Iowa City Math Circle Handouts
Quadrilaterals

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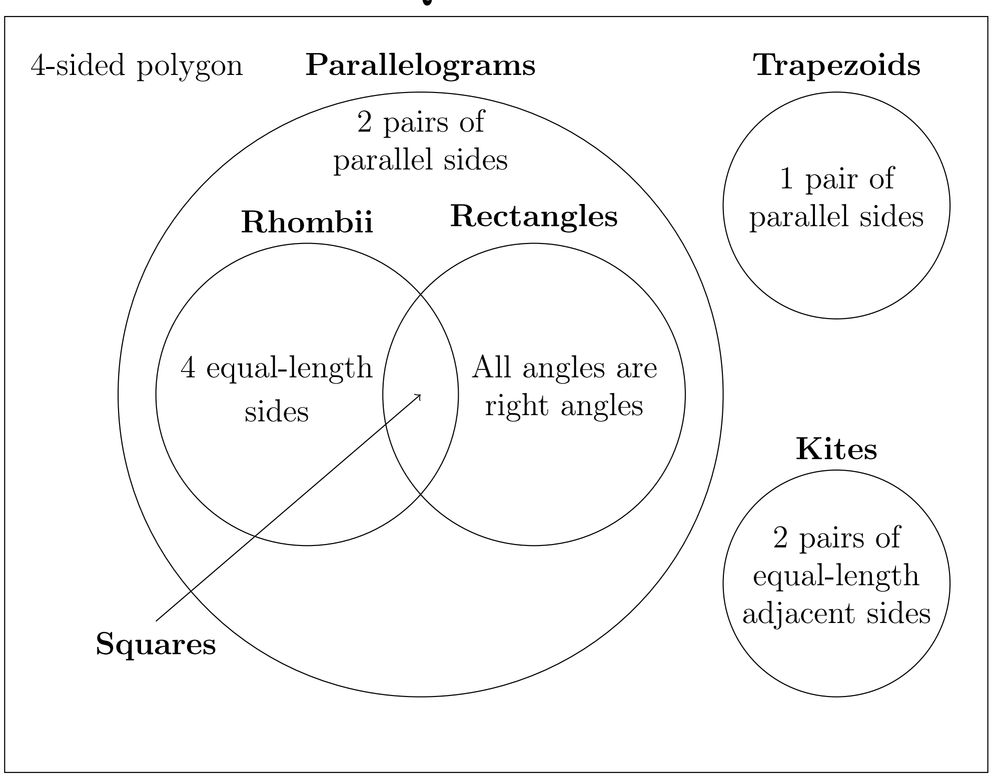
A *quadrilateral* is a polygon with four sides. There are several types of quadrilaterals that have special properties and frequently come up in geometry problems. We'll discuss each type in depth later on in the chapter. For now, the following figure gives a brief summary of these different types of quadrilaterals using Venn diagrams, showing their very interconnected relationships.

Quadrilaterals



In this chapter, we define a trapezoid as having *exactly* one pair of parallel sides, so parallelograms cannot be trapezoids. Likewise, we define a kite as having 2 pairs of equal-length adjacent sides, but these these two lengths must be *distinct*. Therefore, we don't classify a square as a kite. These previous two points are important to note in the context of this book and in math competitions, because the definitions of trapezoids and kites vary from source to source.

An important property of all quadrilaterals is that its interior angles $360°$. Why? Let's take a look at the following example which gives the proof.

Example 0.1. Prove that the sum of the interior angles of any quadrilateral is $360°$.

Solution. We can draw a *diagonal* of the quadrilateral (a line segment with endpoints as two, non-adjacent vertices of the quadrilateral) to form two triangles. Since all the angles in the two triangles add up to the angles in the quadrilateral, and the sum of the angles of both triangles is $360°$, the sum of the interior angles of a quadrilateral is $360°$. △

Not only is the sum of the interior angles is $360°$, but the sum of the exterior angles is also $360°$. This holds in general for all polygons, as a matter of fact.

Another principle of the quadrilateral is that the sum of any three sides cannot be smaller then the fourth side (this is analogous to the Triangle inequality). The following example gives a proof of this result that uses the Triangle inequality.

Example 0.2. (Quadrilateral Inequality) Let $ABCD$ be a quadrilateral with $AB = a$, $BC = b$, $CD = c$, $DA = d$, and $AC = e$. Show that $a + b + c > d$.

