + s/\omega_{p_1}\right)} = \frac{\omega_{po}}{\omega_{z_1}}\frac{\left(s_{z_1}/s + 1\right)}{\left(1 + s/\omega_{p_1}\right)} = G_0\frac{\left(s_{z_1}/s + 1\right)}{\left(1 + s/\omega_{p_1}\right)}$$

Dimension of a gain



Testing with Mathcad®

- We wanted a magnitude of 1 at 2.7 kHz



Testing with SPICE

parameters

i=(1+fc^2/fp1^2)*(1+fc^2/fp2^2)
j=(1+fz1^2/fc^2)*(1+fc^2/fz2^2)
Wpi=sqrt(i/j)*G*fz1*2*pi

e=(Wp1-Wz1)*(Wp1-Wz2)
f=Wp1*Wz1+Wp1*Wz2-Wz1*Wz2

Td=e/(f*Wp1)
N=((Wp1^2)/f)-1
Ti=((Wz1+Wz2)/(Wz1*Wz2))-(1/Wp1)
kpf=(Wpi/Wz1)-(Wpi/Wp1)+(Wpi/Wz2)

parameters

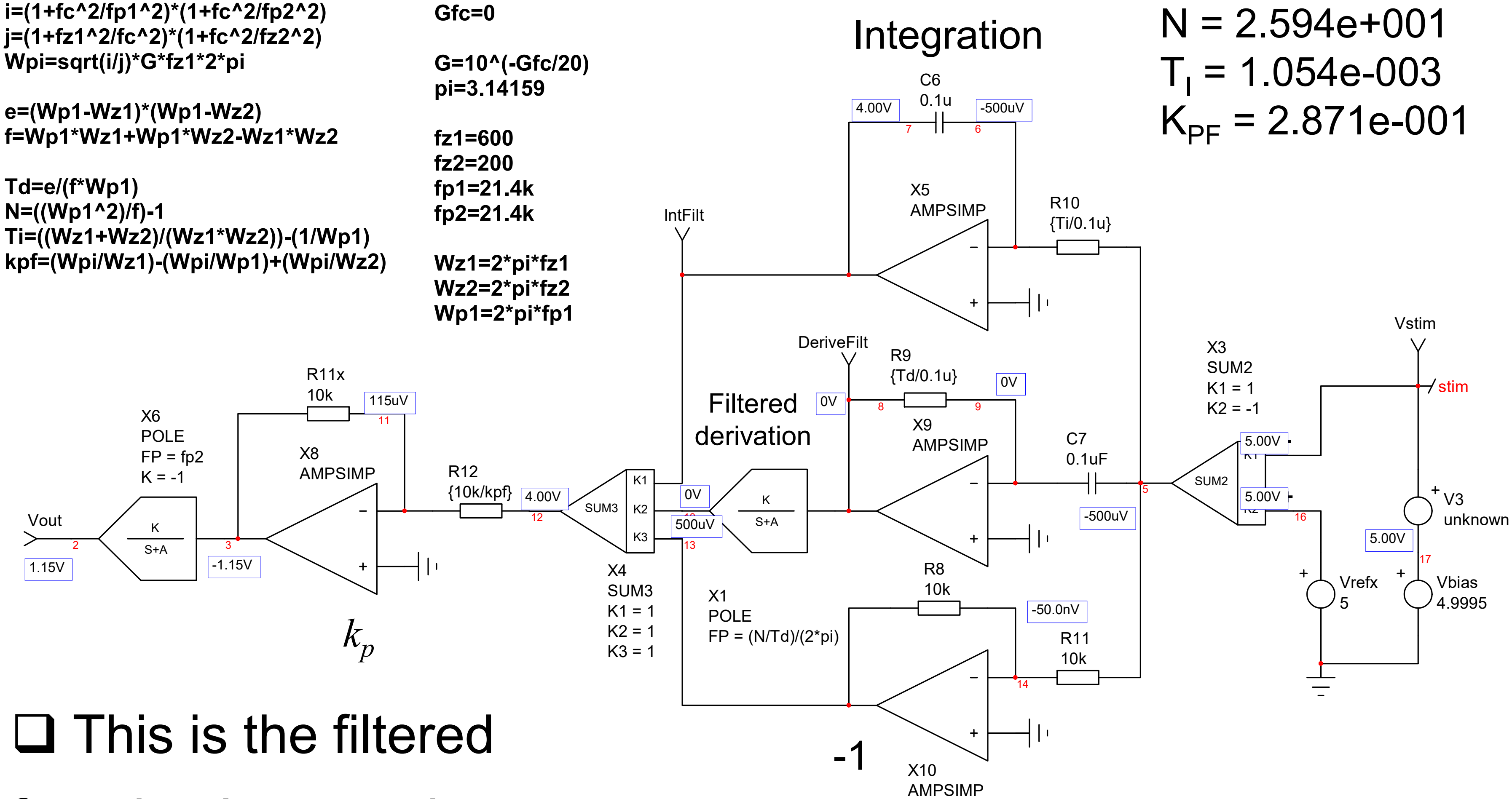
fc=2740
Gfc=0

G=10^(-Gfc/20)
pi=3.14159

fz1=600
fz2=200
fp1=21.4k
fp2=21.4k

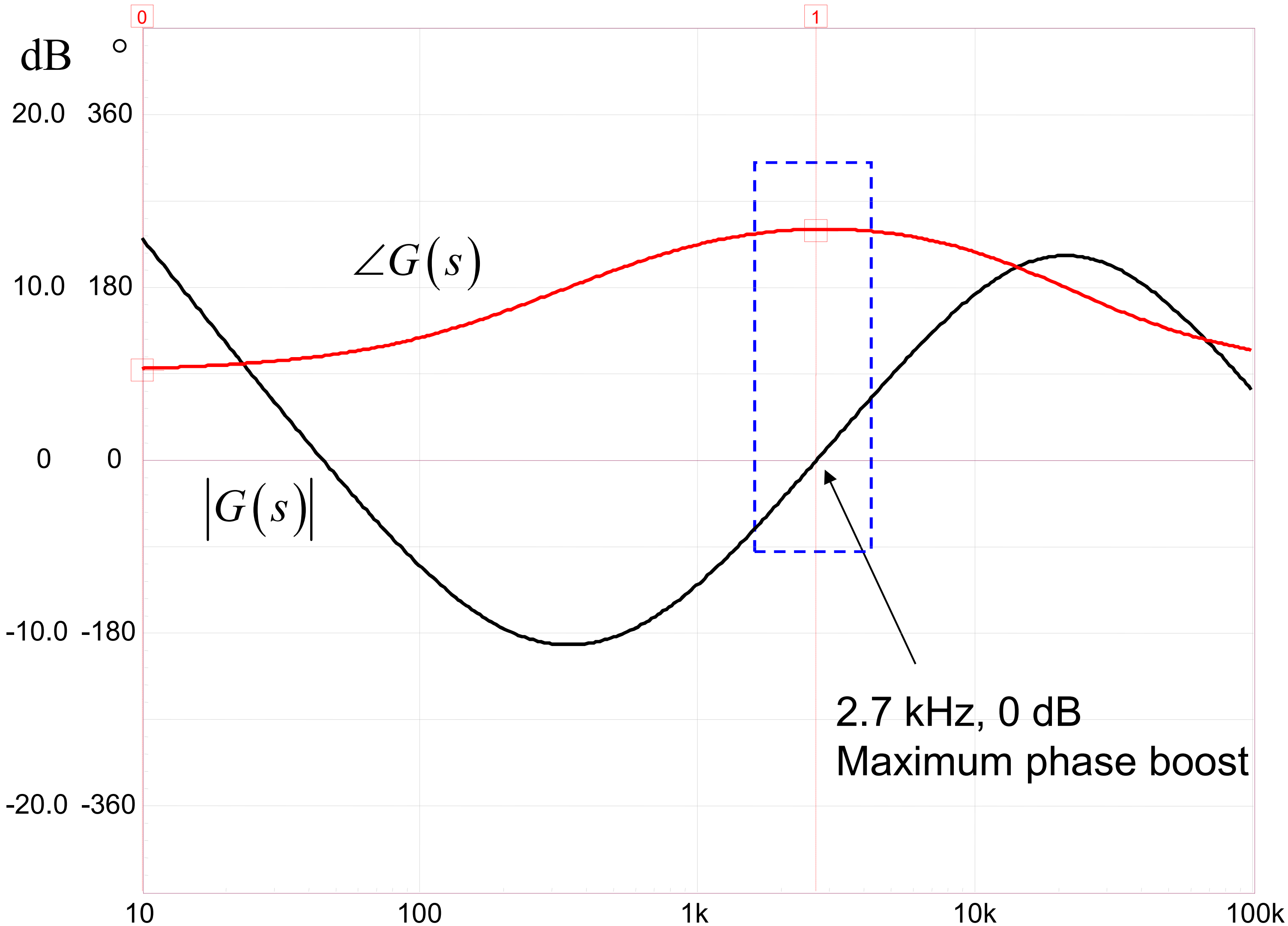
Wz1=2*pi*fz1
Wz2=2*pi*fz2
Wp1=2*pi*fp1

T_D = 1.929e-004
N = 2.594e+001
T_I = 1.054e-003
K_{PF} = 2.871e-001



- This is the filtered form implementation

Testing with SPICE



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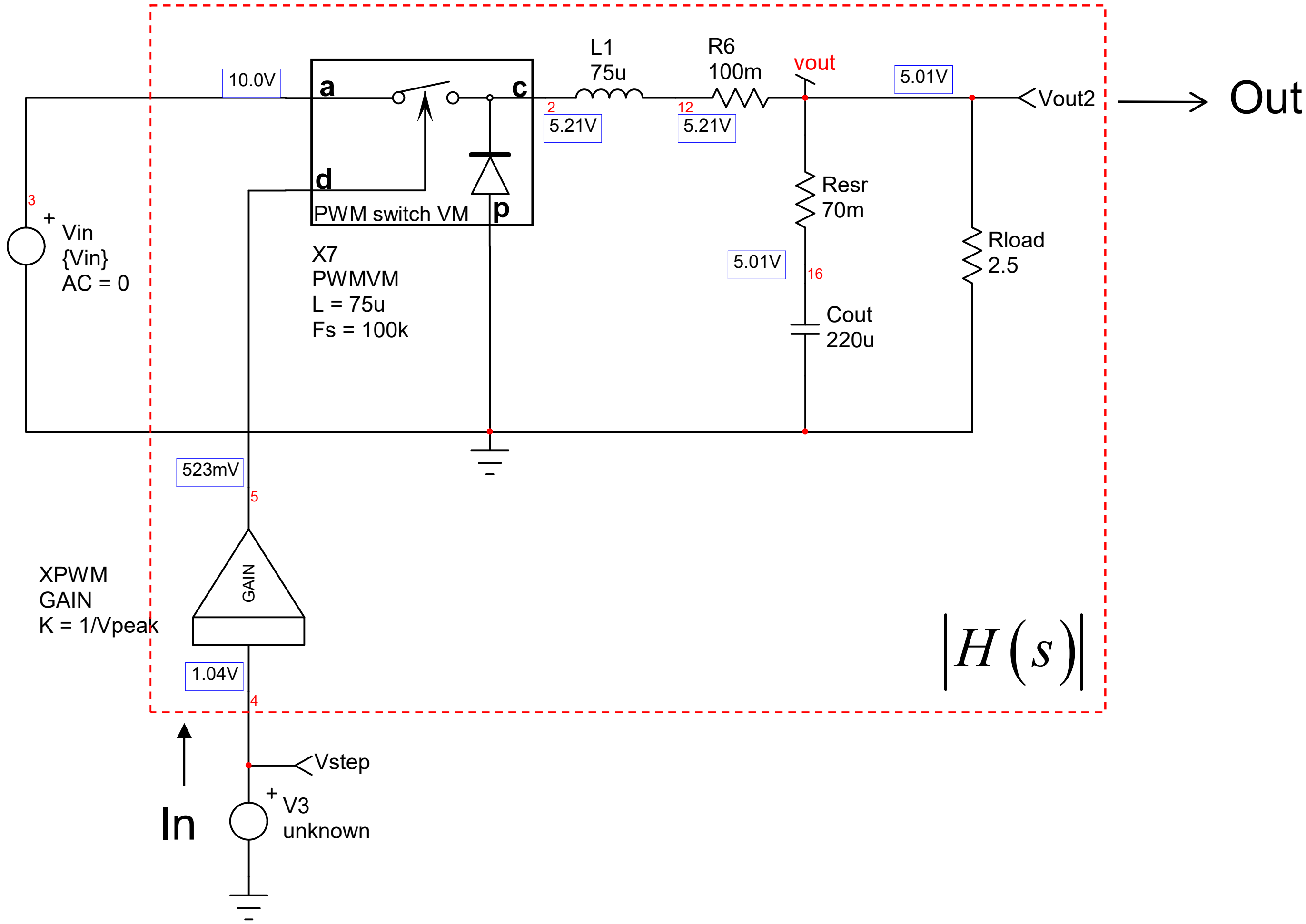
Stabilizing a Buck with a PID

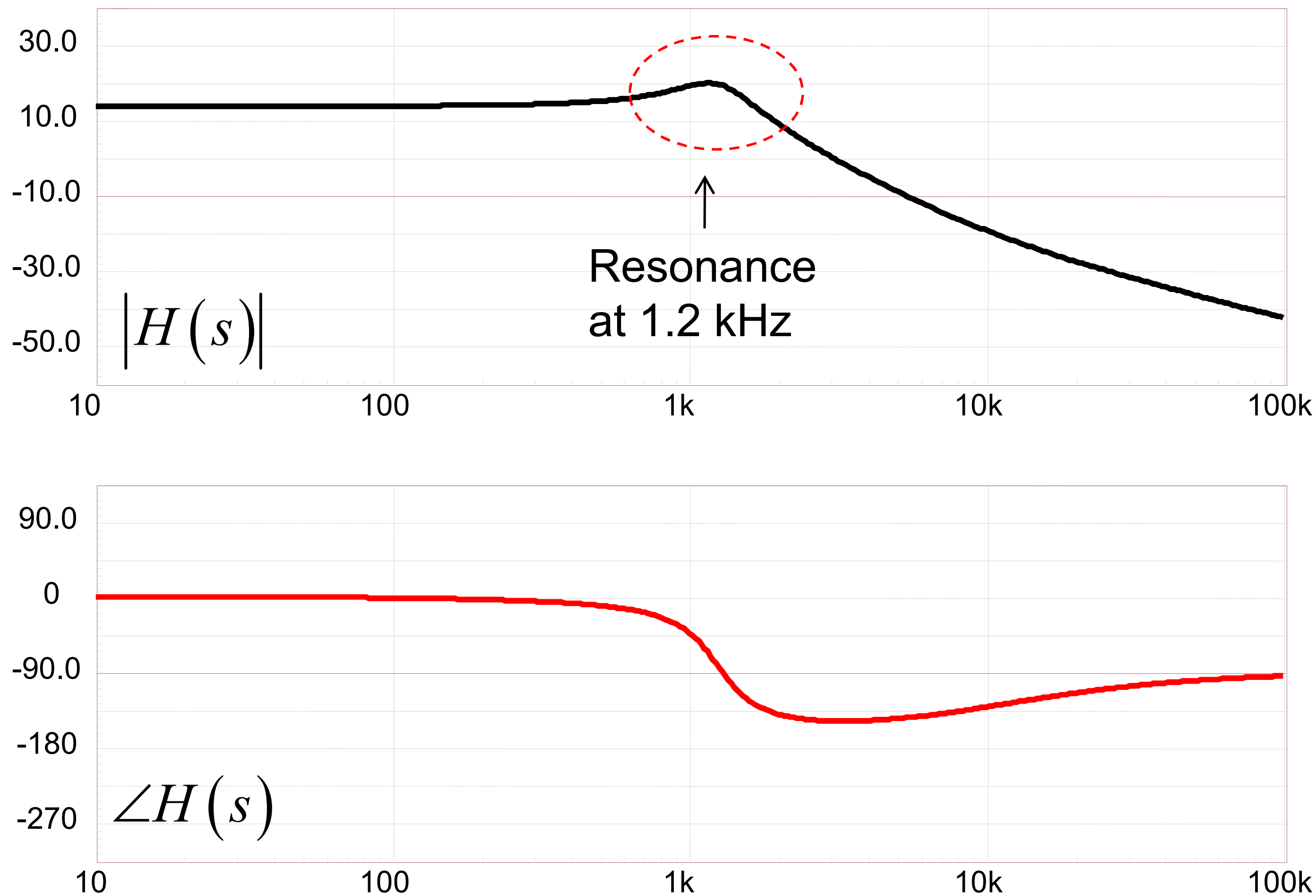
- ❑ We will use a PID to stabilize a voltage-mode Buck converter



The Plant

Small-Signal Response of the Buck

- The transfer function shows a resonance at 1.2 kHz

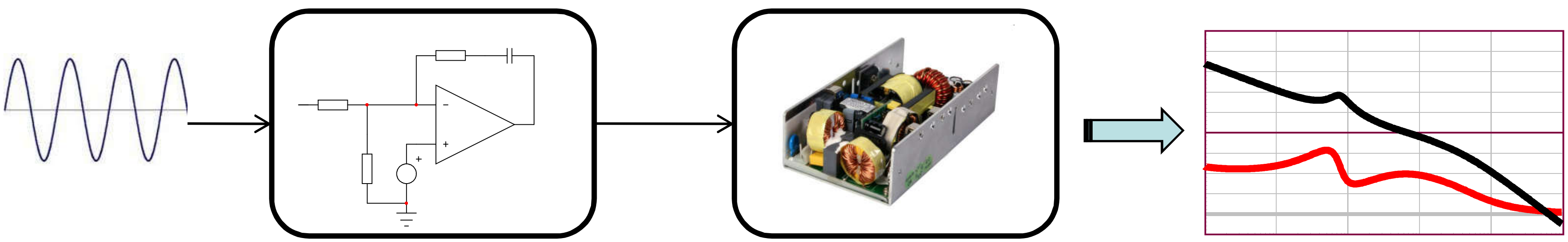


Compensating the Buck – Method 1

- We will explore three different methods for compensation:

1. Shape closed-loop gain to make it a 2nd-order system
2. Place poles and zeros to crossover at 10 kHz
3. Shape the output impedance only

- Method 1 – derive the open-loop gain first



$G(s)$ $H(s)$ $T_{OL}(s) = G(s)H(s)$

Compensating the Buck – Method 1

- The loop gain expression is that of the PID and $H(s)$

$$T_{OL}(s) = \frac{\overbrace{1 + s\left(\frac{\tau_d}{N} + \tau_i\right) + s^2\left(\frac{\tau_d \tau_i}{N} + \tau_d \tau_i\right)}^{N(s)}}{s\frac{\tau_i}{k_p}\left(1 + \frac{\tau_d}{N}s\right)} H_0 \frac{1 + s/\omega_{z_1}}{\underbrace{\left(\frac{s}{\omega_0}\right)^2 + \frac{s}{Q_0\omega_0} + 1}_{D(s)}}$$

- To simplify the expression, we can neutralize $D(s)$ by $N(s)$

⇨ Place a double zero at the double pole position:

$$1 + s\left(\frac{\tau_d}{N} + \tau_i\right) + s^2\left(\frac{\tau_d \tau_i}{N} + \tau_d \tau_i\right) = \left(\frac{s}{\omega_0}\right)^2 + \frac{s}{Q_0\omega_0} + 1$$

Compensating the Buck – Method 1

- The loop gain expression is now well simplified:

$$T_{OL}(s)=\frac{H_0\left(1+s/\omega_{z_1}\right)}{s\frac{\tau_i}{k_p}\left(1+\frac{\tau_d}{N}s\right)}$$

- For a unity feedback control system, the closed-loop gain is:

$$T_{CL}(s)=\frac{\dfrac{H_0\left(1+s/\omega_{z_1}\right)}{s\frac{\tau_i}{k_p}\left(1+\frac{\tau_d}{N}s\right)}}{1+\dfrac{H_0\left(1+s/\omega_{z_1}\right)}{s\frac{\tau_i}{k_p}\left(1+\frac{\tau_d}{N}s\right)}}=\frac{1+s/\omega_{z_1}}{1+s\left(\frac{1}{\omega_{z_1}}+\frac{\tau_i}{H_0k_p}\right)+s^2\left(\frac{\tau_d\tau_i}{NH_0k_p}\right)}$$

Compensating the Buck – Method 1

- We want a damped second-order response:

$$1+s\left(\frac{1}{\omega_{z_1}}+\frac{\tau_i}{H_0 k_p}\right)+s^2\left(\frac{\tau_d\tau_i}{NH_0 k_p}\right)=1+\frac{s}{\omega_c Q_c}+\left(\frac{s}{\omega_c}\right)^2$$

Closed-loop denominator → left side; Second-order canonical form → right side

- Choose a crossover frequency and a quality factor:

$Q_c = 0.5$ Non-ringing response $\omega_c = 27.3\,\mathrm{rd/s} \rightarrow 10\,\mathrm{kHz}$

Compensating the Buck – Method 1

- Four unknowns, four equations:

$$\frac{1}{\omega_{z_1}}+\frac{T_i}{H_0 k_p}=\frac{1}{\omega_c Q_c} \qquad \frac{T_d T_i}{N H_0 k_p}=\frac{1}{\omega_c{}^2} \qquad \frac{T_d}{N}+T_i=\frac{1}{\omega_0 Q_0} \qquad \frac{T_d T_i}{N_1}+T_d T_i=\frac{1}{\omega_0{}^2}$$

$$T_d=\frac{Q_0 Q_c{}^2\omega_{z_1}{}^2\omega_0{}^2+Q_c{}^2\omega_{z_1}\omega_0\omega_c{}^2+Q_0 Q_c{}^2\omega_c{}^4-Q_c\omega_{z_1}{}^2\omega_0\omega_c-2Q_0 Q_c\omega_{z_1}\omega_c{}^3+Q_0\omega_{z_1}{}^2\omega_c{}^2}{\omega_0\omega_c\left(Q_c\omega_c-\omega_{z_1}\right)\left(Q_c\omega_c{}^2-\omega_{z_1}\omega_c+Q_0 Q_c\omega_{z_1}\omega_0\right)}=1.116\,\text{ms}$$

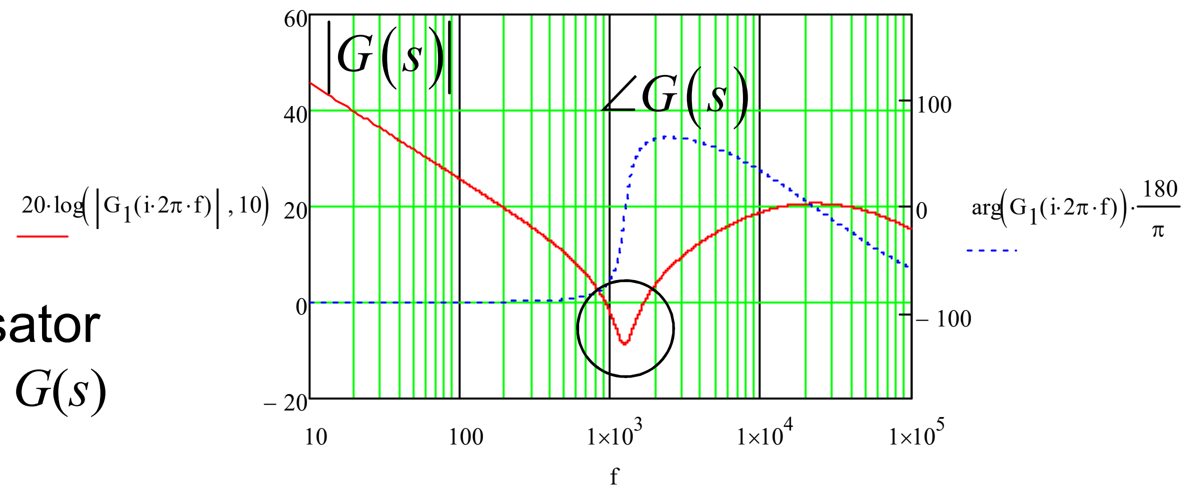
$$N=-\frac{Q_0 Q_c{}^2\omega_{z_1}{}^2\omega_0{}^2+Q_c{}^2\omega_{z_1}\omega_0\omega_c{}^2+Q_0 Q_c{}^2\omega_c{}^4-Q_c\omega_{z_1}{}^2\omega_0\omega_c-2Q_0 Q_c\omega_{z_1}\omega_c{}^3+Q_0\omega_{z_1}{}^2\omega_c{}^2}{\omega_0 Q_c\omega_{z_1}\left(Q_c\omega_c{}^2-\omega_{z_1}\omega_c+Q_0 Q_c\omega_{z_1}\omega_0\right)}=72.4$$

$$T_i=-\frac{Q_c\omega_c{}^2-\omega_{z_1}\omega_c+Q_0 Q_c\omega_{z_1}\omega_0}{Q\omega_{z_1}\omega_0\omega_c-Q_0 Q_c\omega_0\omega_c{}^2}=14.6\,\mu\text{s} \qquad k_p=-\frac{Q_c\omega_{z_1}\left(Q_c\omega_c{}^2-\omega_{z_1}\omega_c+Q_c Q_0\omega_{z_1}\omega_0\right)}{H_0 Q_0\omega_0\left(\omega_{z_1}-Q_c\omega_c\right)^2}=0.178$$

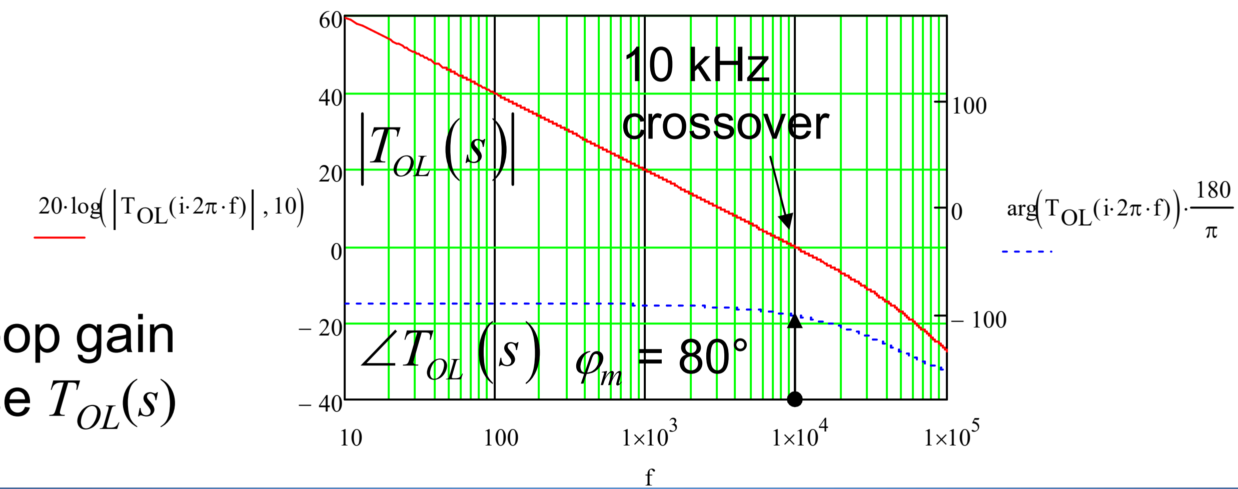
Compensating the Buck – Method 1

- We can now compute our transfer functions in Mathcad®

Compensator response $G(s)$

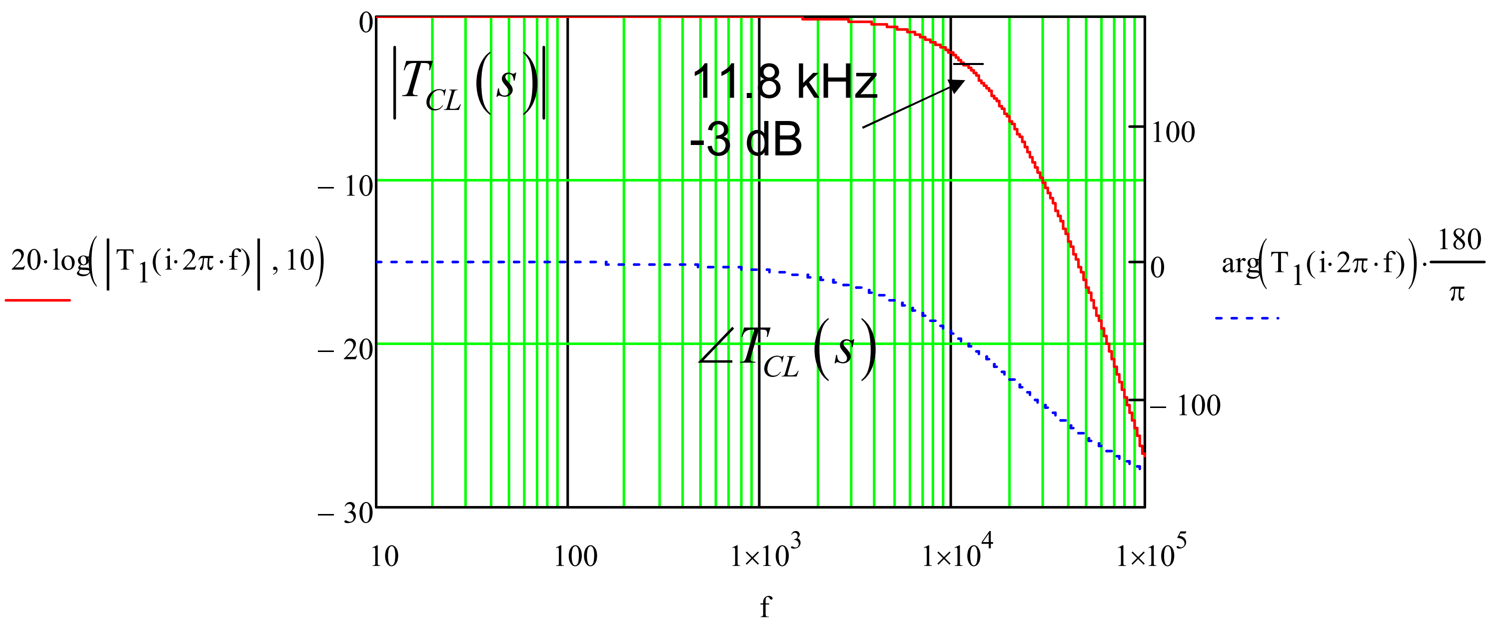


Open-loop gain response $T_{OL}(s)$



Compensating the Buck – Method 1

- The closed-loop system is perfectly compensated



$$T_{CL}(s) = \frac{T_{OL}(s)}{1+T_{OL}(s)}$$

Compensating the Buck – Method 1

- ❑ We can test the compensation with SPICE

parameters

fc=10k
Gfc=-20
Vin=10
Vpeak=2

G=10^(-Gfc/20)
pi=3.14159

fz1=1.2k
fz2=1.2k
fp1=10k
fp2=50k

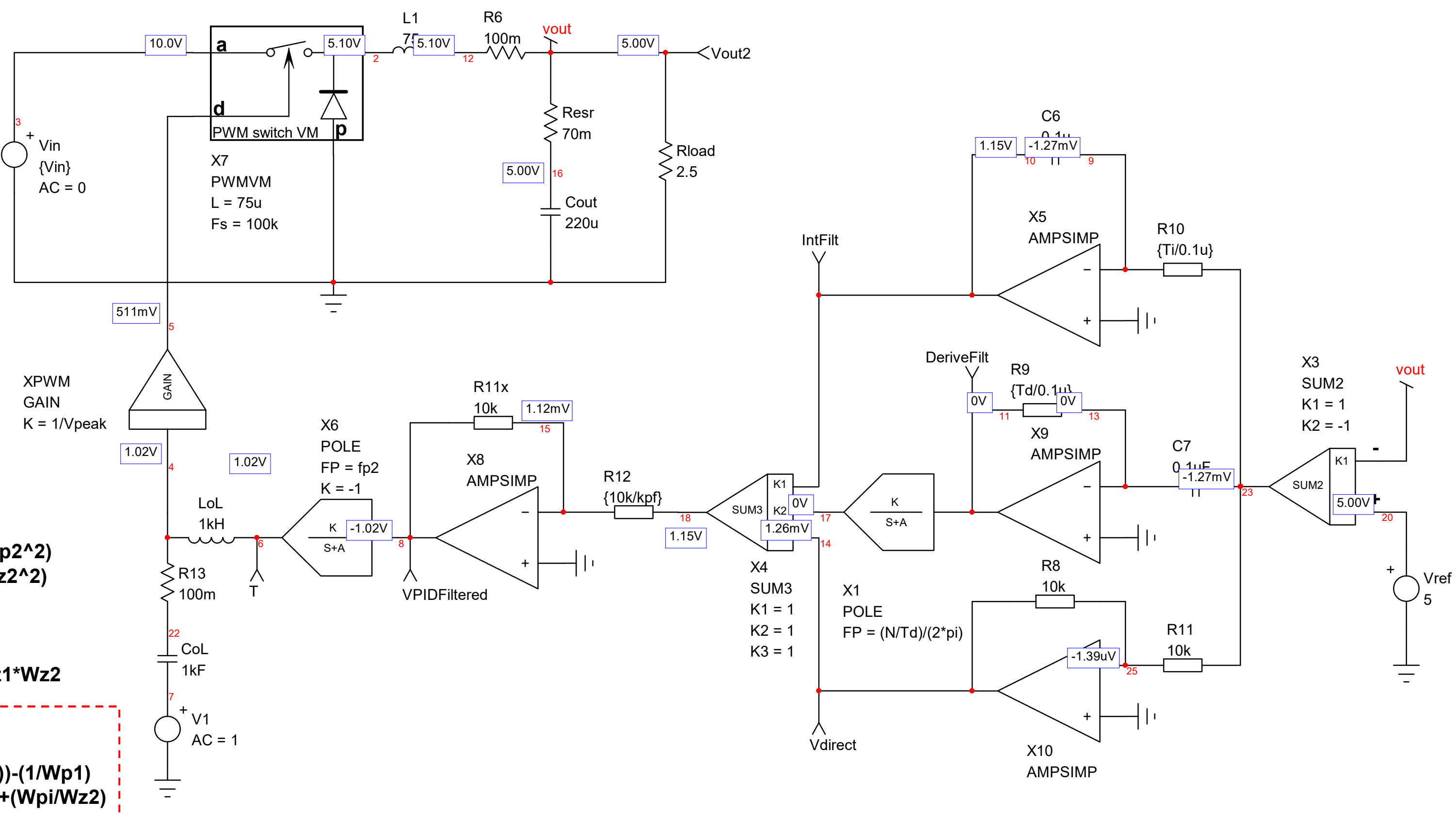
Wz1=2*pi*fz1
Wz2=2*pi*fz2
Wp1=2*pi*fp1

i=(1+fc^2/fp1^2)*(1+fc^2/fp2^2)
j=(1+fz1^2/fc^2)*(1+fc^2/fz2^2)
Wpi=sqrt(i/j)*G*fz1*2*pi

e=(Wp1-Wz1)*(Wp1-Wz2)
f=Wp1*Wz1+Wp1*Wz2-Wz1*Wz2

Td=e/(f*Wp1)
N=((Wp1^2)/f)-1
Ti=((Wz1+Wz2)/(Wz1*Wz2))-(1/Wp1)
kpf=(Wpi/Wz1)-(Wpi/Wp1)+(Wpi/Wz2)

Tdd=1.116m
Tii=14.6u
kpff=0.178
NN=72.4



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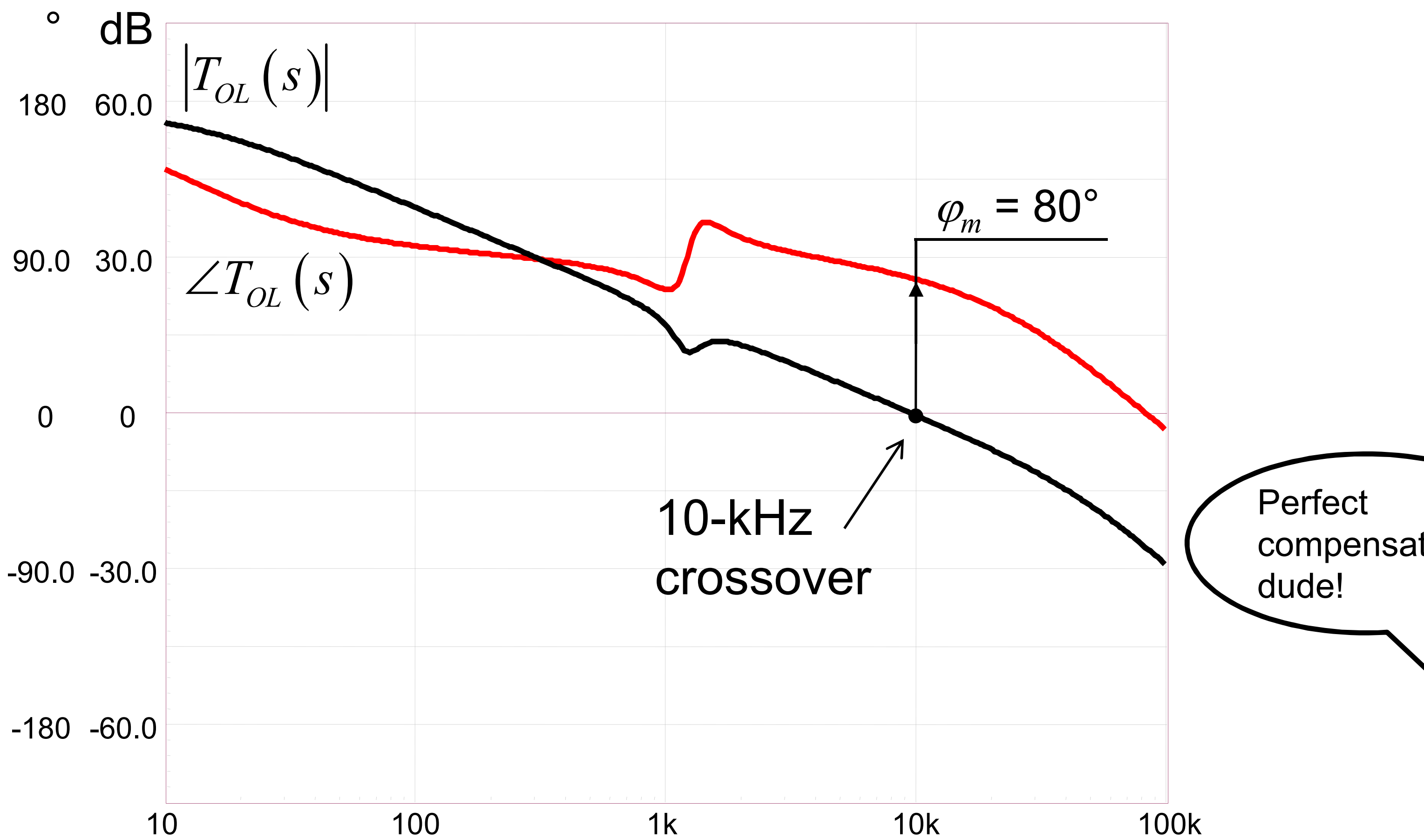
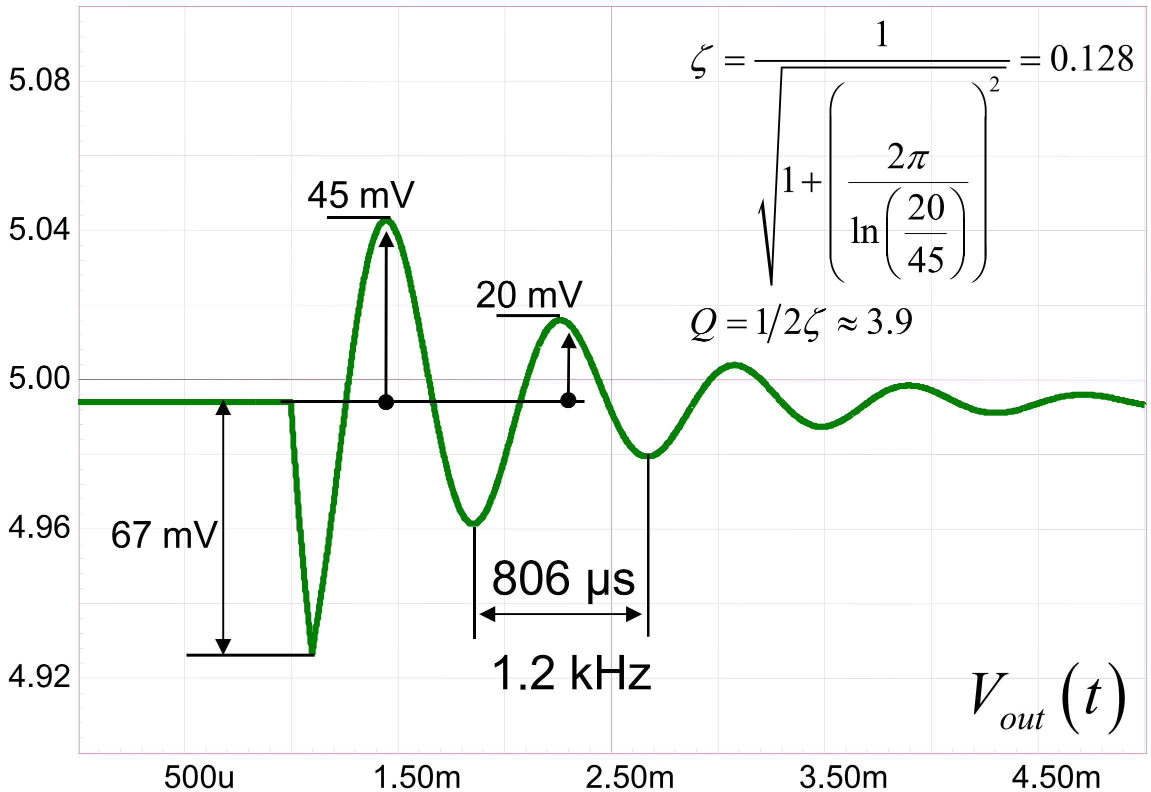
Compensating the Buck – Method 1

