The Self-Made Tapestry
Pattern Formation in Nature
1999
Philip Ball

1. Patterns

There is a recognized class of objects called 'dubiofossils' which are microscopic rock structures whose origin one cannot unambiguously ascribe either to organic or inorganic causes.

Bell wishes to show that pattern and organised complexity of form need not arise from something as complicated as life, but can be created by simple physical laws.

Imposters. Form alone cannot provide unambiguous evidence for the presence of biology.

Look – no hands. Nature's patterns recur in situations that appear to have noting in common.

Form and Life. The Darwinian paradigm considers form to follow the function that best equips the genes for survival. However, D'Arcy Thompson (in Growth and Form) argued that form is dictated by physical and chemical forces and that the dogma of selective forces was not all pervasive. Lamarck argued before Darwin that evolution was a response to the environment. The mapping of the Human genome is naïve in its apparent blindness of the long causal chain between genotype and phenotype. The genes only code for the proteins in a body and not the sugars, lipids, non-protein hormones, oxygen, small inorganics like nitrous-oxide and minerals like calcium. DNA must utilize some of the organizational and pattern forming phenomenon observed in inorganic systems.

Is biology just physics? Ball's views are in accord with Dawkins (including the view that genetics is not deterministic) but Goodwin takes a stronger view of the role of pattern forming systems which only seem to loose strength when they are moved from the specific to the general.

What is form? It is difficult to be mathematically rigorous about either pattern or form. Patterns involve repeating similar elements. Form is the characteristic shape of a class of objects. Patterns are typically extended in space while forms are typically bounded and finite. Symmetry and form are related but not synonymous. A circle has the highest 2D mathematical symmetry but we tend to perceive a circle as featureless (see Weyl's symmetry or Stewart and Golubitsky's Fearful symmetry). Mathematics provides precise definitions. The logarithmic spiral $r=a^{\theta}$ describes the curve of a shell or horn generated by a constant angular speed and a linearly increasing distance – i.e. a simple algorithmic growth law.

Model Making. Scientific models seek to include only that which is important and avoid irrelevant detail. Often a phenomena can be successfully modelled from more than one entirely different perspective. Numerical models enumerate the individual behaviour of a systems parts from which the system behaviour emerges. Analytical models involve mathematical expressions for the relationships between different bulk properties of the system. Models can often generate the complex patterns seen in nature from remarkably few and simple ingredients.

Breaking the monotony. Mathematical symmetry is defined in terms of the maximum number of transformations possible without changing the situation. A spherical soap bubble of perfect symmetry is a result of the random motion of a very large number of particles (i.e. an average). Total randomness engenders perfect symmetry. Patterns and forms that we tend to recognize as such are the result of symmetry breaking. However, the symmetry of a pattern formed by a symmetry-breaking force does not always reflect the symmetry of the force. (example: hexagonal convection cells in an evenly heated pan).

2. Bubbles

Darwin declared that honeycombs were hexagonal latices because that was the most efficient use of the wax. D'Arcy Thompson disagreed.

Water's skin. Bubbles form a hexagonal lattice. Water droplets form spheres because of surface tension which results from a net inwards attraction on the molecules at the surface. The surface molecules are more energetic than the others in the bulk of the liquid. A spherical water droplet minimizes the surface area and hence the energy of the droplet and is hence an equilibrium state. The different faces of a crystal also have a surface tension which determines which face grows more quickly. A cylindrical column of liquid will separate into droplets. Pattern forming instabilities involve symmetry breaking and a characteristic wavelength.

Balloon games. Adding soap to water assists bubble formation by reducing the surface tension by preferentially occupying the surface positions. A bubbles size is determined by a balance between the surface tension and the internal pressure. Bubble formed on a frame adopt an exclusively tensile shape.

A good head. When a foam is formed, the excess liquid drains out and then a coarsening process begins. A wet foam has roughly spherical bubbles packed haphazardly. As the foam dries the walls bend to balance the pressure between adjacent bubbles. The smaller bubble is always convex due to their higher pressure. In equilibrium, three walls meet at 120° in a 2-D packing and 109.5° in a 3-D packing. Hence 2-D packing is hexagonal but there is no regular 3-D packing - the beta-tetrakaidecahdron is close but the Weaire and Phelan identifies a repeat unit made up of 8 cells works better.

Face to face. Bees have to marry 2 2-D stacks of hexagons back to back. There are two stable solutions, one is favoured with wetter foams and the other with drier foams.

Curved spaces. Cell membranes are composed of a double layer of lipids. The equilibrium topology of these 3-D structure can take on a number of configurations which are seen in the shape of blood vessels, a bud to be pinched off, a starfish, or even a doughnut under various conditions.

Bubbles in flatland. Surfactant films on water – Langmuir films, exhibit four phases as the density changes – one each are analogous to a solid and a gas but two are analogous to a liquid (LC and LE). A wide array of patterns can be formed reminiscent of crystals snowflakes, rods, worms etc.