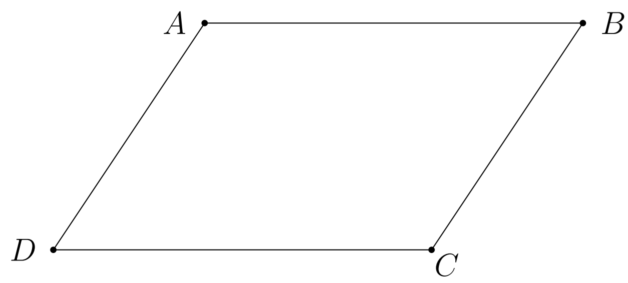
Solution. By the Triangle inequality on triangles ΔABC and ΔADC, we have $a+b > e$ and $e + c > d$, respectively. We can then add c on both sides of the first inequality to get $a + b + c > e + c$. However, since $e + c > d$, we can combine those two to get $a + b + c > d$, as desired. △

The following checkpoint requires you to be able to apply the Quadrilateral inequality.

Checkpoint 0.1. Joy has 30 thin rods, one each of every integer length from 1 cm through 30 cm. She places the rods with lengths 3 cm, 7 cm, and 15 cm on a table. She then wants to choose a fourth rod that she can put with these three to form a quadrilateral with positive area. How many of the remaining rods can she choose as the fourth rod? *Source: AMC 10*

1 Parallelograms

A *parallelogram* is a quadrilateral with exactly two pairs of parallel sides. Notice that these pairs of parallel sides must be opposite, as they cannot intersect at a vertex and still be parallel. The below quadrilateral is classified as a parallelogram because both pairs of opposite sides are parallel.



Next, we'll discuss the main properties of parallelograms in the following theorem.

Theorem 1.1. *The following results hold for any parallelogram $ABCD$, which, by definition, satisfies $AB||CD$ and $BC||AD$.*

1. *Any two adjacent angles of a parallelogram are supplementary (i.e. sum to $180°$).*
2. *Opposite angles of a parallelogram are congruent.*
3. *Opposite sides of a parallelogram are congruent; $AB = CD$ and $BC = AD$.*
4. *The diagonals of a parallelogram bisect each other. In other words, let the diagonals of $ABCD$ intersect at E. Then $AE = EC$ and $BE = ED$.*
5. *The sum of the squares of the sides is equal to the sum of the squares of the diagonals: $AB^2 + BC^2 + CD^2 + DA^2 = AC^2 + BD^2$*
6. *The area of any parallelogram is equal to $b \cdot h$, where b is the length of one side and h is the distance between that side and its opposite side. We call b the base and h the height of the parallelogram. Note that there are 2 different base-height pairs for a given parallelogram, but $b \cdot h$ is always constant.*

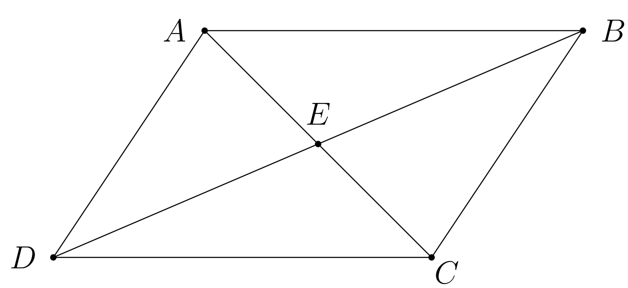


Figure 1a: Theorem 1.1, 1-4

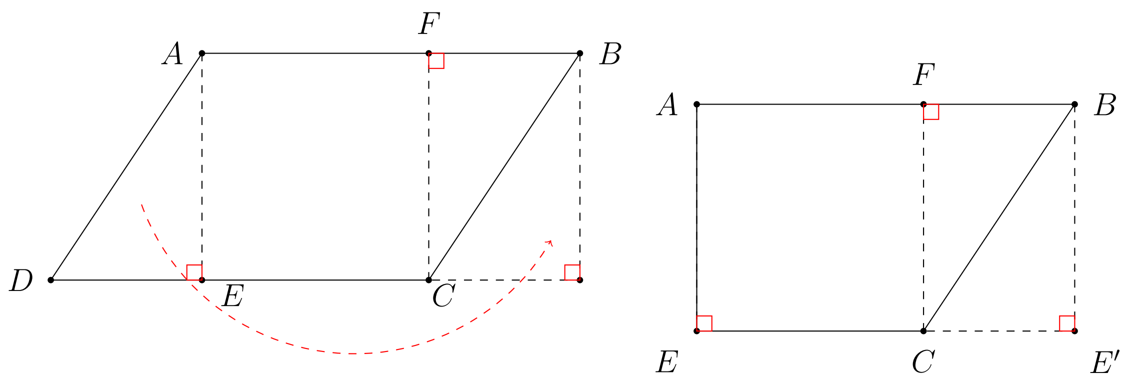


Figure 1b: Theorem 1.1, 5-6

Proof. We'll use the above figures extensively in our proofs.