- We can test the compensation with SPICE



Compensating the Buck – Method 1

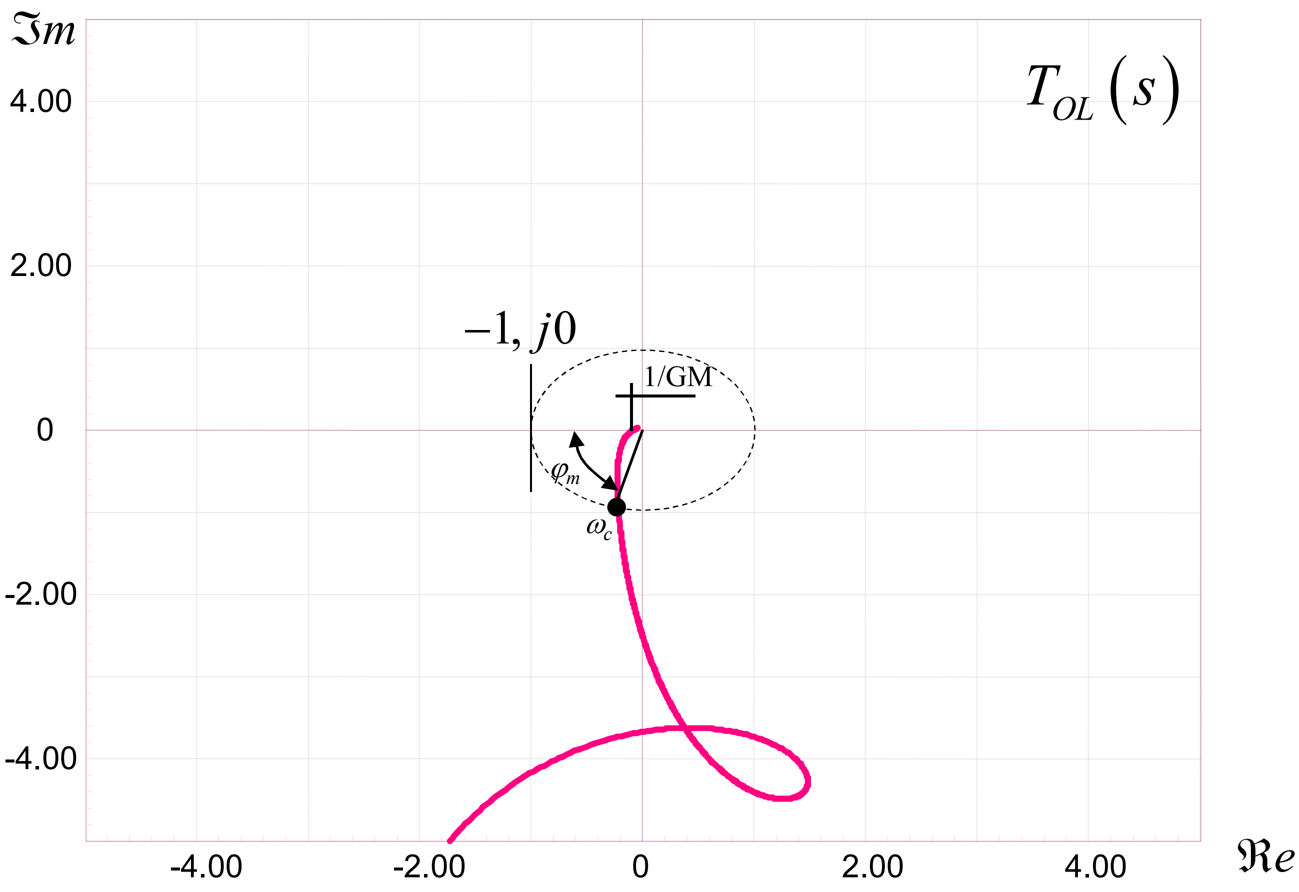
- We have a stable but oscillatory response!



$\Delta I_{out} = 1\text{ A in }100\ \mu\text{s}$

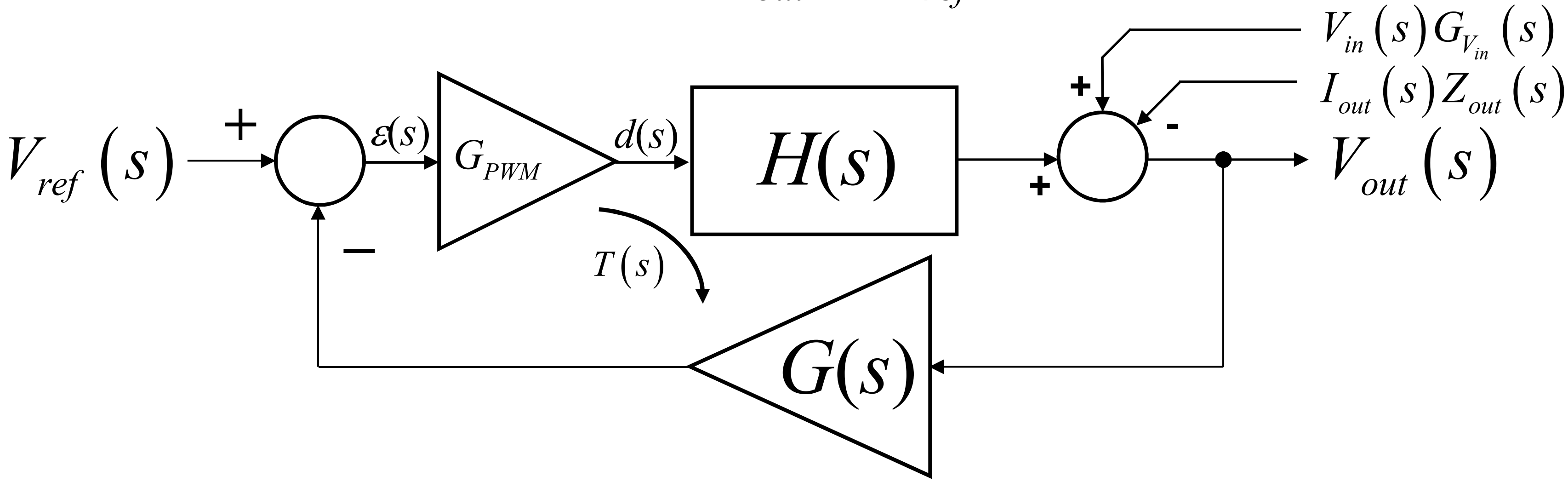
Compensating the Buck – Method 1

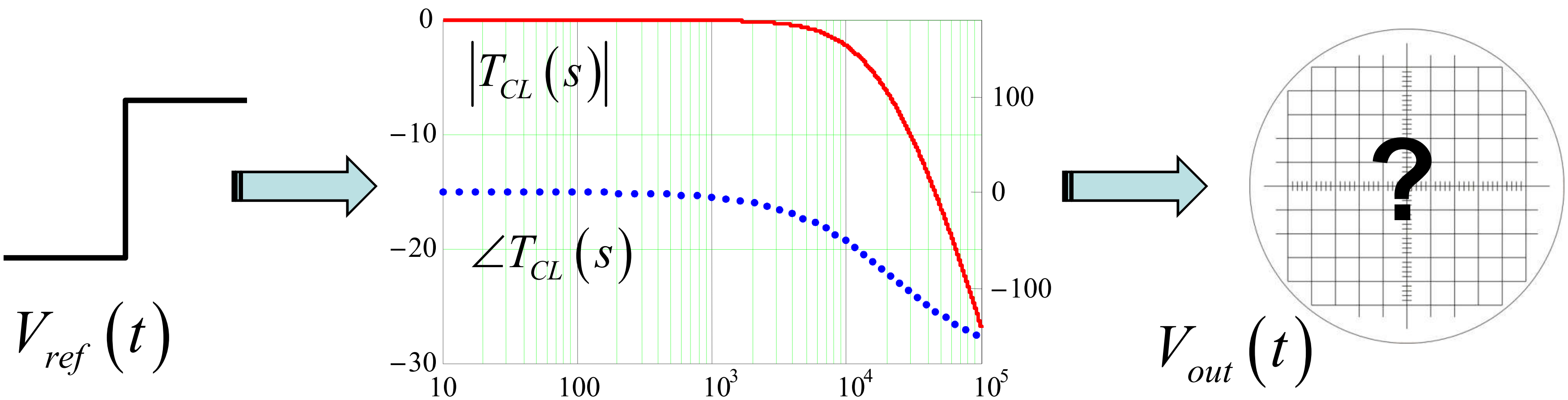
- Bode or Nyquist do not predict the oscillatory response



Compensating the Buck – Method 1

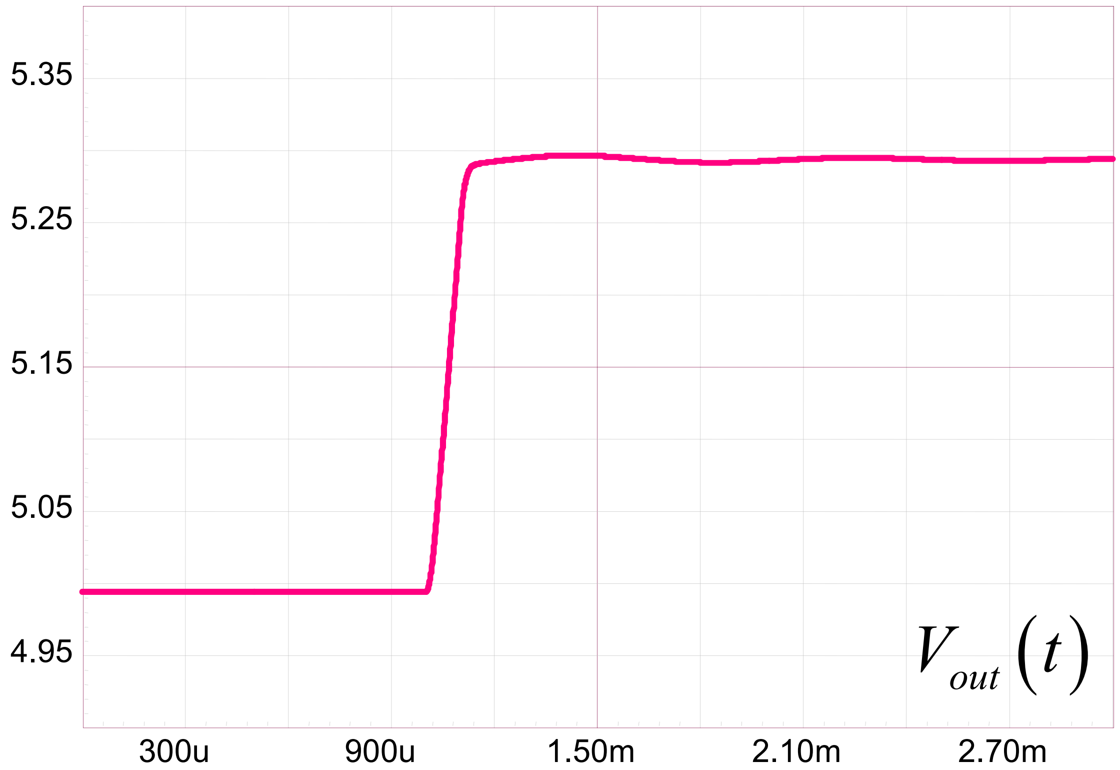
- We compensated the V_{out} to V_{ref} path only!





Compensating the Buck – Method 1

- ❑ The output response is as expected



- ❑ Where is the issue coming from then?

Course Agenda

- ❑ Introduction to Control Systems
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- ❑ **Considering the Output Impedance**
- ❑ Classical Poles/Zeros Placement
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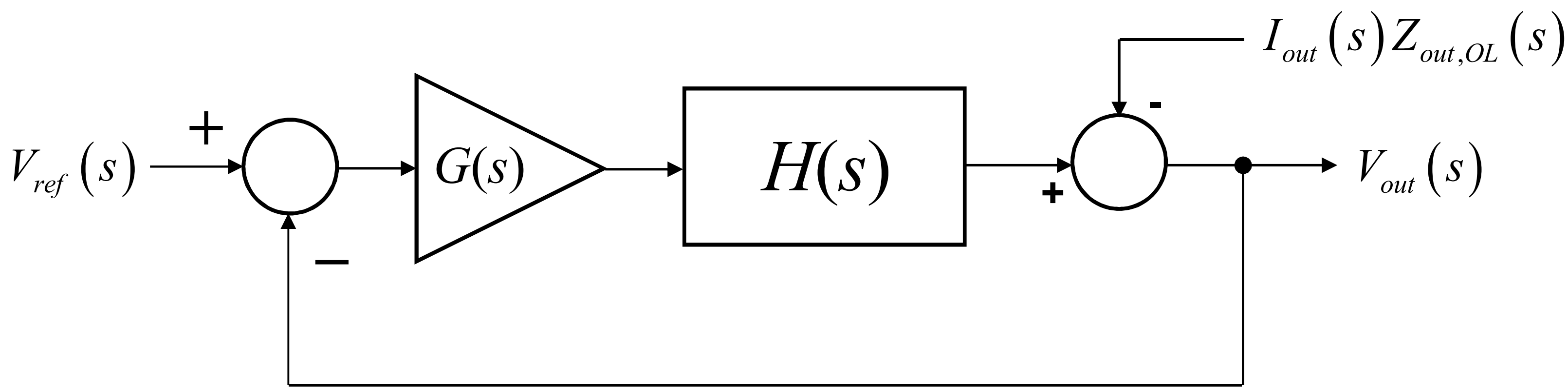
- ❑ What is Delay Margin?
- ❑ Gain Margin is not Enough

Compensating the Buck – Method 1

- In reality, V_{ref} is fixed: "we have a regulator, stupid!"



- Because the system is linear, superposition applies

$$V_{out1}(s) = V_{ref}(s)\frac{T_{OL}(s)}{1+T_{OL}(s)} \qquad V_{out2}(s) = I_{out}(s)Z_{out,OL}(s) - V_{out}(s)T_{OL}(s)$$

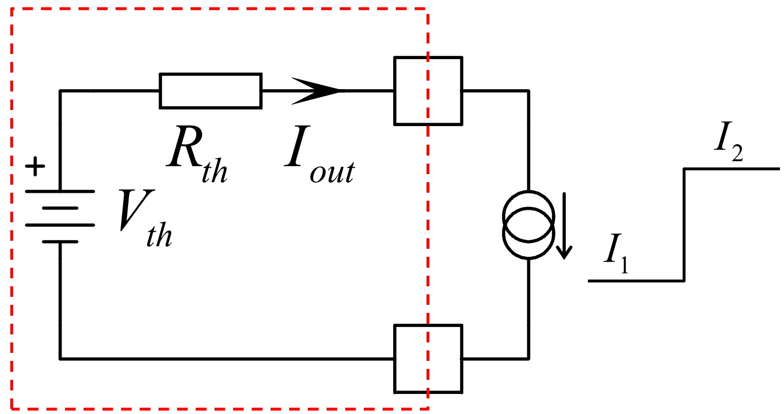
$$V_{out}(s) = V_{out1}(s) + V_{out2}(s) = V_{ref}(s)\frac{T_{OL}(s)}{1+T_{OL}(s)} - I_{out}(s)\underbrace{\frac{Z_{out,OL}(s)}{1+T_{OL}(s)}}_{Z_{out,CL}}$$

- During the load step, $\hat{v}_{ref} = 0$: Z_{out} fixes the response!

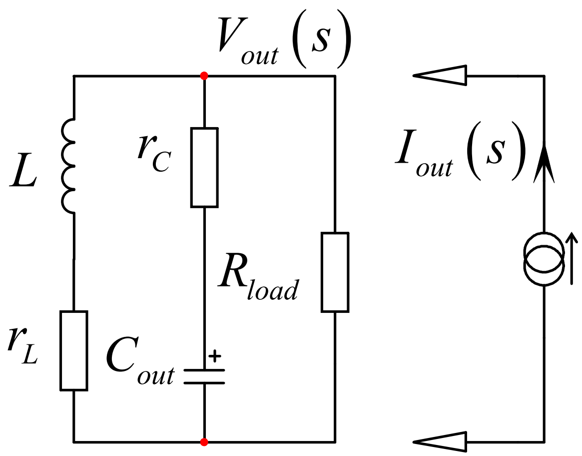
Compensating the Buck – Method 1

- What matters is the output impedance $Z_{out,CL}$

model



- What is the output impedance of a buck converter?



It follows the form

$$Z_{out}(s) = R_0 \frac{N(s)}{D(s)}$$

Ω Ω No dimension

Compensating the Buck – Method 1

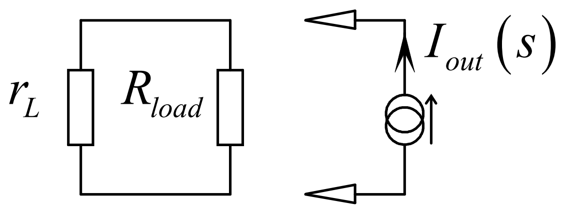
- The output impedance is a transfer function

$$Z_{out}(s) = \frac{V_{out}(s)}{I_{out}(s)}$$

$V_{out}(s)$ ← response

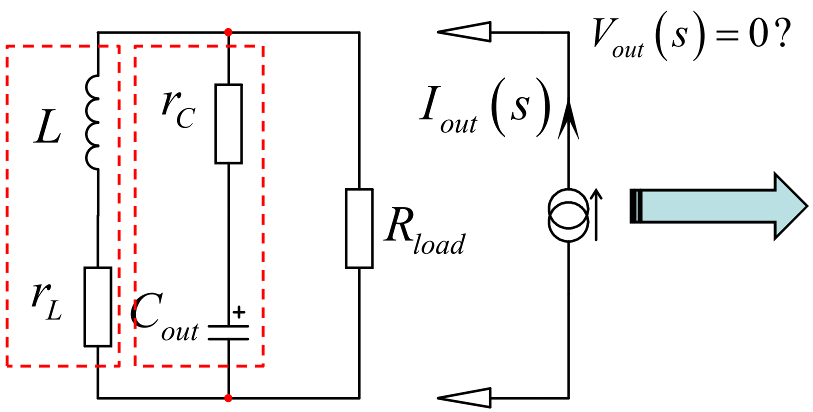
$I_{out}(s)$ ← excitation

- Let's find the term R_0 in dc: open caps, short inductors



$$R_0 = r_L \,||\, R_{load}$$

- The zeros cancel the response



$$sL + r_L = 0 \qquad r_L\left(s\frac{L}{r_L} + 1\right) = 0 \qquad \omega_{z_1} = \frac{r_L}{L}$$

$$\frac{1}{sC_{out}} + r_C = 0 \qquad 1 + sr_C C_{out} = 0 \qquad \omega_{z_2} = \frac{1}{r_C C_{out}}$$

Compensating the Buck – Method 1

- ❑ The denominator is solely dependent on the structure
  - ➢ It is independent from the excitation: set it to zero!
- ❑ There are two storage elements: this is a 2nd–order network

$$D(s) = 1 + a_1 s + a_2 s^2 = 1 + \frac{s}{\omega_0 Q} + \left(\frac{s}{\omega_0}\right)^2$$

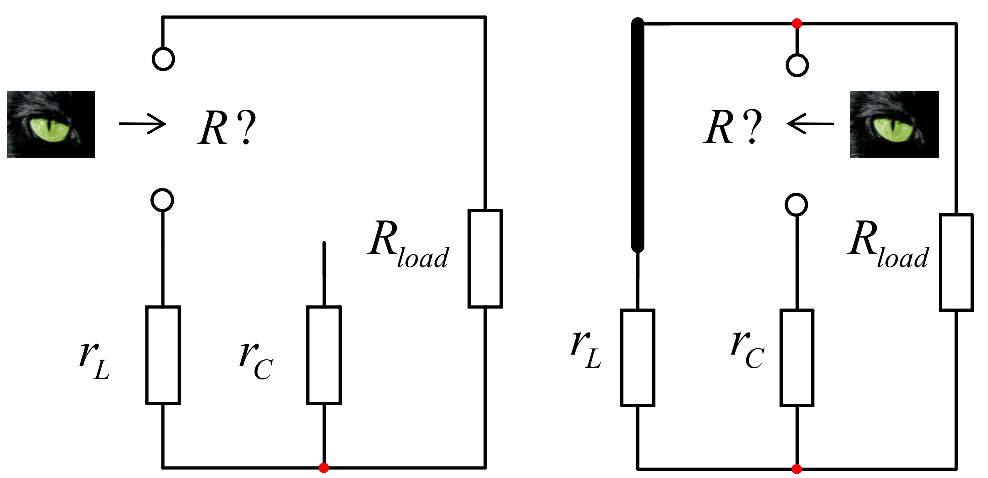
- ❑ D must be dimensionless thus: $a_1 \equiv (Hz)^{-1}$ $a_2 \equiv (Hz)^{-2}$
  - ➢ The two possible terms for a_1 are $\tau_1 + \tau_2$
  - ➢ The two possible terms for a_2 are $\tau_1 \tau'_2$, $\tau'_1 \tau_2$

$$\tau = \frac{L}{R} \text{ or } \tau = RC$$

Compensating the Buck – Method 1

- For a_1 look at the resistance R driving L and C

  - Look at the driving impedance at L while C is in its dc state
  - Look at the driving impedance at C while L is in its dc state



$$\ldots s\left(\frac{L}{r_L + R_{load}} + C\left[\left(r_L \parallel R_{load}\right) + r_C\right]\right)$$

$$\downarrow \qquad\qquad \downarrow$$

$$\tau_1 \qquad\qquad \tau_2$$

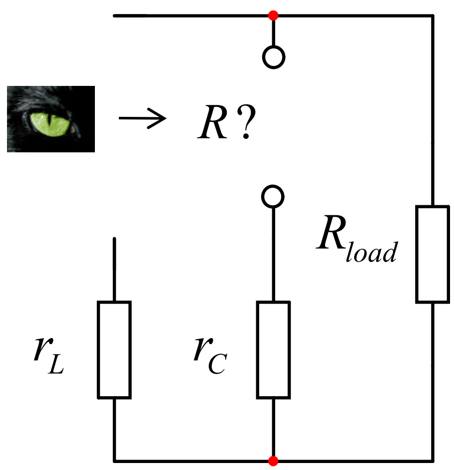
$$R = r_L + R_{load} \qquad\qquad R = \left(r_L \parallel R_{load}\right) + r_C$$

Compensating the Buck – Method 1

- ❑ how τ_1 (involving L) combines with τ'_2 (involving C)?
- ❑ how τ_2 (involving C) combines with τ'_1 (involving L)?

$\Big\}\ a_2$

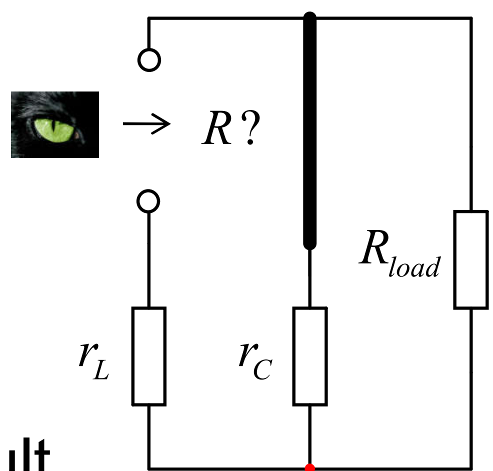
- ➢ Look at the driving impedance at C while L is in its HF state
- ➢ Look at the driving impedance at L while C is in its HF state



If we chose

$$\tau_1 = \frac{L}{r_L + R_{load}}$$

$$\tau'_2 = C\left(r_c + R_{load}\right)$$



If we chose

$$\tau_2 = C\left[\left(r_L \,\|\, R_{load}\right) + r_C\right]$$

$$\tau'_1 = \frac{L}{r_L + \left(R_{load} \,\|\, r_C\right)}$$

Same result

$$\tau_1 \tau'_2 = \tau'_1 \tau_2$$

Compensating the Buck – Method 1

- We have our denominator!

$$D(s) = 1 + s\left(\frac{L}{r_L + R_{load}} + C\left[\left(r_L \parallel R_{load}\right) + r_C\right]\right) + s^2\left(LC\frac{r_C + R_{load}}{r_L + R_{load}}\right)$$

- The complete transfer function is now:

$$Z_{out}(s) = \left(r_L \parallel R_{load}\right)\frac{\left(1 + s\frac{L}{r_L}\right)\left(1 + s r_C C_{out}\right)}{1 + s\left(\frac{L}{r_L + R_{load}} + C\left[\left(r_L \parallel R_{load}\right) + r_C\right]\right) + s^2\left(LC\frac{r_C + R_{load}}{r_L + R_{load}}\right)}$$

See "Fast Analytical Techniques" from Vatché Vorpérian, Cambridge Press

Compensating the Buck – Method 1

- It can be put under the following form:

$$Z_{out}(s) = R_0 \frac{\left(1 + s/\omega_{z_1}\right)\left(1 + s/\omega_{z_2}\right)}{1 + \frac{s}{\omega_0 Q} + \left(\frac{s}{\omega_0}\right)^2}$$

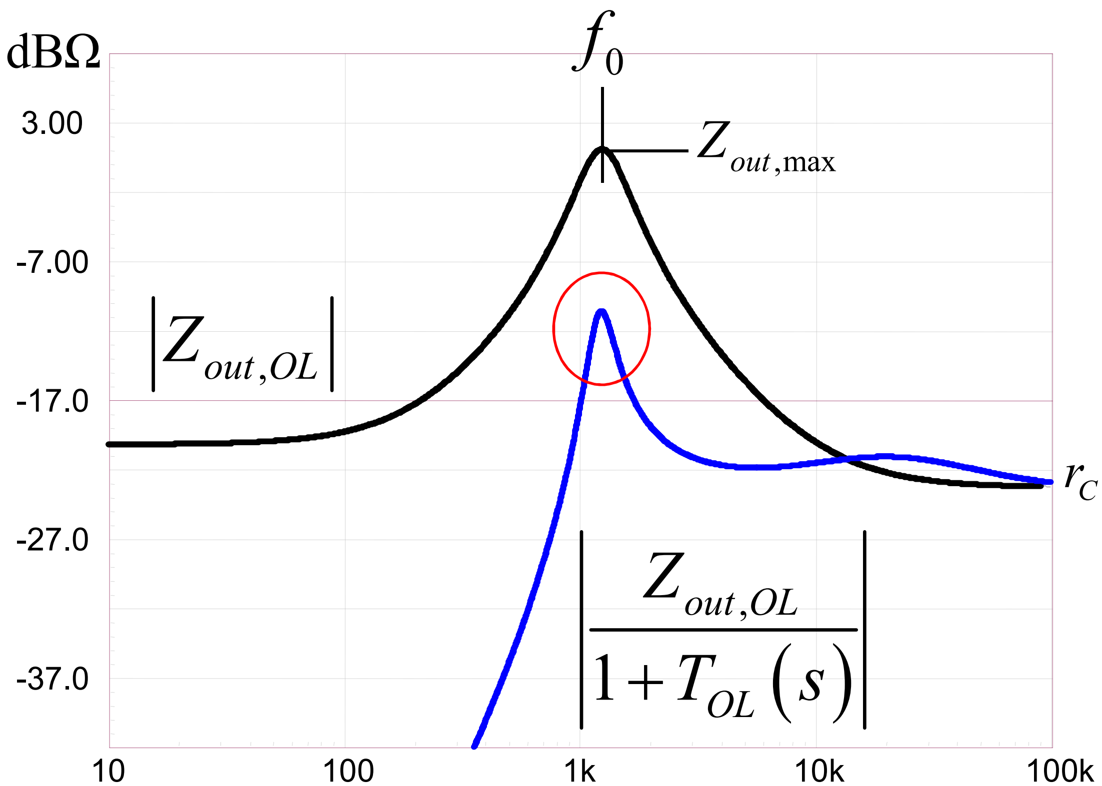
- Where we can identify the terms:

$$R_0 = r_L \,||\, R_{load} \qquad \omega_{z_1} = \frac{r_L}{L} \qquad \omega_{z_2} = \frac{1}{r_C C_{out}}$$

$$\omega_0 = \frac{1}{\sqrt{LC_{out}}}\sqrt{\frac{r_L + R_{load}}{r_C + R_{load}}} \qquad Q = \frac{LC_{out}\omega_0\left(r_C + R_{load}\right)}{L + C_{out}\left(r_L r_C + r_L R_{load} + r_C R_{load}\right)}$$

Compensating the Buck – Method 1

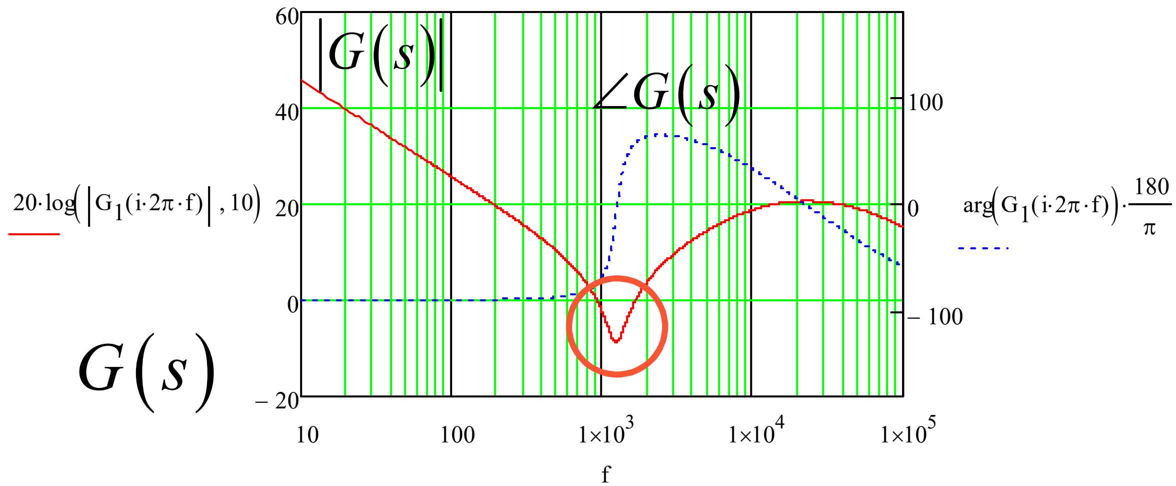
- If we now plot the output impedance, we see peaking
- For an non-oscillatory response, the peaking must be damped!



- It's not, this is where the problem comes from

Compensating the Buck – Method 1

- ❑ We organized a gain deficit right at the resonance!



- ❑ To tame the peaking, we must have gain at f_0
- ❑ How much do we peak at f_0?

$$\left|Z_{out,\max}\left(\omega_0\right)\right| = R_0 \frac{\sqrt{1+\left(\omega_0/\omega_{z_1}\right)^2}\sqrt{1+\left(\omega_0/\omega_{z_2}\right)^2}}{\sqrt{\left(1-\omega_0{}^2\left(LC_{out}\dfrac{r_C+R_{load}}{r_L+R_{load}}\right)\right)^2+\left(\omega_0\left(\dfrac{L}{r_L+R_{load}}+C_{out}\left(r_C+r_L \parallel R_{load}\right)\right)\right)^2}}$$

Compensating the Buck – Method 1

- ❑ We can impose a magnitude to stay below r_C
- ➢ evaluate the needed gain to fulfill this goal:

$$\left|\frac{Z_{out,\max}(f_0)}{1+T_{OL}(f_0)}\right| \le r_C \quad \overset{\approx}{\Longrightarrow} \quad \frac{\left|Z_{out,\max}(f_0)\right|}{\left|T_{OL}(f_0)\right|} \le r_C \quad \Longrightarrow \quad \left|T_{OL}(f_0)\right| \ge \frac{\left|Z_{out,\max}(f_0)\right|}{r_C}$$

Closed-loop output impedance

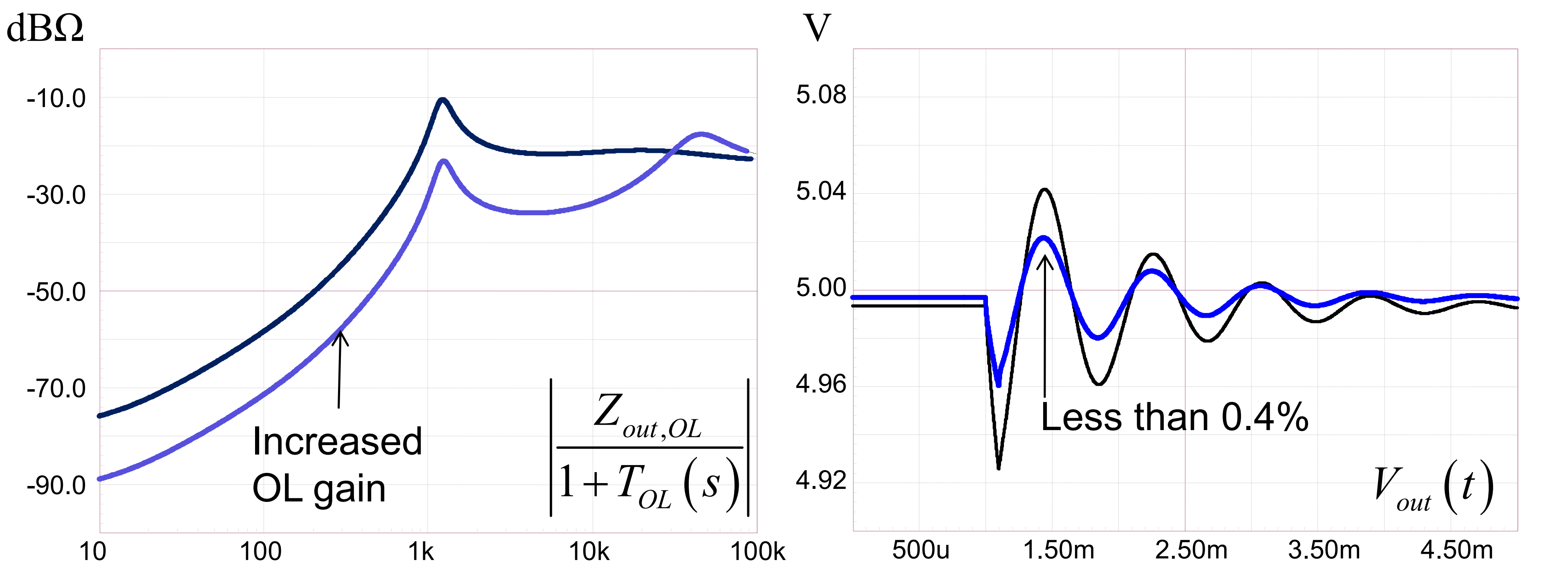
- ❑ Applying the numerical values of the buck:

$$\left|T_{OL}(f_0)\right| \ge \frac{1.12}{70m} \ge 16 \text{ or } 24\,\text{dB}$$

- ❑ Is this enough to obtain a ringing-free response?

Compensating the Buck – Method 1

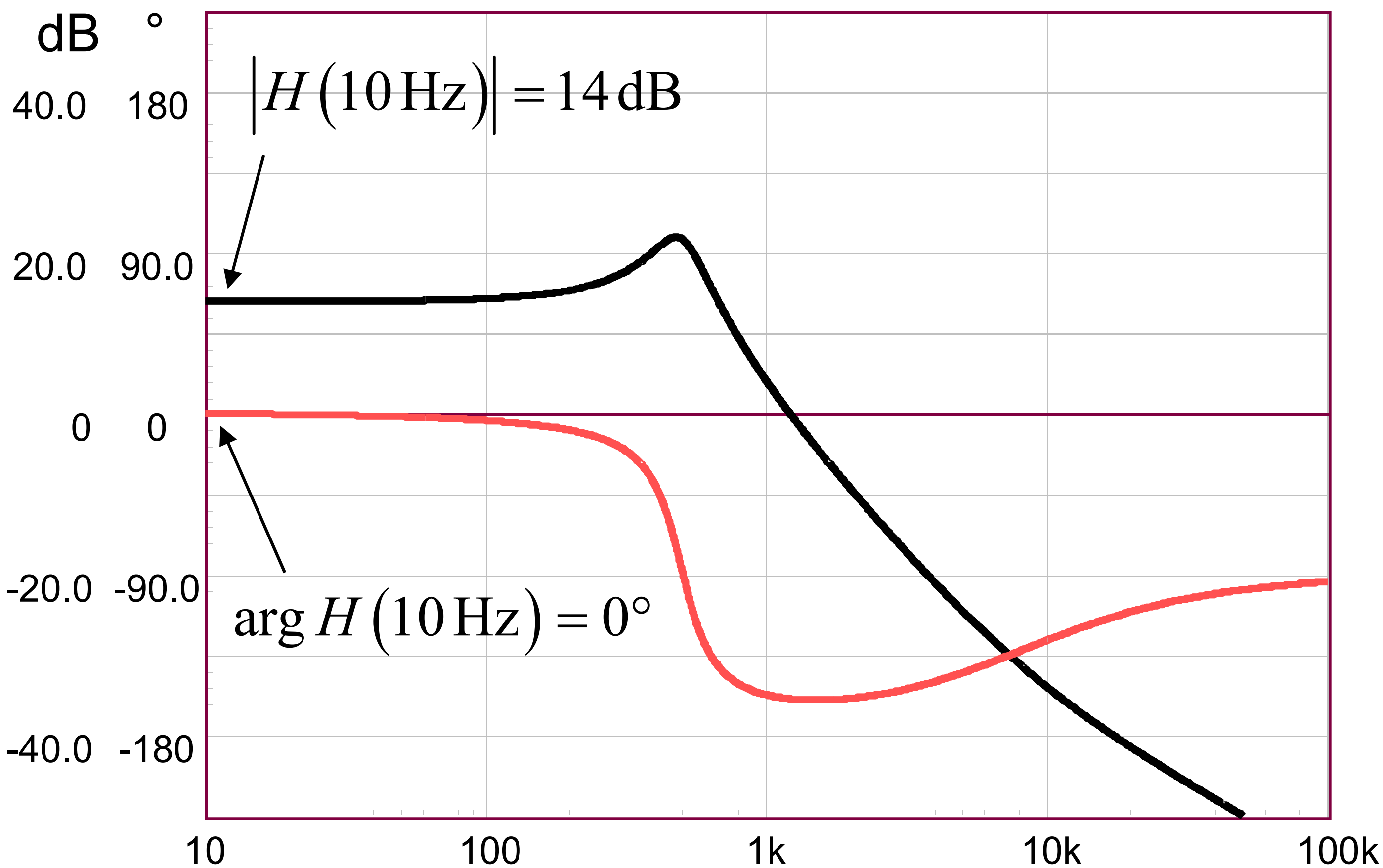
- ❑ No, ringing is reduced but not eliminated



- ❑ The peaking in the output impedance is still there!
- ❑ The notched zeros are the cause for the gain dip at f_0
- ➢ We must find a different compensation method

Another (Bad) Example

- Can we crossover at 10 Hz according to this plot?



