Logical Aspects of Shrub-Depth

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In the last few years there has been an increase of interest in the graph invariant called tree-depth, and its applications in algorithms and logic. Shrub-depth, introduced in 2012 [4], was developed as a dense graph counterpart to tree-depth. As such, it is related to clique-width in a similar way as tree-depth is to tree-width. On the logical side Gajarský and Hliněný [3] were able to prove that the expressive powers of FO and MSO1 coincide on graph classes of bounded shrub-depth, generalizing the result of Elberfeld, Grohe and Tantau [2]. In this work we continue the research of the logical aspects of shrub-depth. Our main result states that the concept of shrub-depth of a graph class is stable under MSO1 interpretations and transductions (more precisely, the shrub-depth value does not grow under any non-copying MSO1 transduction). From that we derive that the integer values of shrub-depth define the lower ω levels of the MSO1 transduction hierarchy of simple graphs, partially answering an open question raised by Blumensath and Courcelle [1].

Reference

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