The plumbers nightmare. When the density of the surfactants are increased sufficiently they are forced into the body of the liquid water where they aggregate into a bewildering array of different structures. As density increases; sausages, hexagonal logs, flat bilayer sheets, and sheets joined by pores. The pores between adjacent sheets segment the entire volume into two isolated volumes (sponge). As the number of pores increase their spacing becomes regular – a tubular crystal. These phases have a *periodic minimal surface* with *zero mean curvature*.

Making the least of things. Tetrahedron spanning films can be stacked together to for bi-continuous periodic labyrinths – the P surface. There are also periodic D-surfaces and G-surfaces.

Cells get cubic. Bi-continuous network are found extensively in cells – more commonly in evolutionarily older cells. Nature has clearly found many uses for these spontaneously formed structures which cram a lot of surface area into a small space.

Fossil foams. Many organisms use membranes as scaffolds for erecting stronger, more rigid superstructures with fantastic architectures. At times these regular structures have been mistaken for inorganic origins.

Test-tube skeletons. There may be engineering applications to natural processes which result in patterns formed from very impure, messy chemical mixtures.

A poor mix. Co-polymers contain more than one sort of monomeric unit and in block co-polymers these different units alternate in blocks along the chain. Block co-polymers have a propensity to form patterns when the two types of polymer do not mix well together and the patterns emerge spontaneously from this delicate interplay of forces.

What do bees know? As it turns out, D'Arcy Thompson was wrong with regard to the formation of the bees' honeycomb. It is not constructed by the spontaneous formation of bubbles in wax at held at the correct temperature. Bees do it the hard way!

3. Waves

Surprisingly, a mixture of chemical compounds can form stripes or beat out a pulse.

Travelling waves. Belousov was attempting to mimic some of the aspects of the metabolic biochemical process glycolysis when he found that the mixture oscillated between two colours with some regularity. His results were ignored because his peers assumed that the results were due to poor experimental error because the 2nd law of thermodynamics would have to be violated to allow a reaction to reverse spontaneously. The effect was extended but did eventually settle down to equilibrium – but there were oscillations on the way. Chemistry was accustomed to only considering well mixed solutions and the endpoint (rather than the path)

Systems such as this do not deliver form for free because a continual supply of energy is required to keep the system away from equilibrium. But it is form from formlessness.

The BZ reaction has two reaction pathways. The first is auto-catalytic which accelerated rapidly (generating one colour) but soon runs low on reactants. The second pathway consumes the coloured products of the first pathway but generates some of the reactants (another colour) for the first pathway. And the process repeats. This limit cycle can be displayed by plotting the concentration of the two set of reactants against each other.

The BZ reaction can also create a pattern in space (spirals in 2D or rolls and spiral rings in 3D) by conducting the reaction in a gel where diffusion of the reactants is limited. This is a ***reaction-diffusion system***.

An ***excitable medium*** can change its state locally when some stimulus reaches a certain threshold. The medium must go through a refractory period once it has been excited during which it cannot be excited again.

There are a variety of other examples including catalytic converters.

Rock Art. Liesegang performed experiments (related to photography) with silver nitrate and potassium chromate in a gelatin gel. Liesegang bands forming in a tall column of this mixture are the result of an oscillatory precipitation at an advancing diffusion from. Similar processes in nature result in the formation of coloured bands in rocks.

Burn up. When a butane-oxygen mix is burnt the oxygen diffuses more rapidly as it it lighter. As the rate of flow is increased the initially uniform temperature flame progressively separates into hot cells (1, 5, 6, 9, 11, 14 ....). The cells are not stable but move continuously and a number of other recognizable patterns can be obtained.

Going wild. The BZ and combustion processes but change stable behaviour abruptly as the conditions are changed gradually. This is known as ***period doubling bifurcation*** when the limit cycle abruptly gains additional loops. As conditions are changed gradually the bifurcations become more closely spaced until the systems becomes chaotic with no discernible periodic behaviour.

Rhythms of life. Heart attacks are an example of a periodic system becoming chaotic. A heart beat propagates electrically through the whole heart in a periodic fashion with a refractory period – an excitable medium. Spiral waves can be set up and observed in a piece of heart tissue and even in a whole heart.

Microbial crosstalk. Bacteria generally operate in isolation but when the environmental conditions become difficult they can clump together. The bacterial emit a chemoattractant which stimulates movement towards the source (also a mechanism in brain development). Slime mould is an example which clumps and forms a fruiting head which produces resilient spores. This chemoattractant mechanism can set up an excitable medium.

Bacterial black holes. Colonies of growing bacteria can form a variety of radial and concentric patterns. As the colony expands out from a central point in a wave-front, the food becomes scares due to local over population and the bacteria begin to emit a chemoattractant. Any small irregularity in the ring will cause the symmetry to break and a clump will form setting off a domino effect around the ring. If there is also a repulsive effect between clumps then they form along radial lines. Th clumps loosely represent a singularity dubbed 'chemotactic collapse' where the positive feedback of the attraction maximizes.

In the beginning. Chemical waves can be observed on frogs eggs. Spiral galaxies may be another example.

4 Bodies

“Natural selection ..... is a tool for retrospective rationalization, and rarely if ever for prediction.” pg 78

The surface markings on pelts and shells are immediately recognizable as patterns (with a potential for survival value) but body plans might also be considered as biological patterns, albeit very complex and refined ones, that derive from the sequential and hierarchical sub-patterning of simpler patterns.