1. This is just a direct consequence of the opposite sides of a parallelogram being parallel.

2. From the opposite sides being parallel, we get $\angle CAD = \angle BCA$ and $\angle BAC = \angle ACD$. Summing these two equations, we get $\angle BAD = \angle BCD$. Likewise, we can obtain $\angle ABC = \angle ADC$. Thus, opposite angles of a parallelogram are congruent.

3. From simple ASA congruence, we get that ΔABC is congruent to ΔCDA. From this congruence, we get the desired result.

4. From simple ASA congruence, we get that ΔABE is congruent to ΔCDE. From this congruence, we get the desired result.

5. From Figure 1.1b, we have $AD^2 = DE^2 + AE^2$, $BC^2 = BF^2 + CF^2$, $AC^2 = EC^2 + AE^2$, and $BD^2 = (CD + BF)^2 + AE^2$ by the Pythagorean theorem. Therefore,

$$
\begin{aligned}
&(AC^2 + BD^2) - (AB^2 + BC^2 + CD^2 + DA^2) \\
&= EC^2 + AE^2 + (CD + BF)^2 + AE^2 - (AB^2 + BC^2 + CD^2 + DA^2) \\
&= EC^2 + (AE^2 + BF^2) + CD^2 + 2CD \cdot BF + AE^2 - (AB^2 + BC^2 + CD^2 + DA^2) \\
&= EC^2 + 2AB \cdot BF + AE^2 - (AB^2 + DA^2) \\
&= EC^2 + 2AB \cdot BF + AE^2 - (AB^2 + (AE^2 + BF^2)) \\
&= EC^2 + 2AB \cdot BF - (AB^2 + BF^2) \\
&= EC^2 - (AB - BF)^2 \\
&= EC^2 - EC^2 \\
&= 0
\end{aligned}
$$

   Hence, we have $AC^2 + BD^2 = AB^2 + BC^2 + CD^2 + DA^2$ as desired.

6. Notice that we can effectively cut off triangle ΔADE from the rest of parallelogram $ABCD$ and paste it on the other side so that $\overline{AD}$ lines up with $\overline{BC}$ and DE becomes an extension to CE. We can do this because ΔADE is congruent to ΔCBF and $\Delta BCE'$ by the congruences detailed above. By doing this, we have transformed the original parallelogram into rectangle $ABE'E$, having the same area. So the area of the rectangle, and thus the parallelogram, is $AB \cdot BE$ (we will discuss the area of a rectangle later on in the chapter). This is just $b \cdot h$, so we are done.

□

There are a handful of useful properties in parallelograms, so the best way to get them down is to solve many problems. Let's try the following example.

Example 1.1. In $\triangle ABC$, $AB = AC = 28$ and $BC = 20$. Points D, E, and F are on sides $\overline{AB}$, $\overline{BC}$, and $\overline{AC}$, respectively, such that $\overline{DE}$ and $\overline{EF}$ are parallel to $\overline{AC}$ and $\overline{AB}$, respectively. What is the perimeter of parallelogram $ADEF$? *Source: AMC 10*

Solution. Because EF is parallel to AB, triangle ECF and BCA are similar, so ECF is an isosceles triangle. Therefore, $EF = FC$. We can now find half of the perimeter by finding the sum of two adjacent sides of $ADEF$. We will use AF and and FE. $AF + FE = AF + FC = AC = 28$, so half of the perimeter of $ADEF$ is 28. Therefore, the perimeter of $ADEF$ is 56. △

Example 1.2. A parallelogram has 3 points at $(2, 1)$, $(4, 2)$, and $(7, 7)$. Find the two possible fourth points of this parallelogram.

Solution. Lets first label the points in this parallelogram. We can call the point at $(2, 1)$ A, $(4, 2)$ B, and $(7, 7)$ C. We can then use the fact that opposite sides are parallel and congruent to each other. We can use the slope and the length of the lines to figure out where D is located at. The slope of AB is $\frac{1}{2}$ and has a rise(change in x) of 2. Line CD must have the same properties as AB, so the two possible x positions of D is 5 and 9. Using the slope, this results in the possible positions of D as $(5, 6)$ and $(9, 8)$. △

Checkpoint 1.1. A street has parallel curbs 40 feet apart. A crosswalk bounded by two parallel stripes crosses the street at an angle. The length of the curb between the stripes is 15 feet and each stripe is 50 feet long. Find the distance, in feet, between the stripes. *Source: AMC*

1.1 Rhombii

A *rhombus* is a parallelogram where all four sides have the same length. Rhombii share all the properties of a parallelogram, but they also have a few additional properties. The diagonals of a rhombus are perpendicular, and this means you can compute the area of a rhombus with the length of two diagonals; if the diagonal lengths are a and b, the area is $\frac{1}{2}ab$. The diagonals of a rhombus split it into four congruent right triangles - therefore, the diagonals also bisect the angles of the rhombus.