- We have no phase lag at 10 Hz, a type 1 could do?

Rolling-off the BW at Low Frequencies

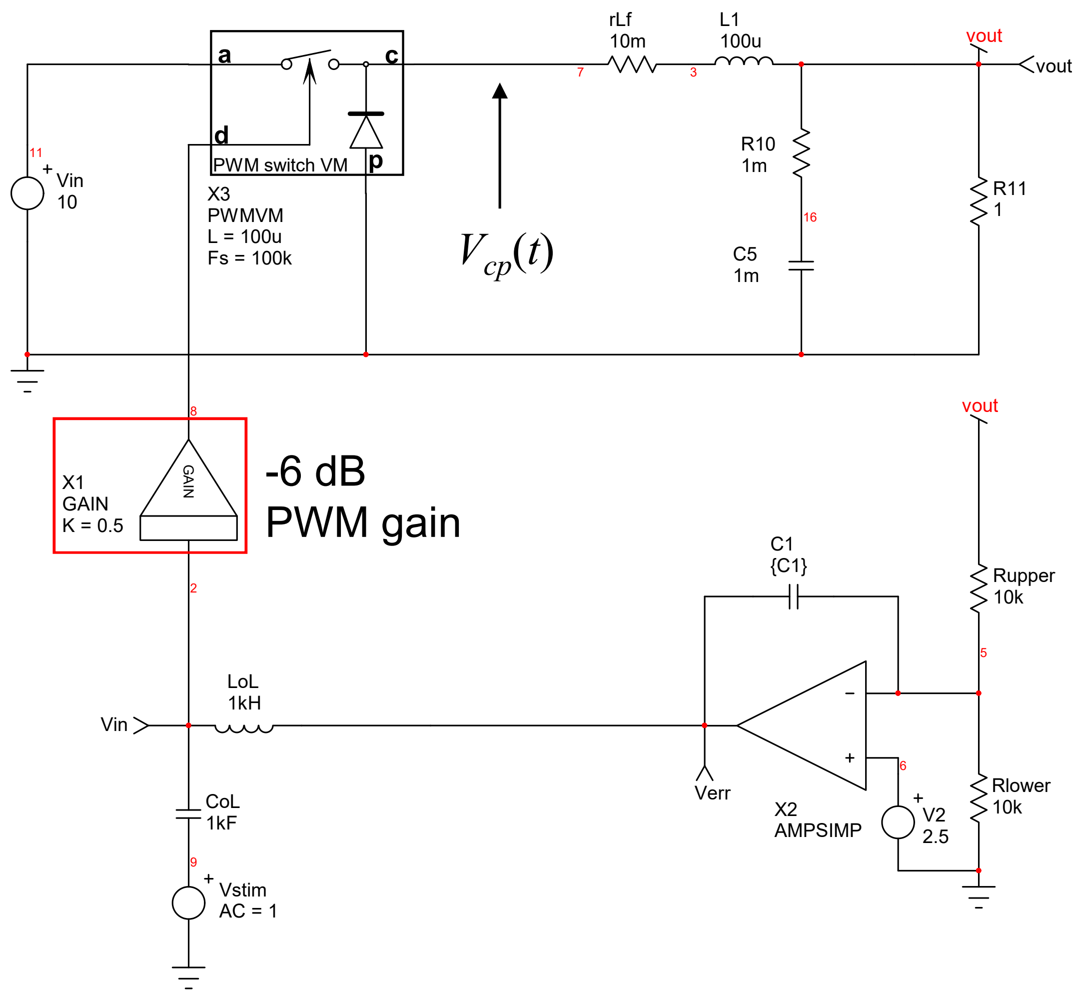
❑ SPICE gives us the open-loop gain snapshot

parameters

Vout=5V
Rupper=10k
fc=10
Gfc=14

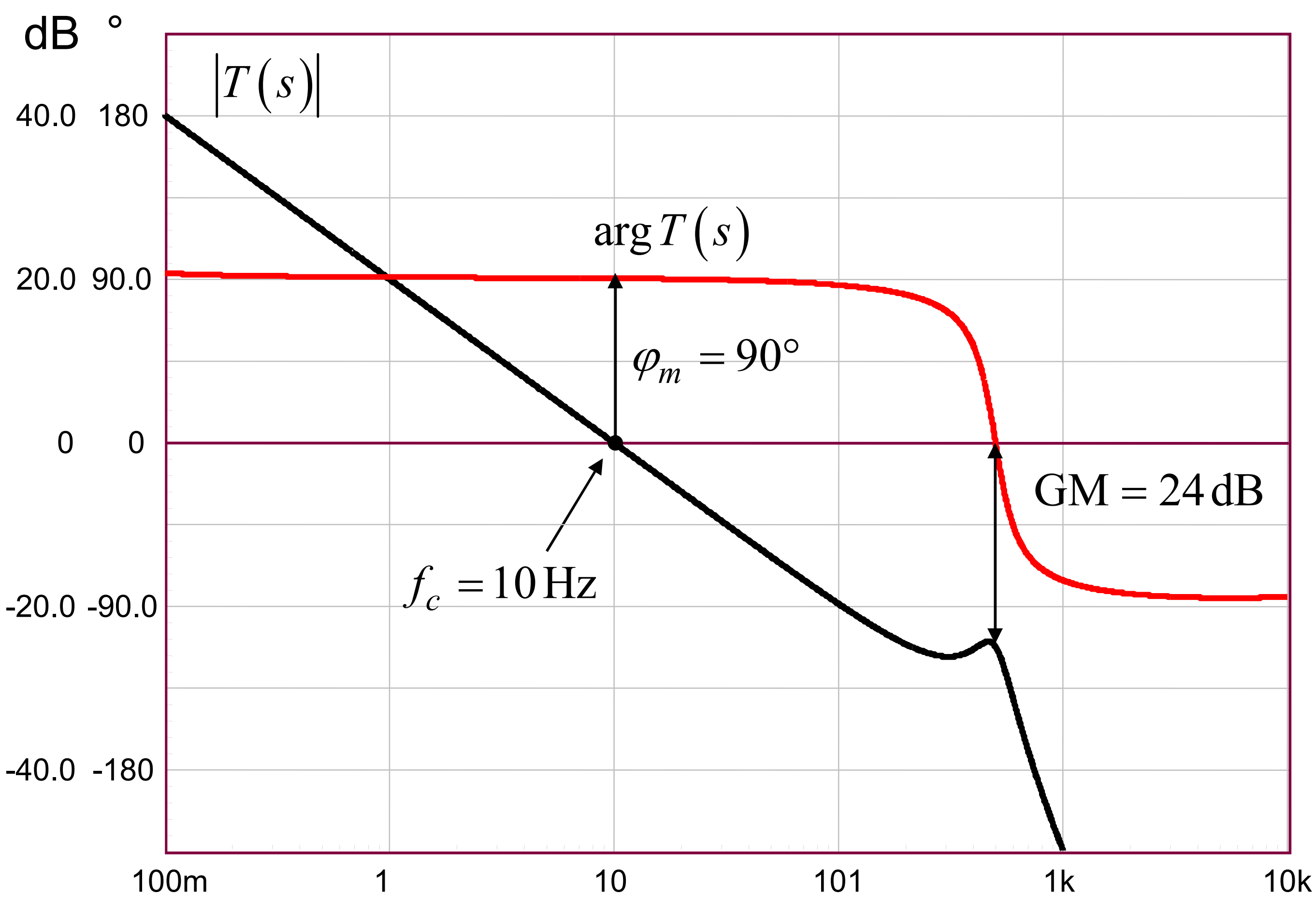
G=10^(-Gfc/20)
pi=3.14159

C1=1/(2*pi*fc*G*Rupper)
fp0=1/(2*pi*C1*Rupper)



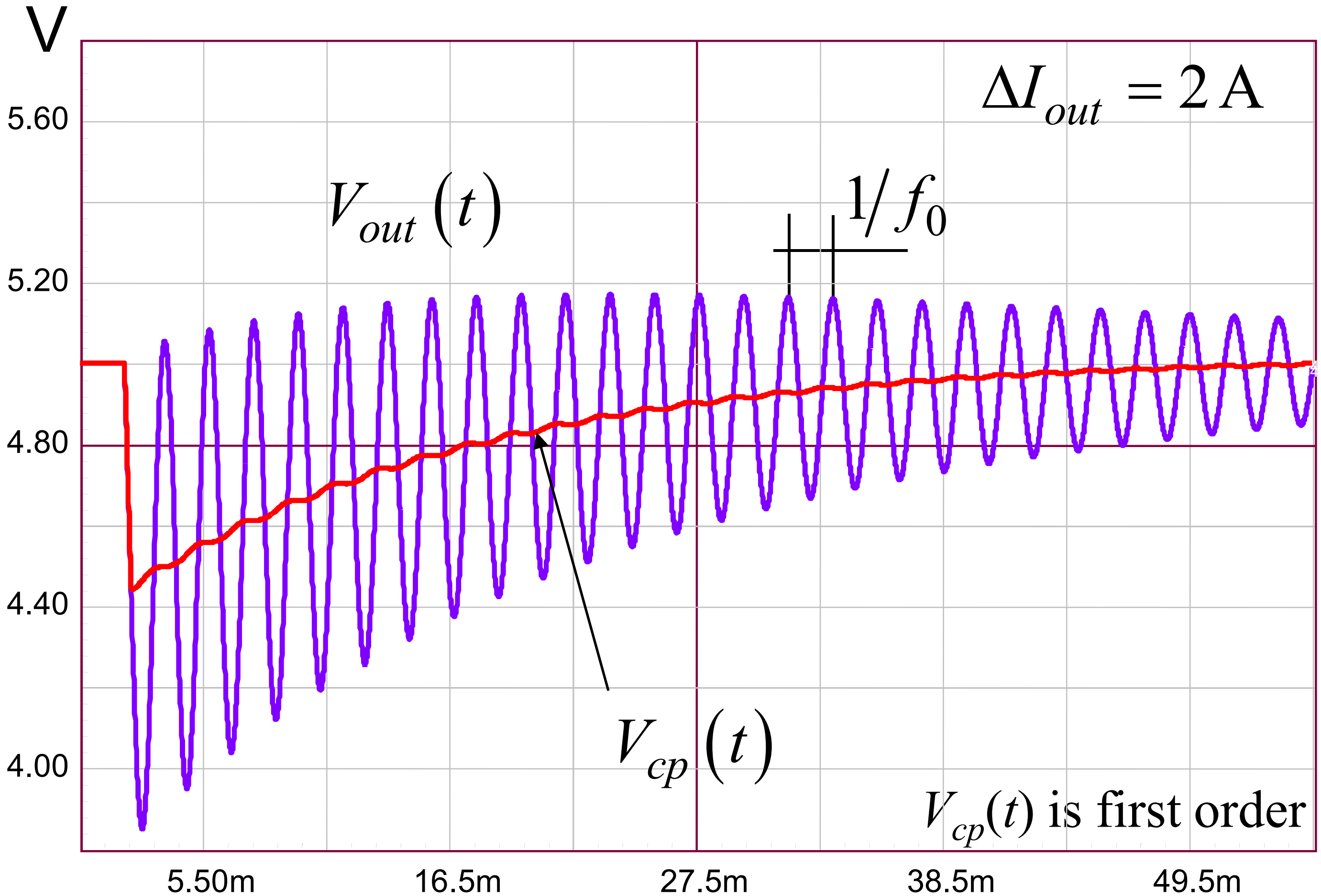
The Open-Loop Gain Looks Good…

- The type 1 confirms our 0-dB crossover frequency



As Expected: It is Ringing!

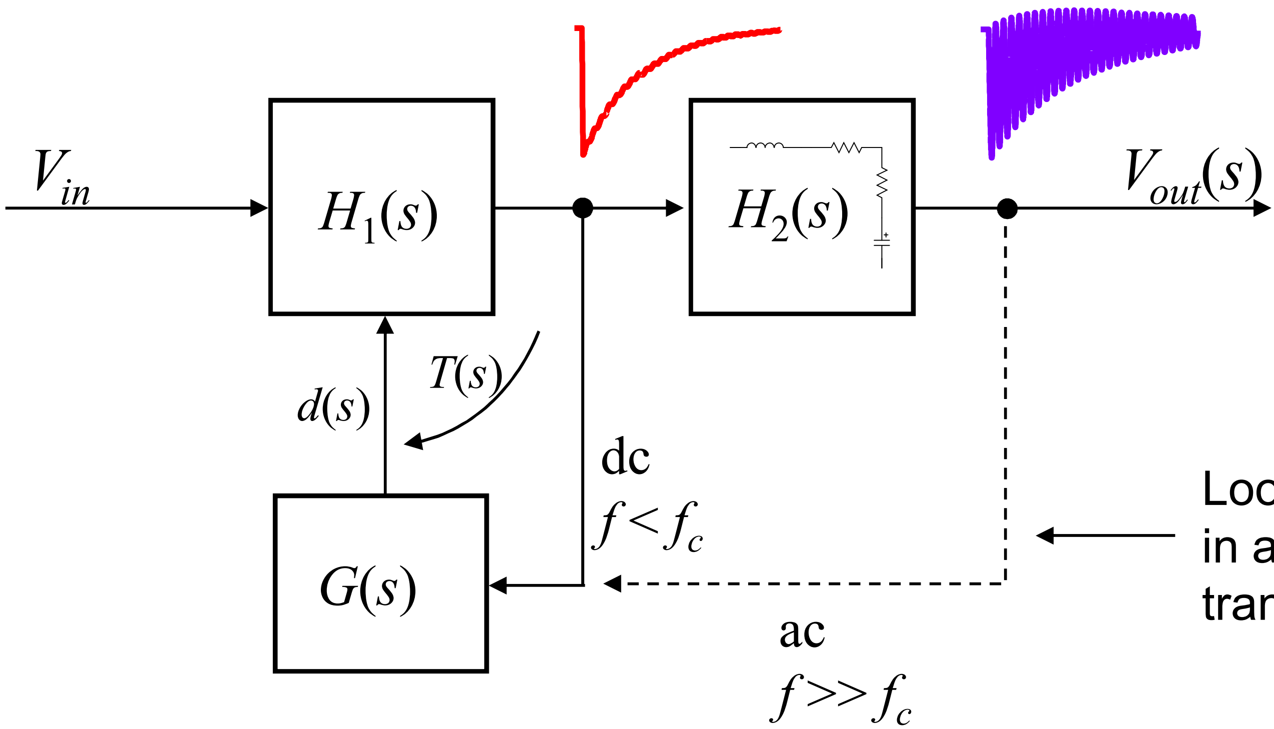
- The load step reveals a ringing ac output



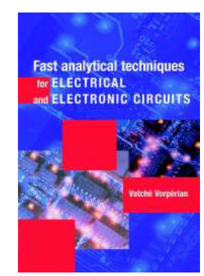
Munch

Good dc Coupling, Weak ac Coupling

- H_1 is stable per Bode analysis, but H_2 is out of the loop…



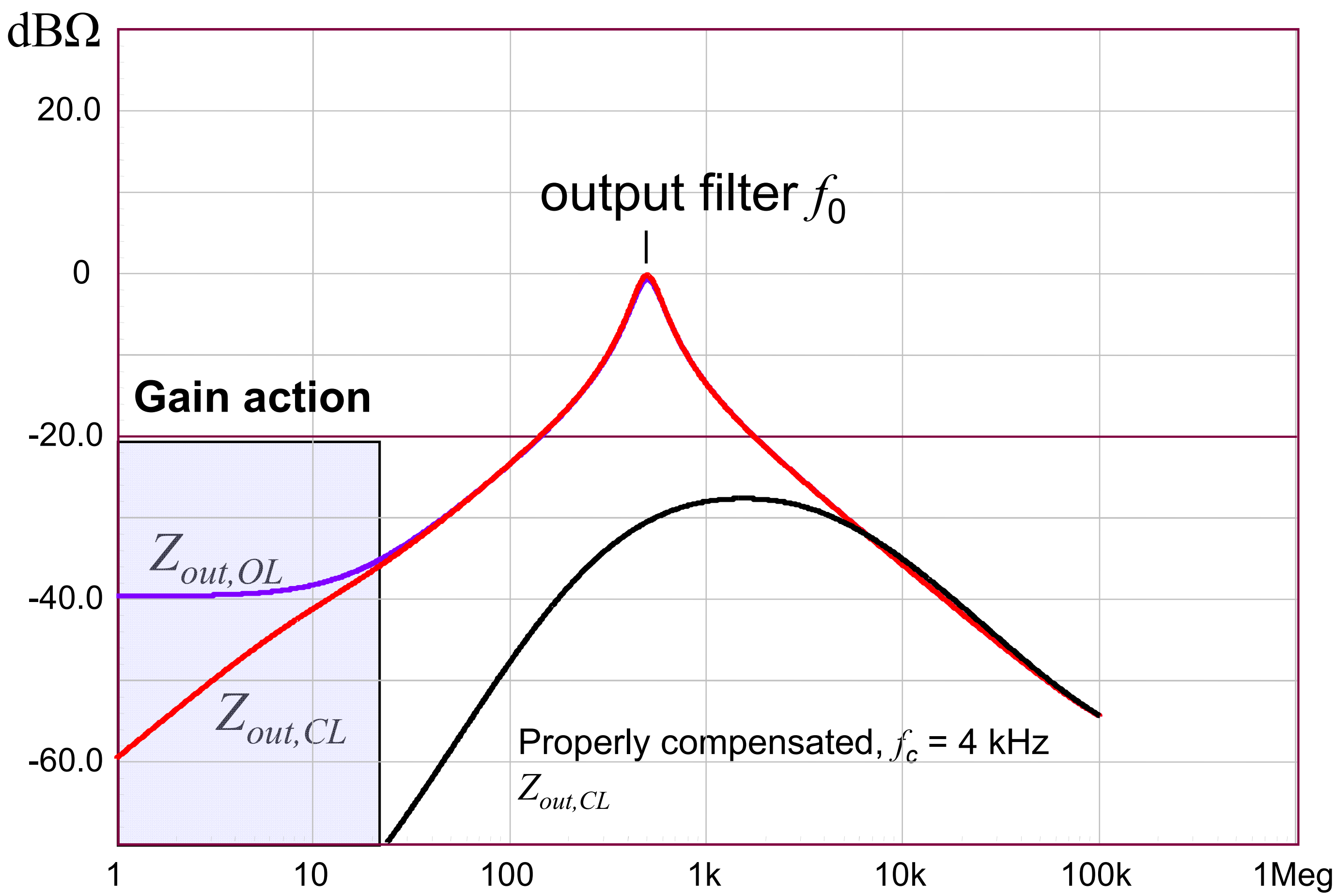
- The dc is fed back via the loop but not the ac…
- Oscillations are NOT due to the loop!



"Fast Analytical Techniques for Electrical and Electronic Circuits", V. Vorpérian, Cambridge Press, 2002

Again, an Undamped RLC Network…

- No gain at resonance: the RLC network runs open loop



- The system cannot reduce the Q at the resonant frequency

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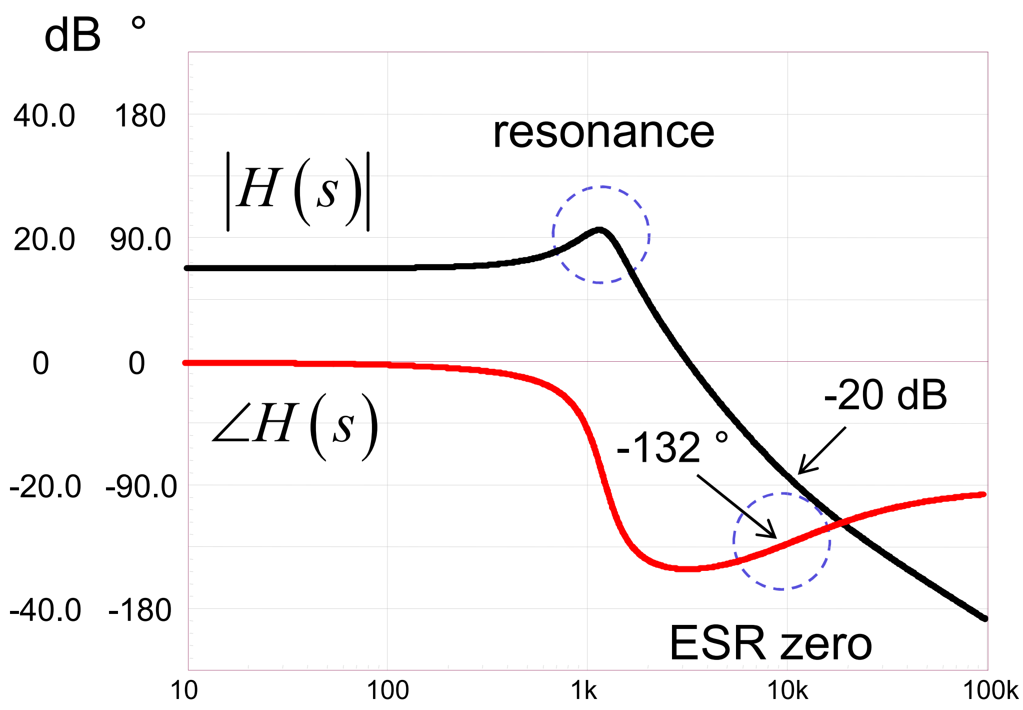
- ❑ Gain Margin is not Enough

Compensating the Buck – Method 2

- ❑ In this method, we will focus on two parameters:
- ✓ crossover frequency f_c
- ✓ phase margin φ_m
- ❑ First, look at the ac response of the power stage:



Compensating the Buck – Method 2

- The peaking in H brings a severe phase lag at f_0
  - stay away from f_0 , pick f_c at least 10 times above (10 kHz)
  - extract the phase/magnitude of H at 10 kHz:

$$\left|H\left(10\,\mathrm{kHz}\right)\right| = -20\,\mathrm{dB} \qquad \angle H\left(10\,\mathrm{kHz}\right) = -132°$$

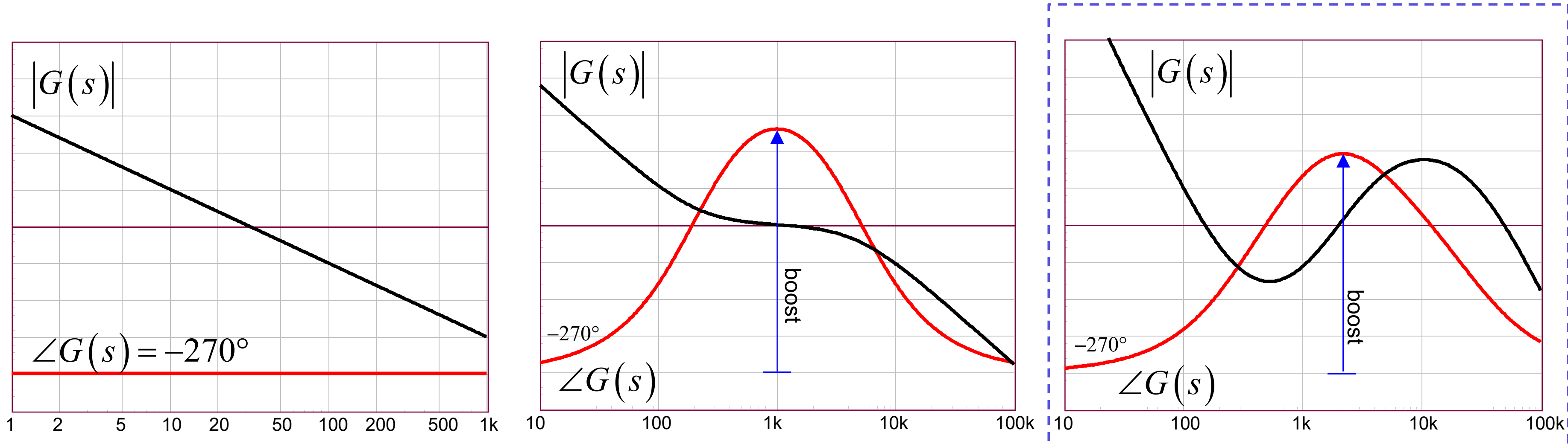
- The compensator G must shape the loop gain T_{OL} by
  - providing a high dc gain for precision: place an origin pole
  - reducing the phase lag at 10 kHz to provide a φ_m of 70°

What compensation type do we need?

Compensating the Buck – Method 2

- ❑ The origin pole brings a permanent phase lag of 90°
  - ➢ added to the op amp inversion of -180°, we have -270°
  - ➢ total phase (op amp and *H*) must be -360 + 70° = -290°
  - ➢ the needed phase boost at f_c is thus:

$$boost = \varphi_m - \angle H(f_c) - 90 = 70 + 132 - 90 = 112°$$



Type 1 – no boost | Type 2 – up to 90° | Type 3 – up to 180°

Compensating the Buck – Method 2

- type 3: an origin pole, a double zero and 2 poles
  - this is our PID compensator!

$$G(s)=-\frac{\left(1+\frac{s}{\omega_{z_1}}\right)\left(1+\frac{s}{\omega_{z_2}}\right)}{\frac{s}{\omega_{po}}\left(1+\frac{s}{\omega_{p_1}}\right)\left(1+\frac{s}{\omega_{p_2}}\right)}=-\frac{\omega_{po}}{\omega_{z_1}}\frac{\left(1+\frac{\omega_{z_1}}{s}\right)\left(1+\frac{s}{\omega_{z_2}}\right)}{\left(1+\frac{s}{\omega_{p_1}}\right)\left(1+\frac{s}{\omega_{p_2}}\right)}$$

- The magnitude is derived as:

$$\left|G(f)\right|=\frac{f_{po}}{f_{z_1}}\frac{\sqrt{1+\left(\frac{f_{z_1}}{f}\right)^2}\sqrt{1+\left(\frac{f}{f_{z_2}}\right)^2}}{\sqrt{1+\left(\frac{f}{f_{p_1}}\right)^2}\sqrt{1+\left(\frac{f}{f_{p_2}}\right)^2}}$$

- The argument is found to be:

$$\arg G(f)=\arg N-\arg D$$

$$\arg N=\arctan\left(-\frac{f_{z_1}}{f}\right)-\pi+\arctan\left(\frac{f}{f_{z_2}}\right)$$

$$\arg D=\arctan\left(\frac{f}{f_{p_1}}\right)+\arctan\left(\frac{f}{f_{p_2}}\right)$$

Compensating the Buck – Method 2

- Place the double zero at f_0 ,the second pole at $F_{sw}/2$
- The 0-dB crossover pole is adjusted to provide +20 dB at f_c
- The first pole is adjusted to provide the right φ_m

$$\arg G(f_c) = \arctan\left(-\frac{f_{z_1}}{f_c}\right) - \pi + \arctan\left(\frac{f_c}{f_{z_2}}\right) - \arctan\left(\frac{f_c}{f_{p_1}}\right) - \arctan\left(\frac{f_c}{f_{p_2}}\right)$$

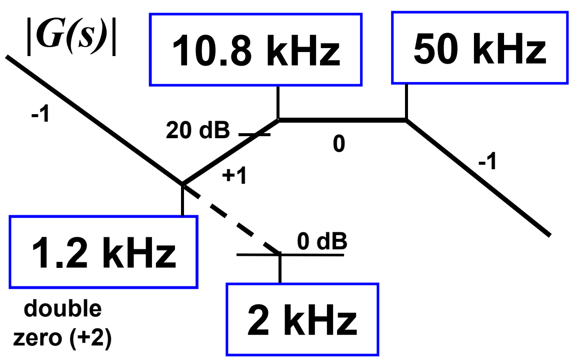
$$\Rightarrow f_{p_1} = -\frac{f_c}{\tan\left(\arg G + \tan^{-1}\left(\frac{f_c}{f_{p_2}}\right) + \tan^{-1}\left(\frac{f_{z_1}}{f_c}\right) - \tan^{-1}\left(\frac{f_c}{f_{z_2}}\right)\right)} = 10.8\,\text{kHz}$$

Compensating the Buck – Method 2

- The 0-dB crossover pole is adjusted to give 20 dB at 10 kHz

$$f_{po} = \left|G\left(f_c\right)\right| f_{z_1} \frac{\sqrt{1+\left(\frac{f_c}{f_{p_1}}\right)^2}\sqrt{1+\left(\frac{f_c}{f_{p_2}}\right)^2}}{\sqrt{1+\left(\frac{f_{z_1}}{f_c}\right)^2}\sqrt{1+\left(\frac{f_c}{f_{z_2}}\right)^2}} \approx 2\,\text{kHz}$$

- The final configuration is as follows:



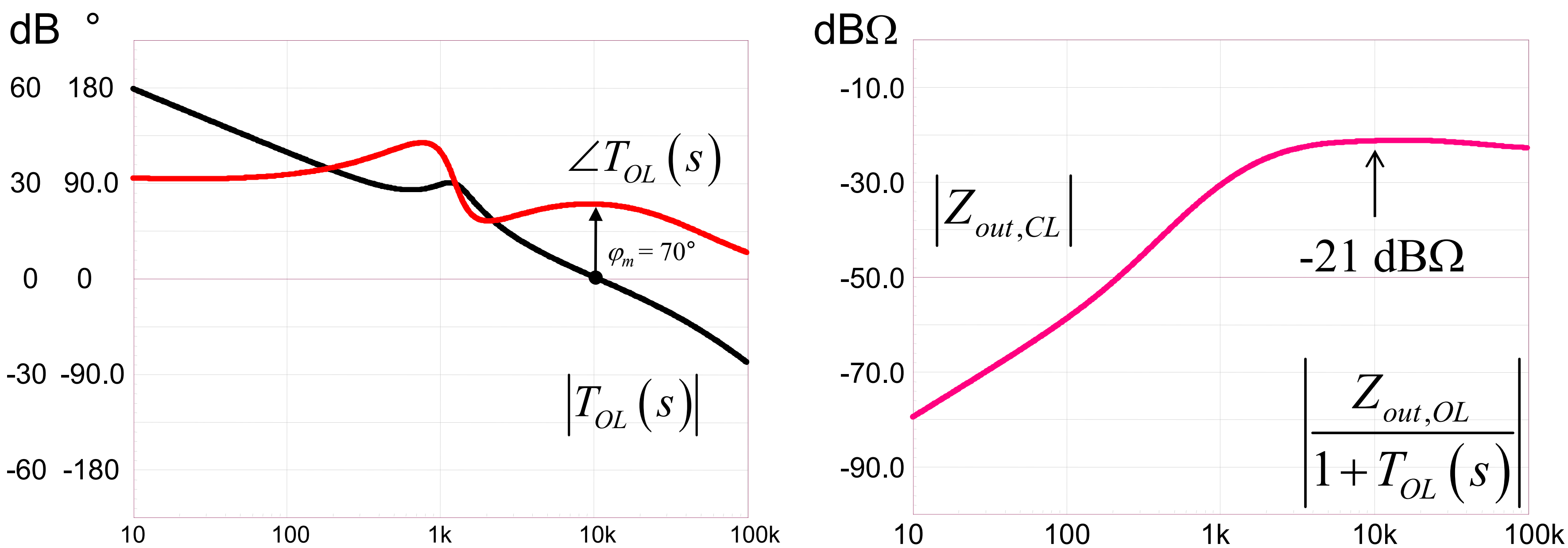
$$\left|T_{OL}\left(1.2\,\text{kHz}\right)\right| = \left|G\left(1.2\,\text{kHz}\right)\right|\left|H\left(1.2\,\text{kHz}\right)\right| = 37\,\text{dB}$$

It should be ok to damp Z_{out}

Compensating the Buck – Method 2

- Enter the PID coefficients from the poles/zeros positions

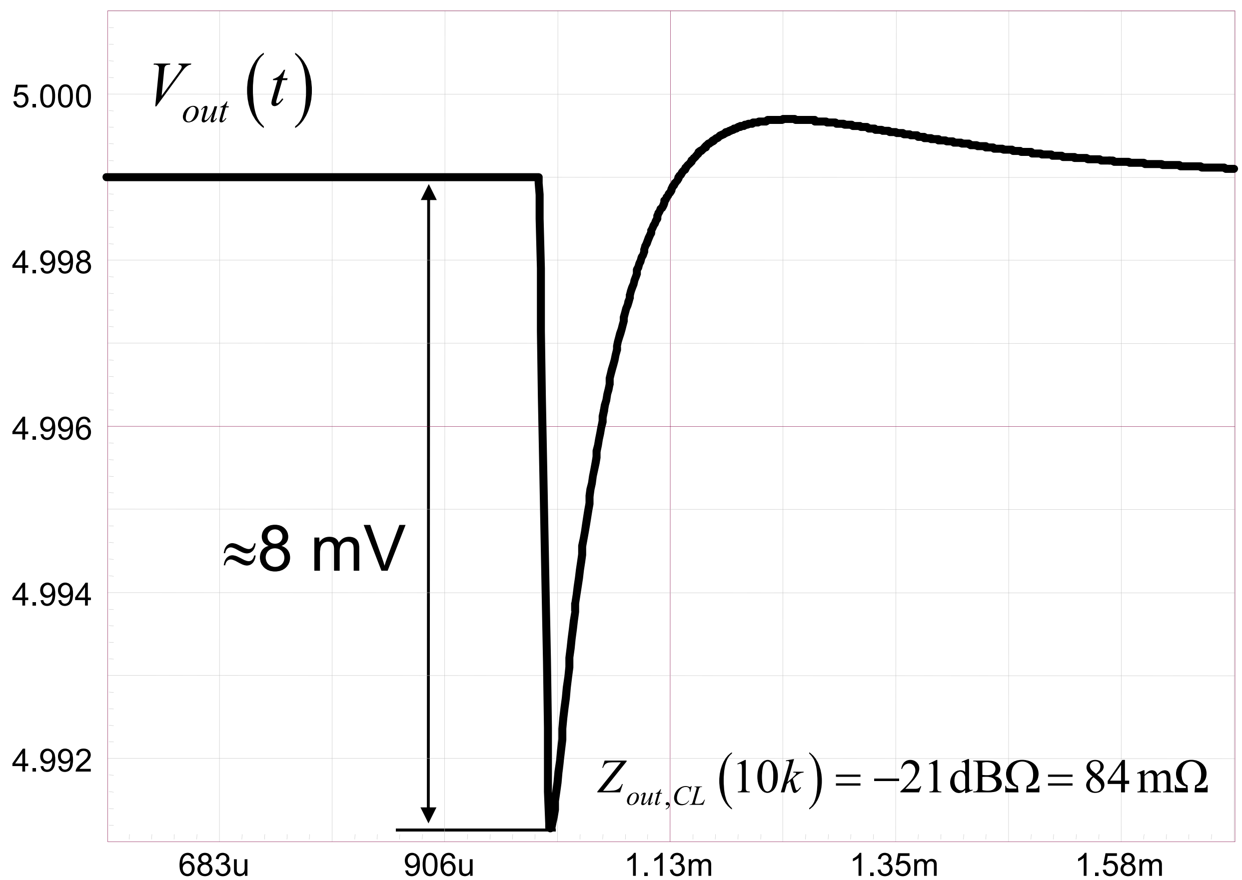
$\tau_d = 55.5u \qquad \tau_i = 250u \qquad N = 3.76 \qquad k_p = 3.1$



- More than 30 dB at f_0 and absolutely no peaking in Z_{out}

Compensating the Buck – Method 2

- The transient response with a 0.1-A load step is excellent

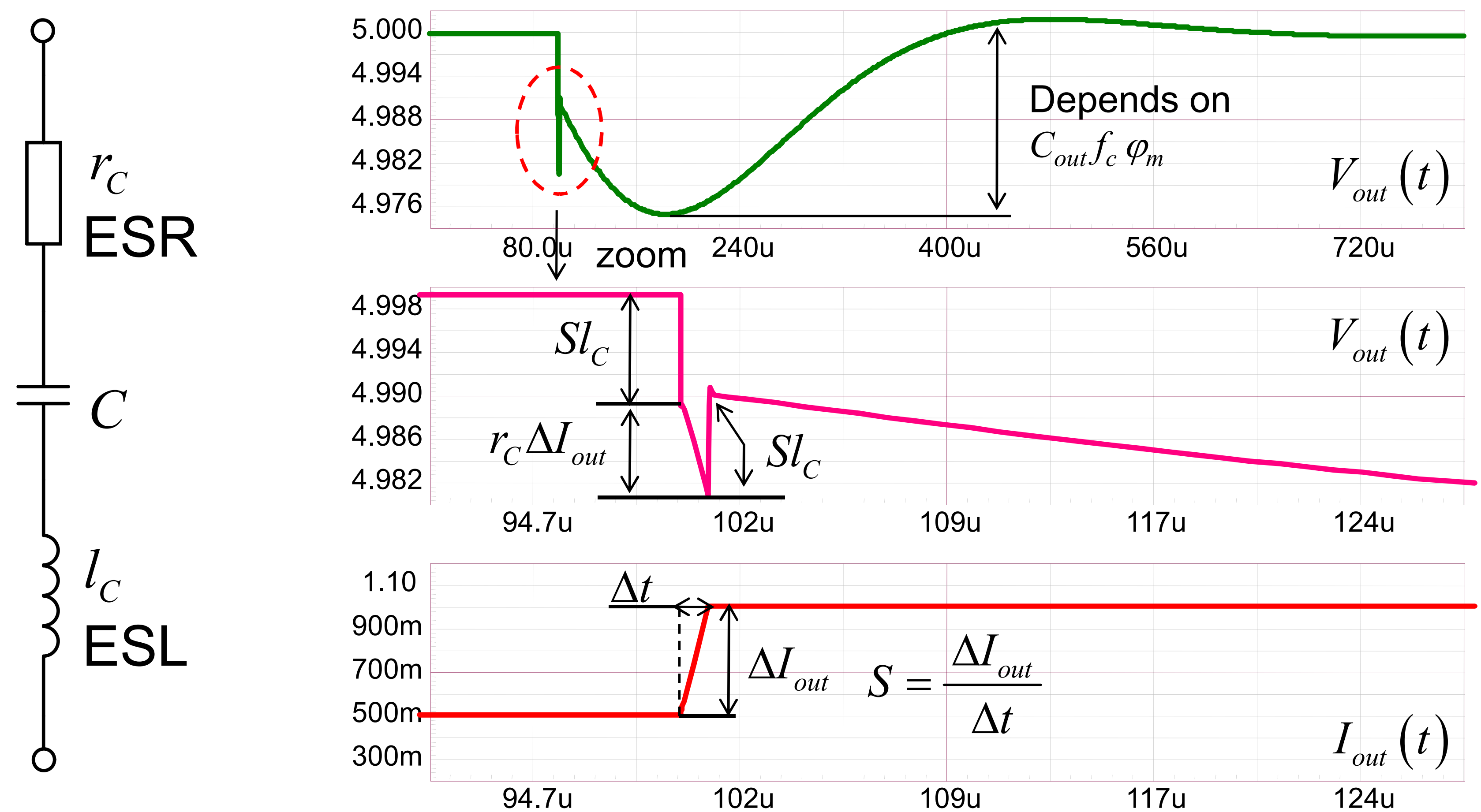


Course Agenda

- ❑ Introduction to Control Systems
- ❑ Shaping the Error Signal
- ❑ How to Implement the PID Block?
- ❑ The PID at Work with a Buck Converter
- ❑ Considering the Output Impedance
- ❑ Classical Poles/Zeros Placement
- ❑ **Shaping the Output Impedance**
- ❑ Quality Factor and Phase Margin
- ❑ What is Delay Margin?
- ❑ Gain Margin is not Enough