Frozen waves. "Turing's ultimate dream was to make a thinking machine, an artificial brain. His interest in brain structure and development led him to ponder on broader questions of biological development, and ultimately to the issue of morphogenesis. In 1952 he published a paper describing a hypothetical chemical reaction that could generate spontaneous symmetry breaking, leading to stable spatial patterns, in an initially uniform mixture of chemical compounds. This, he suggested, might provide a model for how patterning takes place in an initially spherical fertilized egg. Entitled 'The chemical basis of morphogenesis', this paper is undoubtedly one of the most influential in the whole of theoretical biology." pg 79

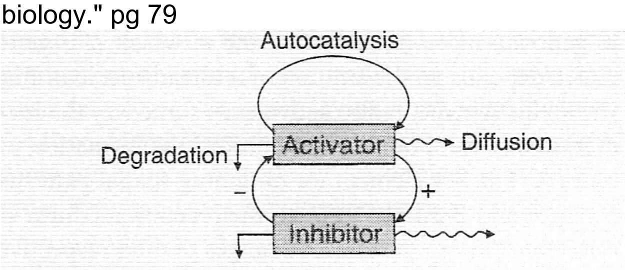


Fig. 4.2 How an activator–inhibitor scheme works. The activator generates more of itself by autocatalysis, and also activates the inhibitor. The inhibitor disrupts the autocatalytic formation of the activator. Meanwhile, the two substances diffuse through the system at different rates, with the inhibitor migrating faster.

Short ranged activation and long range inhibition are the principal elements of Turing's patterns. Turing's patterns are maintained – the symmetry is broken – only so long as the system is driven away from equilibrium. They arise spontaneously from a homogeneous medium in a symmetry-breaking instability as the driving force away from equilibrium is increased.

Making striped paint. The mechanism that gives rise to BZ chemical waves is not the same as the instability that leads to Turing waves; the Former are travelling while the latter are stationary.

A hypothetical version of Turing's mechanism was proposed in 1968 and 1974. A practical reaction was demonstrated in 1990 by De Kepper and over large areas by Ouyang and Swinney in 1991. The patterns changed abruptly with gradual changes in experimental conditions. Both periodic and disordered patterns are possible (spots and stripes) and there is a more or less uniform separation between the features.

Skin Deep. Turing's mechanism is attractive in an evolutionary sense as it economizes on the amount of (genetic) information needed to produce the pattern to just the blueprint for making the activator and inhibitor substances at the right stage in development.

The form of the pattern is constrained by the shape of the area on which it is forming so Murray studied tail patterns which are spotted or striped. He found that the dimensions of the tail in the embryo determined the characteristics of the patterns. He also successfully modelled the transition where the zebra's leg joins the body. A more sophisticated model was required to model the giraffes blotches with travelling activator and inhibitor.

Shell stripes that run along or around the growth axis, while superficially similar, are in fact frozen time histories of quantitatively different patterning processes: one in which a spatially periodic pattern along the growing edge remains in place as the shell grows, and the other in which bursts of pigmentation occur uniformly along the entire growth edge followed by periods of growth without pigmentation. Radial stripes add new stripes to maintain the distance between stripes. Environmental interruptions to growth can cause patterns to halt and restart.

The striped patterns on an angel fish are not fixed at an early stage of development but stay more or less the same as the fish grows. When the stripes get to wide it will be split or unzipped by a new stripe indication that the stripes are a ongoing dynamic process.

The patterning on butterfly and moth wings is established during pupation. These patterns can be broadly classified by the nymphalid ground plan which depicts the most common basic elements observed from which a huge number of real patterns can be derived by selecting omitting or distorting the individual elements. The pattern can be strongly modified by the pattern of veins. This classification of pattern elements helps immeasurably because it enables researchers to focus on handful of basic symmetry systems before worrying about how they have been distorted in a particular instance.

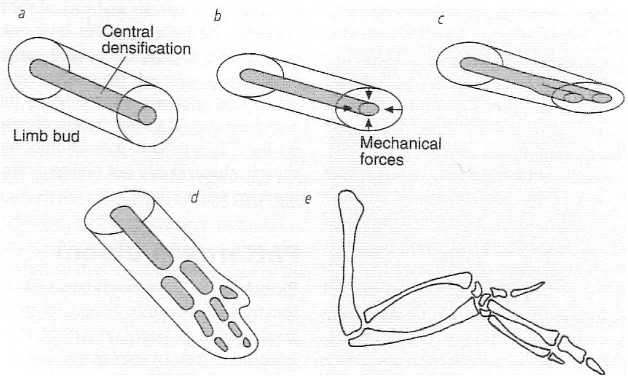
The nymphalid ground plan provides a kind of template onto which all actual patterns can be mapped, so that the underlying nature of the pattern elements can be discerned. Then this pattern is regarded as an assembly of autonomous wing cells, each of which is itself a collection of pattern elements such as stripes and eye spots which are induced by 'organizing centres', sources and sinks of morphogens. The morphogens are assumed to diffuse through the wing cell, throwing biochemical switches where they surpass some critical threshold. And these organizing centres can lie only at the wing cell midpoints or at their edges (at veins or wing-tips). The sources and sinks are placed by an activator-inhibitor scheme.

