Communications on Applied Mathematics and Computation
https://doi.org/10.1007/s42967-020-00073-7

ORIGINAL PAPER

The Spectral Radii of Intersecting Uniform Hypergraphs

Peng-Li Zhang[1] · **Xiao-Dong Zhang**[1]

Received: 10 November 2019 / Revised: 10 April 2020 / Accepted: 17 April 2020

Abstract

The celebrated Erdős–Ko–Rado theorem states that given $n \geqslant 2k$, every intersecting k-uniform hypergraph G on n vertices has at most $\begin{pmatrix} n-1 \\ k-1 \end{pmatrix}$ edges. This paper states spectral versions of the Erdős–Ko–Rado theorem: let G be an intersecting k-uniform hypergraph on n vertices with $n \geqslant 2k$. Then, the sharp upper bounds for the spectral radius of $\mathcal{A}_\alpha(G)$ and $\mathcal{Q}^*(G)$ are presented, where $\mathcal{A}_\alpha(G) = \alpha\mathcal{D}(G) + (1-\alpha)\mathcal{A}(G)$ is a convex linear combination of the degree diagonal tensor $\mathcal{D}(G)$ and the adjacency tensor $\mathcal{A}(G)$ for $0 \leqslant \alpha < 1$, and $\mathcal{Q}^*(G)$ is the incidence $\mathcal{Q}$-tensor, respectively. Furthermore, when $n > 2k$, the extremal hypergraphs which attain the sharp upper bounds are characterized. The proof mainly relies on the Perron–Frobenius theorem for nonnegative tensor and the property of the maximizing connected hypergraphs.

Keywords Erdős–Ko–Rado theorem · Intersecting hypergraph · Tensor · Spectral radius

Mathematics Subject Classification 05C65 · 05C50

1 Introduction

A family $\mathcal{F}$ of sets is called intersecting if $A \cap B \neq \emptyset$ for any two $A, B \in \mathcal{F}$. An intersecting family of sets is trivial if all of its members share a common element. One essential problem in extremal set theory is to study the properties of intersecting families.

For integers $1 \leqslant k \leqslant n$ and a set X of n elements, $[n] = \{1, 2, \cdots, n\}$. In 1961, Erdős et al. in their paper [9] obtained the well-known theorem: when $n \geqslant 2k$, every intersecting family of k-subsets on n-element X has at most $\binom{n-1}{k-1}$ members. Moreover, when $n > 2k$, the

This work is supported by the National Natural Science Foundation of China (Nos. 11971311, 11531001), and the Montenegrin-Chinese Science and Technology Cooperation Project (No. 3-12).

✉ Xiao-Dong Zhang
xiaodong@sjtu.edu.cn

Peng-Li Zhang
zpengli@sjtu.edu.cn

[1] School of Mathematical Sciences, MOE-LSC, SHL-MAC, Shanghai Jiao Tong University, Shanghai 200240, China

Published online: 29 June 2020

extremal family is unique (up to isomorphism): it consists of all the k-subsets of X that contains a fixed element, and it is easy to see that the extremal family is trivial.

The Erdős–Ko–Rado theorem, as one of the most fundamental results in extremal combinatorics, provides information about systems of intersecting sets and has many interesting applications and extensions, such as the Hilton–Milner theorem, see [16, 18, 19] for a full account.

Also there have been outstanding work on intersecting families satisfying certain properties; for example, Frankl [10] presented some sharp upper bounds on the size of intersecting families with certain maximum degree which extended the Hilton–Milner theorem. Furthermore, intersecting families from the point of view of the minimum vertex degree have been investigated in [12, 13, 23]. For more new results and progress on intersecting families, readers are referred to a survey paper [14] and an excellent book [16].