Example 1.3. Find the length of the other diagonal if a rhombus has side length 10 and one of the diagonal lengths is 12.

Solution. Because the diagonals meet up at a right angle, we can form a right triangle with half of the diagonals as the legs and the side length as the hypotenuse. Such a right triangle will have side lengths 6, x, and 10 as the hypotenuse. We can then see x is 8, so the full length of that diagonal is $2x = 16$. △

Example 1.4. A rhombus has one side of length 6 and a 60° angle. Compute its area.

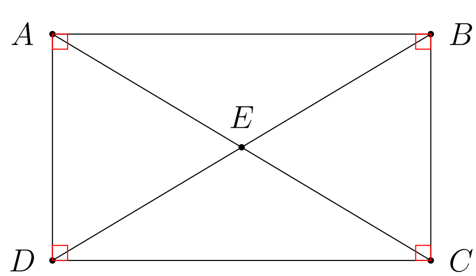
Solution. First we note that if a rhombus has one side of length 6, then all of its side must also be 6. In addition, we know that two angles have measure 60°, as opposite angles are equal. Now, consider the diagonal connects the two non-60° vertices. This diagonal splits the rhombus in half, such that the two 60° angles are on different halves of the rhombus. Each of these halves is an equilateral triangle! Why? Because we know the triangle is isosceles, and the the vertex angle is 60°, hence, the two base angles must also be 60°. Therefore, the area of our rhombus would be twice the area of an equilateral triangle with side length 6. Recall that the area of an equilateral triangle with side length s is $\frac{\sqrt{3}}{4}s^2$. Using this formula, we find the area of the rhombus to be $2 \cdot \frac{\sqrt{3}}{4} \cdot 6^2 = \boxed{18\sqrt{3}}$. △

1.2 Squares and Rectangles

A *rectangle* is a parallelogram with all right angles. The following theorem gives us a few nice properties of rectangles, in addition to those that they share in common with all parallelograms such as opposite sides being equal.

Theorem 1.2. *The following properties hold for all rectangles.*

1. *The two diagonals of any rectangle have equal length.*
2. *The area of any rectangle is the product of two of its adjacent sides.*



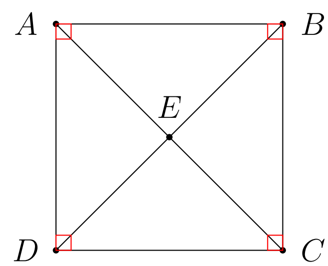
Proof. We will utilize the above figure to prove the theorem.

1. Applying the Pythagorean theorem to triangles ΔBCD and ΔABC, we get $AD^2 + CD^2 = AC^2$ and $BC^2 + DC^2 = BD^2$. But since all rectangles are parallelograms, we have that $BC = AD$ and $AB = DC$ (opposite sides are congruent). Therefore, $AD^2 + CD^2 = BC^2 + DC^2$, so $AC = BD$.

2. We can split up the rectangle by diagonal AC into two right triangles. Each right triangle has area $\frac{AB \cdot BC}{2}$, so the total area if $AB \cdot BC$, as desired.

□

A *square* is a rhombus with all right angles. Additionally, a square can also be classified as a rectangle with all sides having equal length. As a result, the area of a square with side length s will be s^2 and the perimeter of a square will be $4s$. Furthermore, we have that the diagonals of a square bisect the interior angles at their endpoints, creating 45-45-90 isosceles right triangles that are extremely useful.

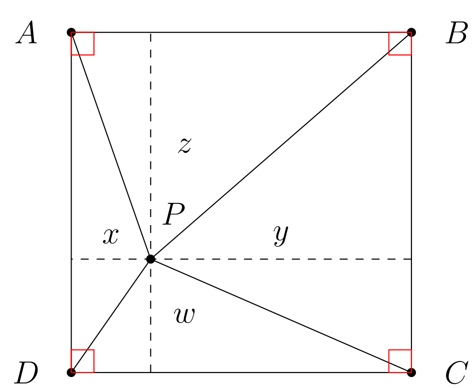


Example 1.5. On a square $ABCD$ with side length 12, let point E lie on the square such that E is 3 units away from line AB and 5 points away from line AD. What is the length of EC?