And natural selection has driven this whole flexible scheme to mimic other poisonous butterflies or leaves.

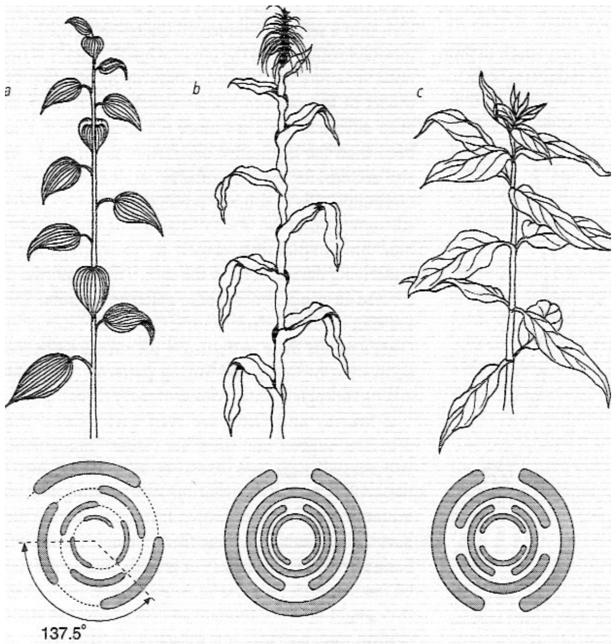
Written on the body. The reference grid of a fertilized egg is apparently painted by diffusing chemicals. Each of the chemical morphogenes has a limited potential by itself to structure an egg, but several of them, launched from different sources provide a criss-cross of diffusional gradients. They suffice to break the symmetry of the egg and to sketch out the fundamentals of the body plan.

The most extensively studied of development systems , the fruit fly, Drosophila melanogaster is unusual because it does not become compartmentalized into many cells separated by membranes until there is some 6000 nuclei in the egg and the body plan is already laid out. Most other multicellular animals develop cell by cell which presents a barrier to the simple diffusion of a morphogene. The initial positioning of this structure in the anterior of the fruit fly egg occurs prior to fertilization of the egg. No Turing mechanism is involved in mapping out the subsequent symmetry breaking patterns.

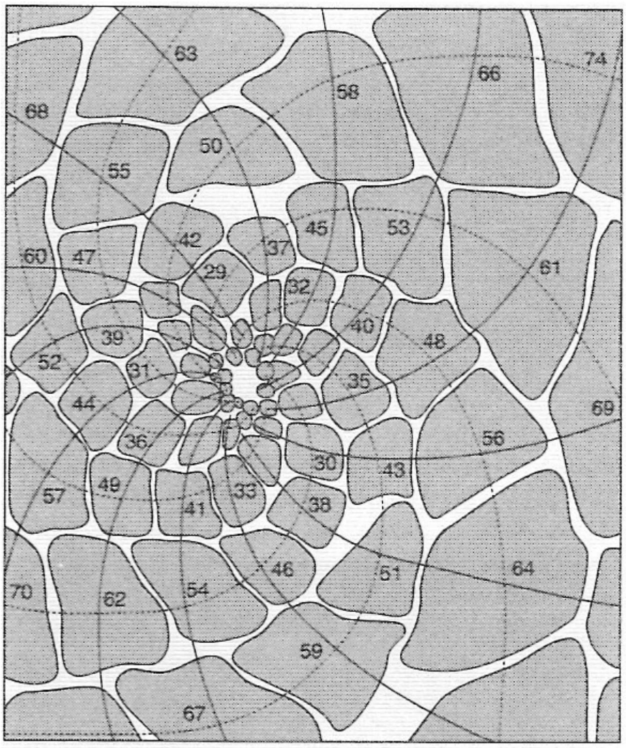
Murray and Oster have postulated a model for structuring and patterning of body plan at a relatively late stage of development involving spontaneous instabilities similar to chemical Turing patterns. Their mechanochemical model of morphogenesis involves a cylindrical developing bone, becoming unstable and flattening into an oval until it separates longitudinally into parallel bones (bifurcated ulna and then digits), followed by further latitudinal instabilities to separate the bones at the joints. This basic body plan is see across a diverse range of large animals.



Patterns in bloom. Three distinct patterns can be identified in the arrangement of leaves around plant stems (philotaxis): spiral, distichous and whorled.



If a cross-section is taken from above, the positions of successive primordia trace out two systems of spirals in a spiral opposite directions. This is particularly visible when they develop into a flower head instead of leaves. The growth order of the leaves along each of the two lines are subsequent numbers in the Fibonacci sequence (3,5), (5,8) or (8,13).



And this fact carries with it all of the remarkable relations of the Fibonacci sequence.

5. Branches

We can recognize qualitatively similar branching patterns in a huge variety of different physical and organic systems.