Compensating the Buck – Method 3

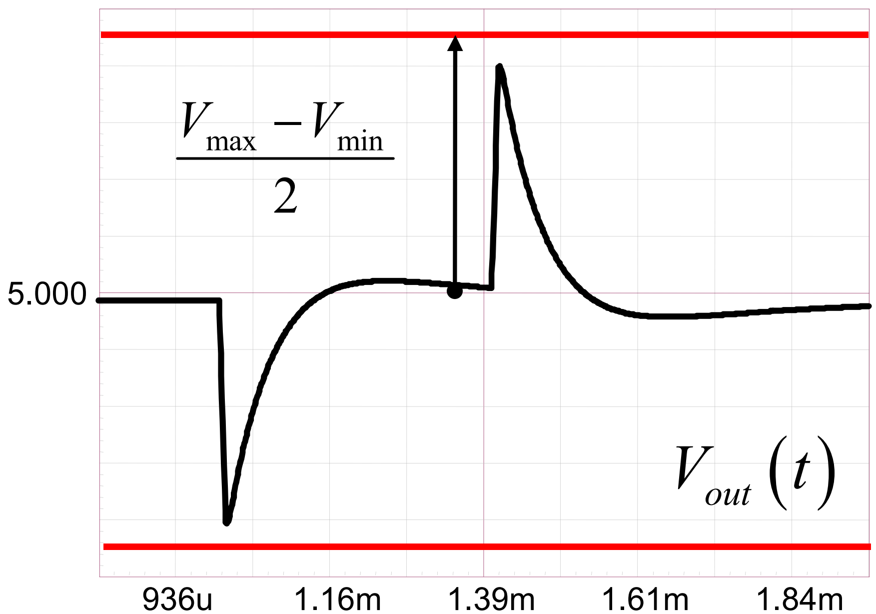
- ❑ The capacitor stray elements affect the transient response

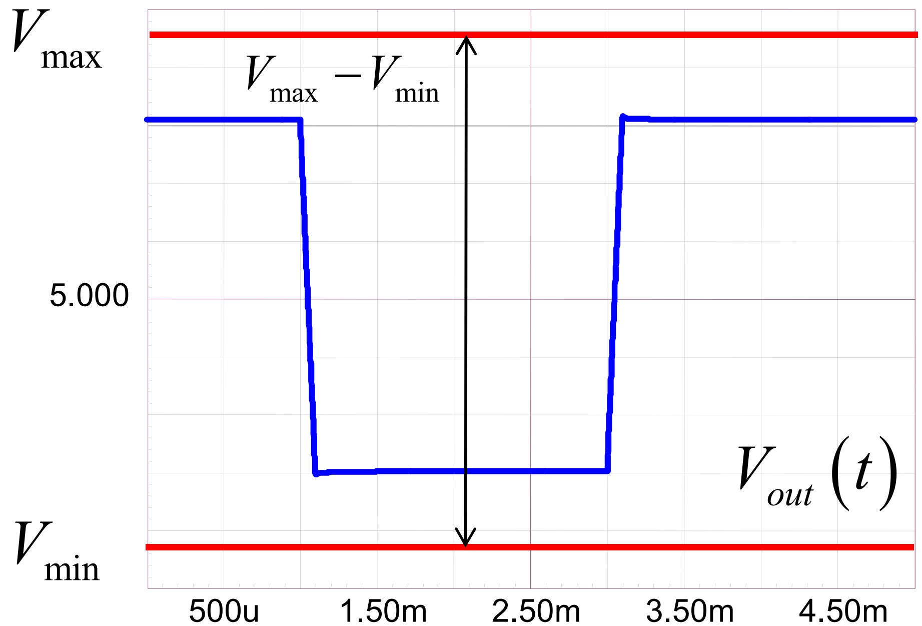


R. Redl et al."Optimizing the Load Transient Response of the Buck Converter", APEC 1998

Compensating the Buck – Method 3

- ❑ During a step load, the converter fights the current change





❑ Traditional compensation:
➢ inductive output impedance

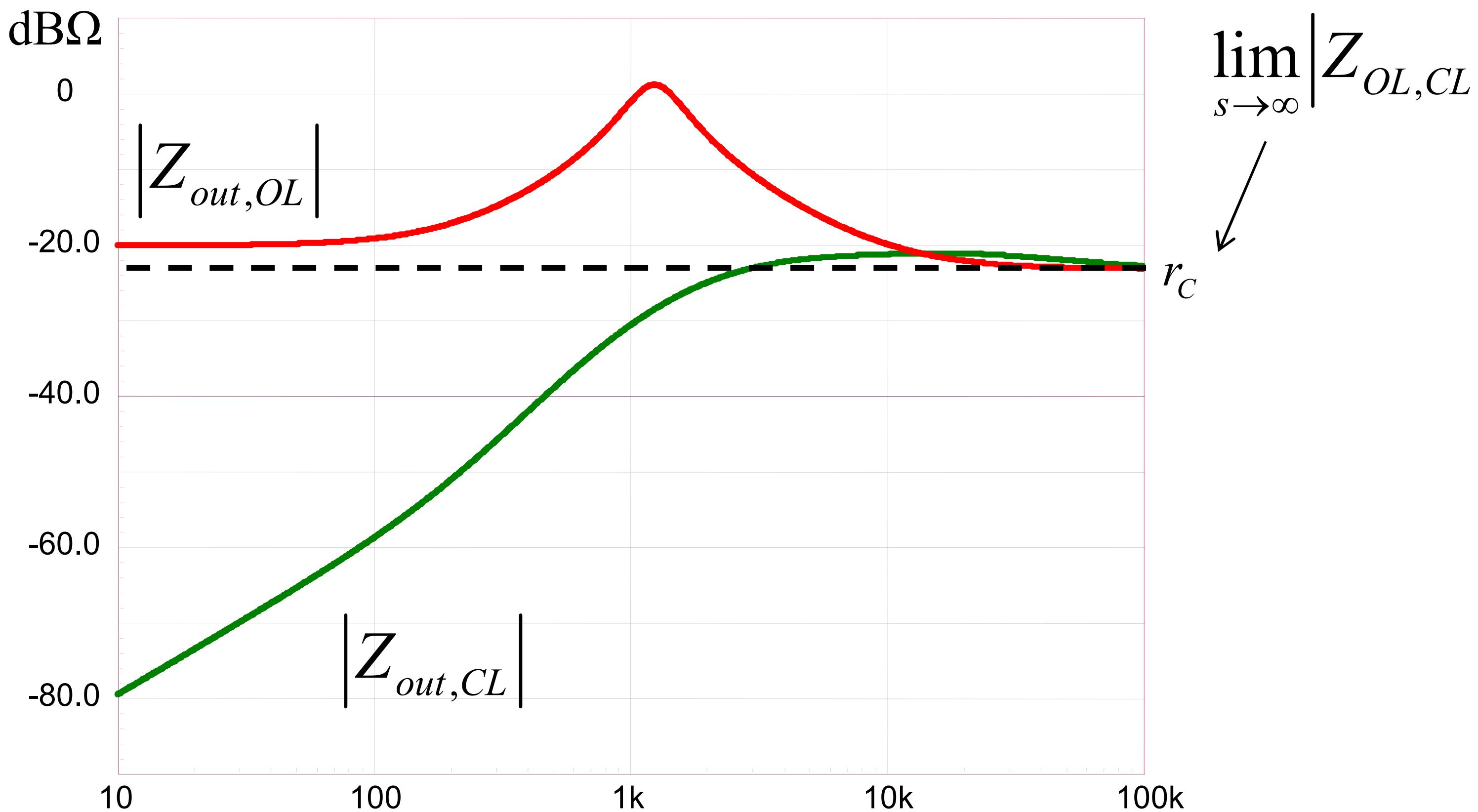
⇨ Limited excursion

❑ Adaptive Voltage Positioning
➢ resistive output impedance

⇨ Full-span excursion

Compensating the Buck – Method 3

- Whatever the gain, Z_{out} meets the open-loop value beyond f_c



- Make the output impedance equal to r_C along the freq. range

Compensating the Buck – Method 3

- How to force the output impedance to be resistive?

$$Z_{out,CL} = \frac{Z_{out,OL}}{1+T(s)} = R_0 \frac{\left(1+\frac{s}{\omega_{z_1}}\right)\left(1+\frac{s}{\omega_{z_2}}\right)}{\left(\frac{s}{\omega_0}\right)^2 + \frac{s}{\omega_0 Q} + 1} \frac{1}{1 + H_0 \frac{1+\frac{s}{\omega_{z_1}}}{\left(\frac{s}{\omega_0}\right)^2 + \frac{s}{\omega_0 Q} + 1} G(s)}$$

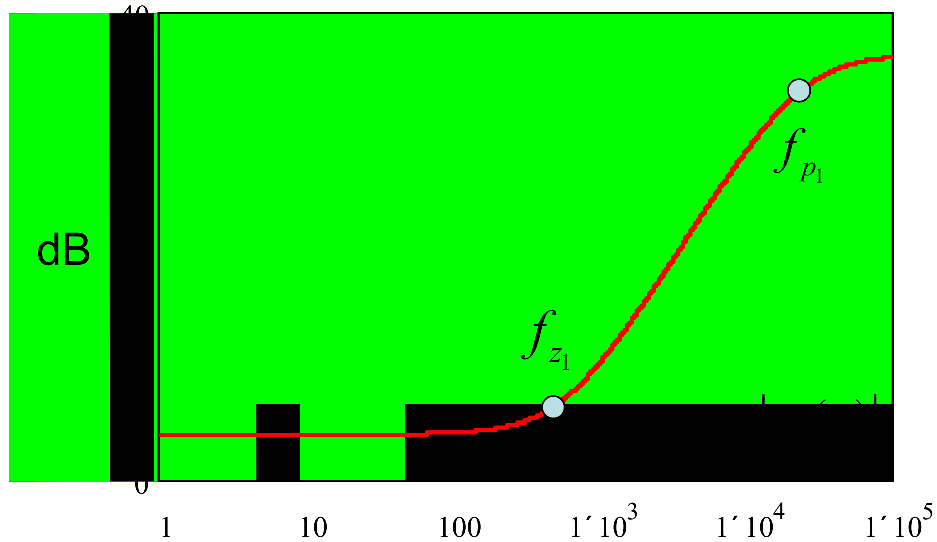
Extract $G(s)$ to have:

$$Z_{out,CL} = r_C$$

Compensating the Buck – Method 3

- Once $G(s)$ is extracted, what filter is that?

$$G(s)=\frac{R_0\left(1+\frac{s}{\omega_{z_2}}\right)}{r_C G_0}-\frac{\left(\frac{s}{\omega_0}\right)^2+\frac{s}{Q\omega_0}+1}{G_0\left(1+\frac{s}{\omega_{z_1}}\right)}$$



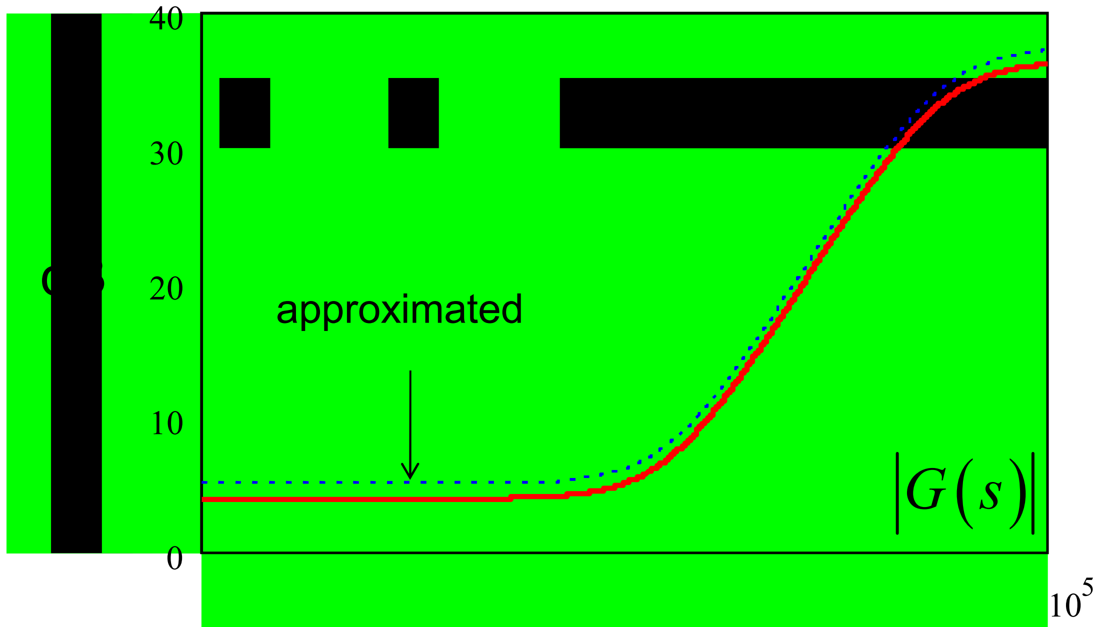
$$G(s)=K_0\frac{1+\frac{s}{\omega_{z_G}}}{1+\frac{s}{\omega_{p_G}}}$$

Compensating the Buck – Method 3

- Some parameter identification is now needed:

$$K_0 = \frac{r_L - r_C}{H_0 r_C} = 1.8 \qquad a = \frac{r_L}{\omega_{z_1}\omega_{z_2}} - \frac{r_C}{\omega_0^{\ 2}} = 47.7p \qquad b = r_L\left(\frac{1}{\omega_{z_1}} + \frac{1}{\omega_{z_2}}\right) - \frac{r_C}{Q\omega_0} = 74.2u$$

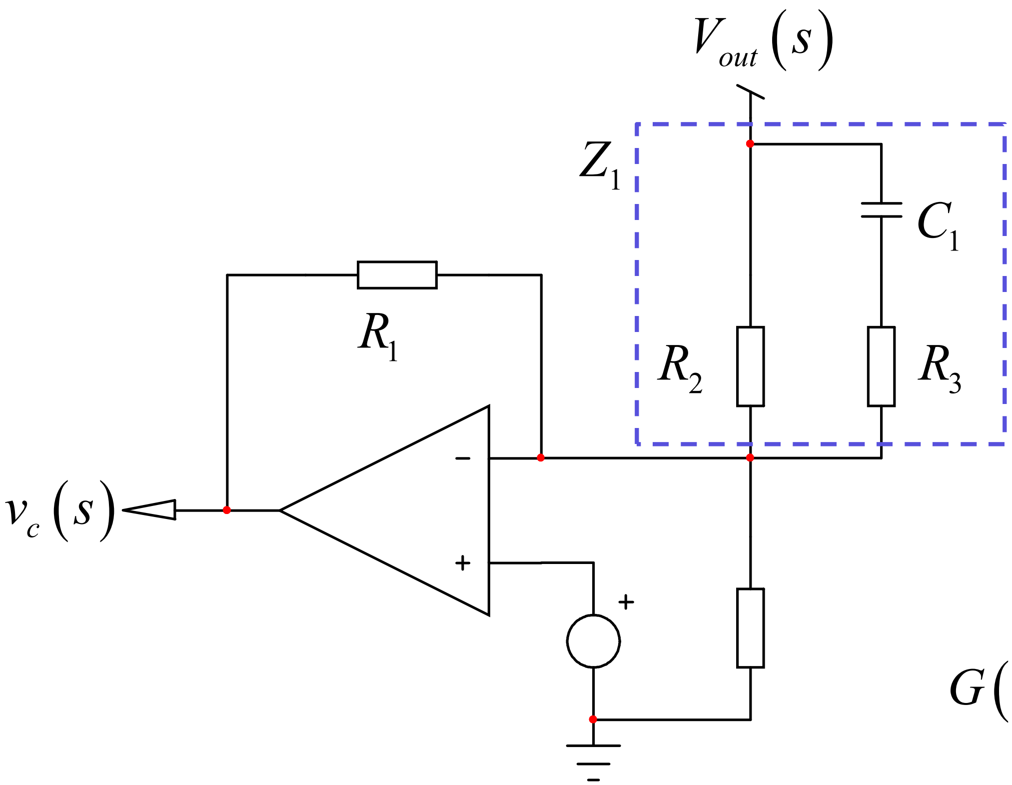
$$c = r_L - r_C = 0.27 \qquad f_{p_G} = f_{z_1} = 24\,\mathrm{kHz} \qquad f_{z_G} = \frac{b - \sqrt{b^2 - 4ac}}{4\pi a} = 580\,\mathrm{Hz}$$



YAO et al. ,"Design Considerations for VRM Transient Response Based on the Output Impedance", IEEE Proceedings, 2003

Compensating the Buck – Method 3

- The following op amp architecture will do the job



$$\frac{v_c(s)}{V_{out}(s)} = -\frac{R_1}{Z_1}$$

$$Z_1 = \frac{R_2\left(R_3 + \frac{1}{sC_1}\right)}{R_2 + \left(R_3 + \frac{1}{sC_1}\right)}$$

$$G(s) = -\frac{R_2}{R_1}\frac{\left[1 + sC_1\left(R_2 + R_3\right)\right]}{1 + sR_3C_1}$$

$$\Rightarrow \quad K_0 = \frac{R_2}{R_1} \qquad \omega_{z_1} = \frac{1}{C_1\left(R_2 + R_3\right)} \qquad \omega_{p_1} = \frac{1}{R_3C_1}$$

Compensating the Buck – Method 3

- A test fixture is assembled using a buck averaged model

parameters

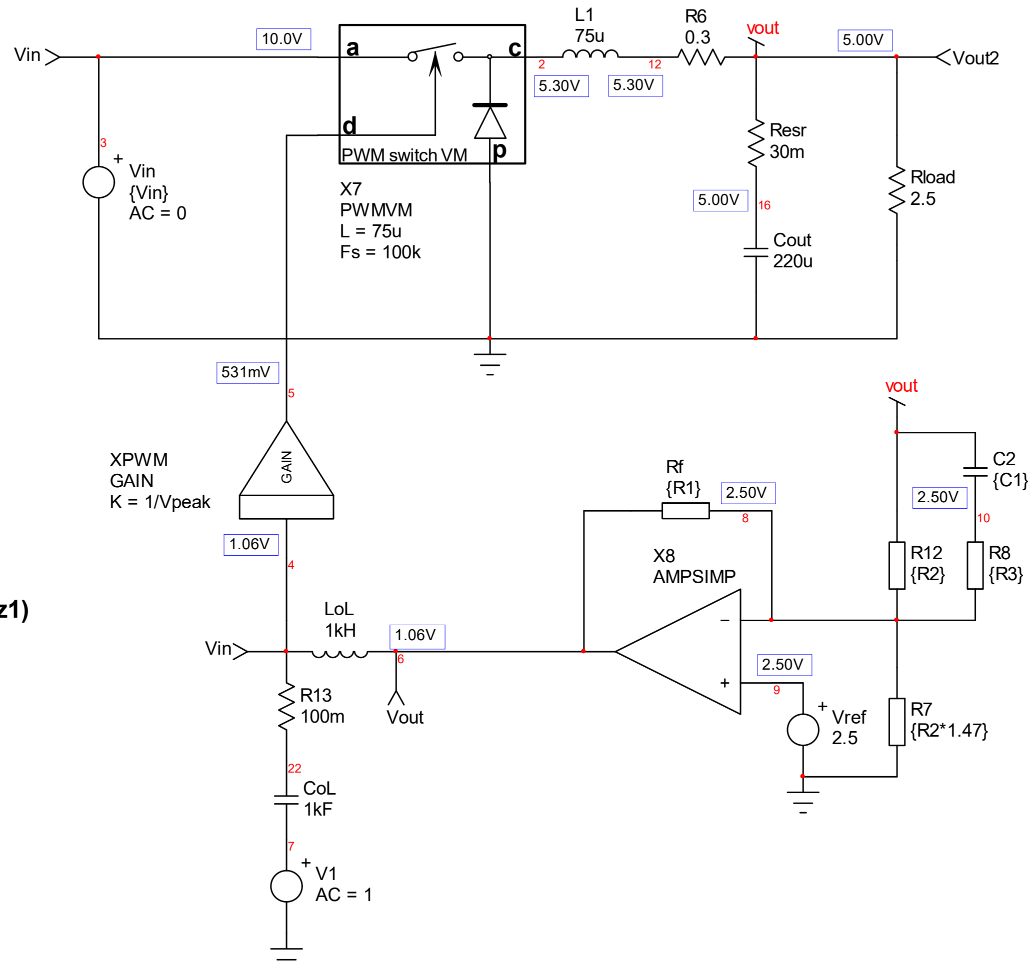
Vin=10
Vpeak=2

pi=3.14159

fz1=580
fp1=24k

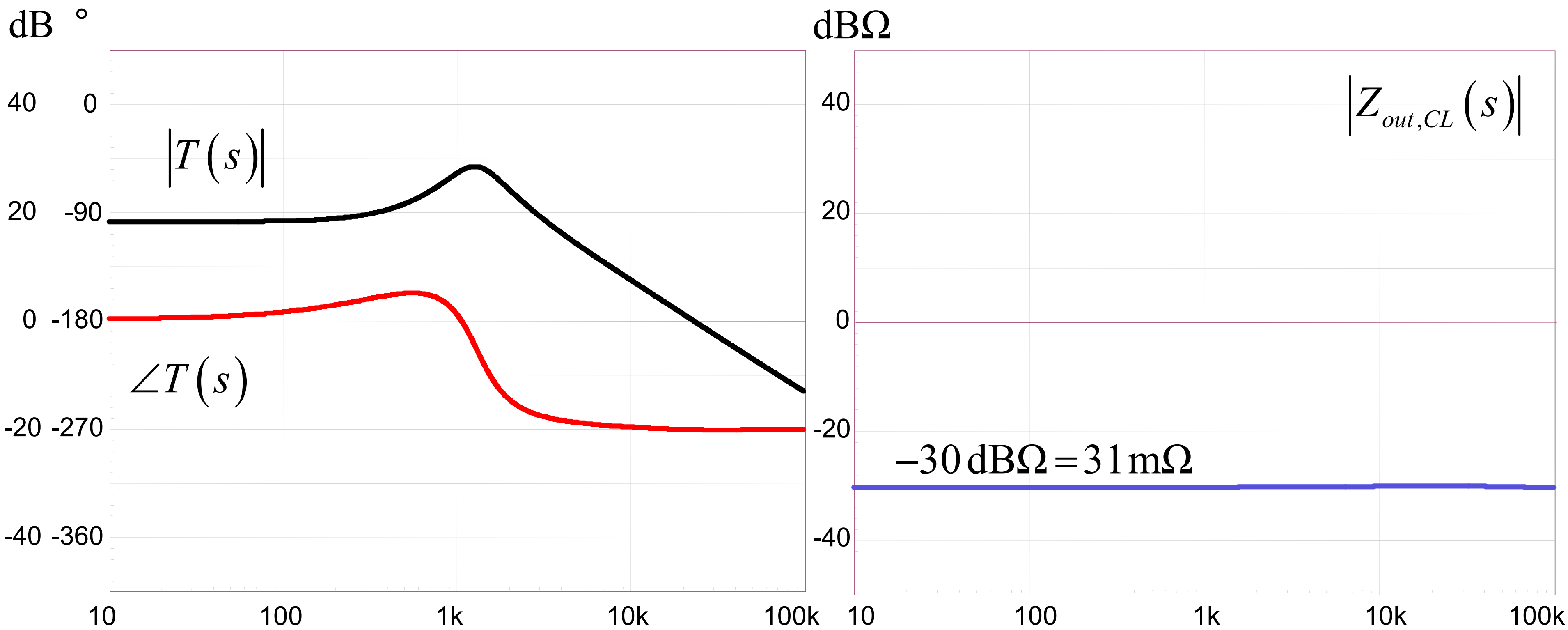
Wz1=2*pi*fz1
Wp1=2*pi*fp1
kv=1.8

R1=10k
R2=R1/kv
R3=R1*Wz1/(Kv*(Wp1-Wz1))
C1=(Kv*(Wp1-Wz1))/(R1*Wp1*Wz1)



Compensating the Buck – Method 3

❑ After stabilization we have good margins with a 20-kHz f_c



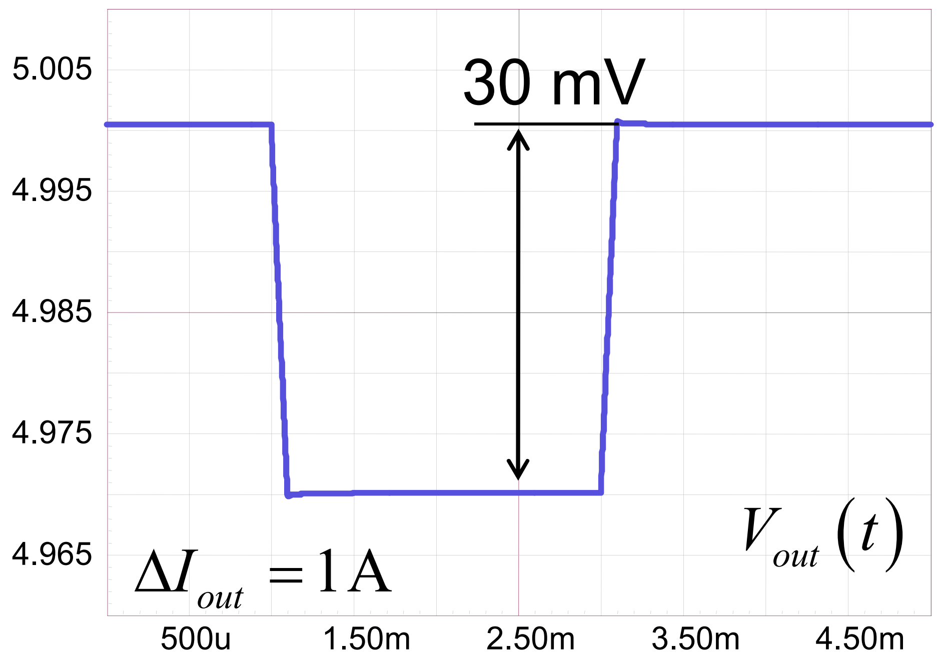
❑ The output impedance is purely resistive!

❑ But the dc gain is low: line and load regulation problems!

Compensating the Buck – Method 3

- Also, the gain expression K_0 can be a problem:

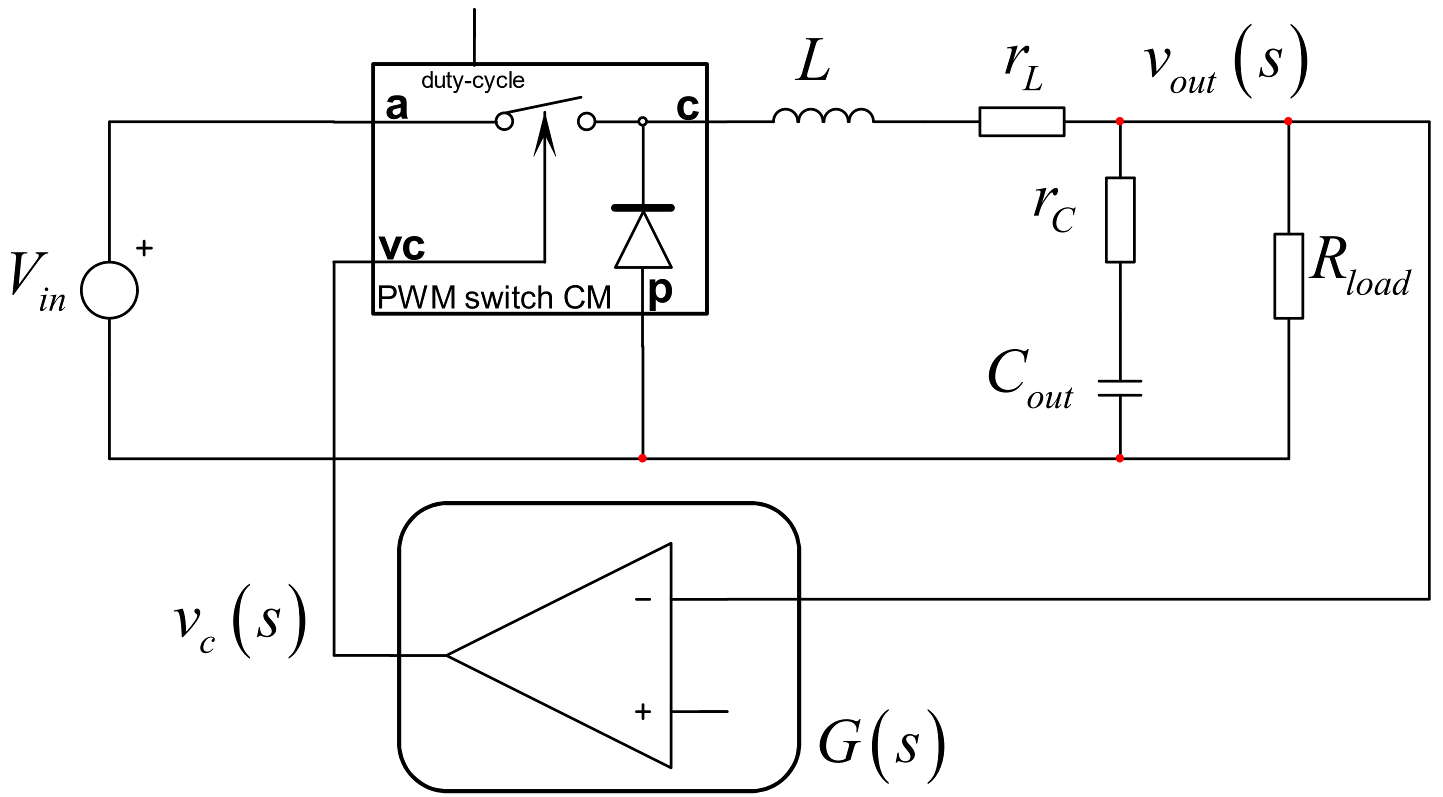
$$K_0 = \frac{r_L - r_C}{G_0 r_C} > 0 \rightarrow r_L > r_C$$



- VM, fixed frequency, is not the best for Z_{out} resistive shaping

Compensating the Buck – Method 3

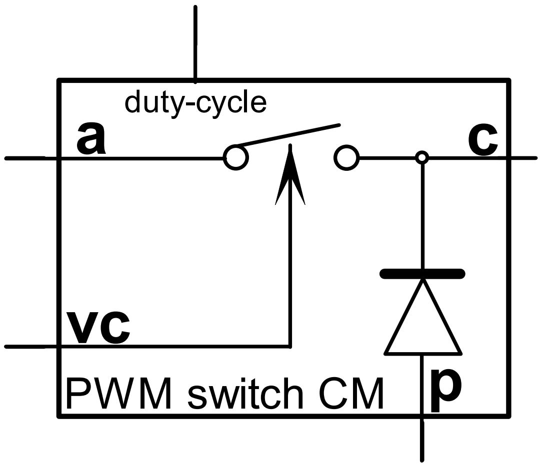
- Going current-mode, fixed frequency is one way to go

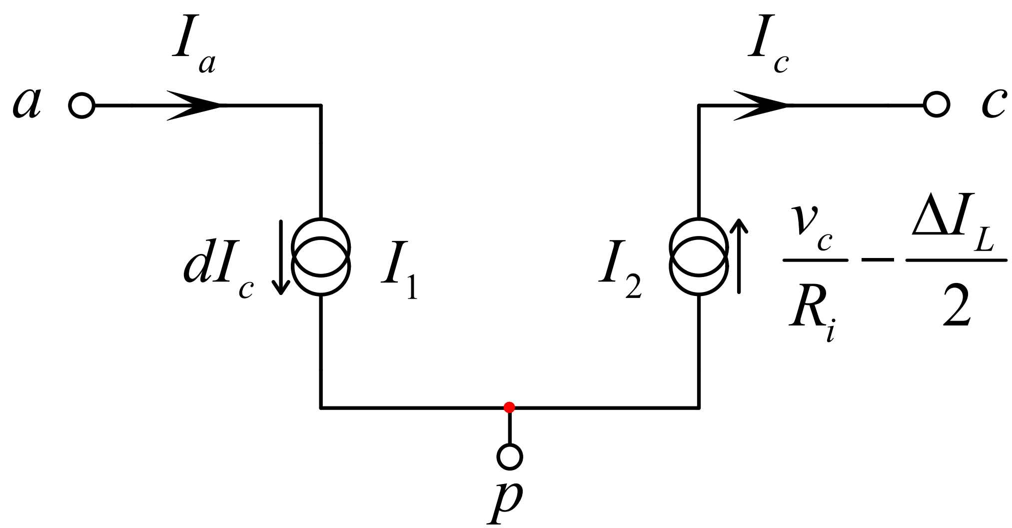


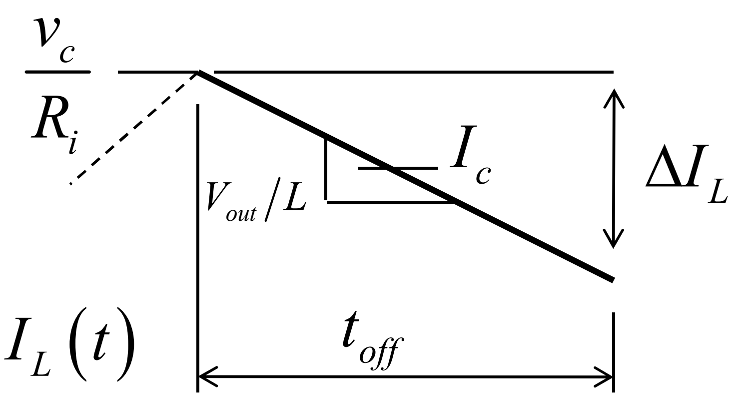
- Use the PWM switch model in current-mode for Z_{out}

Compensating the Buck – Method 3

- ❑ The large-signal model combines two current-sources







$$I_2 = \frac{v_c}{R_i} - \frac{(1-D)T_{sw}}{2L} V_{out}$$

Compensating the Buck – Method 3

- For ac study, we must obtain a small-signal model

$$I_2\left(V_{out},V_c\right)=\frac{V_c}{R_i}-\frac{\left(1-D\right)T_{sw}}{2L}V_{out}=\frac{V_c}{R_i}-\frac{\left(1-\frac{V_{out}}{V_{in}}\right)T_{sw}}{2L}V_{out}$$

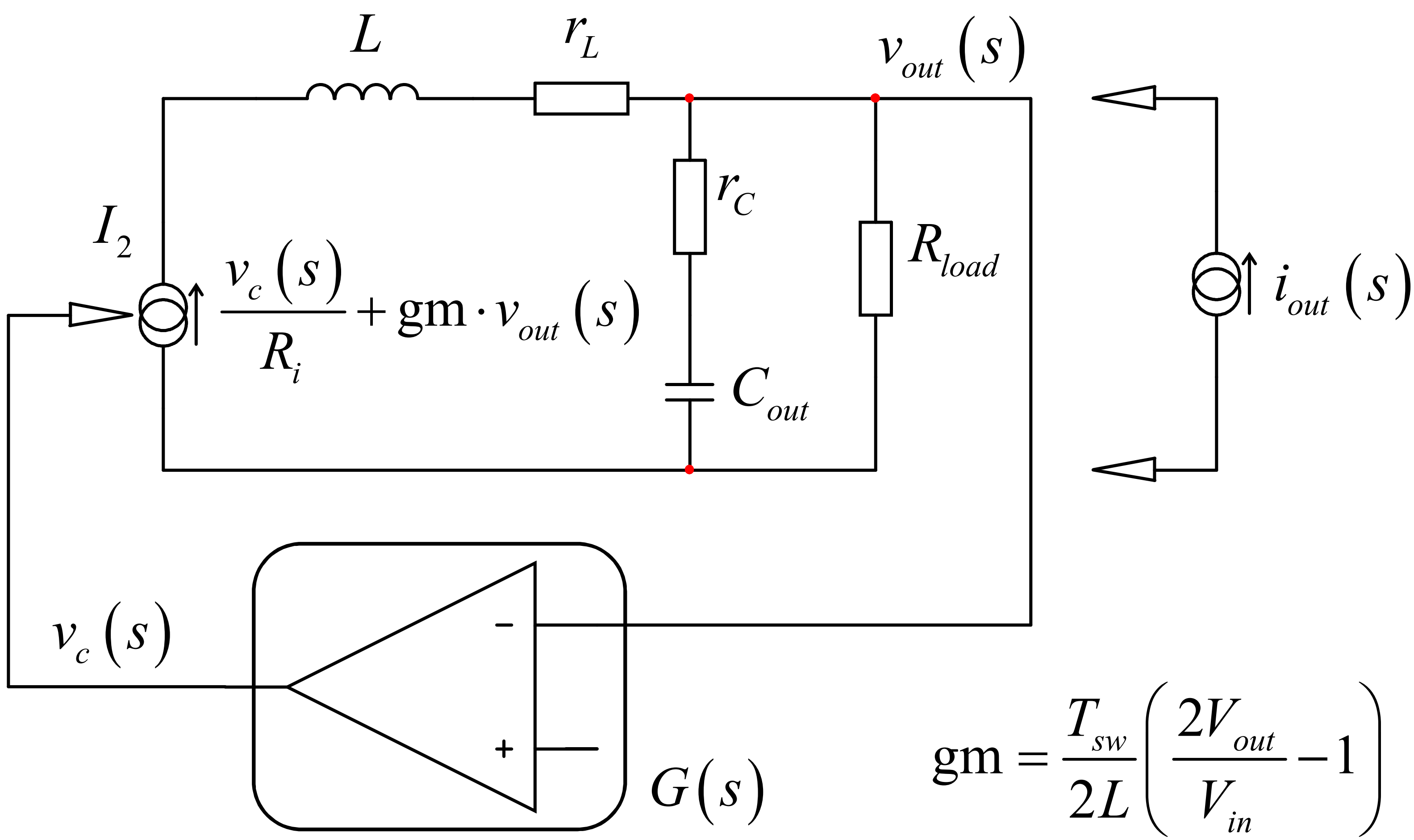
- Calculate the partial derivative coefficients to v_{out} and v_c

$$\frac{\partial I_2}{\partial V_c}\hat{v}_c+\frac{\partial I_2}{\partial V_{out}}\hat{v}_{out}=\frac{\hat{v}_c}{R_i}+\underbrace{\frac{T_{sw}}{2L}\left(\frac{2V_{out}}{V_{in}}-1\right)}_{\text{gm}}\hat{v}_{out}$$