Solution. Since $ABCD$ is a square, the distances between AB and CD, and AD and BC are 12. Since E is 3 units away from AB, it is 9 units away from CD. (we can simply subtract 9 from the 12). Similarly, E is 7 units away from BC. Now, we can calculate EC with the Pythagorean theorem. If we draw point F on BC such that EF and BC are perpendicular, we find that EF is 9 because CD is 9. Now, with right triangle EFC, we can find that EC is $\sqrt{81+49}$ or $\sqrt{130}$. △

Often when you see a problem that involves a rectangle or square, you will have to use triangles in some sort of way to solve the problem. It is very common that such problem will require the use of the Pythagorean theorem, and here is a useful theorem that revolves around it.

Theorem 1.3. *(British flag theorem) Let P be a point chosen in the interior of rectangle $ABCD$. Then $AP^2 + CP^2 = BP^2 + DP^2$.*



Proof. Let the distance from point P to side AD be x, to side BC be y, to side AB be z, and to side CD be w. From the Pythagorean Theorem, we know:

$$x^2 + z^2 = AP^2$$
$$y^2 + z^2 = BP^2$$
$$y^2 + w^2 = CP^2$$
$$x^2 + w^2 = DP^2$$

Then it follows that $AP^2 + CP^2 = x^2 + z^2 + y^2 + w^2 = BP^2 + DP^2$, completing the proof. □

Checkpoint 1.2. A circle is inscribed in a square, then a square is inscribed in this circle, and finally, a circle is inscribed in this square. What is the ratio of the area of the smaller circle to the area of the larger square? *Source: AMC*

2 Trapezoids

A *trapezoid* is a quadrilateral with exactly one pair of parallel sides. By drawing the diagonals and seeing them as transversals between the two parallel sides, we can get several congruent angles.

Example 2.1. Trapezoid $ABCD$ has base $AB = 20$ units and base $CD = 30$ units. Diagonals AC and BD intersect at X. If the area of trapezoid $ABCD$ is 300 square units, what is the area of triangle BXC? *Source: MATHCOUNTS*

Solution. First we find the height of the trapezoid. Plugging the given values into the area formula, we get $300 = \frac{1}{2}h(20 + 30) \implies h = 12$. To find the area of triangle BXC, we can first find the area of BDC (which is $\frac{1}{2} \cdot 30 \cdot 12 = 180$)and subtract the area of CXD.

Notice that the diagonals of the trapezoid form two transversals with the bases. This means that triangles ABX and CXD are similar from AA similarity. Consequently, the height of CXD will be in a ratio of $\frac{30}{20} = \frac{3}{2}$ with the height of ABX, so the height of CXD will be $\frac{3}{5} \cdot 12$. Then the area of CXD is $\frac{1}{2} \cdot 12 \cdot \frac{3}{5} \cdot 30 = 108$. Subtracting CXD's area from the area of BDC gives us that the area of BXC is $180 - 108 = \boxed{72}$. △

The pair of sides that are not parallel in a trapezoid are known as *legs*. The *midsegment* of a trapezoid is the segment formed by connecting the midpoints of the two legs.

Theorem 2.1. *The midsegment of a trapezoid with bases a and b is:*

1. *parallel to the bases*
2. *of length $\frac{a+b}{2}$*
3. *located halfway in-between the two bases.*

Proof. We can prove this with coordinate geometry. WLOG, let one of the trapezoid's bases be on the x-axis, with one point at $(0,0)$ and the other point at $(a,0)$. Then let the other base have coordinated (x,h) and $(x+b,h)$. The midpoints of the legs will be $(\frac{x}{2},\frac{h}{2})$ and $(\frac{x+a+b}{2},\frac{h}{2})$. Since the midsegment has a slope of 0 and is $\frac{h}{2}$ units above the $x-axis$, it is (1) parallel to the bases and (3) located halfway in between. It also has (2) length $\frac{(x+a+b)-x}{2}=\frac{a+b}{2}$. □

Theorem 2.2. *The area of a trapezoid with base lengths a and b and height h is $\frac{1}{2}h(a+b)$.*