When crystals grow slowly under a shallow gradient then the atoms can pass in and out of solution to find the lowest energy arrangement. Rapid crystal grow has a different result as each atom sticks where it first touches governed by the rate of diffusion of the particle. Wittel and Sander modelled this situation as diffusion-limited aggregation (DLA). Vincent Fleury suggested a different model

In rapid Crystal grow bumps grow into branches because there is an instability which amplifies the growth on small irregularities.

Crystals have a fractal dimension. The number of particles in a cluster grows in proportion to the area of the cluster. $N \propto r^d$ Where d is the fractal dimension. d=1.71 for 2d DLA and d=2.43 for 3d DLA.

The Mandelbrot set is deterministic,platonic, symmetrical and of the like which is not seen in nature. The Mandelbrot set and DLA clusters are both fractals because they are both scale invariant but beware, not all branched structures are fractal and being fractal is a description of what not how.

Viscous fingering occurs as a low viscosity fluid advances into a fluid with higher viscosity. The Saffman-Taylor instability describes the higher pressure gradient around a bulge that gets steeper the sharper the bulge, driving more rapid growth. The equations are analogous to those describing DLA, however, viscous fingering differs from DLA in that the fingers have a characteristic length scale defined by the average width of the fingers. The fingers define a more or less periodic wavelength around the perimeter. This is due to surface tension which is absent from DLA models. Noise or randomness can influence growth patterns in pronounced ways.

Underlying symmetry can introduce order to crystal growth such as the hexagonal snowflake, or cubic salt crystal. It is not known why the 6 arms of a snowflake so closely resemble each other or why thy are flat.

Regularly branched crystals analogous to a single snowflake are can be found in many other solidifying materials. Dendrites form when the melt cools rapidly and they form a whole range of parabolic advances caused by the speed at which the latent heat is conducted away from the solidifying tip. The rate of conduction increases where the temperature gradient is steepest which is near the tightest radius tip. The underlying anisotropy (non-equivalence of direction) of the crystal can dominate an result in fingered growth rather than branched.

The basis for the similarity between the 6 branches of a snowflake may be related to crystalline oscillation but it should be noted that the arms are only similar, not identical.

Platonic shapes like a circle conform to a static description while branching structures require an algorithmic description, either deterministic or with random elements.

Branching examples include trees and blood-vessels. Blood-vessels include closed loops which are due to the branching structure forming under the influence of attracting and repelling chemotaxis in tissue which itself is growing. Heart rate, metabolic rate and distribution system sizes scale with power laws which are a multiple of ¼ rather than the 1/3 that would be expected for a 3d volume. Bacteria colonies, despite their more active capabilities, have growth structures limited by nutrient supply and diffusion speed which resemble DLA type structures. Occasionally, a colony changes its growth pattern drastically at one single point where a successful mutation has occurred. Also, at very low nutrient levels the growth patterns change when the bacterial alter their behaviour to survive, possibly by using chemotactic repulsion.

6. Breakdowns

Cracks run because the stresses around a hole are far higher than in the surrounding material – the chemical bonds break one at a time. Ductile materials release the stress by releasing dislocations into the surrounding material.

The speed of crack growth has three stages; a rapid acceleration up to an appreciable fraction of the speed of sound in the material, a long smooth straight crack still accelerating, and finally wild fluctuation and rough edges when a critical speed is exceeded. At the critical speed the greatest stress is at right angles to the direction of propagation.

Closer inspection possible by carefully controlling the crack speed shows a transition from straight propagation to a regular sinusoidal crack where the period is determined by the width of the material. When the experiment was performed on a tube (quasi infinite width) the tube cracked into a spiral.

Cracks are fractal structures.

In dielectric breakdown (sparks) the gradient of the electric field is higher about the tips so the spark is more likely to advance from there.

Cracks in thin sheets like drying mud and semi-conductor materials have been successfully modelled with very small polystyrene balls. There is a threshold speed at which the direction of tip growth becomes unpredictable.

A river network grows in the opposite direction to the fluid flow and it can be considered to be a crack growth. Horton stated that the number of streams of order n (number of tributaries) is roughly proportional to c^{-n} but c is not really a constant ($0.5 < c < 0.6$) and Kirchner showed that almost every conceivable kind of branched network (including random) obeys Hortons laws. . [note: this may be why Mandelbrot's fractal landscape forgeries are convincing]

River networks grow fastest a the tips because that is where the rate of flow is highest; until two tips approach each other and reduce the volume. Invasion percolation describes the movement of a fluid thru a porous medium. The tip grows at the weakest pore and only rarely will tips join.

Rodriguez-Iturbe proposed that a network evolves in such a way as to minimize the total rate at which the mechanical potential energy of the water flowing thru the network is expended.

Self avoidance in river networks collapses in deltas where the processes of sediment scouring, transport and deposition dominates the development of the network.

The river networks are self-organizing into stable patterns that remain statistically static despite constant change in the detail.