- Update the schematic with this linear source
  - as R, L and C are also linear, Laplace applies

Compensating the Buck – Method 3

- We look at the output impedance closed-loop



$$gm = \frac{T_{sw}}{2L}\left(\frac{2V_{out}}{V_{in}} - 1\right)$$

Compensating the Buck – Method 3

- We can apply the superposition theorem:

$$v_{out}(s) = \left(\frac{v_c(s)}{R_i} + \mathrm{gm} \cdot v_{out}(s)\right)\left(R_{load} \,||\, \left(r_C + \frac{1}{sC_{out}}\right)\right) \qquad i_{out} \text{ is } 0$$

$$v_{out}(s) = -i_{out}(s)\left(R_{load} \,||\, \left(r_C + \frac{1}{sC_{out}}\right)\right) \qquad I_2 \text{ is } 0$$

- Considering $v_c(s) = -G(s)v_{out}(s)$ we have:

$$v_{out}(s) = v_{out}(s)\left(\mathrm{gm} - \frac{G(s)}{R_i}\right)\left(R_{load} \,||\, \left(r_C + \frac{1}{sC_{out}}\right)\right) - i_{out}(s)\left(R_{load} \,||\, \left(r_C + \frac{1}{sC_{out}}\right)\right)$$

$$v_{out}(s)\left(1 - \left(\mathrm{gm} - \frac{G(s)}{R_i}\right)\left(R_{load} \,||\, \left(r_C + \frac{1}{sC_{out}}\right)\right)\right) = -i_{out}(s)\left(R_{load} \,||\, \left(r_C + \frac{1}{sC_{out}}\right)\right)$$

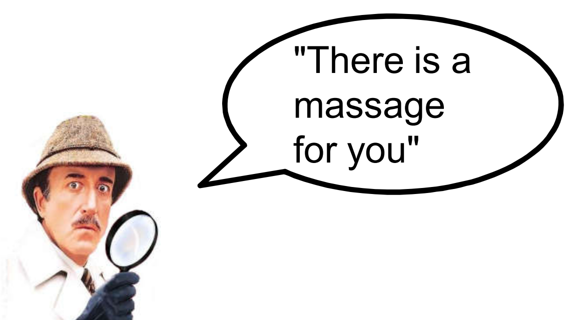
Compensating the Buck – Method 3

- The output impedance is thus:

$$Z_{out,CL}(s) = -\frac{\hat{v}_{out}(s)}{\hat{i}_{out}(s)} = \frac{\left(R_{load} \parallel \left(r_C + \frac{1}{sC_{out}}\right)\right)}{1 - \left(\text{gm} - \frac{G(s)}{R_i}\right)\left(R_{load} \parallel \left(r_C + \frac{1}{sC_{out}}\right)\right)}$$

- Giving a small massage, we obtain:

$$Z_{out,CL}(s) = \frac{R_{load} R_i}{R_i + G(s) R_{load} - R_i R_{load} \cdot \text{gm}} \frac{1 + s r_C C_{out}}{1 + sC_{out}\left(\frac{R_i R_{load} + R_i r_C + G(s) R_{load} r_C - R_i R_{load} r_C \text{gm}}{R_i + G(s) R_{load} - R_i R_{load} \cdot \text{gm}}\right)}$$



Compensating the Buck – Method 3

- Factoring and re-arranging, we have:

$$Z_{out,CL}(s) = R(s)\frac{1+s/\omega_{z_1}}{1+s/\omega_{p_1}} \quad R(s) = \frac{R_{load}R_i}{R_i + G(s)R_{load} - R_iR_{load}\cdot \mathrm{gm}} \quad \omega_{z_1} = \frac{1}{r_C C_{out}}$$

$$\omega_{p_1} = \frac{1}{C_{out}\left(\frac{R_iR_{load} + R_ir_C + G(s)R_{load}r_C - R_iR_{load}r_C\mathrm{gm}}{R_i + G(s)R_{load} - R_iR_{load}\cdot \mathrm{gm}}\right)}$$

- Now make $Z_{out,CL}(s)$ = r_C and extract $G(s)$:

$$G(s) = \frac{R_i\left(R_{load} - r_C + R_{load}r_C\mathrm{gm}\right)}{r_C R_{load}} \frac{1 + sC_{out}\frac{R_{load}r_C{}^2\mathrm{gm} - r_C{}^2}{R_{load} - r_C + R_{load}r_C\mathrm{gm}}}{1 + sr_C C_{out}}$$

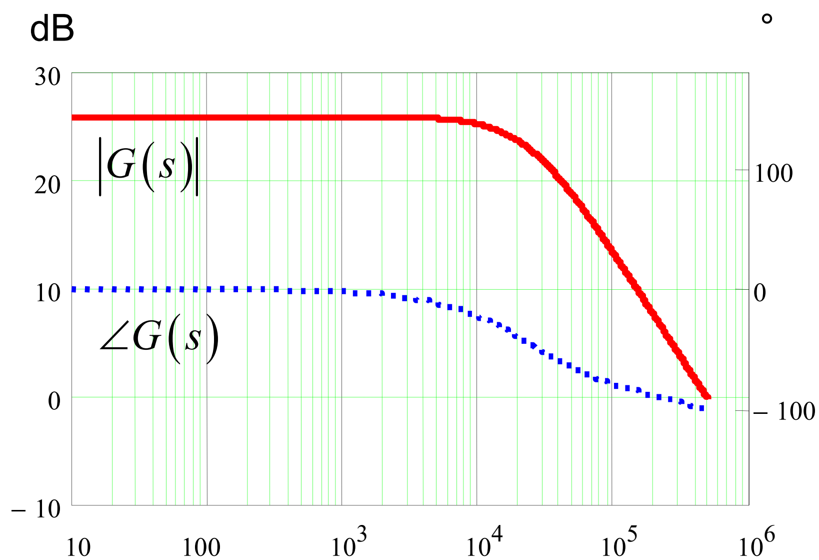
Compensating the Buck – Method 3

- The compensator brings a single pole/zero response

$$G(s) = G_0 \frac{1 + s/\omega_{z_1}}{1 + s/\omega_{p_1}} \qquad G_0 = \frac{R_i\left(R_{load} - r_C + R_{load} r_C \mathrm{gm}\right)}{r_C R_{load}} \approx \frac{R_i}{r_C} \qquad \omega_{p_1} = \frac{1}{C_{out} r_C}$$

$$\omega_{z_1} = \frac{1}{C_{out} \dfrac{R_{load} r_C{}^2 \mathrm{gm} - r_C{}^2}{R_{load} - r_C + R_{load} r_C \mathrm{gm}}} \approx \frac{1}{C_{out} \dfrac{-r_C{}^2}{R_{load}}}$$

Very high frequency can be neglected



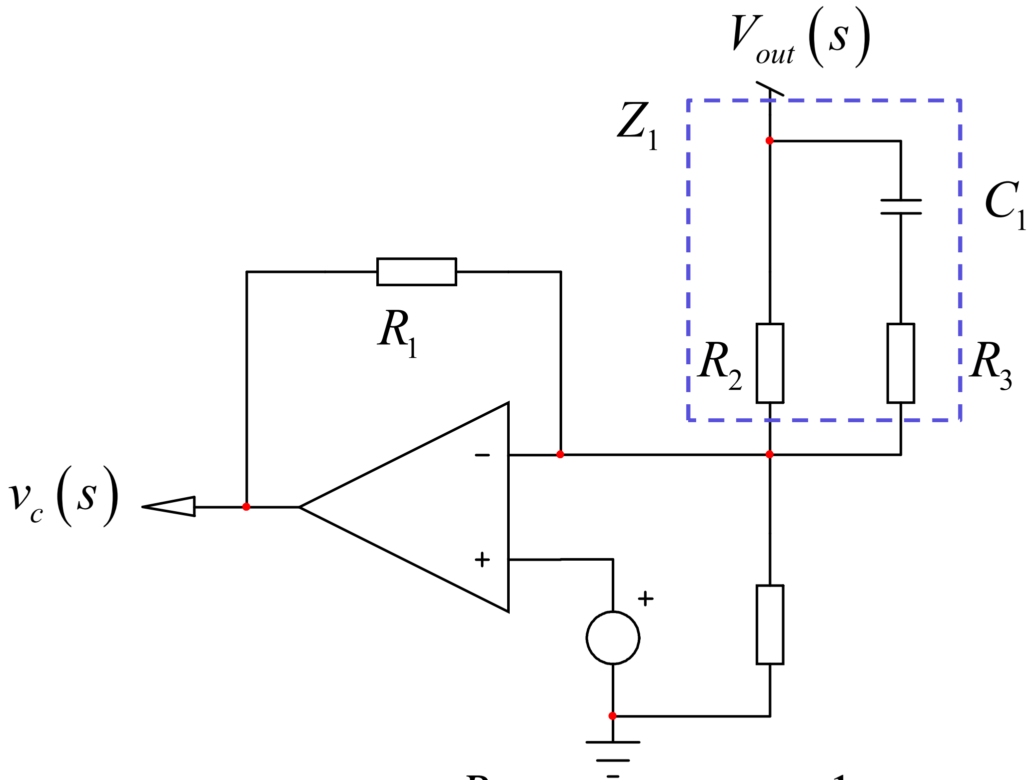
$R_i = 0.6\,\Omega$

$r_C = 30\,\mathrm{m}\Omega$

$C_{out} = 220\,\mu\mathrm{F}$

Compensating the Buck – Method 3

- ❑ Sub-harmonic oscillations at $F_{sw}/2$ can cause peaking
- ➢ Place a zero at πF_{sw} as recommended by YAO and al.



$$\frac{v_c(s)}{V_{out}(s)} = -\frac{R_1}{Z_1}$$

$$Z_1 = \frac{R_2\left(R_3 + \frac{1}{sC_1}\right)}{R_2 + \left(R_3 + \frac{1}{sC_1}\right)}$$

$$G(s) = -\frac{R_2}{R_1}\frac{\left[1 + sC_1\left(R_2 + R_3\right)\right]}{1 + sR_3C_1}$$

$$G_0 = \frac{R_2}{R_1} \qquad \omega_{z_1} = \frac{1}{C_1\left(R_2 + R_3\right)} \qquad \omega_{p_1} = \frac{1}{R_3C_1}$$

YAO et al. ,"Design Considerations for VRM Transient Response Based on the Output Impedance", IEEE Proceedings, 2003

Compensating the Buck – Method 3

- Run the SPICE simulation with the CM PWM switch model

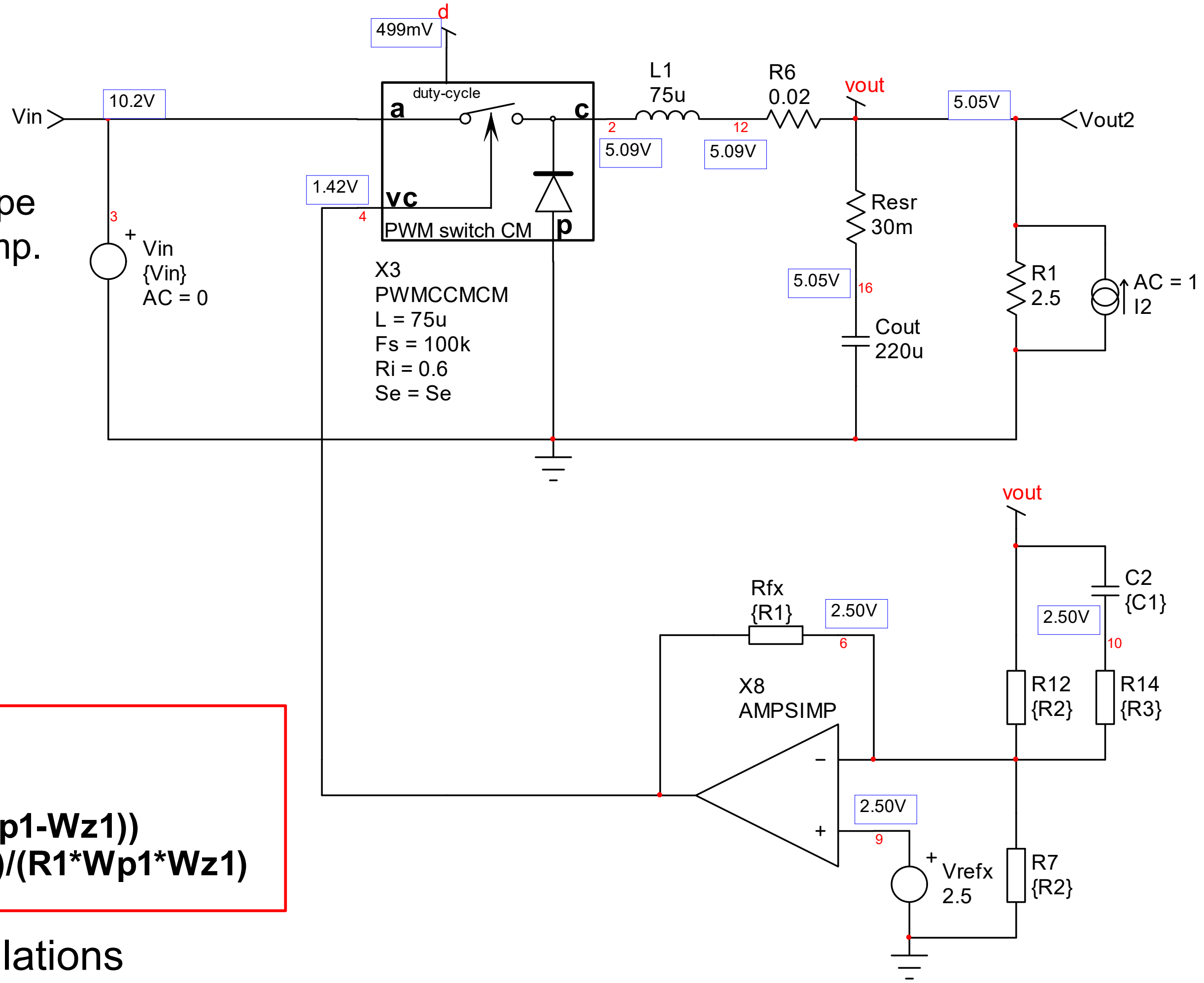
parameters

Vin=10.18
se=20k → Slope comp.
Fsw=100k
pi=3.14159

pi=3.14159
Ri=0.6
Cout=220u
rC=30m
wp=1/(rC*Cout)

Wz1=pi*Fsw
Wp1=1/(rC*Cout)
G=Ri/rC

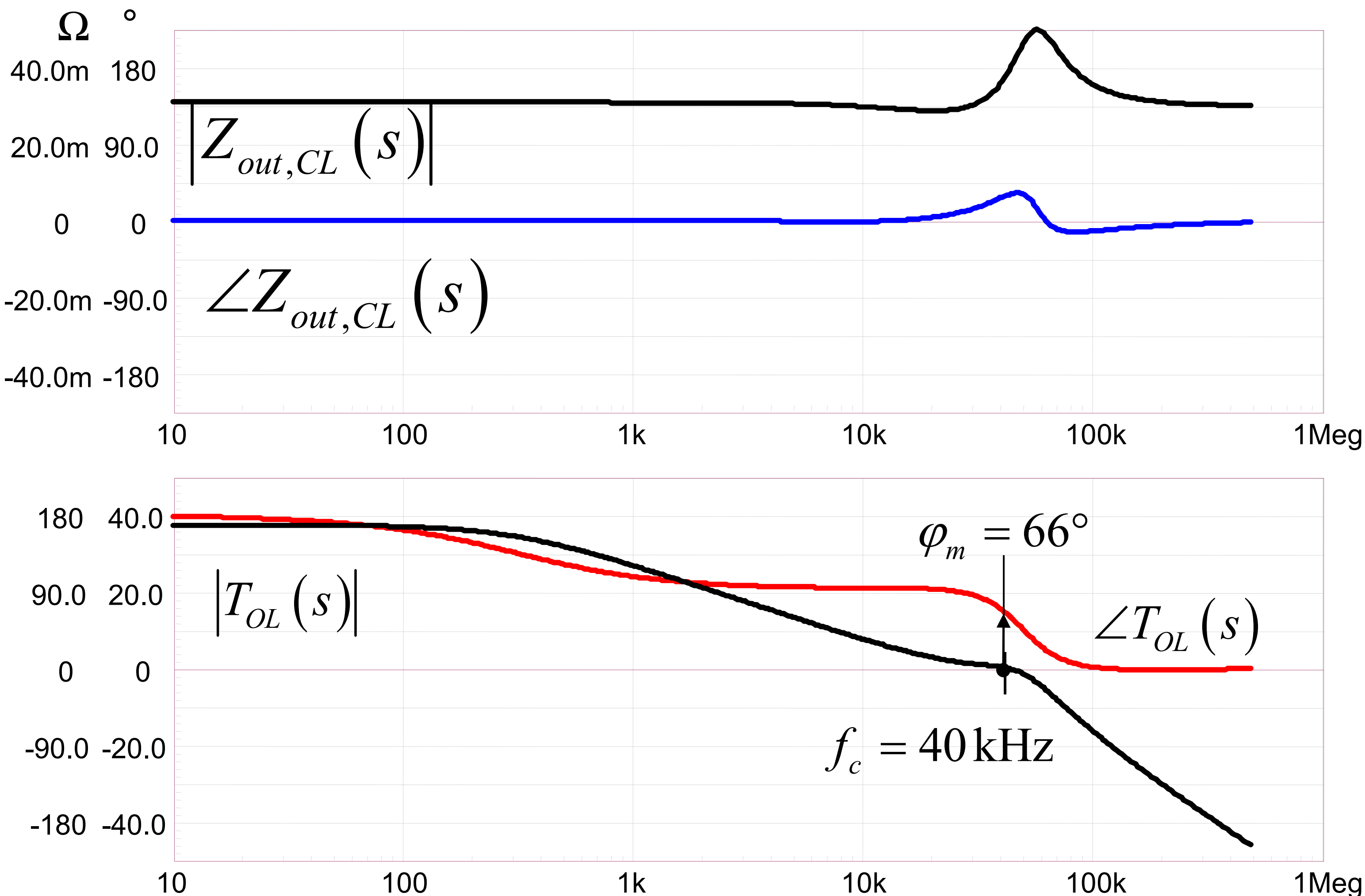
R1=10k
R2=R1/G
R3=R1*Wz1/(G*(Wp1-Wz1))
C1=(G*(Wp1-Wz1))/(R1*Wp1*Wz1)



Automated calculations

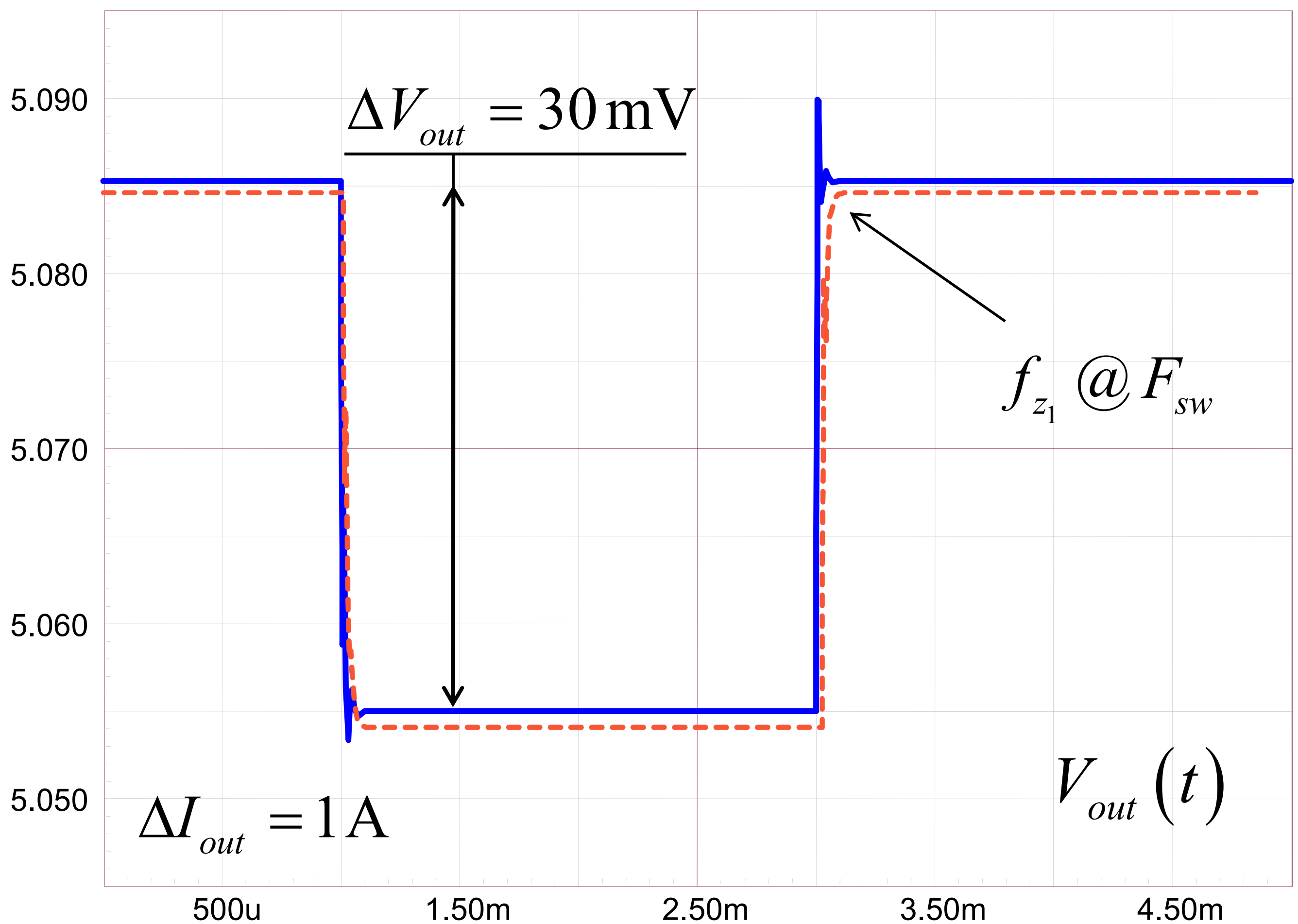
Compensating the Buck – Method 3

- The output impedance is resistive, the system is stable.



Compensating the Buck – Method 3

- The output response is a square signal as expected



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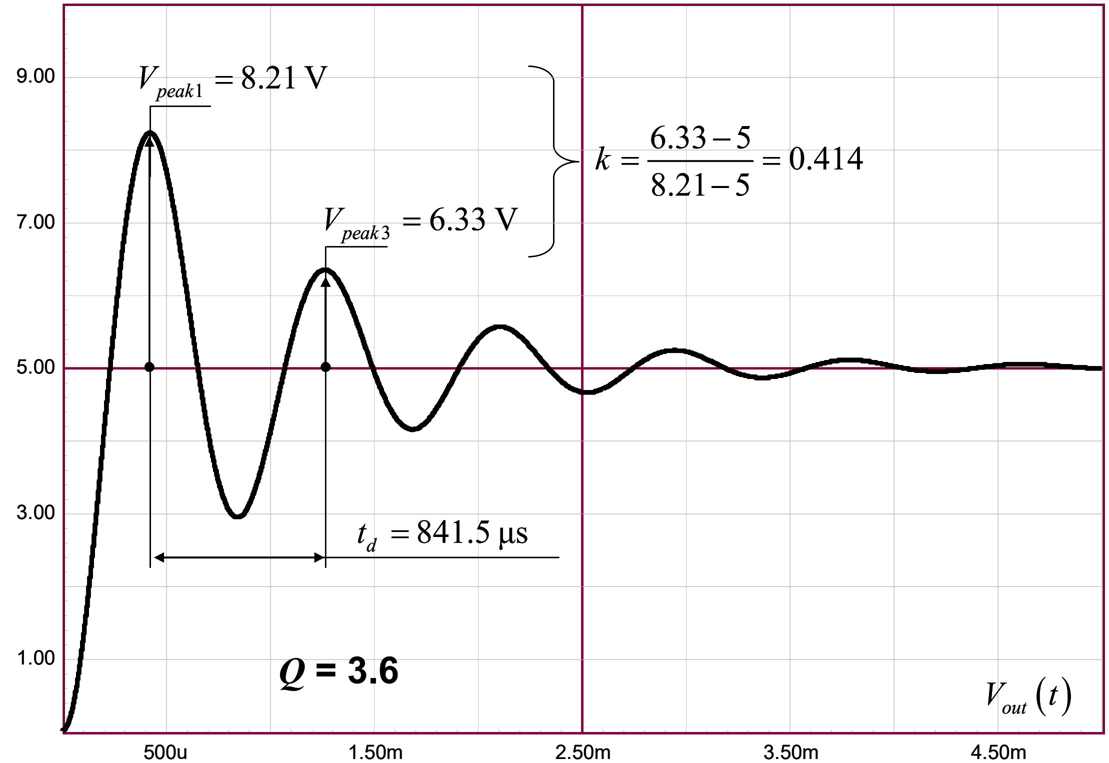
Output Impedance and Quality Factor

- How to get Q and φ_m from the available signals?



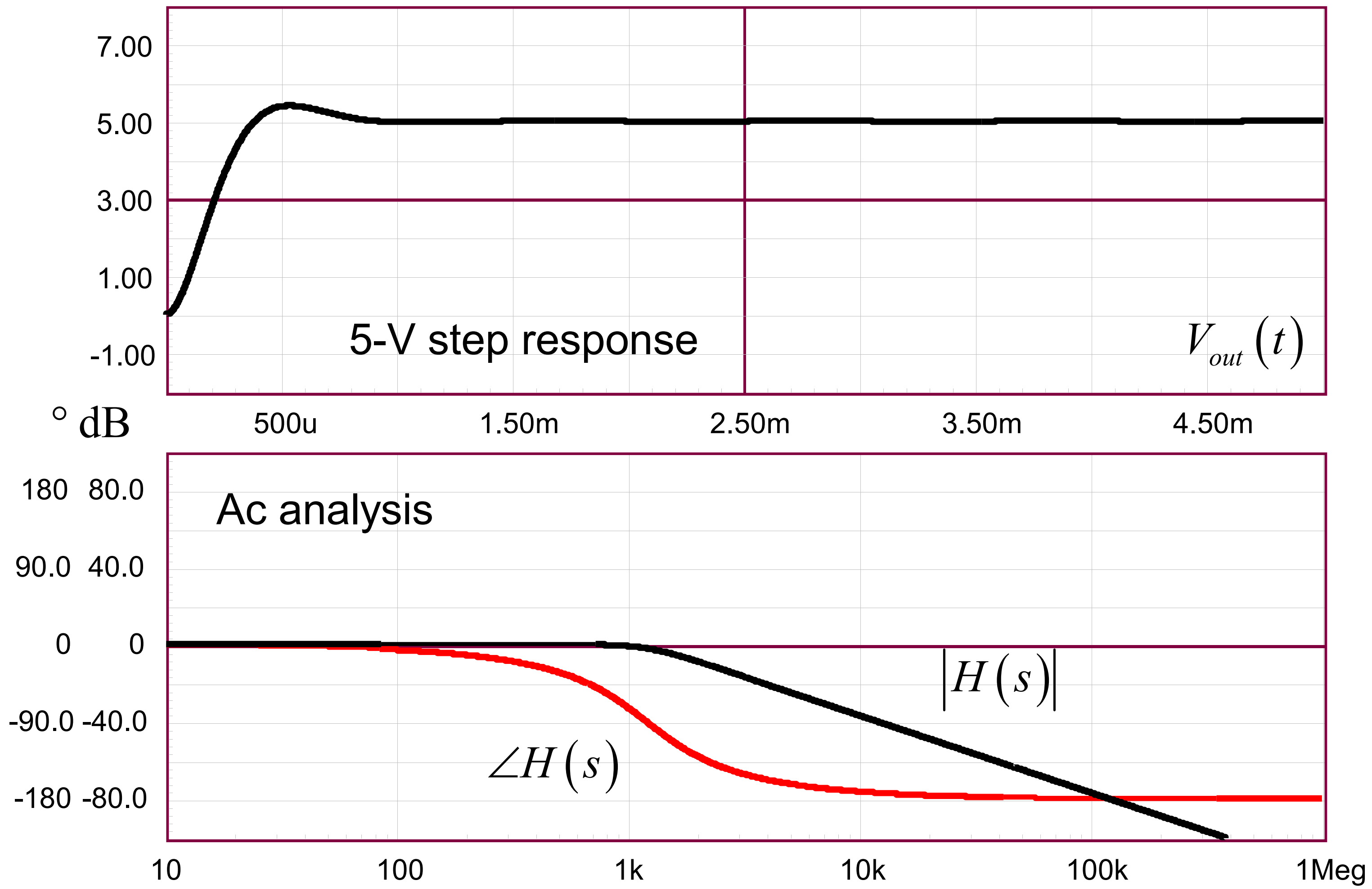
You need enough signal and ringing to extract the data

Must remain small-signal!

$$Q = \sqrt{\left(\frac{\pi}{\ln k}\right)^2 + \frac{1}{4}} \qquad \zeta = \frac{1}{\sqrt{\left(\frac{2\pi}{\ln k}\right)^2 + 1}} \qquad f_0 = \frac{1}{t_d\sqrt{1-\zeta^2}}$$

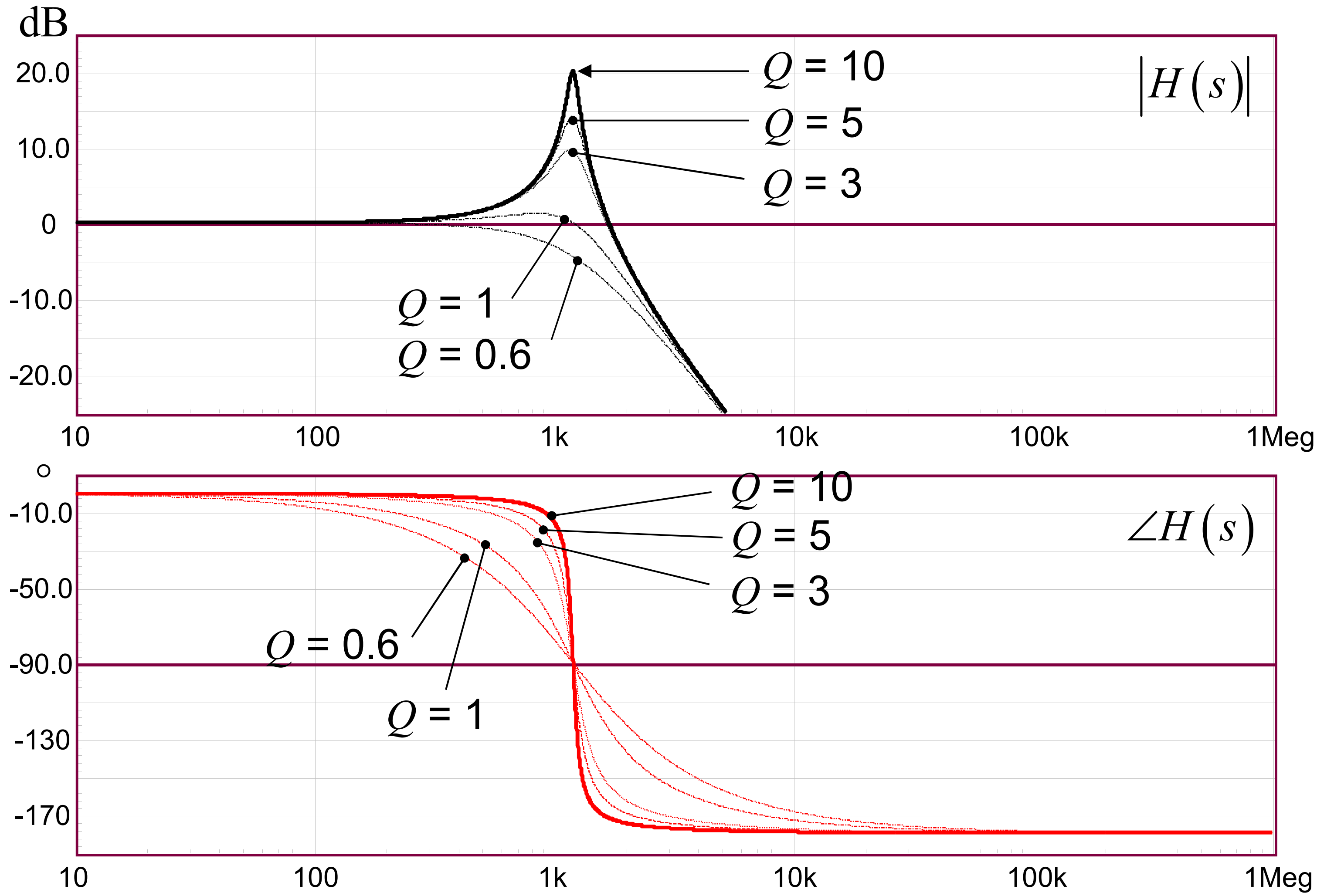
Output Impedance and Quality Factor

- Extraction is difficult with flat ac 2nd-order responses



Output Impedance and Quality Factor

- The phase drops at a different pace as Q changes



Output Impedance and Quality Factor

- Is there a link between Q and the phase rate of change?

Group delay ⇒ $$\tau_g = -\frac{d\varphi(\omega)}{d\omega} \longrightarrow \text{[s]}$$

- Let's apply the definition to a 2nd-order network:

$$H(s) = \frac{1}{1 + \frac{s}{\omega_0 Q} + \left(\frac{s}{\omega_0}\right)^2} \quad \xrightarrow{s = j\omega} \quad H(j\omega) = \frac{1}{\underbrace{1 - \frac{\omega^2}{\omega_0^2}}_{a} + j\underbrace{\frac{\omega}{\omega_0 Q}}_{b}}$$

$$|H(\omega)| = \sqrt{a^2 + b^2} = \frac{1}{\sqrt{\left(1 - \frac{\omega^2}{\omega_0^2}\right)^2 + \left(\frac{\omega}{\omega_0 Q}\right)^2}}$$

$$\angle H(\omega) = \tan^{-1}\left(\frac{b}{a}\right) = \tan^{-1}\left[\frac{\omega}{\omega_0 Q}\frac{1}{\left(1 - \frac{\omega^2}{\omega_0^2}\right)}\right]$$

Output Impedance and Quality Factor

- We can apply τ_g definition to the argument

$$\tau_g = -\frac{d\tan^{-1}\left[\frac{\omega}{\omega_0 Q}\frac{1}{\left(1-\frac{\omega^2}{\omega_0^{\;2}}\right)}\right]}{d\omega} = \frac{Q\omega_0\left(\omega^2+\omega_0^{\;2}\right)}{Q^2\omega^4-2Q^2\omega^2\omega_0^{\;2}+Q^2\omega_0^{\;4}+\omega^2\omega_0^{\;2}}$$

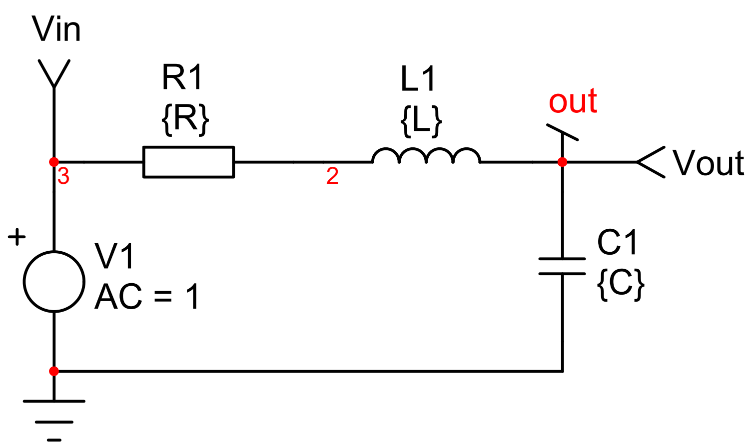
$$\omega = \omega_0 \quad \Rightarrow \quad \tau_g = \frac{2Q}{\omega_0}$$

- At ω_0 the following formula links Q to the group delay

$$Q = \frac{\tau_g\omega_0}{2} = \tau_g\pi f_0$$

Output Impedance and Quality Factor

- Let's apply the theory to a classical case, the *RLC* filter

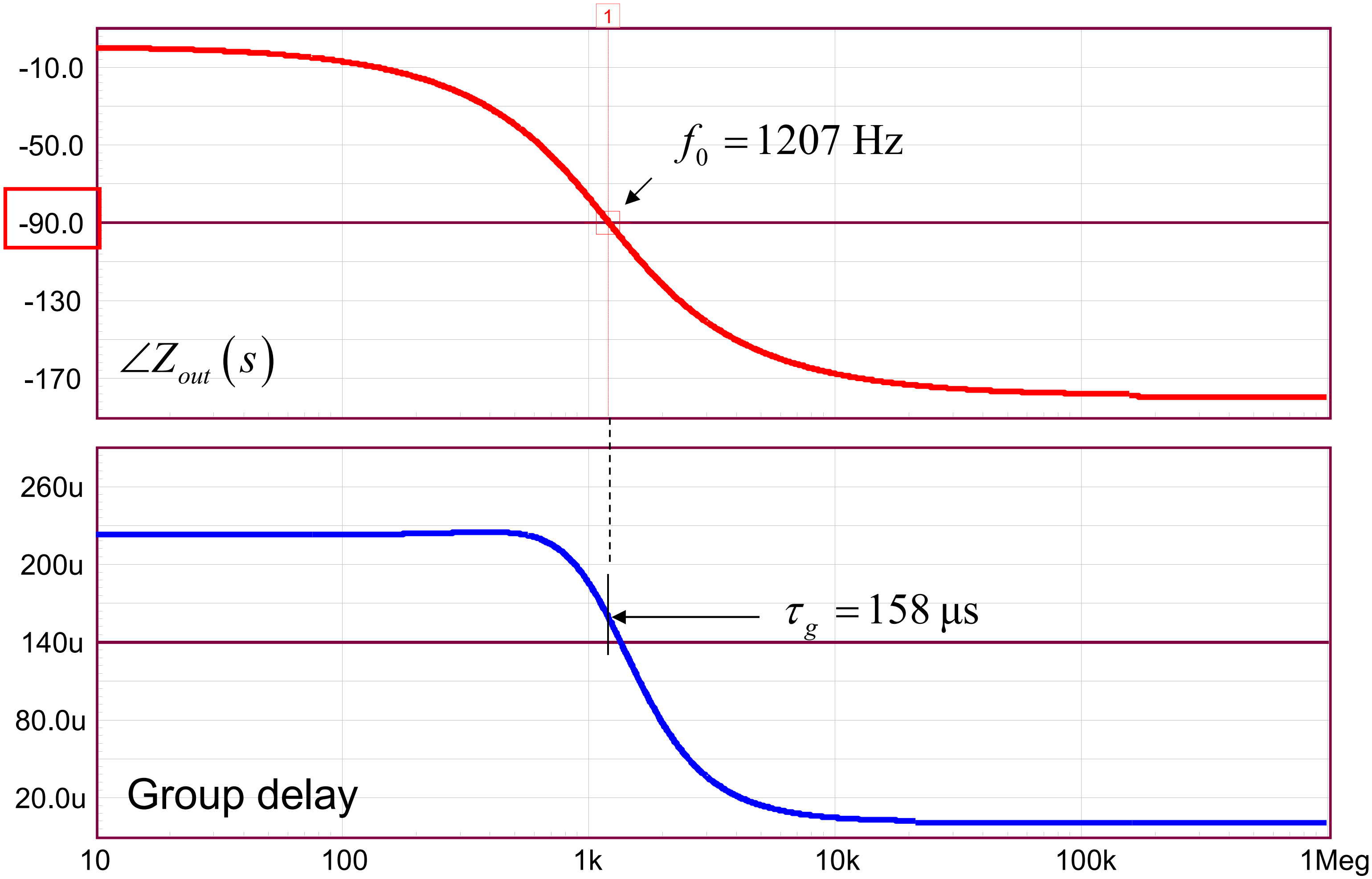


parameters

Vp=5
f0=1.2k
L=10u
C=1/(4*3.14159^2*f0^2*L)
w0=(L*C)^-0.5
Q=0.6
R=L*w0/Q
R2=1/(Q*C*w0)
Q1=(sqrt(L/C))/R
Dzeta=(R/2)*sqrt(C/L)
Dzeta1=R/(2*L*w0)
Q3=1/(2*Dzeta)
per=1/(f0*sqrt(1-Dzeta^2))
tp=1/(2*f0*sqrt(1-Dzeta^2))