Proof. Without loss of generality, let DC be the shorter base and AB be the longer base in trapezoid $ABCD$.We can extend sides AD and BC so that they intersect at point E above the trapezoid. We first note that ΔEDC and ΔEAB are similar by AA similarity - both triangles share angle AEB ($\angle AEB=\angle DEC$) and because $DC||AB$, we have that $\angle EDC=\angle EAB$. Now, let us define $a=DC$, $b=AB$, h as the height of the trapezoid, and h_1 as the height of ΔEDC. By similarity, we have that the ratio between the height and the base of each triangle should be the same, giving us $\frac{h_1}{a}=\frac{(h_1+h)}{b}$ (where the LHS is the ratio for ΔEDC and the RHS is the ration for ΔEAB). Cross-multiplying, we obtain $h_1b=h_1a+ha$, or $h_1=\frac{ha}{b-a}$.
Now to find the area of the trapezoid, we can simply compute

$$\begin{aligned}
\text{Area}[\Delta EAB-\Delta EDC] &= \text{Area}[\Delta EAB]-\text{Area}[\Delta EDC] \\
&= \frac{1}{2}(h_1+h)b-\frac{1}{2}h_1a \\
&= \frac{1}{2}\left(\frac{ha}{b-a}+h\right)b-\frac{1}{2}\frac{ha}{b-a}a \\
&= \frac{1}{2}\frac{ha}{b-a}(b-a)+\frac{1}{2}hb \\
&= \frac{1}{2}h(a+b)
\end{aligned}$$

□

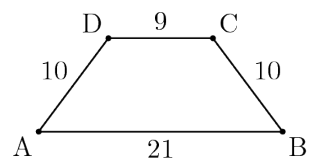
Checkpoint 2.1. In trapezoid $ABCD$, the parallel sides AB and CD have lengths of 8 and 20 units, respectively, and the altitude is 12 units. Points E and F are the midpoints of sides AD and BC, respectively. What is the area of quadrilateral $EFCD$ in square units? *Source: MATHCOUNTS*

A trapezoid with legs of equal length is an *isosceles trapezoid.* Isosceles trapezoids have congruent diagonals and have congruent base angles (angles formed by each leg and one of the bases). As a result of this angle property, the sum of the angles at one base and its corresponding angle at the other base, formed by the same leg, is 180°.

Theorem 2.3. *In a isosceles trapezoid $ABCD$, with base AB having length a, and base CD having length b, let E denote the intersection of the diagonals. Then, the ratio of triangles $ABE:CDE:ADE:BCE$ is $a^2:b^2:ab:ab$. Note that the areas of triangle BCE and ADE are the same.*

Proof. □

Checkpoint 2.2. The isosceles trapezoid shown has side lengths as labeled. How long is segment AC? *Source: MATHCOUNTS*



3 Kites

A *kite* is a quadrilateral with two pairs of equal-length adjacent sides. The diagonals of a kite are perpendicular to each other. In addition, the longer diagonal bisects the shorter diagonal. The diagonals split the kite into four right triangles - specifically, two pairs of congruent right triangles.

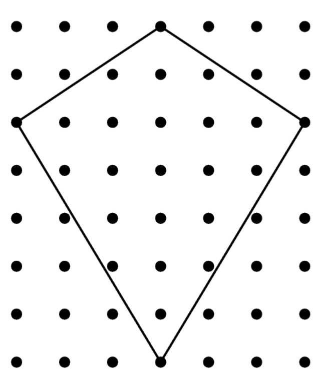
Theorem 3.1. *The area of a kite with diagonal lengths d_1 and d_2 and is $\frac{1}{2}d_1d_2$.*

Example 3.1. The parabolas $y = ax^2 - 2$ and $y = 4 - bx^2$ intersect the coordinate axes in exactly four points, and these four points are the vertices of a kite of area 12. What is $a + b$? *Source: AMC*

Solution. The y-intercept of $y = ax^2 - 2$ is -2, and the y-intercept of $y = 4 - bx^2$ is 4, so one diagonal of the kite has length 6. This means the other diagonal must have length $\frac{12}{6 \cdot \frac{1}{2}} = 4$.

Since both parabolas are centered about the y-axis, they must intercept the x-axis at the same two points. The two points must be $(-2, 0)$ and $(2, 0)$. So $0 = 4a - 2 \implies a = 0.5$, and $0 = 4 - 4b \implies b = 1$, giving us $a = 0.5$ and $b = 1$, so the answer is $\boxed{1.5}$. △

Checkpoint 3.1. To promote her school's annual Kite Olympics, Genevieve makes a small kite and a large kite for a bulletin board display. The kites look like the one in the diagram. For her small kite Genevieve draws the kite on a one-inch grid. For the large kite she triples both the height and width of the entire grid.