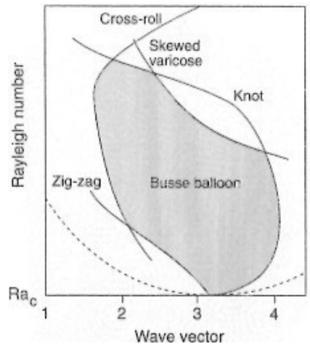
The landscape (mountains and coastlines) resulting from erosion is also fractal. Fractal landscapes are not isotropically self-similar but are instead said to be self-affine (the appropriate scaling in different directions varies)

7. Fluids

“Traditional Western artists have seldom faced up squarely to the challenge of change. It's not easy to paint something that is never still. Yet to the traditional Chinese artist, that can be the whole point of the exercise – to capture the fundamental forms of motion.” pg 165

“Turbulence is the graveyard of theories” David Ruelle.

Rayleigh investigated the motion of fluid trapped between two horizontal plates when heated from below. When fluid is heated gently the convection initially forms into rolls. When the Rayleigh number is increased beyond 1708 the convection pattern becomes bimodal (two sets of perpendicular rolls). The dimensionless Rayleigh number represents the balance between buoyancy and viscosity. The Navier-Stokes equations of fluid motion are relatively simple, but difficult to solve. Rayleigh analysis predicts the points of stability by more sophisticated analysis is required to predict the nature of the convection. In experimental conditions a wide range of patterns are influenced by boundary conditions.



Benard had earlier conducted a similar investigation but with a free surface and observed hexagonal convection cells due to the changes in surface tension with temperature with a threshold defined by the Marangoni number. Interestingly the surface tension causes the surface to deform with depressions over the up-flow.

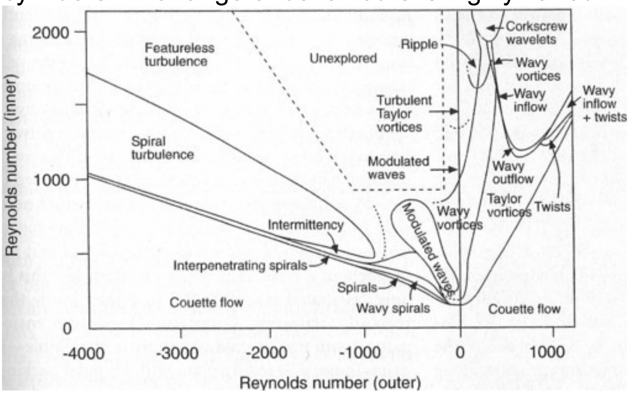
Convections in the earth's mantle appears to operate in two layers with a boundary at 660km depth where there is probably a change in the crystalline structure of the mantle material. Iceland has the dubious distinction of sitting over a mantle plume. Mantle plumes can be up to 2000km in diameter and smaller if from a shallower origin. The cold material probably sinks in sheets.

Large scale convections in the earth's atmosphere traces out three hemispheric convection cells; the Hadley cell between the equator and about 30° latitude, the Ferrel cell at mid-attitudes and the polar cell over the poles. The large scale pattern of ocean convection is called the ocean thermohaline (heat-salt) circulation. The freeze-thaw cycles in the northern tundra set up visible polygonal cells of stones on flat ground and stripes on sloping ground. The sun's surface shows convective patterns changing on a minute by minute basis.

The Reynolds number is essentially the ratio of the forces driving the flow to the forces retarding it (viscous drag). Flow around a cylinder is laminar for Re<4, laminar with two trailing eddies for 4<Re<40, vortex shedding commences and the wake acquires a wavy nature (Re=40), which crest (Re=50), and break up downstream (Re=200) and become fully turbulent up to Re=300000 when the boundary layer around the cylinder also becomes turbulent.

The Kelvin-Helmholtz instability describes a wavy disturbance caused by shear between two layers of flow. A small wave at a point of shear creates a pressure difference (lift) which further accentuates the wave.

The Taylor-Couett cell investigates the behaviour of a fluid trapped in the annulus between two rotating cylinders. The range of behaviours is highly varied.



Zaslavsky has looked at flows driven by a force that varies periodically in both time and space which is relevant to plasmas. Two dimensional flow can break up into a series of circulating cells with 4,5,6 or even 10 fold symmetry. Three dimensional flow chaotic regions in the flow can be arranged regularly in cells.

Even chaotic flows can have characteristic statistical forms. Fractal dimension allows us to characterise patterns with a precise numerical parameter, and thus distinguish between different classes of pattern. In turbulence the velocities at different points will tend to be correlated and these correlations can be very long ranged. Big eddies can transfer their energy into ever smaller eddies in an energy cascade such that the eddies appear at all relevant length scales. The fluid has a kind of scale invariance.

The statistical picture of turbulence has been valuable, but it works best when the system is more or less the same, on average, at every point. But this is not a good description of turbulence as there can be long lived structures or even periodic patterns. Describing turbulence in terms of statistical scaling laws does not contain all of the information.

Jupiter's Great Red Spot is a persistent feature of a turbulent environment absorbing smaller vortices.