- ✓ plot the ac response
- ✓ calculate the group delay
- ✓ see if we can find *Q*

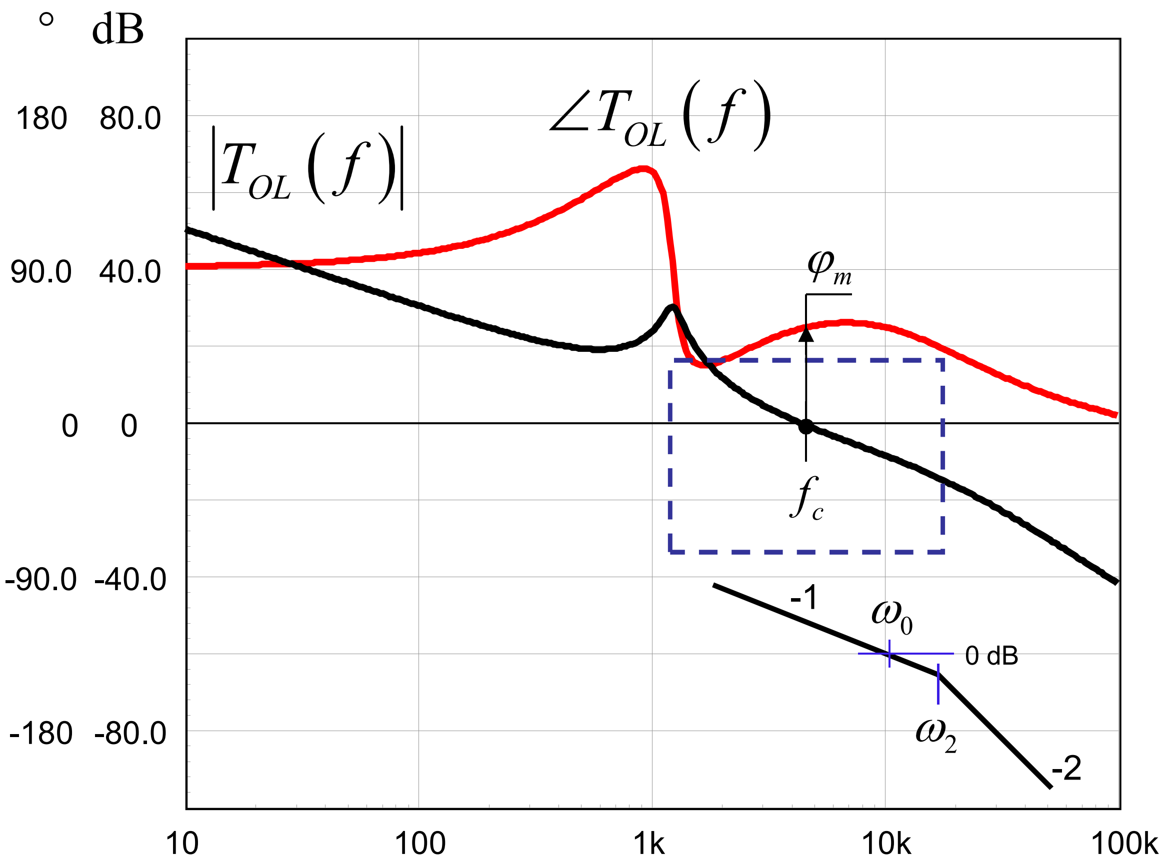
Output Impedance and Quality Factor



$$Q = \tau_g \pi f_0 = 1207 \times 158u \times 3.14159 = 0.599$$

Output Impedance and Quality Factor

❑ Knowing Q can also lead us to the phase margin φ_m



Open-loop ac analysis

Approximation of $T(s)$ around f_c

$$T_{OL}(s) = \frac{1}{\left(\frac{s}{\omega_0}\right)\left(1 + \frac{s}{\omega_2}\right)}$$

Output Impedance and Quality Factor

- If we consider the open-loop gain around f_c only…

$$T_{OL}(s) = \frac{1}{\left(\frac{s}{\omega_0}\right)\left(1+\frac{s}{\omega_2}\right)}$$

Closed-loop ⟹ Unity return

$$\frac{T_{OL}(s)}{1+T_{OL}(s)} = \frac{1}{\frac{s^2}{\omega_0\omega_2}+\frac{s}{\omega_0}+1}$$

$$\frac{1}{\frac{s^2}{\omega_0\omega_2}+\frac{s}{\omega_0}+1} = \frac{1}{\frac{s^2}{\omega_c^{\;2}}+\frac{s}{\omega_c Q_c}+1}$$

Identify ⟹

$$Q_c = \sqrt{\frac{\omega_0}{\omega_2}} \qquad \omega_c = \sqrt{\omega_0\omega_2}$$

Q_c: Closed-loop data; ω_c: Open-loop data

- We want to link Q_c and the crossover frequency

$$T_{OL}(s) = \frac{1}{\left(\frac{s}{\omega_0}\right)\left(1+\frac{s}{\omega_2}\right)}$$

⟹ $\omega_0 = Q_c^{\;2}\omega_2$

$$T_{OL}(s) = \frac{1}{\left(\frac{s}{Q_c^{\;2}\omega_2}\right)\left(1+\frac{s}{\omega_2}\right)}$$

Output Impedance and Quality Factor

- Calculate the T_{OL} magnitude at crossover with Q_c in:

$$\left|\frac{1}{\left(\frac{j\omega_c}{Q_c{}^2\omega_2}\right)\left(1+\frac{j\omega_c}{\omega_2}\right)}\right| = \frac{Q_c{}^2\omega_2{}^2}{\sqrt{\omega_c{}^2\omega_2{}^2+\omega_c{}^4}}$$

Solve ω_c

$$\left|T_{OL}(\omega_c)\right| = 1$$

$$\omega_c = \frac{\omega_2\sqrt{\left(\sqrt{1+4Q_c{}^4}-1\right)}}{\sqrt{2}}$$

- Derive the argument of T_{OL}, simplify it:

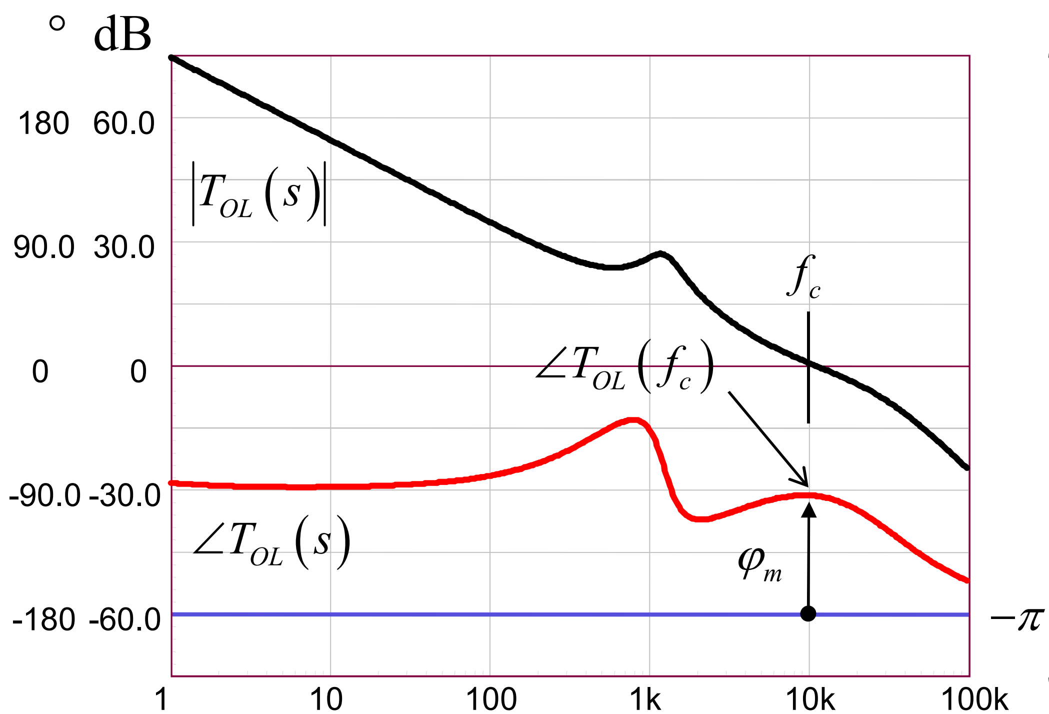
$$\arg T_{OL}(s) = \arg\frac{1}{\left(\frac{s}{\omega_0}\right)\left(1+\frac{s}{\omega_2}\right)} = \arg\left(\frac{jQ_c{}^2\omega_2{}^2}{-\omega_c\omega_2 - j\omega_c{}^2}\right) = \arg\left(-\frac{1}{-j\frac{\omega_c\omega_2}{Q_c{}^2\omega_2{}^2}+\frac{\omega_c{}^2}{Q_c{}^2\omega_2{}^2}}\right)$$

$$\arg T_{OL}(s) = \arg(-1) - \tan^{-1}\left(-\frac{\frac{\omega_c\omega_2}{Q_c{}^2\omega_2{}^2}}{\frac{\omega_c{}^2}{Q_c{}^2\omega_2{}^2}}\right) = -\pi - \tan^{-1}\left(-\frac{\omega_2}{\omega_c}\right)$$

Output Impedance and Quality Factor

❑ If we substitute ω_c by its definition, we have:

$$\arg T_{OL}(s) = -\pi - \tan^{-1}\left(-\sqrt{\frac{2}{\sqrt{(1+4Q_c^4)}-1}}\right) = -\pi + \tan^{-1}\left(\sqrt{\frac{2}{\sqrt{(1+4Q_c^4)}-1}}\right)$$



$$\angle T_{OL}(f_c) - \varphi_m = -\pi$$

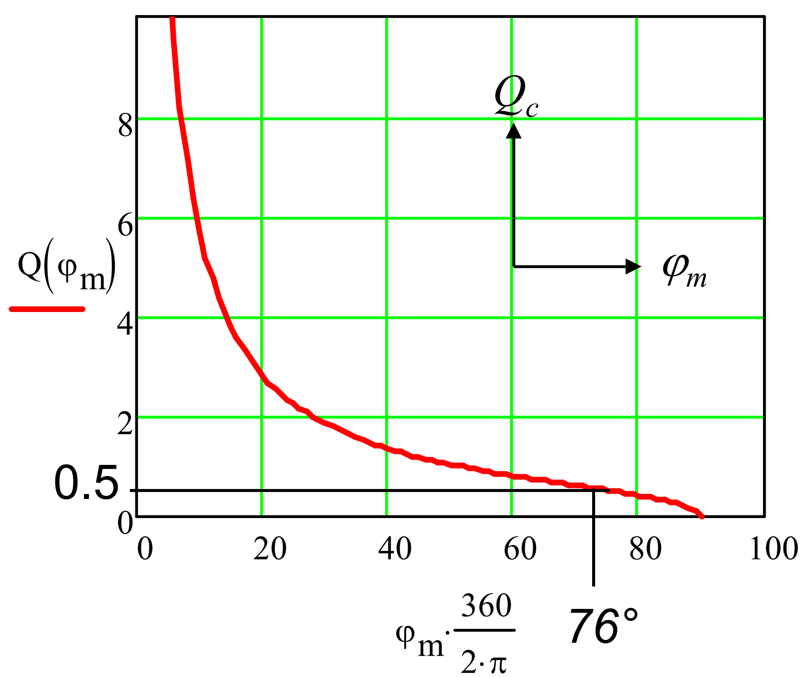
$$\varphi_m = \angle T_{OL}(f_c) + \pi$$

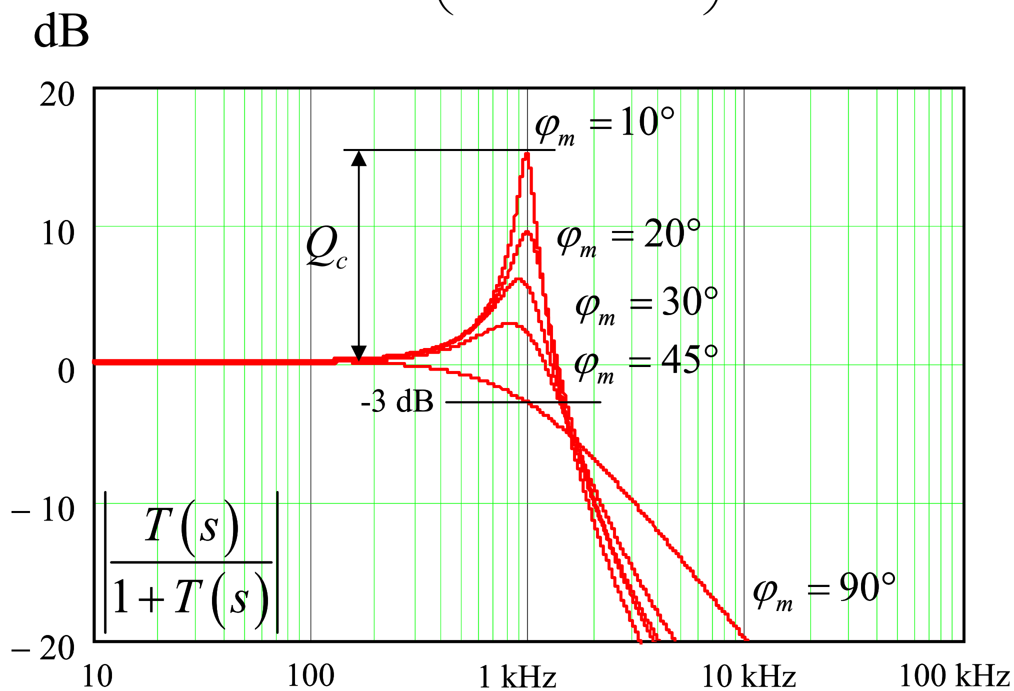
$$\varphi_m = \tan^{-1}\left(\sqrt{\frac{2}{\sqrt{(1+4Q_c^4)}-1}}\right)$$

Output Impedance and Quality Factor

- We can now extract the closed-loop quality coefficient:

$$Q_c = \frac{\sqrt[4]{1+\tan(\varphi_m)^2}}{\tan(\varphi_m)} = \frac{\sqrt{\cos(\varphi_m)}}{\sin(\varphi_m)} \Rightarrow \varphi_m = \cos^{-1}\left(\frac{\sqrt{4Q_c^{\,4}+1}-1}{2Q_c^{\,2}}\right)$$

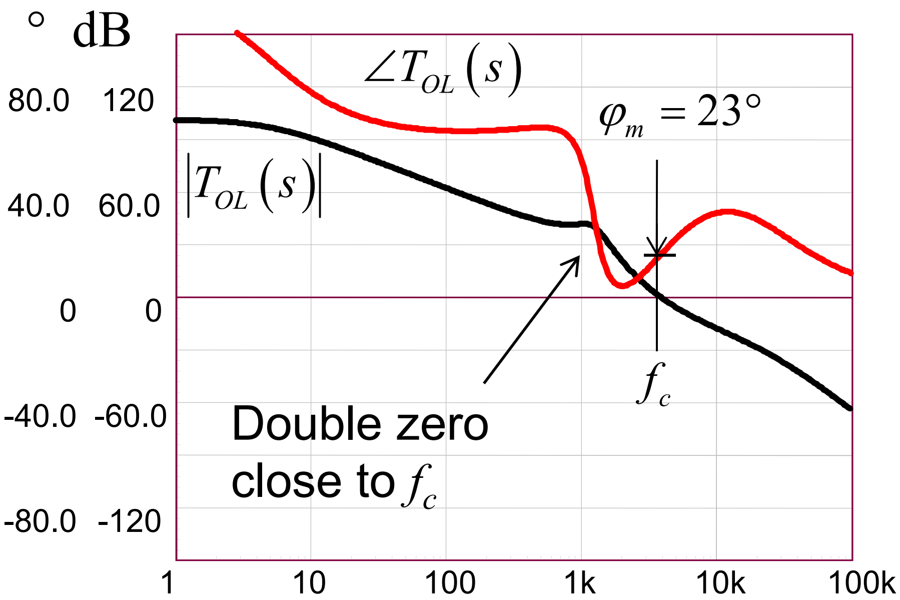


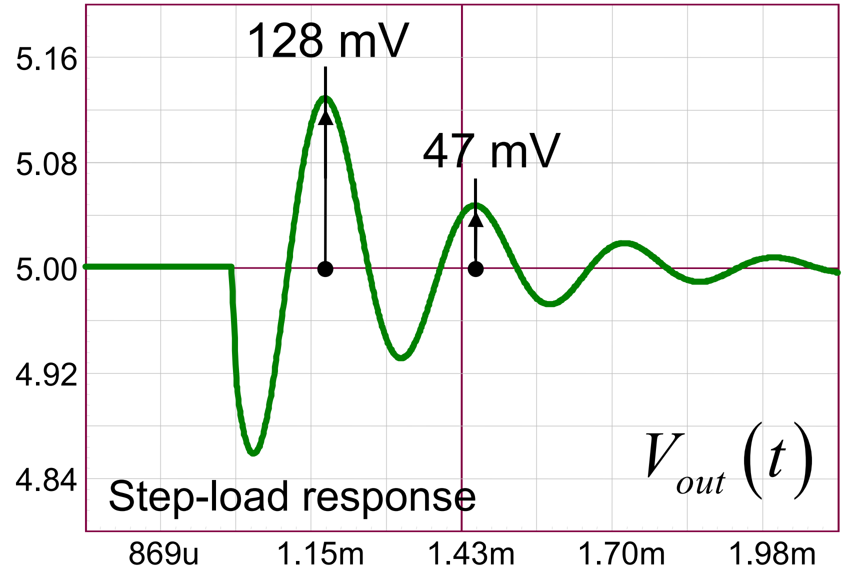


Closed-loop gain

Output Impedance and Quality Factor

- The formula considers the vicinity of f_c only: precision?





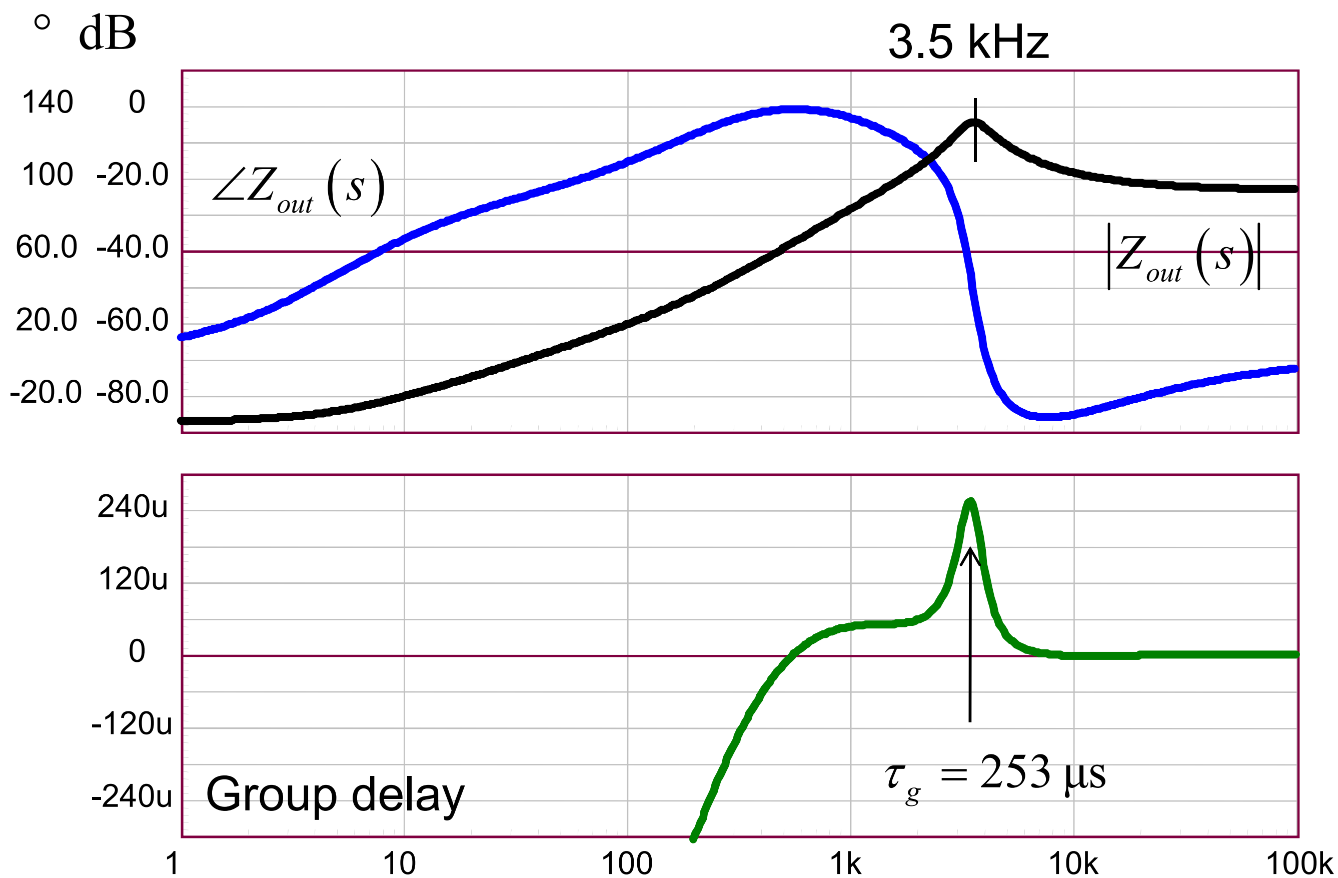
$$Q_c = \sqrt{\left(\frac{\pi}{\ln\left(\frac{47}{128}\right)}\right)^2 + \frac{1}{4}} = 3.1$$

$$\varphi_m = \cos^{-1}\left(\frac{\sqrt{4Q_c^{\,4}+1}-1}{2Q_c^{\,2}}\right) \approx 18°$$

"Revisiting the Response of Closed Loop of PWM Converters ", S. Ben-Yaakov, IEEE Apec 2008

Output Impedance and Quality Factor

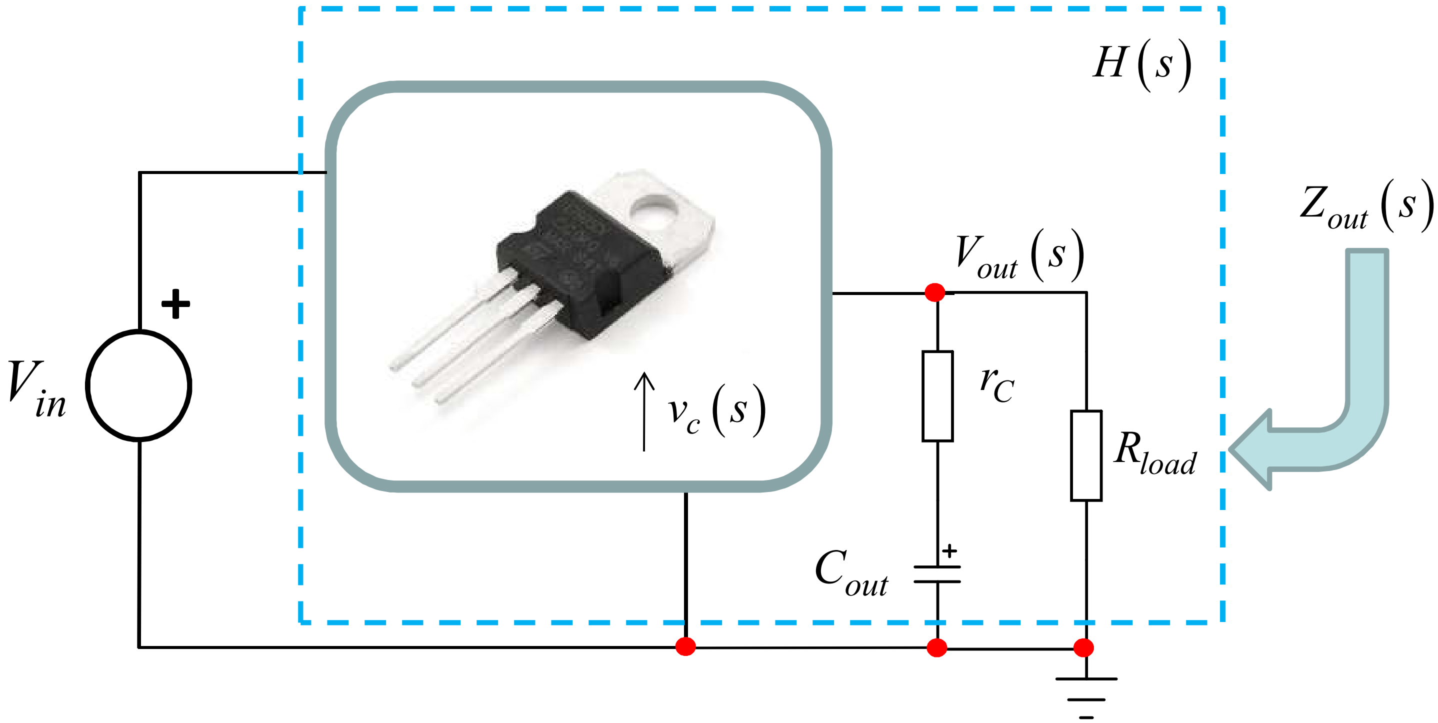
- We can ac sweep the output impedance also and check τ_g



$Q = \tau_g \pi f_0 = 3.5k \times 253u \times 3.14159 \approx 2.8$ ⇨ $\varphi_m \approx 20.4°$

Output Impedance and Quality Factor

- We can apply the technique to simple linear regulators



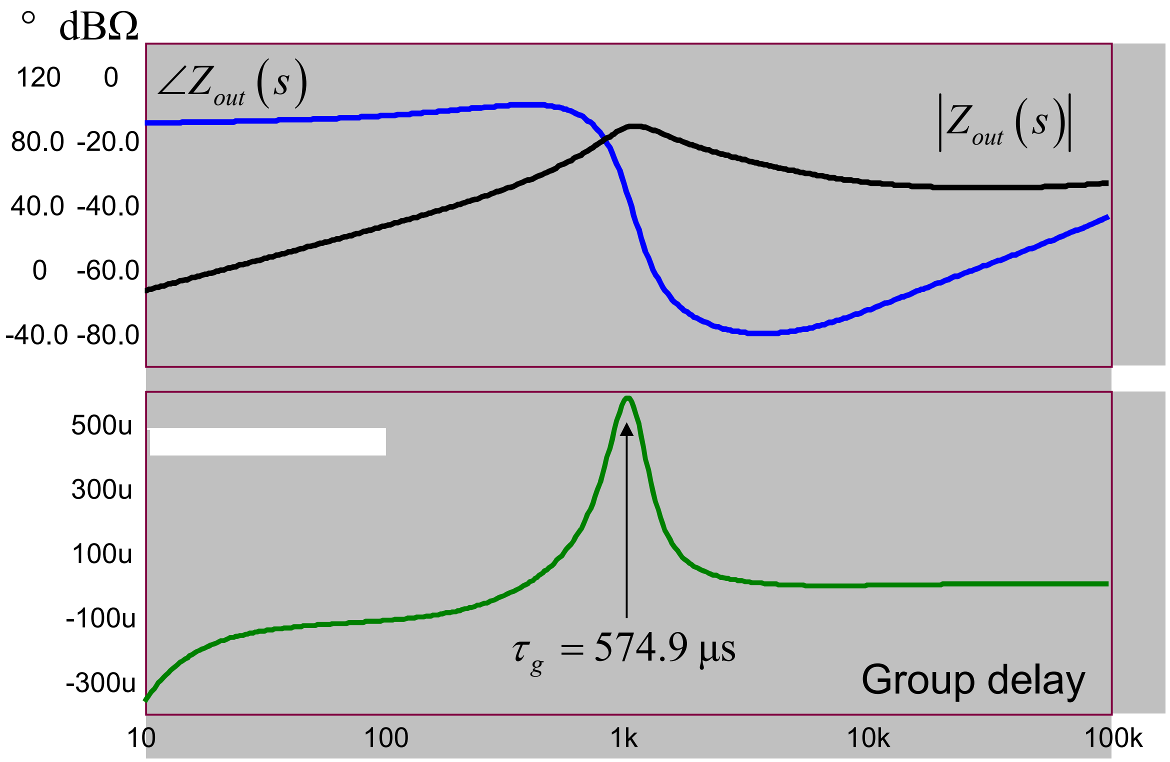
- You have no means to perform open-loop analysis

- Plot its output impedance with a network analyzer…

S. Sandler, C. Hymowitz, "New Technique for Non-Invasive Testing of Regulator Stability", PET Magazine, September 2011

Output Impedance and Quality Factor

- Plot magnitude and phase of Z_{out} and compute τ_g

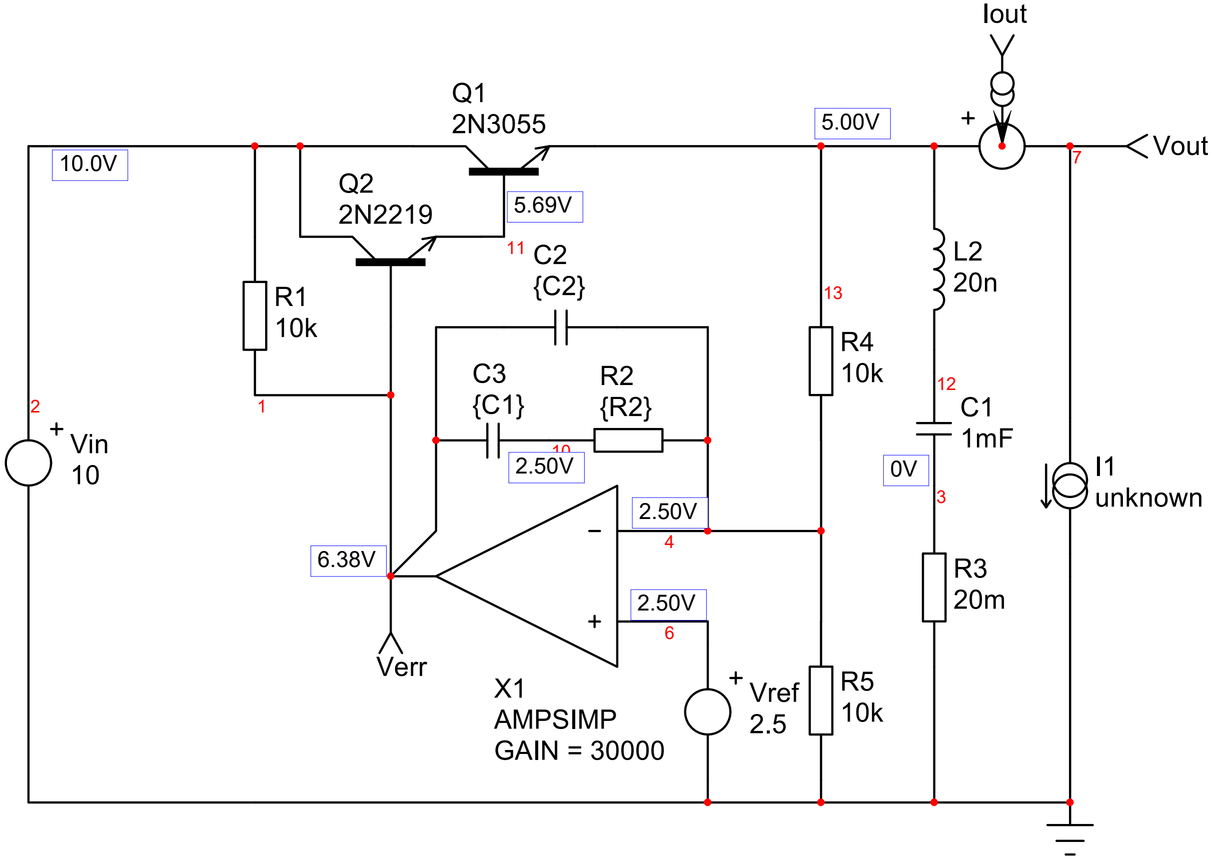


$$Q = \tau_g \pi f_0 = 1k \times 574.9u \times 3.14159 = 1.8$$

$$\varphi_m = \cos^{-1}\left(\frac{\sqrt{4Q^4+1}-1}{2Q^2}\right) = \cos^{-1}\left(\frac{\sqrt{4\times 1.8^4+1}-1}{2\times 1.8^2}\right) = \cos^{-1}(857m) \approx 31°$$

Output Impedance and Quality Factor

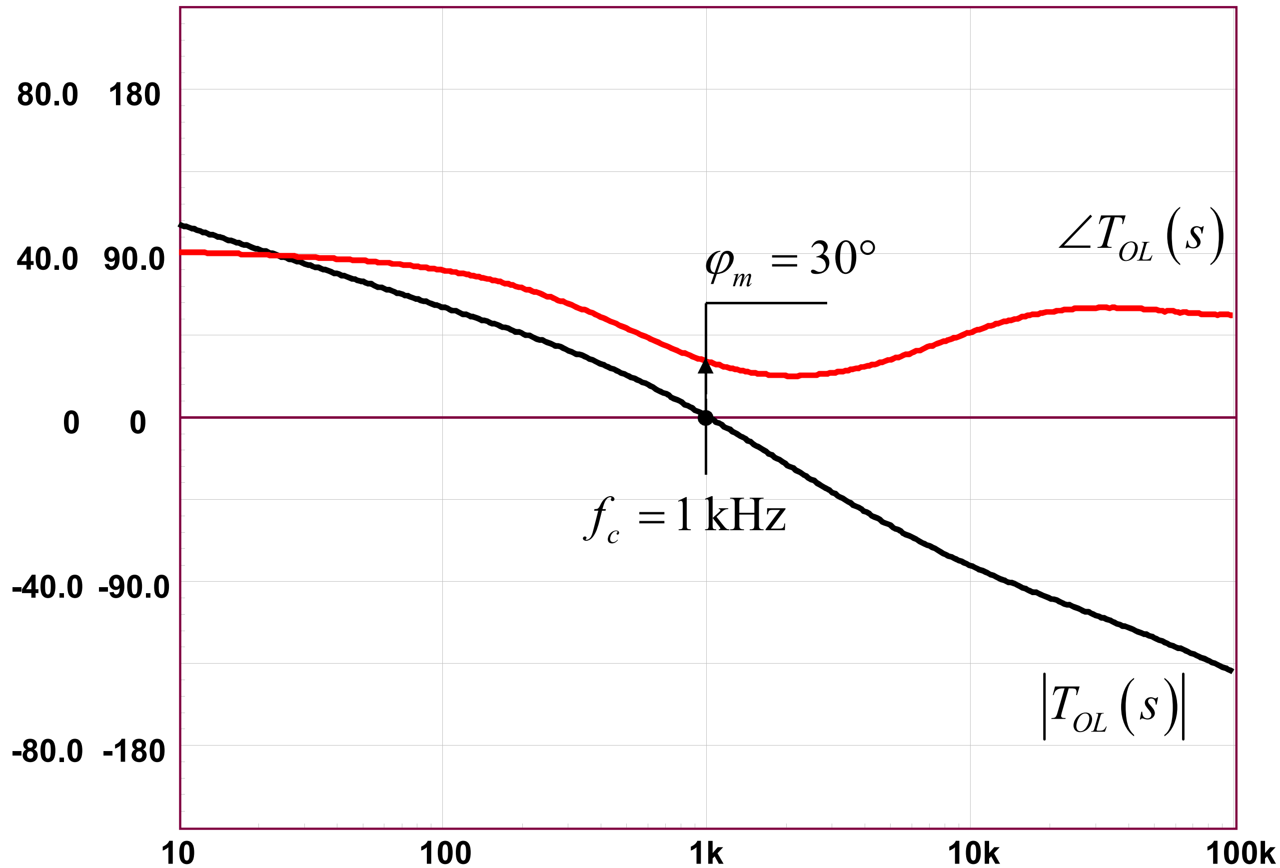
- ❑ The LDO circuit is made of the following elements



- ❑ We can open its loop and plot the open-loop gain $T_{OL}(s)$

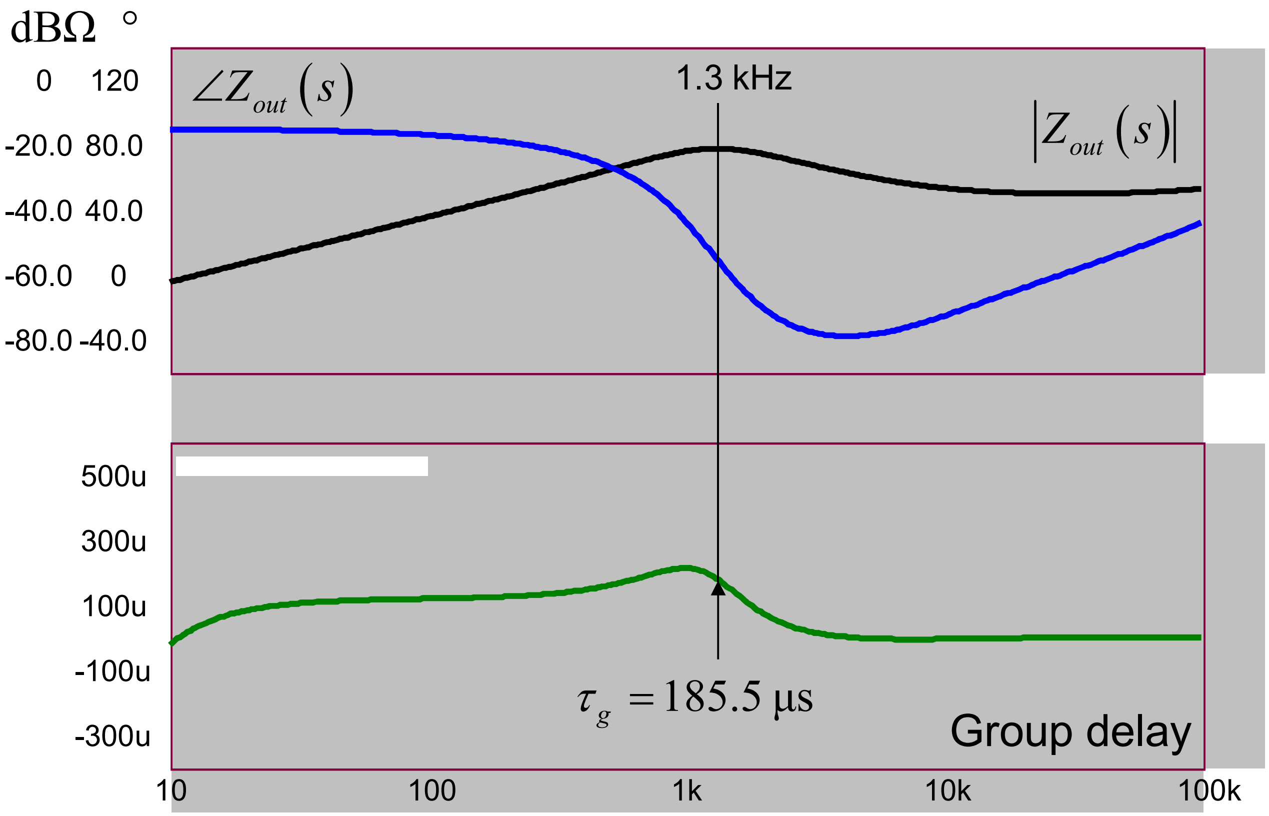
Output Impedance and Quality Factor

- The open-loop plot confirms the experimental results



Output Impedance and Quality Factor

- The phase margin is increased to 60°



$$Q = \tau_g \pi f_0 = 1.3k \times 185.5u \times 3.14159 \approx 0.76$$

$$\varphi_m = \cos^{-1}\left(\frac{\sqrt{4Q^4+1}-1}{2Q^2}\right) = \cos^{-1}\left(\frac{\sqrt{4\times 0.76^4+1}-1}{2\times 0.76^2}\right) = \cos^{-1}(457m) \approx 63°$$