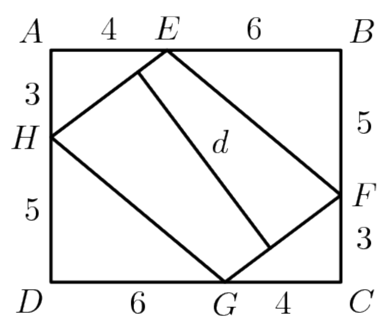
The large kite is covered with gold foil. The foil is cut from a rectangular piece that just covers the entire grid. How many square inches of waste material are cut off from the four corners? *Source: AMC*

4 Exercises

1. $\star$ A parallelogram has adjacent sides of lengths s units and $2s$ units forming a 45-degree angle. The area of the parallelogram is $8\sqrt{2}$ square units. What is the value of s? Express your answer in simplest radical form. *Source: MATHCOUNTS*

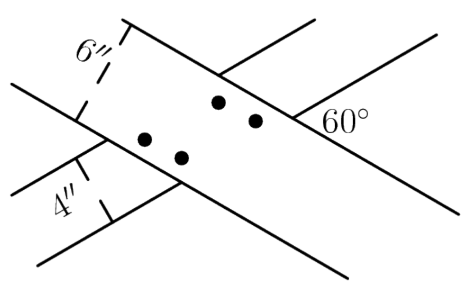
2. $\star$ A rectangle $ABCD$ has an area of 1200. Point E is drawn on the midpoint of AB, and point F is drawn on. CD such that $DF = 2 \cdot CF$. What is the area of quadrilateral $AECF$?

3. $\star$ In the figure, $ABCD$ is a rectangle and $EFGH$ is a parallelogram. Using the measurements given in the figure, what is the length d of the segment that is perpendicular to $\overline{HE}$ and $\overline{FG}$? *Source: AMC*



4. $\star$ In $\triangle ABC$, $AB = AC = 28$ and $BC = 20$. Points D, E, and F are on sides $\overline{AB}$, $\overline{BC}$, and $\overline{AC}$, respectively, such that $\overline{DE}$ and $\overline{EF}$ are parallel to $\overline{AC}$ and $\overline{AB}$, respectively. What is the perimeter of parallelogram $ADEF$? *Source: AMC*

5. $\star$ Two boards, one four inches wide and the other six inches wide, are nailed together to form an X. The angle at which they cross is 60 degrees. If this structure is painted and the boards are separated what is the area of the unpainted region on the four-inch board? (The holes caused by the nails are negligible.) Express your answer in simplest radical form. *Source: MATHCOUNTS*



6. ⋆ If the diagonals of a quadrilateral are perpendicular to each other, the figure would always be included under the general classification:

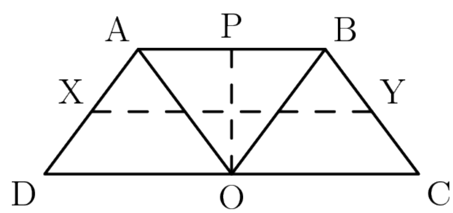
(A) rhombus **(B)** rectangles **(C)** square **(D)** isosceles trapezoid

(E) none of these

Source: AHSME

7. ⋆ In trapezoid $ABCD$ the lengths of the bases AB and CD are 8 and 17 respectively. The legs of the trapezoid are extended beyond A and B to meet at point E. What is the ratio of the area of triangle EAB to the area of trapezoid $ABCD$? Express your answer as a common fraction. *Source: MATHCOUNTS*

8. ⋆ Three congruent isosceles triangles DAO, AOB and OBC have $AD = AO = OB = BC = 10$ and $AB = DO = OC = 12$. These triangles are arranged to form trapezoid $ABCD$, as shown. Point P is on side AB so that OP is perpendicular to AB.

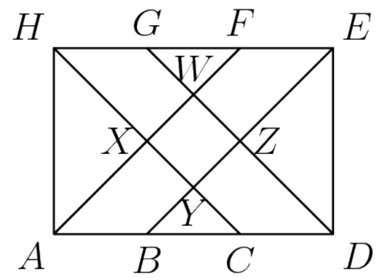


Point X is the midpoint of AD and point Y is the midpoint of BC. When X and Y are joined, the trapezoid is divided into two smaller trapezoids. The ratio of the area of trapezoid $ABYX$ to the area of trapezoid $XYCD$ in simplified form is $p : q$. Find $p + q$. *Source: AoPS*

9. ⋆ $PQRS$ is a trapezoid with an area of 12 and bases PQ and RS. RS is twice the length of PQ. What is the area of $\triangle PQS$? *Source: CEMC*