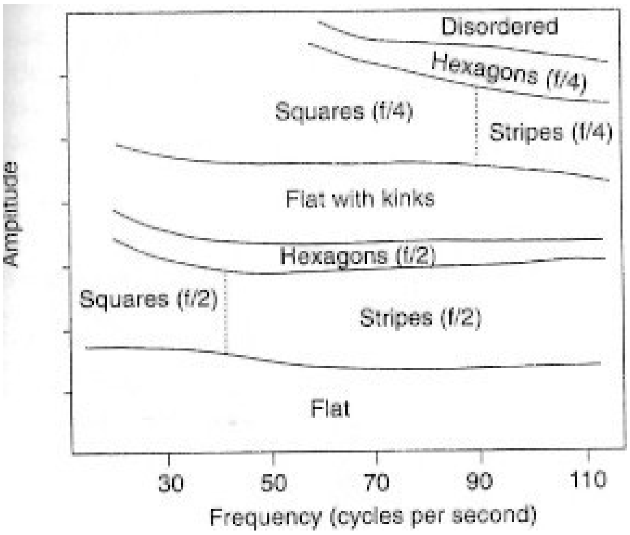
8. Grains

A granular substance is composed of solid grains but is able to liquid-like behaviour but with the ability to resist shear stresses and support heavy loads.

“If there's one thing that has become clear, it is that granular media are seldom predictable. Shaking together different kinds of grains can either ensure good mixing or have the opposite effect of causing them to segregate according to their size. Sound waves can bend through a right angle as they travel through sand, while the stress below a sand pile has a minimum where the pile is highest. The pressure at the bottom of a tall column of sand does not depend on its height, and this is why a sand glass is a good timekeeper-the sand leaks away at a steady rate even though the column gets smaller. If water were like this, the pressure at the sea bed would be no greater than that a few metres below the surface. “ pg 199

When a vessel of grains is shaken, large grains move upwards due to a ratcheting effect, but there is also a “convection cycle” set up due to friction with the walls on the upwards stroke so that the central grains move up faster.

Studies of a very thin layer of tiny bronze spheres in a shallow sealed container, pumped free of air and vibrated rapidly vertically display a wide range of behaviours.



These behaviours can be seen in remarkable simple models involving only randomisation (analogous to diffusion) and energy dissipation (analogous to reaction).

Often at an amplitude just below the onset of a global pattern, a single element of the pattern can form when initiated by some perturbation. Sometimes the elements link together into chains.

Avalanches of mixtures of grains will segregate into stripes when the grains have both a different size and a different angle of repose.

Grains rolled in a drum will mix if the drum is less than half full, opposing wedges will mix if the drum is half full, and the inner core will remain unmixed if the drum is over half full. In a long cylinder, grains with a different angle of repose will separate out into equal width bands due to the amplification of small initial inhomogeneities.

When grains of sand are added to a pile, the addition of a single grain can result in an avalanche of any magnitude. The frequency of an avalanche of a certain size decreases in inverse proportion to its size. Similar approximate 1/f laws are found in electrical discharge, the sun's luminosity, and different types of music. Note in music that there are statistical similarities that completely overlook drastic and important differences.

Sand piles are constantly seeking the least stable state rather than the most stable as in many other systems. These 'critical states' are found in a diversity of systems such as magnets, liquid phase transitions, and Big Bang theory. It is extremely difficult to maintain a fluid at its critical point as it is extremely sensitive to perturbations. However, sand piles seem to constantly return to their critical states rather than escape from them.

Power law behaviour is also seen in earthquakes, volcanic activity and fossil records.

The Bell curve is an example of a common statistical outcome from a random source.

“..self-organised critical states have a scale invariance just like that of fractals, and the spatial distributions of their component elements .... can be truly fractal. This is potentially important, as there was previously no known general mechanism for generating fractal structures.” pg 215

Investigations into rice grains verified self-organising critical behaviour for long-grain rice and highlighted importance of gaining sufficient data to distinguish between a power-law relation and a stretched exponential (short-grain rice). So self-organising critical behaviour exists but is not universal.

When sand dunes move it is the large scale pattern, independent of the constituent grains. Meinhardt suggests that dunes are akin to an activator-inhibitor system showing dislocations, terminations and bifurcations. But there must be other mechanisms because dunes are too diverse. There are characteristic sizes. Each dune forms a wind shadow which inhibits dune formation. The smaller ripples on dunes travel faster than the larger ones. Longitudinal dunes form parallel to the prevailing wind direction, Barchan dunes form perpendicular and there are also star dunes. Werner created a single model that produces all of the major dune types which represent attractors adjusted by wind variability.

9. Communities

Humans produce patterns both wittingly and unwittingly.

The Lotka-Volterra mechanism described the dynamic equilibrium in a simple predator-prey population model. It is too simple to provide an accurate prediction of real populations but it underpins the cyclical or often chaotic nature of population dynamics.

One complication is that populations are generally not homogeneous (contagious distribution) and the local density differences can play a significant role due the the sensitivity of the initial conditions. Viewed as in activator/inhibitor model the Lotka-Volterra participants can have differing radius of influence (e.g. the inhibition of a fox has a larger range than the activation of rabbits.) The patchiness in real population is not 'noise' but can be the core of the dynamic, to the extent that is the populations were homogeneous then they would collapse. Some communities require space to spread out and organise into patchy communities and this is frustrated by human artefacts such as roads.