Course Agenda

- ❑ Introduction to Control Systems
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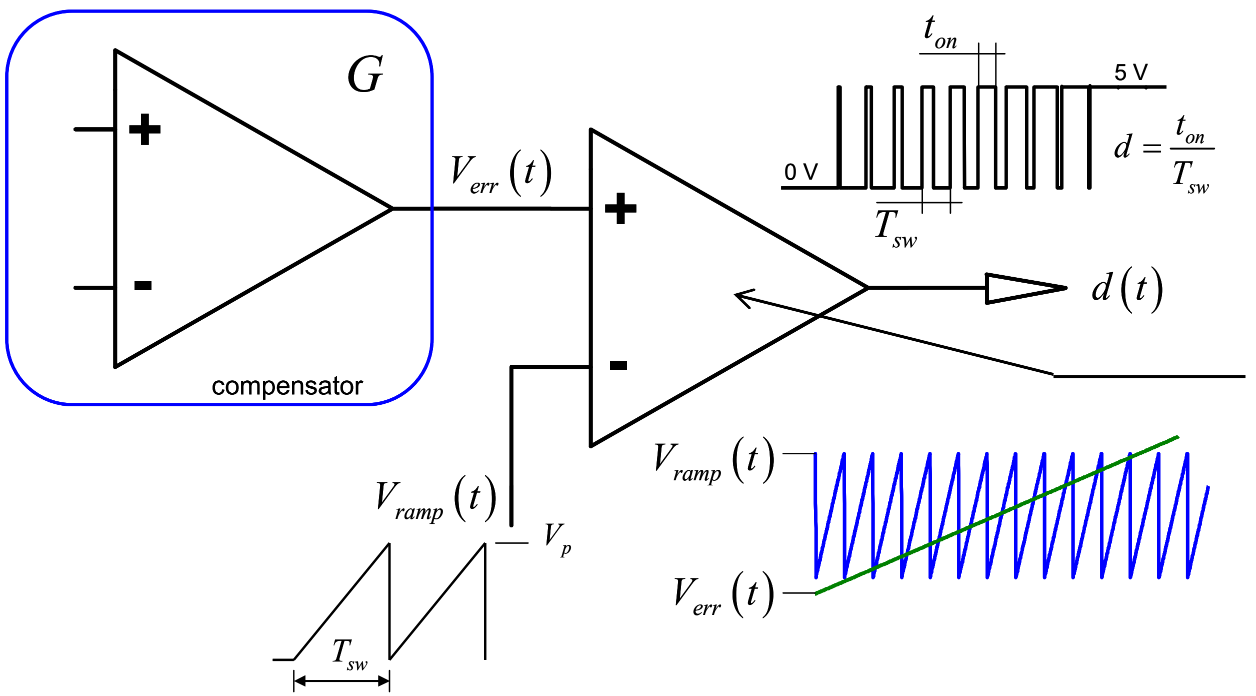
- ❑ The PID at Work with a Buck Converter
- ❑ Considering the Output Impedance
- ❑ Classical Poles/Zeros Placement
- ❑ Shaping the Output Impedance
- ❑ Quality Factor and Phase Margin
- ❑ **What is Delay Margin?**
- ❑ Gain Margin is not Enough

Considering a Delay in the Loop

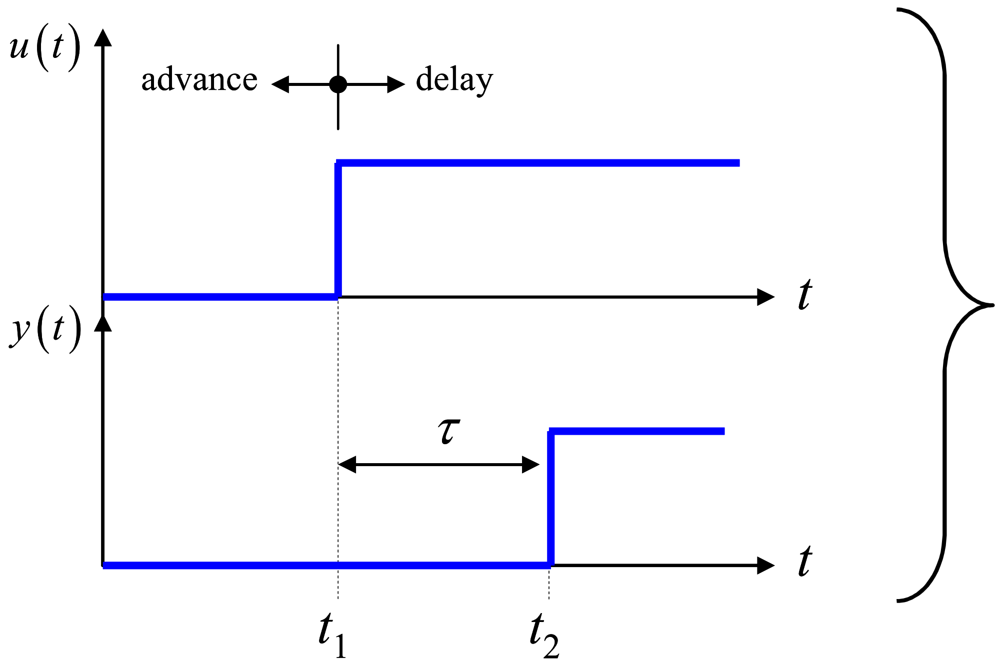
- Before a decision is actually executed, a delay occurs
- The delay can be digital (computation time) or analogue
- A typical delay is the duty-ratio conversion in a VM converter



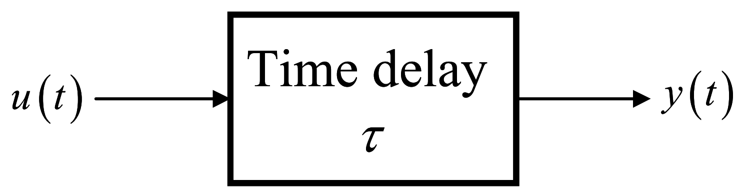
Prop. delay in the 50 – 100 ns range

A Delay is a Time-Domain Shift

- The output signal is the input signal that occured τ s before

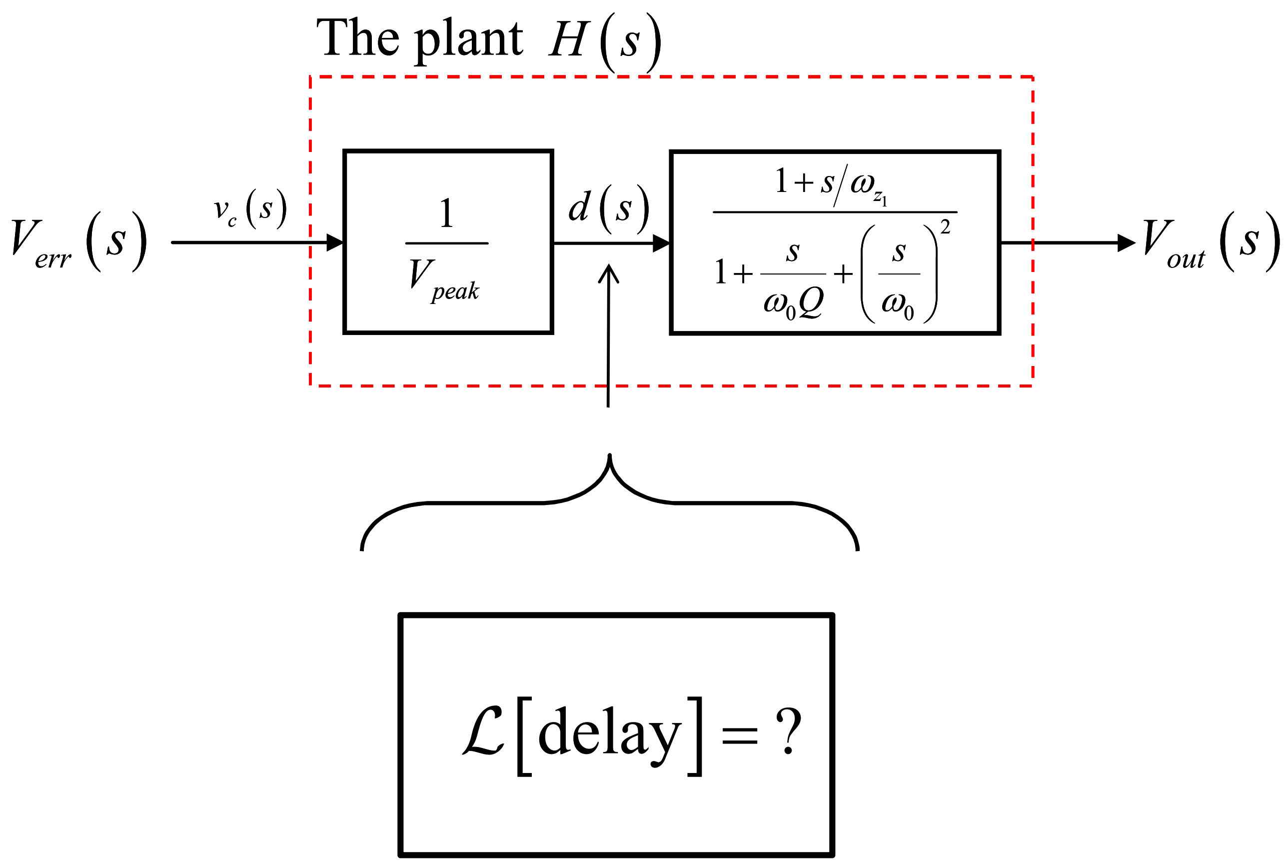


$$y(t) = u(t - \tau)$$



Deriving the Delay

- To account for the delay, we need its Laplace expression

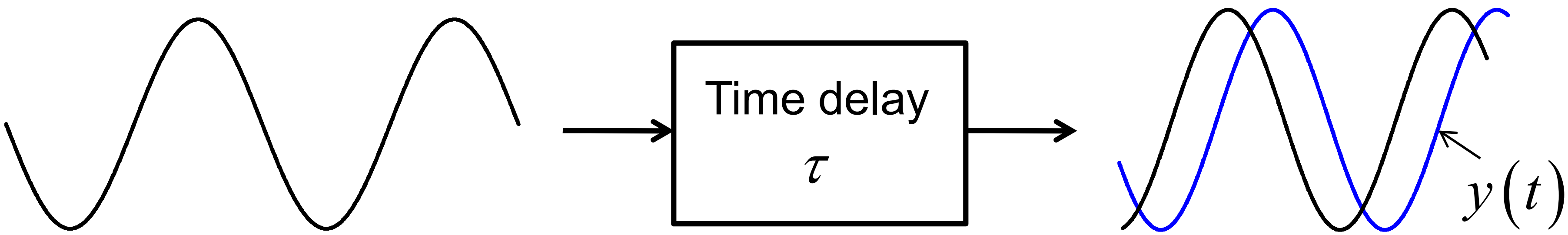


Deriving the Delay

- To account for the delay, we need its Laplace expression

$$\mathcal{L}\left[y(t)\right]=\mathcal{L}\left[u(t-\tau)\right] \Rightarrow ?$$

- Let's start with a sinewave phasor expression



$$u(t)=Ae^{j\omega t}$$

$$y(t)=Ae^{j\omega(t-\tau)}$$

$$e^{a}e^{b}=e^{a+b} \Rightarrow y(t)=Ae^{j\omega(t-\tau)}=Ae^{j\omega t}e^{-j\omega\tau}=u(t)e^{-j\omega\tau}$$

Deriving the Delay

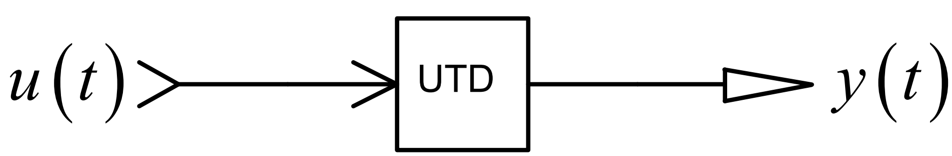
- Let's take the Laplace transform of the new expression

$$Y(s) = U(s)e^{-s\tau} \quad \Longrightarrow \quad \frac{Y(s)}{U(s)} = \boxed{e^{-s\tau}} \quad \xrightarrow{s = j\omega} \quad \frac{Y(j\omega)}{U(j\omega)} = e^{-j\omega\tau}$$

- Euler phasor formula uses the argument as the exponent

$$e^{-j\omega\tau} \to e^{j\varphi} \left. \begin{matrix} \arg e^{-j\omega\tau} = -\omega\tau \\ \left| e^{-j\omega\tau} \right| = 1 \end{matrix} \right\}$$

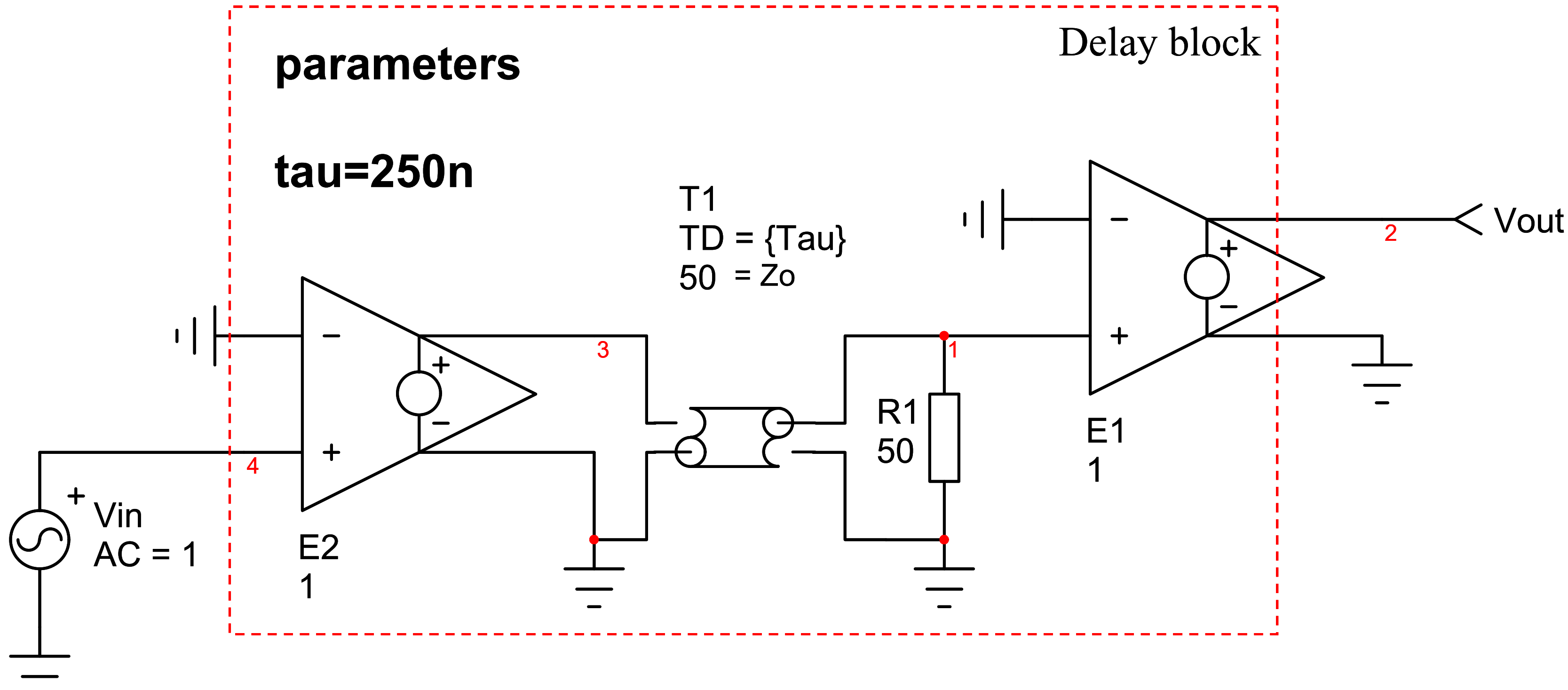
- A delay block is a simple delay line!



K = 1, t_d = 250 ns

Building the Delay

- A delay line can time-shift the input signal

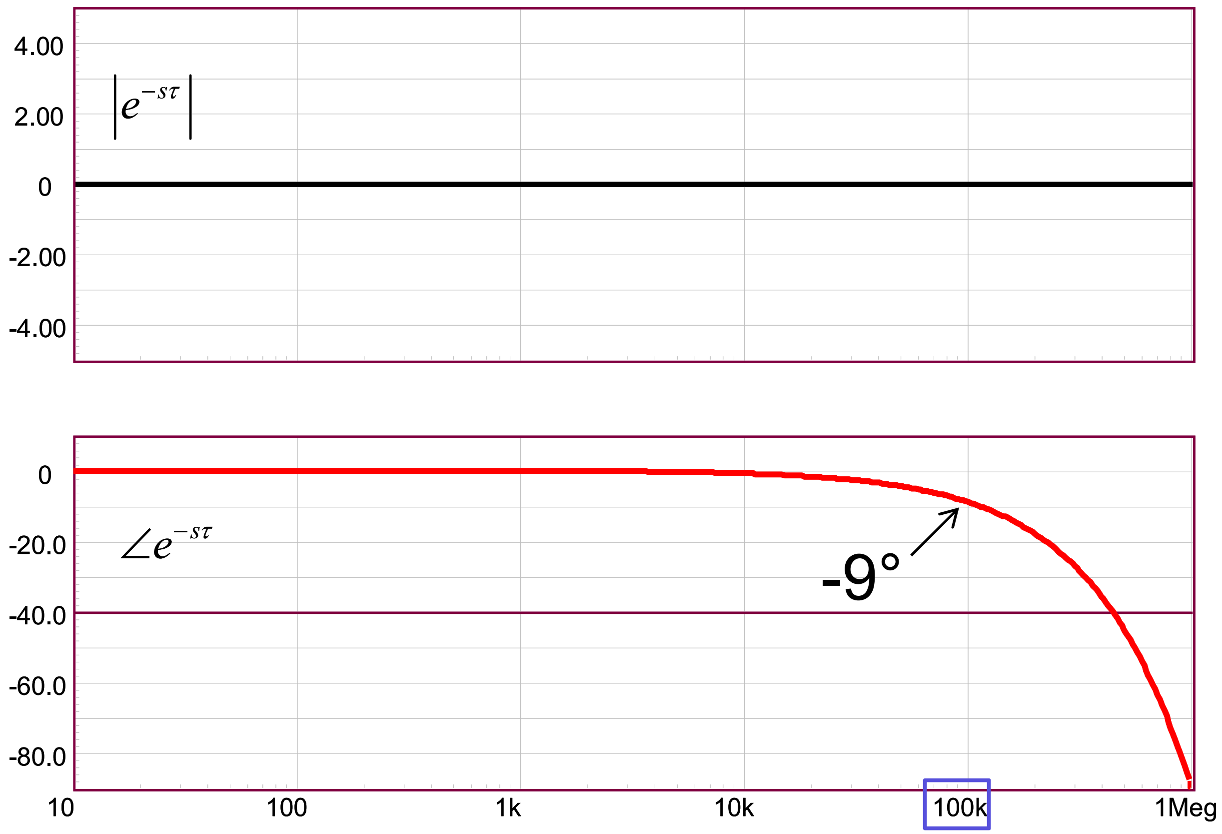


- Best simulation practice is to buffer the input and the output

D. Adar, S. Ben-Yaakov, "Generic Average Modeling and Simulation of Discrete Controllers ", APEC Anaheim, 2001

Building the Delay

❑ A Bode plot confirms the 0-dB magnitude over frequency

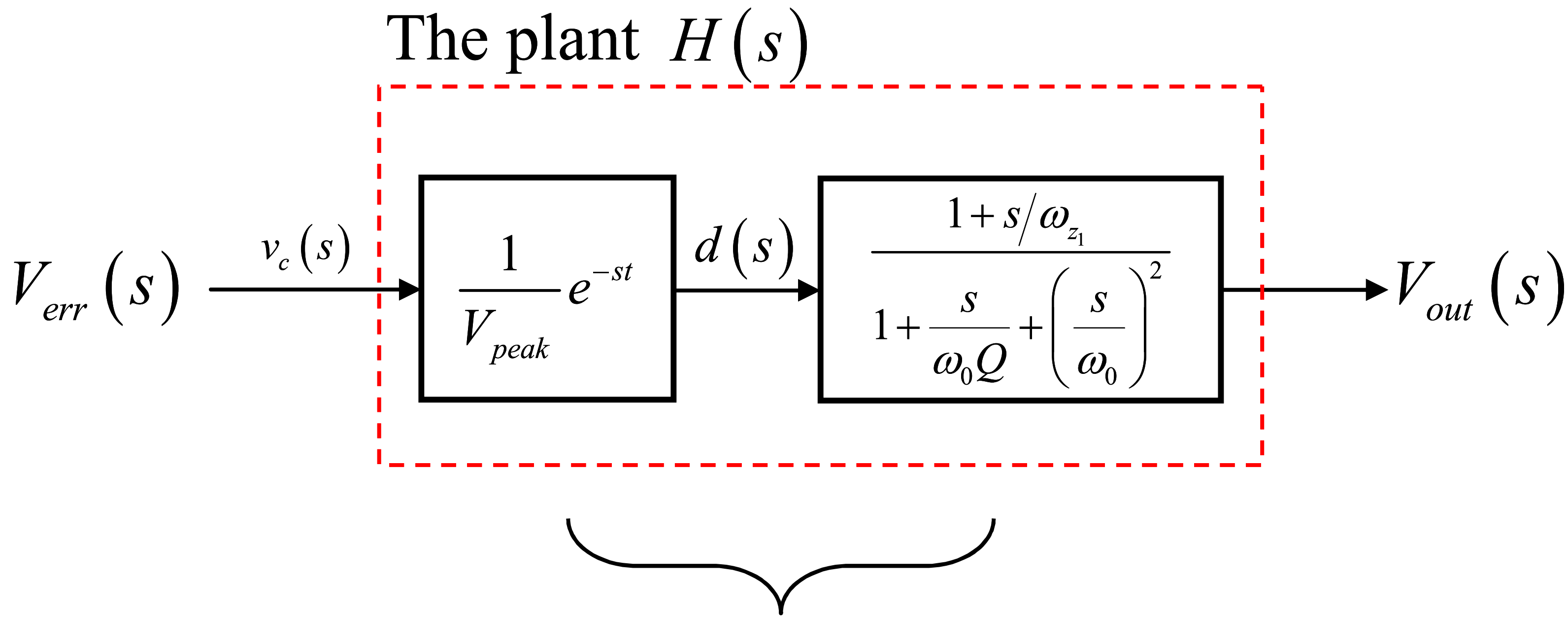


Magnitude is flat

$$\varphi_{100\,\text{kHz}} = -\omega\tau = -100k \times 6.28 \times 250n \times \frac{180}{\pi} = -9°$$

Adding the Delay in the Laplace Domain

- ❑ We can now update our transmission chain with the delay



$$H(s)=\frac{e^{-s\tau}}{V_{peak}}\frac{1+s/\omega_{z_1}}{1+\frac{s}{\omega_0 Q}+\left(\frac{s}{\omega_0}\right)^2}$$

Adding the Delay in the Laplace Domain

- ❑ How do you deal with the term e^{-st} in the transfer function?
- ❑ You don't: replace it with a poles/zeros combination!
  - ➢ A pole will bring phase shift as frequency increases

$$\frac{1}{1+s/\omega_\tau} \Rightarrow \arg = -\tan^{-1}\left(\frac{\omega}{\omega_\tau}\right)$$

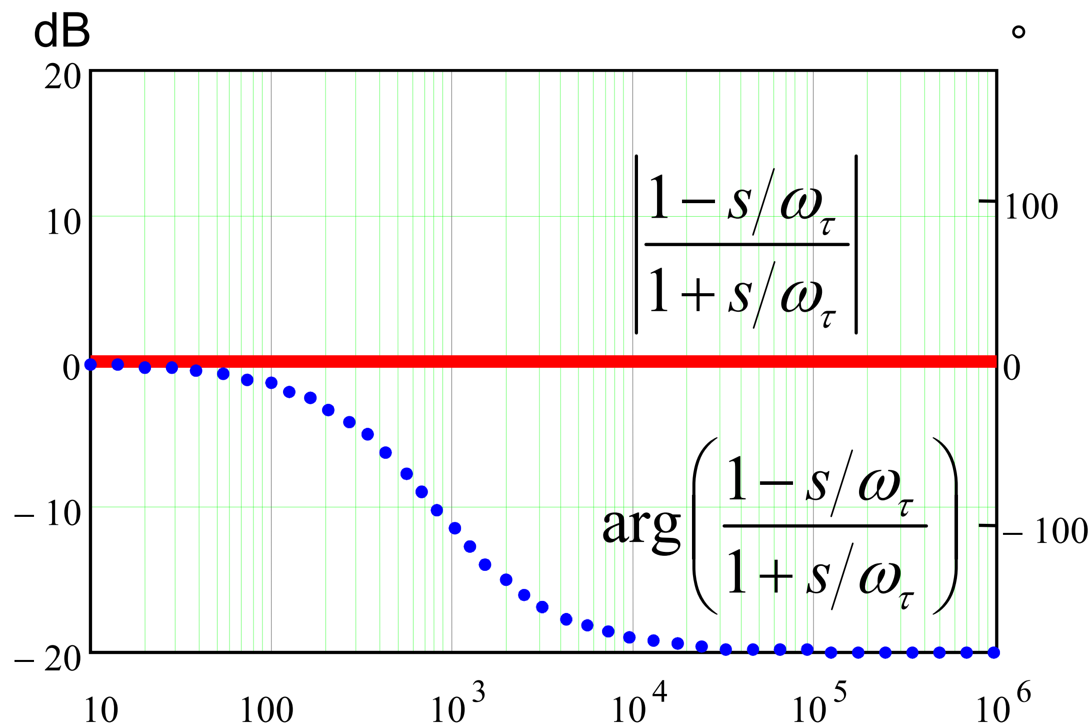
- ❑ But you still need to compensate the transmittance decrease
  - ➢ A zero will do but now, all is neutralized!

$$\left.\frac{1+s/\omega_\tau}{1+s/\omega_\tau}\right\} \quad \text{mag} = \frac{\sqrt{1+\left(\frac{\omega}{\omega_\tau}\right)^2}}{\sqrt{1+\left(\frac{\omega}{\omega_\tau}\right)^2}} = 1 \qquad \arg = \tan^{-1}\left(\frac{\omega}{\omega_\tau}\right) - \tan^{-1}\left(\frac{\omega}{\omega_\tau}\right) = 0$$

Calling the RHP Zero for Help

- ❑ Replacing the LHP zero by a RHP zero does the job
  - ❖ Both pole and zero magnitude neutralize each other
  - ❖ The RHPZ phase lags and cumulates with that of the pole

$$e^{-st} \approx \frac{1 - s/\omega_\tau}{1 + s/\omega_\tau}$$



Mapping the Delay to the Pole/Zero Position

- Both arguments must be equal:

$$\arg\left(e^{-s\tau}\right) = \arg\left(\frac{1-s/\omega_\tau}{1+s/\omega_\tau}\right) \Rightarrow -\omega\tau = \arg\left(1-s/\omega_\tau\right) - \arg\left(1+s/\omega_\tau\right)$$

- Replacing s by $j\omega$:

$$-\omega\tau = \tan^{-1}\left(-\frac{\omega}{\omega_\tau}\right) - \tan^{-1}\left(\frac{\omega}{\omega_\tau}\right) = -2\tan^{-1}\left(\frac{\omega}{\omega_\tau}\right)$$

- Use the arctangent Taylor series equivalent:

$$\tan^{-1} x \approx x - \frac{x^3}{3} + \frac{x^5}{5} + \ldots \Rightarrow -\omega\tau \approx -2\left[\frac{\omega}{\omega_\tau} - \frac{\left(\frac{\omega}{\omega_\tau}\right)^3}{3} + \frac{\left(\frac{\omega}{\omega_\tau}\right)^5}{5}\right]$$

$$-\frac{\left(\frac{\omega}{\omega_\tau}\right)^3}{3} + \frac{\left(\frac{\omega}{\omega_\tau}\right)^5}{5} \approx 0$$

We Have the Padé Approximation

- Solving for $\omega\tau$ gives us…

$$\omega_\tau = \frac{2}{\tau}$$

- Substituting $\omega\tau$ in our first expression

$$e^{-s\tau} \approx \frac{1-\frac{s\tau}{2}}{1+\frac{s\tau}{2}}$$

- This is the 1st-order Padé approximant of an exponential

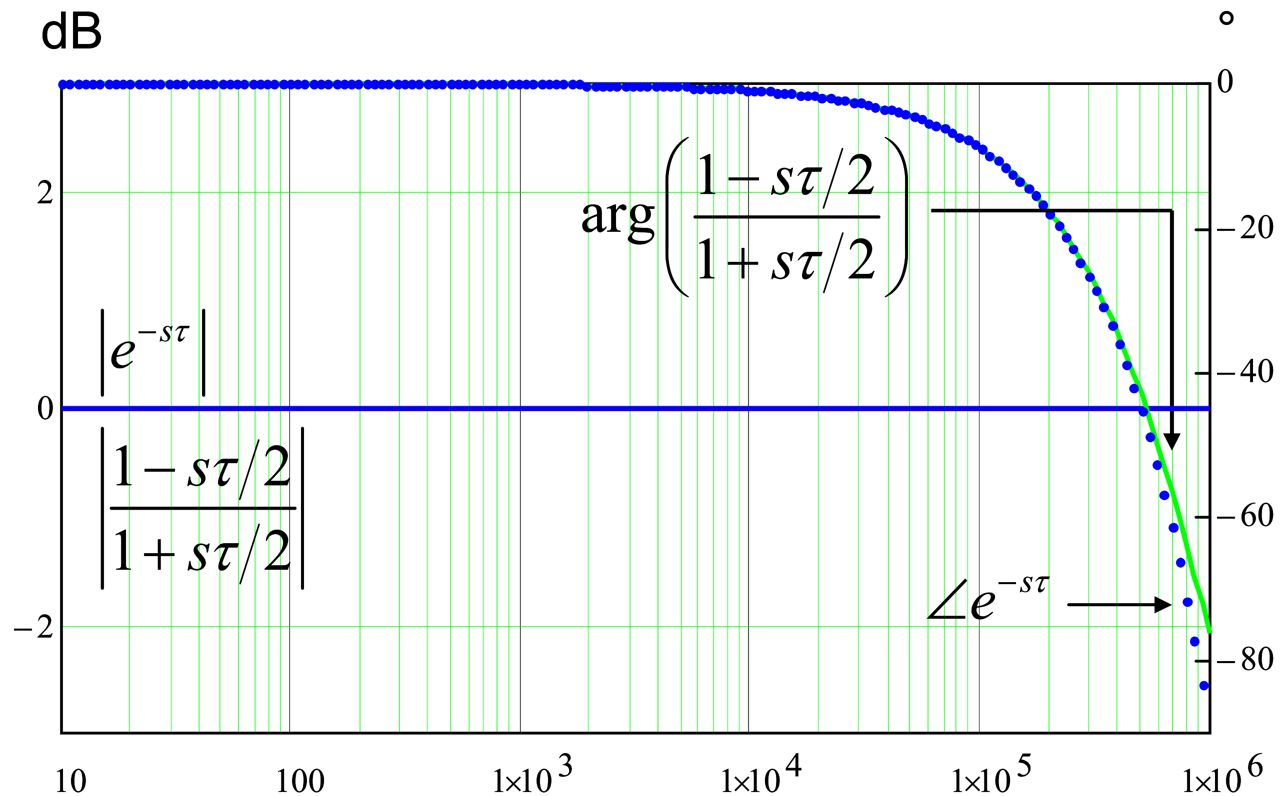
$$e^{x} \approx \frac{1+\frac{x}{2}}{1-\frac{x}{2}} \xrightarrow{\text{2nd-order}} e^{x} \approx \frac{1+\frac{1}{2}x+\frac{1}{12}x^2}{1-\frac{1}{2}x+\frac{1}{12}x^2}$$

Henri Padé
1863-1953

http://en.wikipedia.org/wiki/Padé_approximant

Padé Approximant Frequency Response

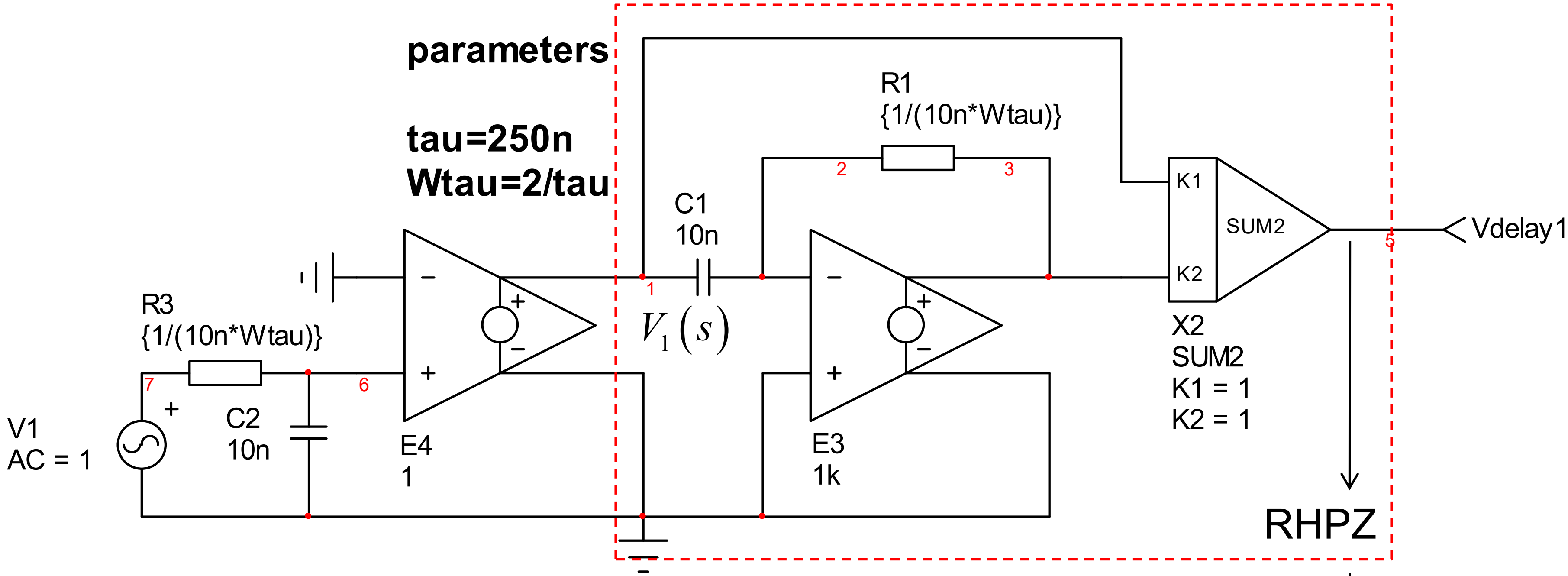
- When frequency increases, a phase deviation occurs



Use higher order approximants to improve precision

How to Avoid the Delay Line?

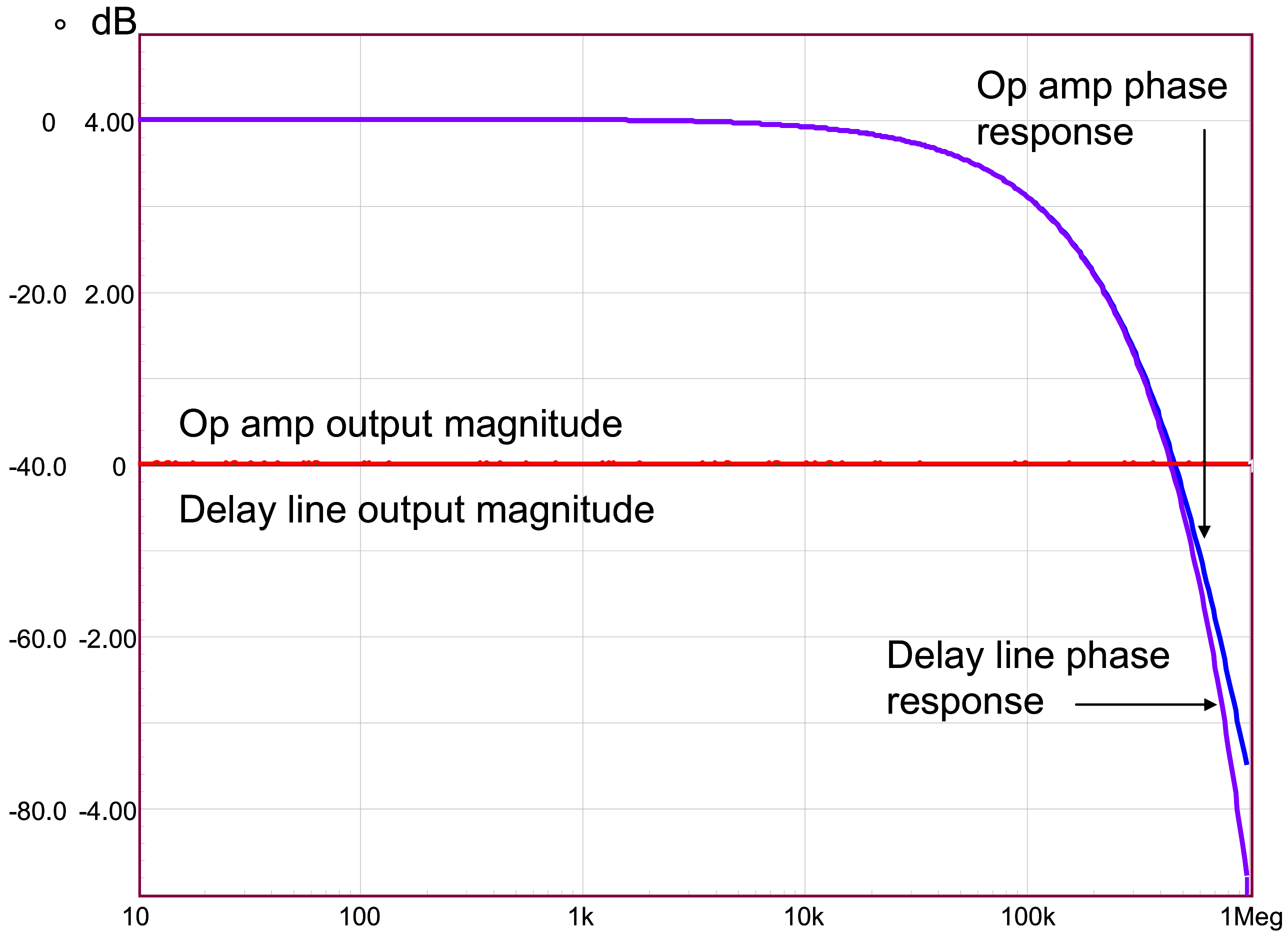
- A delay line adds computational burden in simulations
- Is there any simpler circuit that could be used?



$$V_{out}(s) = V_1(s) - V_1(s)\frac{R_1}{1/sC_1} = V_1(s)[1 - sR_1C_1]$$

How to Avoid the Delay Line?