10. ⋆ Trapezoid $ABCD$ has bases AB and CD. The ratio of the area of triangle ABC to the area of triangle ADC is $7 : 3$. If $AB + CD = 210$ cm, how long is segment $\overline{AB}$? *Source: MATHCOUNTS*

11. ⋆ In rectangle $ADEH$, points B and C trisect $\overline{AD}$, and points G and F trisect $\overline{HE}$. In addition, $AH = AC = 2$. What is the area of quadrilateral $WXYZ$ shown in the figure? *Source: AMC*

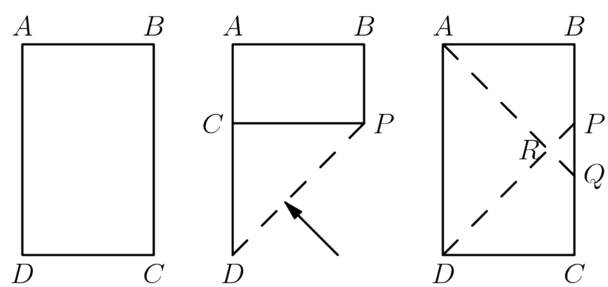


12. ⋆⋆ An isosceles trapezoid has legs of length 30 cm each, two diagonals of length 40 cm each and the longer base is 50 cm. What is the trapezoid's area in sq cm? *Source: MATHCOUNTS*

13. ⋆⋆ In trapezoid $ABCD$ with bases $\overline{AB}$ and $\overline{CD}$, we have $AB = 52$, $BC = 12$, $CD = 39$, and $DA = 5$. What is the area of $ABCD$? *Source: AMC*

14. ⋆⋆ In trapezoid $ABCD$, $\overline{AB}$ and $\overline{CD}$ are perpendicular to $\overline{AD}$, with $AB + CD = BC$, $AB < CD$, and $AD = 7$. What is $AB \cdot CD$? Express your answer as a common fraction. *Source: AMC*

15. ⋆⋆ A rectangular piece of paper $ABCD$ is folded so that edge CD lies along edge AD, making a crease DP. It is unfolded, and then folded again so that edge AB lies along edge AD, making a second crease AQ. The two creases meet at R, forming triangles PQR and ADR. If $AB = 5$ cm and $AD = 8$ cm, what is the area of quadrilateral $DRQC$, in cm^2? *Source: CEMC*



16. ⋆⋆ For how many of the following types of quadrilaterals does there exist a point in the plane of the quadrilateral that is equidistant from all four vertices of the quadrilateral?

    - a square
    - a rectangle that is not a square
    - a rhombus that is not a square
    - a parallelogram that is not a rectangle or a rhombus
    - an isosceles trapezoid that is not a parallelogram

Source: AMC

17. $\star\star$ In trapezoid $ABCD$ we have $\overline{AB}$ parallel to $\overline{DC}$, E as the midpoint of $\overline{BC}$, and F as the midpoint of $\overline{DA}$. The area of $ABEF$ is twice the area of $FECD$. What is AB/DC? *Source: AMC*

18. $\star\star$ Let points $A = (0,0,0)$, $B = (1,0,0)$, $C = (0,2,0)$, and $D = (0,0,3)$. Points E, F, G, and H are midpoints of line segments $\overline{BD}$, $\overline{AB}$, $\overline{AC}$, and $\overline{DC}$ respectively. What is the area of rectangle $EFGH$? *Source: AMC*

19. $\star\star\star$ Rhombus $ABCD$ has side length 2 and $\angle B = 120°$. Region R consists of all points inside the rhombus that are closer to vertex B than any of the other three vertices. What is the area of R? *Source: AMC*

20. $\star\star\star$ Let $ABCD$ be an isosceles trapezoid with $AB = 1$, $BC = DA = 5$, and $CD = 7$. Let P be the intersection of diagonals $\overline{AC}$ and $\overline{BD}$, and let Q be the foot of the altitude from D to $\overline{BC}$. Let $\overline{PQ}$ intersect $\overline{AB}$ at R. Compute $\sin \angle RPD$.

21. $\star\star\star$ The solutions to the equations $z^2 = 4 + 4\sqrt{15}i$ and $z^2 = 2 + 2\sqrt{3}i$, where $i = \sqrt{-1}$, form the vertices of a parallelogram in the complex plane. The area of this parallelogram can be written in the form $p\sqrt{q} - r\sqrt{s}$, where p, q, r, and s are positive integers and neither q nor s is divisible by the square of any prime number. What is $p + q + r + s$? *Source: AMC*