The stable (patchy) patterns formed by population models can be stable under considerable levels of external noise. The noise can to smear the sharp patterns and obscure the finer period doublings but in other situations the presence of noise can have a significant influence on the patterns formed.

Humans like to think that they plan and take altruistic actions but Richard Dawkins evolution and John von Neumann's game theory demonstrate otherwise (example prisoners dilemma with tit-for-tat and Pavlov). When a very simple forms of cooperation and defection are played dynamically in a CA then shifting population patterns form dependent of the reward level for cooperation and defection. Conway's GOL displays an array of stable forms and the proportion of live and dead cells fluctuate in accordance with a 1/f rule. CA is fascinating but it is not clear that their lessons extend to nature.

Urban sprawl exhibits many of the same characteristics as bacterial populations. Cities have fractal dimensions of around 1.7 and can be successfully modelled with a DLA approach which relaxes the 'stick-where-you-hit' rule or the DBM (dielectric break down model) which introduces a kind of outwards growth pressure. Better models again introduce correlated growth which recognises that local growth promotes local growth. These models have long range global attraction exponentially declining from the core and short range local correlations on recent growth

10. Principles

There is not likely to be a single unifying explanation of complex patterning systems but there is enough in common to make a mockery of traditional boundaries between scientific disciplines. The ready availability of computing power and the maturation of a field theory of theoretical physics have underpinned understanding of pattern.

Competing forces lie at the heart of beauty and complexity in pattern formation where the forces are finely balanced and small changes have large effects (the edge of chaos).

Symmetry breaking is the process where a highly symmetrical (bland) medium progressively breaks up into patterns with lower and lower symmetry. The pattern emerges throughout the medium triggered by the smallest instability.

Non-equilibrium situations are the basis for almost all of the pattern-forming systems in this book. Engineering thermodynamics describes equilibrium states but not the transition process (while entropy is being generated). Non-equilibrium thermodynamics began by considering small linear deviations from equilibrium and then larger non-linear deviations. As a non-linear system is driven further from equilibrium it undergoes a bifurcation at which the evolution of the steady-state splits into two branches.

Dissipative structures are supported away from equilibrium by a continuous through-flow of energy and generation of entropy. Dissipative structures differ from crystalline structures which are at equilibrium and the spacial scale of the dissipative structure bears no resemblance to the size of the constituent particles. These structures possess an attractor (possibly chaotic/strange/fractal) which is robust in the face of perturbations. By contrast, a conservative structure alters as a result of perturbation (physical impact of an orbiting celestial body)

Symmetry breaking processes bear a close resemblance to phase transitions. Phase transitions are spontaneous global instabilities that set in when a **threshold** is crossed and may involve symmetry breaking. Bifurcations occur at a point of instability where an abrupt, apparently random, choice is made between two alternatives. First order phase transitions (such as water freezing) do not necessarily involve phase transition, often display hysteresis and the two phases can coexist at the equilibrium point. Second order phase transitions (such as the curie point of magnetisation) involves an abrupt but continuous phase change, always involve symmetry breaking, the coexistence of two phases is impossible, and no hysteresis. This is the distinction between sub-critical and supercritical bifurcations.

Bifurcations can be explored via a linear stability analysis. While the equations that govern a system may be known, particular solutions to the equation are required to understand the system behaviour. At a bifurcation, a previously stable solution becomes unstable while two additional new stable solutions appear.

Patterns forming in equilibrium systems may have a choice of patterns (e.g. alternate crystalline forms) but the one which minimises free energy will be selected preferentially. Landauer has shown that there cannot even I principle be any all encompassing minimisation criteria that can be applied to pattern selection out of equilibrium. The specific details of the system, including the nature of any randomising 'noise' that it experiences must be considered.

However, symmetry <u>often</u> breaks progressively, first into waves in one dimension and then into triangles, squares or hexagons in two dimensions. The size of the features in the pattern can relate to the rate of diffusion between inhibition/activation signals but other factors are influential. Patterns can also oscillate dynamically.

As patterning systems are pushed further from the stability threshold there tends to be an increase in the number of defects which continues until the recognisable pattern becomes the typical distance between elements or defects.

When the pattern medium is not infinite then the boundaries can have an influence.

When patterns are growing it is unclear if pattern dominance is maintain by the speed of growth or the maximum generation of entropy. The characteristics of impinging noise can also have an influence on pattern dominance.

Patterns typically become more complex with increasing driving force.

Correlations in behaviour over scales very significantly larger that the size of the constituent particles is typical of pattern formation and phase transitions. When systems approach a critical instability then the system becomes scale invariant and each element feels the influence of more and more of the rest of the system as the system become highly sensitive (or poised) as a whole.

However a phase transition has a characteristic length scale which is related to the size of the particles while the patterning system have an emergent characteristic scale.

Most of the patterns discussed have been essentially deterministic but others are stochastic and often distinguished by power laws which follow something close to 1/f.

“It is likely, however, that self-organised criticality and power law behaviour are the exception rather than the rule. Nonetheless, it is clear that noise, power law behaviour, scale invariance, avalanche behaviour and fractal forms are intimately connected in some deep way that remains to be fully explored and unravelled.” pg 266