- Good phase response of the analogue circuit



Delay Margin versus Phase Margin

- The characteristic equation is affected by the delay:

$$\chi(s) = 1 + T(s) = 0 \quad \Longrightarrow \quad \chi(s) = 1 + e^{-s\tau} T(s) = 0$$

Unity return denominator $T(s)$

- The stability condition for the magnitude does not change:

$$\left| e^{-s\tau_{\max}} T(\omega_c) \right| = 1 \quad \Longrightarrow \quad \left| e^{-s\tau} \right| = 1 \quad \Longrightarrow \quad \left| T(\omega_c) \right| = 1$$

- The stability condition for the argument does change:

$$-\pi = \arg\left(e^{-s\tau_{\max}}\right) + \arg T(\omega_c) = -\omega\tau_{\max} + \arg T(\omega_c)$$

$$\varphi_m = \pi + \arg T(\omega_c)$$

Solving for τ max

$$\tau_{\max} = \frac{\varphi_m}{\omega_c}$$

The max acceptable delay in the loop

Delay Margin versus Phase Margin

- The delay margin is thus defined by:

$$\Delta\tau = \tau_{\max} - \tau$$

← Current delay (τ)

← Maximum acceptable delay ($\tau_{\max}$)

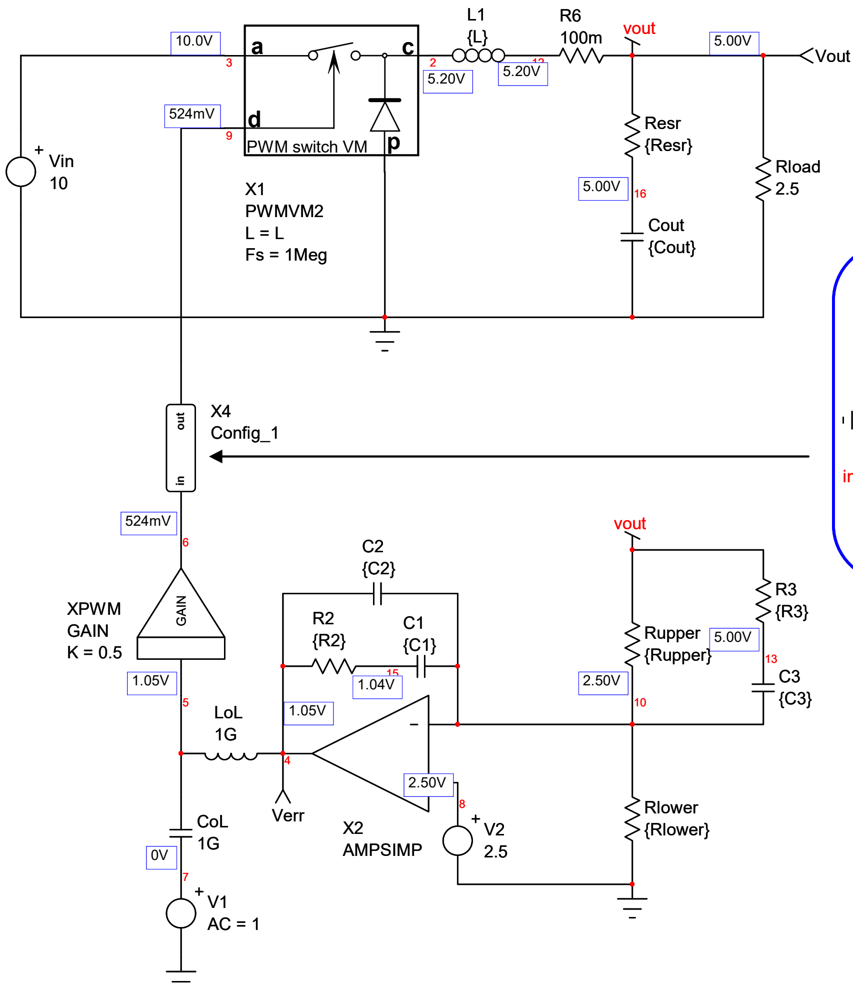
1. A buck converter features a 250-ns internal delay
2. At a 100-kHz crossover, the phase margin is 49.5°
3. The maximum delay, accounting for 250 ns, is:

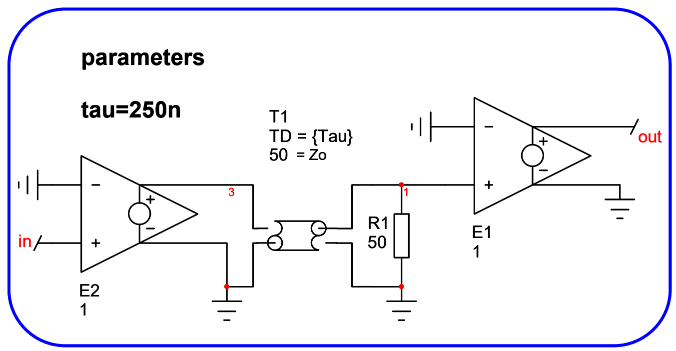
$$\tau_{\max} = \frac{\varphi_m}{\omega_c} = \frac{49.5}{2\pi \times 100k}\frac{\pi}{180} = 1.375\ \mu\text{s}$$

$$\Delta_\tau = 1.375 - 0.250 = 1.125\,\mu\text{s}$$ → Maximum acceptable extra delay

Checking via Simulation

❑ The delay is simply added in series with the PWM block

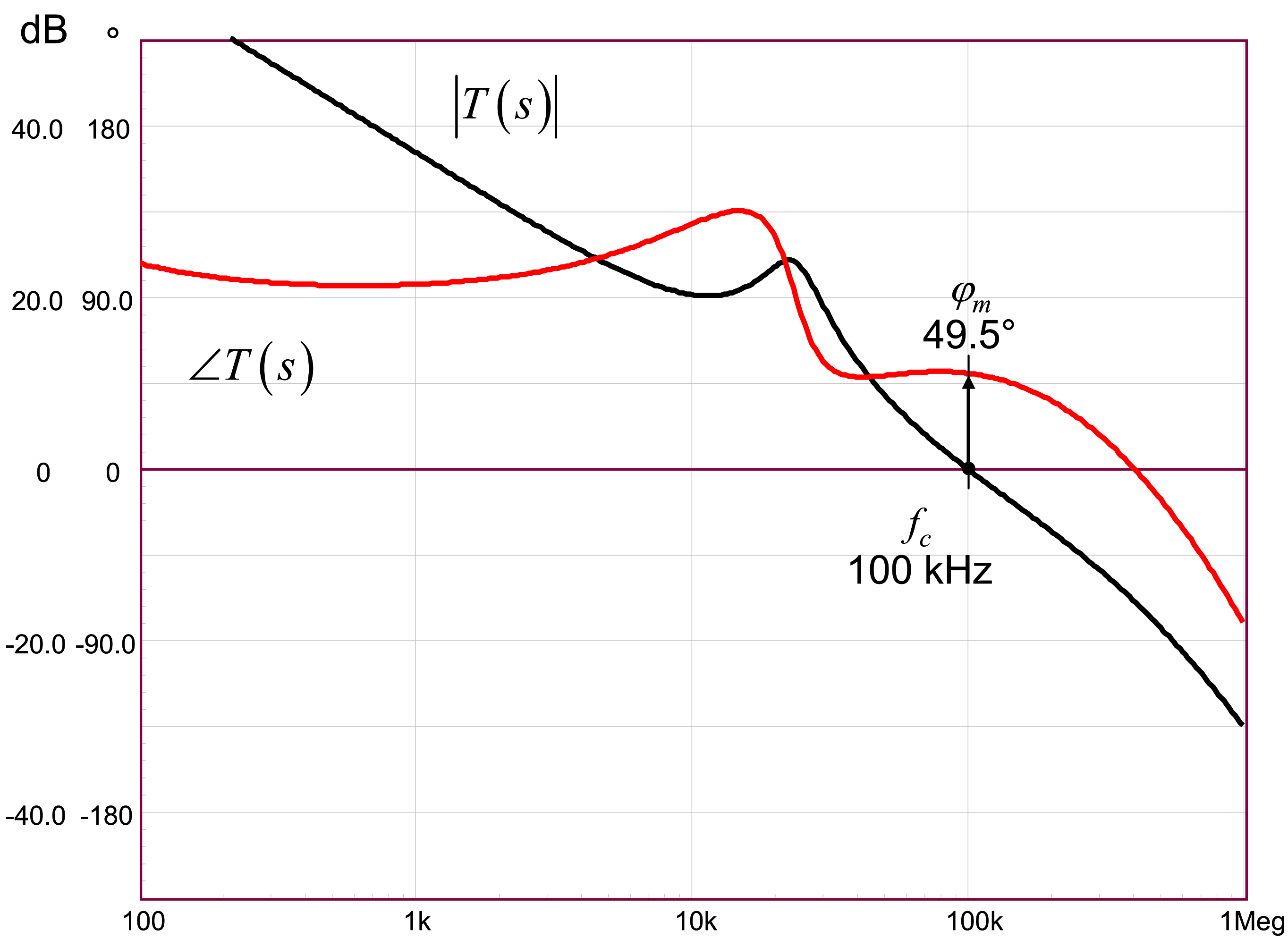




Delay built with an adapted transmission line

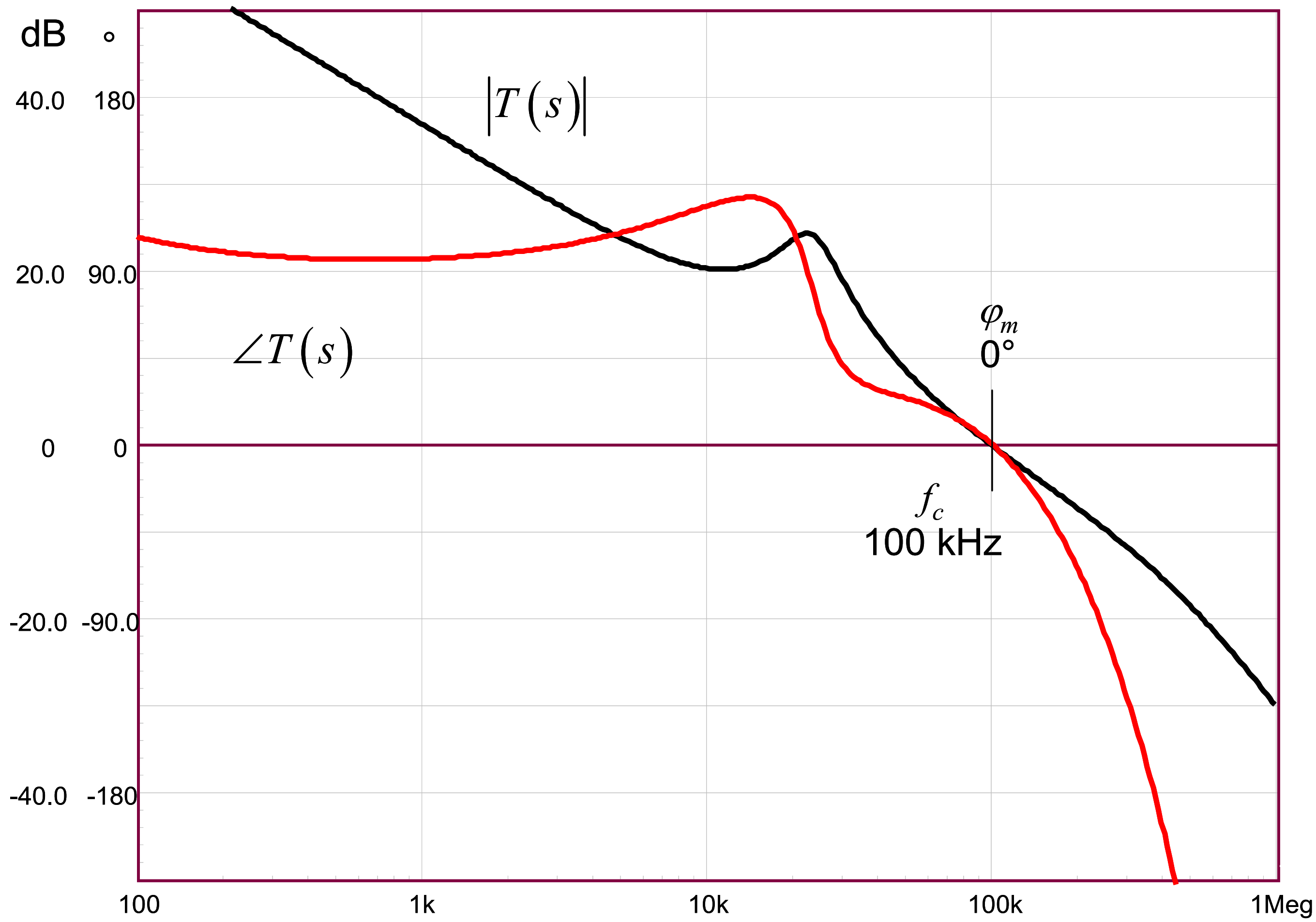
Checking via Simulation

- ❑ With a 250-ns delay, the phase margin is acceptable



Checking via Simulation

- If we add 1.125 µs to 250 ns, no phase margin at all!



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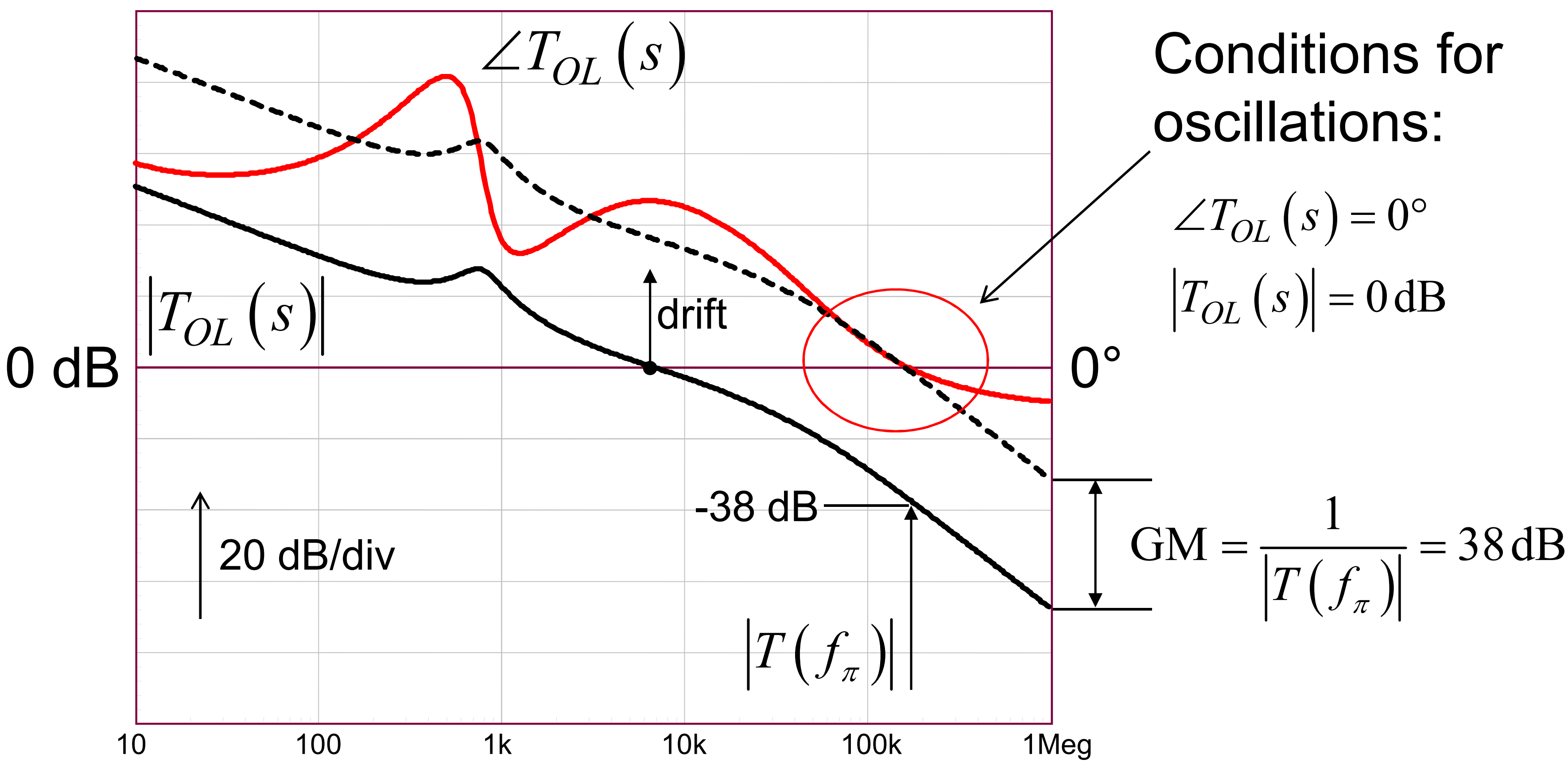
- ❑ **Gain Margin is not Enough**

Gain Margin Defines the Robustness

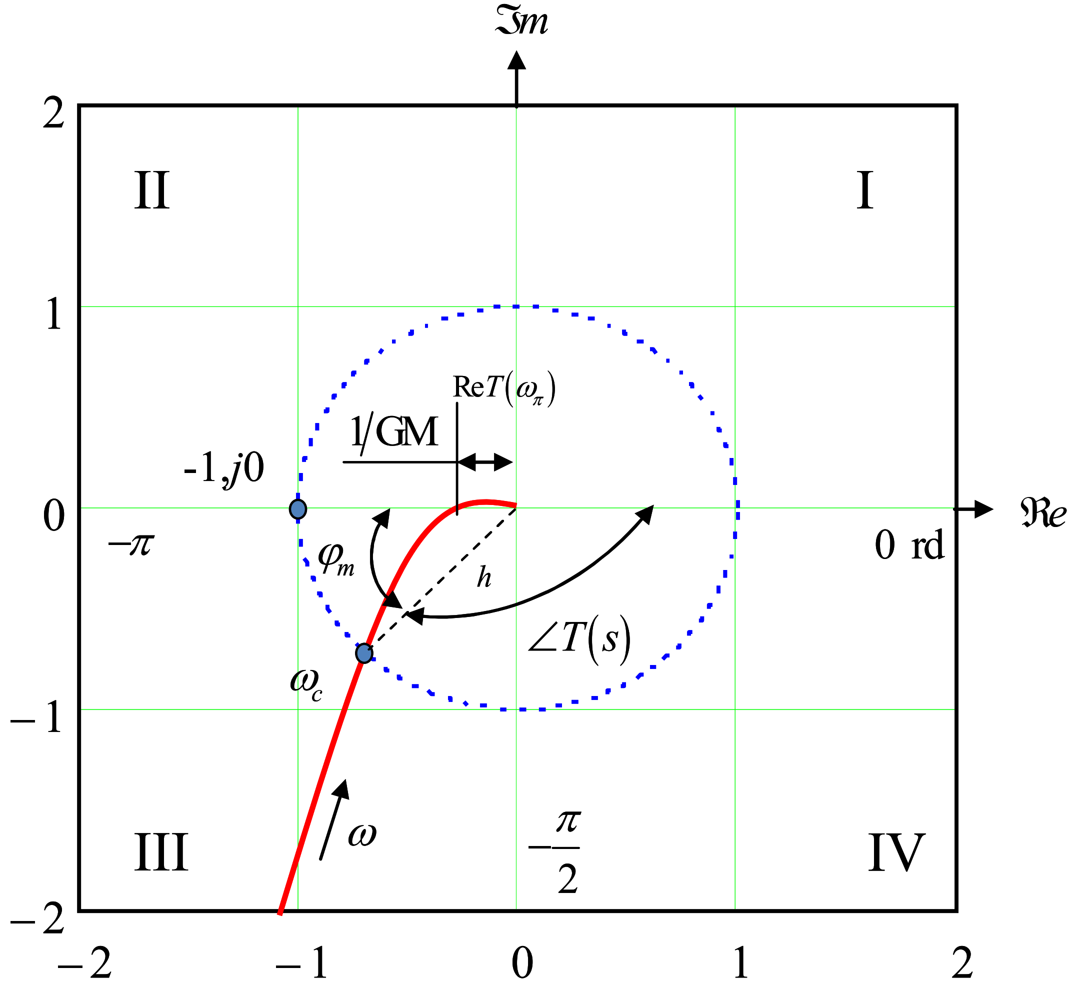
- GM defines the robustness of a system to gain variations



- If the gain drifts up by 38 dB, we have oscillations

Gain Margin Defines the Robustness

- In Bode representation but also in Nyquist



$$\angle T(s) = \pi \text{ or } -\pi$$

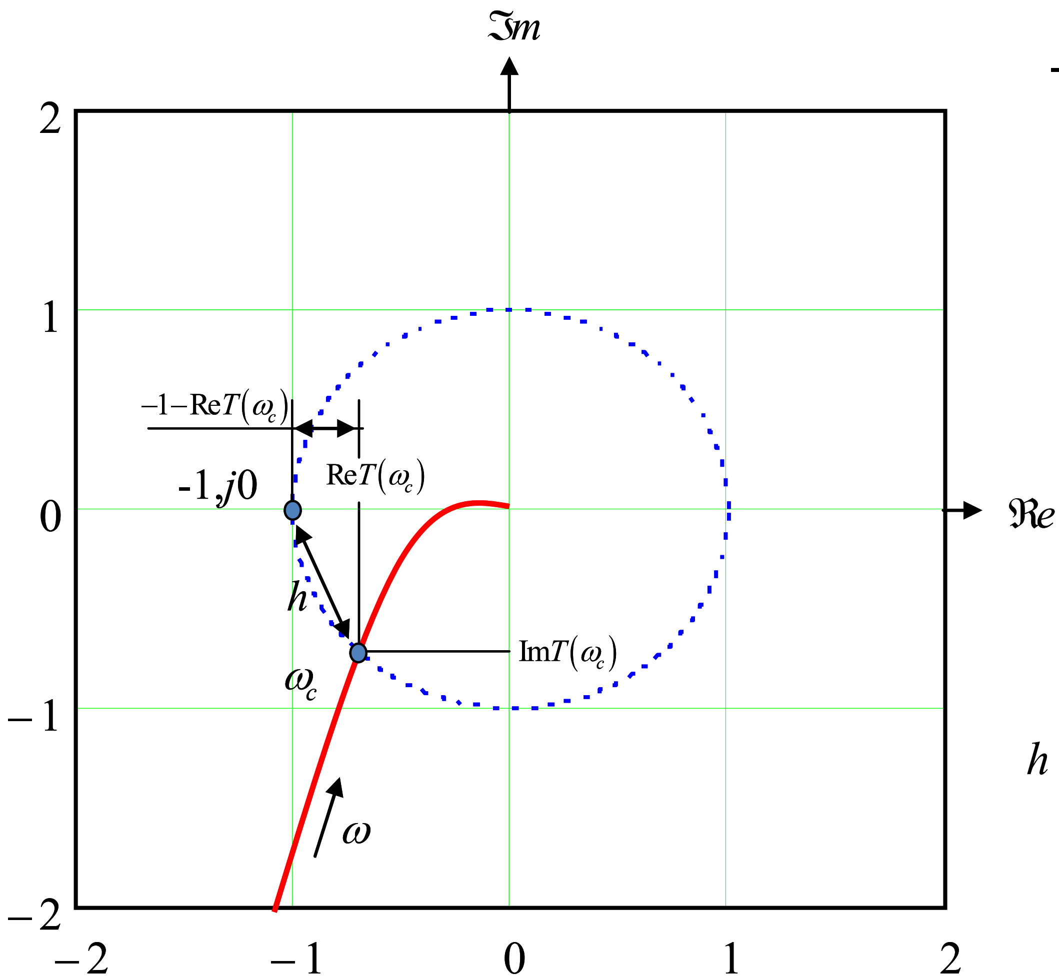
$$\operatorname{Im} T(s) = 0$$

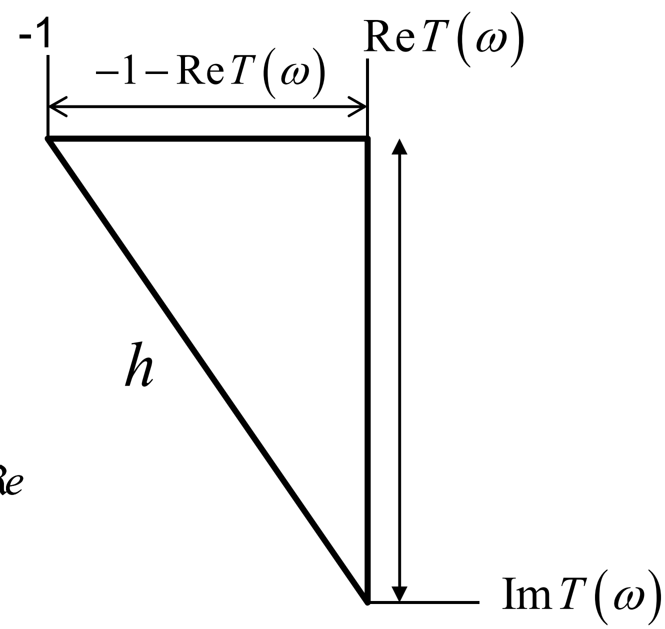
$$\mathrm{GM} = \frac{1}{\sqrt{\operatorname{Re} T(f_\pi)^2 + \operatorname{Im} T(f_\pi)^2}}$$

$$\frac{1}{\mathrm{GM}} = \operatorname{Re} T(f_\pi)$$

Gain Margin in Nyquist

- What really matters is the distance to the "-1" point



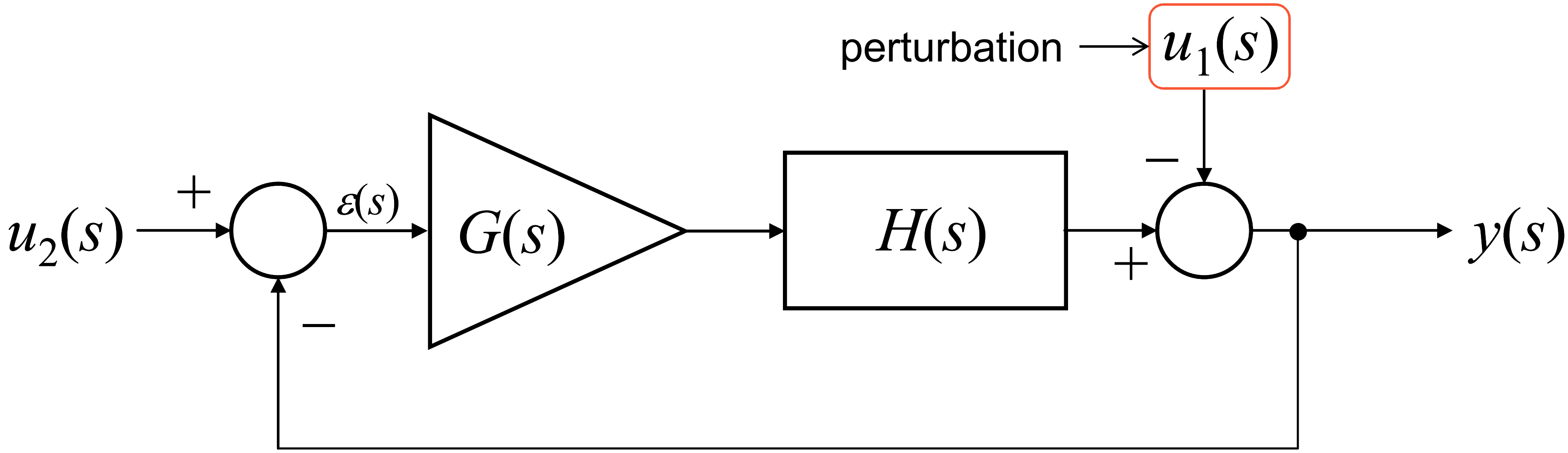


$$h=\sqrt{\left[-\left(1+\mathrm{Re}\,T(\omega)\right)\right]^2+\mathrm{Im}\,T(\omega)^2}$$

$$h=\left|1+T(s)\right|$$

Rejecting the Perturbation

- A closed-loop system rejects the incoming perturbation u_1



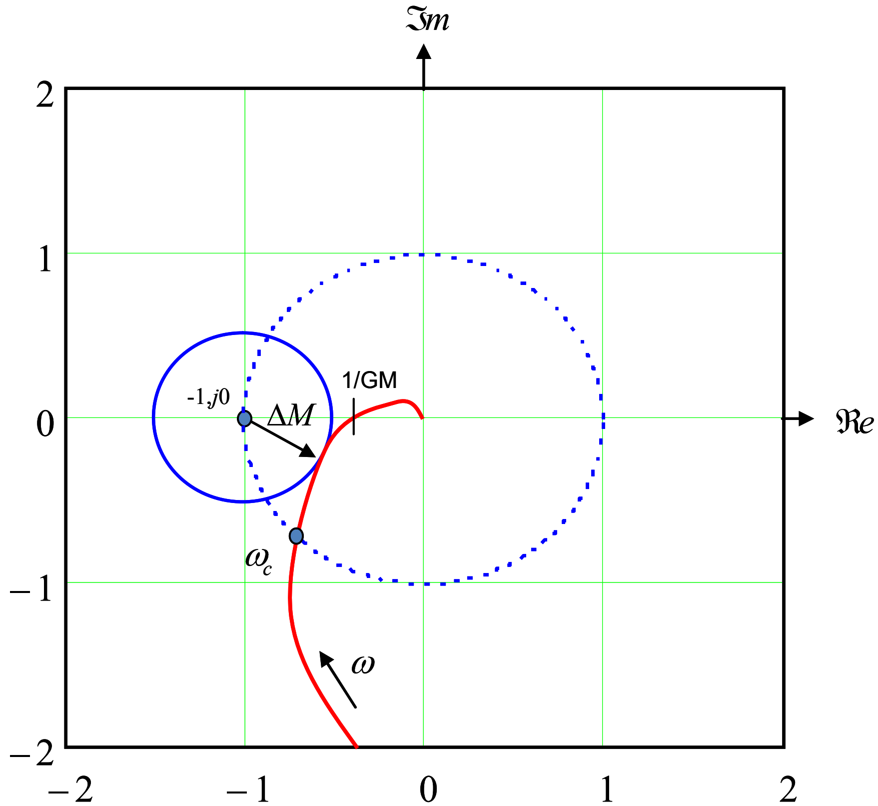
$$y(s) = u_2(s)\frac{T(s)}{1+T(s)} - u_1(s)\frac{1}{1+T(s)}$$

Sensitivity function S

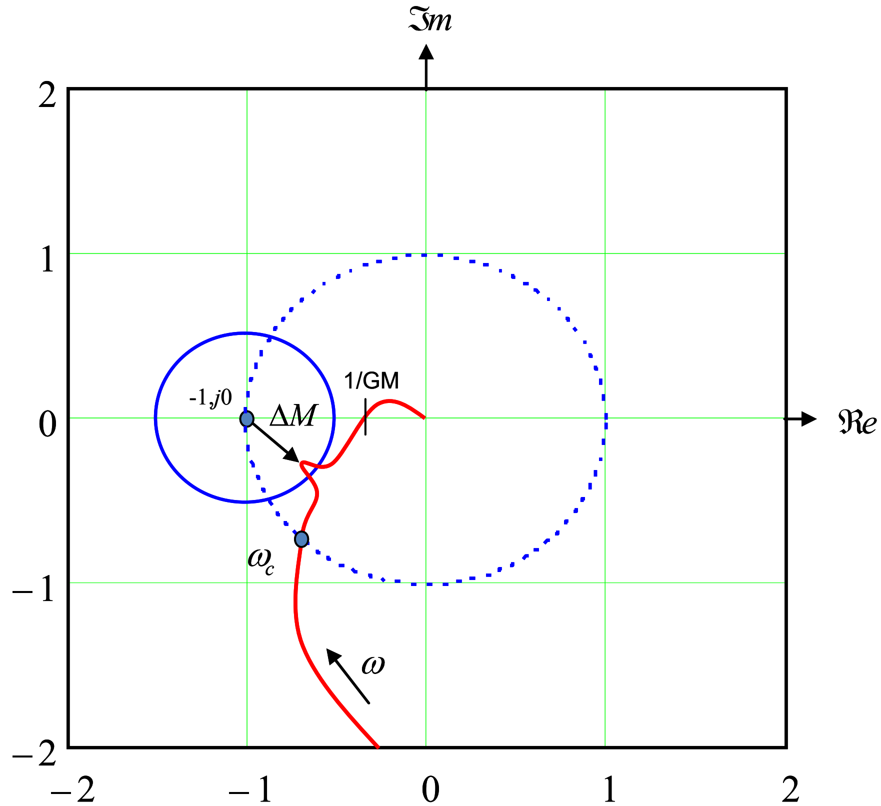
$$h = |1+T(s)| = \frac{1}{S}$$

Watch for a Peak in S

- The trajectory must keep away from the "-1" point
- A 0.5-radius circle detects a peak in S: modulus margin

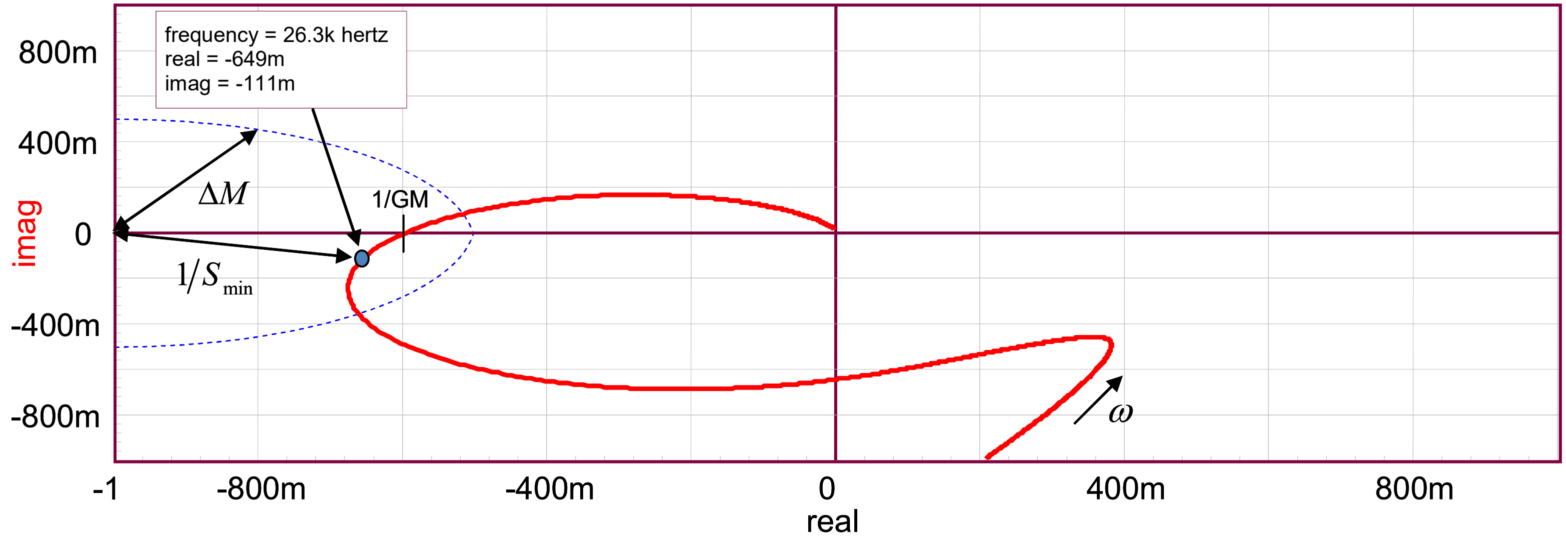


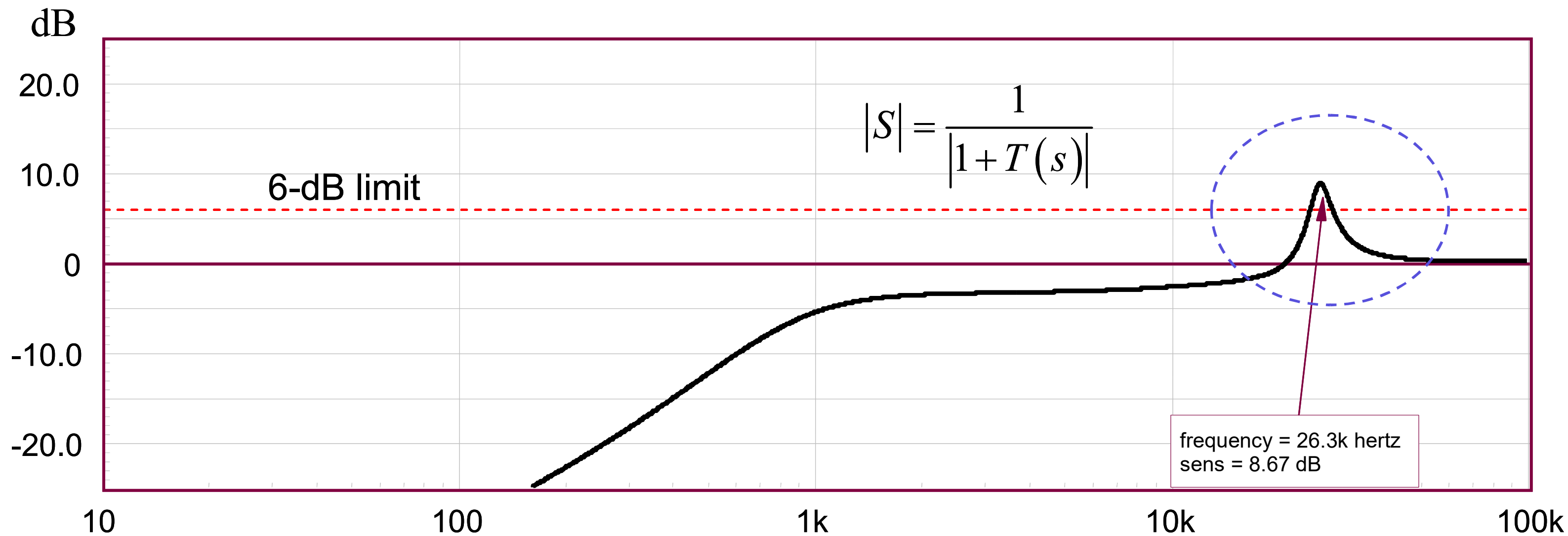
Spec limit



Margin violation

Bode can also Show the Modulus Margin





Conclusion

- ❑ Switching or linear power supplies are regulators
- ❑ Applying a pure mathematical compensation brings problems
- ❑ Engineering judgment found output impedance guilty
- ❑ Lack of sufficient gain at resonance brings oscillations
- ❑ Standard poles/zeros placement method gives good results
- ❑ For high-speed dc-dc converters, resistive shaping rules
- ❑ Q to phase margin approximation requires engineering judgment
- ❑ Less known delay and modulus margins are useful figures!