Let G be a hypergraph on n vertices with a vertex set $V(G)$ and an edge set $E(G)$. The elements of $V = V(G)$, labeled as $\{v_1, \cdots, v_n\}$, are referred to as vertices and the elements of $E = E(G)$ are called edges. If $|e| = k$ for each $e \in E(G)$, then G is said to be a k-uniform hypergraph. For $k = 2$, it refers to the ordinary graph. For a vertex $v_i \in V(G)$, we denote $E_{v_i}(G) = \{e \in E(G) | v_i \in e\}$, which is the set of edges containing the vertex v_i. The degree d_{v_i} of a vertex $v_i \in V(G)$ is defined as $d_{v_i} = |e_j : v_i \in e_j \in E(G)|$. A hypergraph is d-regular if $d_{v_1} = \cdots = d_{v_n} = d$. A complete k-uniform hypergraph is defined to be a hypergraph $G = (V(G), E(G))$ with the edge set consisting of all k-subsets of $V(G)$. Clearly, it is a $d = \begin{pmatrix} n-1 \\ k-1 \end{pmatrix}$-regular hypergraph. Moreover, two vertices are said to be adjacent if there is an edge that contains both of these vertices. Two edges are said to be adjacent if their intersection is not empty. A vertex v is said to be incident to an edge e if $v \in e$.

A walk W of length l in G is a sequence of alternate vertices and edges: $v_0 e_1 v_1 e_2 \cdots e_l v_l$, where $\{v_i, v_{i+1}\} \subseteq e_{i+1}$ for $i = 0, 1, \cdots, l-1$. If $v_0 = v_l$, then W is called a circuit. A walk of G is called a path if no vertices or no edges are repeated. A circuit G is called a cycle if no vertices or edges are repeated except $v_0 = v_l$. A hypergraph G is said to be connected if every two vertices are connected by a path. If $l > 1$ and $v_0 = v_l$, then this path of length l is called a cycle of length l. We assume that G is simple throughout this paper, which means that $e_i \neq e_j$ if $i \neq j$. More information on hypergraphs can be referred to both in Berge [1] and Bretto [2].

A hypergraph $G = (V(G), E(G))$ is called intersecting if there is at least one element in any two edges of $E(G)$. Hence, for an intersecting family $\mathcal{F}$ on the n-element X, there is an intersecting hypergraph $G = (X, \mathcal{F})$ corresponding to this intersecting family.

Let $S_{n,k,1}$ be the k-uniform hypergraph on n vertices which all edges share exactly a common vertex u, such that the hypergraph obtained by deleting u from each edge $e \in E(S_{n,k,1})$ is a complete $(k-1)$-uniform hypergraph on $n-1$ vertices.

We may view a family $\mathcal{F}$ of k-subsets of X as a k-uniform hypergraph G with the vertex set X and the edge set $\mathcal{F}$. The celebrated Erdős–Ko–Rado theorem can be stated as the following.

Theorem 1.1 *For two integers $n \geqslant 2k$, every intersecting k-uniform hypergraph G on n vertices has at most $\begin{pmatrix} n-1 \\ k-1 \end{pmatrix}$ edges. Moreover, when $n > 2k$, the equality holds if and only if $G = S_{n,k,1}$.*

To present spectral versions for the Erdős–Ko–Rado theorem, we first introduce some notations of tensors. For integers $k \geqslant 2$ and $n \geqslant 2$, a real tensor (also called hypermatrix)

$\mathcal{T}=(t_{i_1\cdots i_k})$ of order k and dimension n refers to a multidimensional array with entries $t_{i_1\cdots i_k}$, such that

$$t_{i_1\cdots i_k}\in\mathbb{R}\quad \text{for all } i_j\in[n]=\{1,2,\cdots,n\} \text{ and } j\in[k].$$

The tensor $\mathcal{T}$ is called symmetric if $t_{i_1\cdots i_k}$ is invariant under any permutation of its indices $i_1,i_2,\cdots,i_k$.

For $k\geqslant 2$, let G be a k-uniform hypergraph with $V(G)=[n]$. The adjacency tensor of G (see [6]) is defined as the k-th order n-dimensional tensor $\mathcal{A}(G)=(a_{i_1\cdots i_k})$, where

$$a_{i_1\cdots i_k}=\begin{cases}\frac{1}{(k-1)!} & \text{if } \{i_1,\cdots,i_k\}\in E(G),\\ 0 & \text{otherwise.}\end{cases}$$

Clearly, the adjacency tensor is always nonnegative and symmetric. Recently, Keevash et al. [24] gave an adjacency spectral version for the Erdős–Ko–Rado theorem.

Theorem 1.2 ([24]) *For any $k\geqslant 2$, there is an n_0, such that the following holds for $n\geqslant n_0$. Let G be an intersecting k-uniform hypergraph on n vertices. Then,*

$$\rho(\mathcal{A}(G))\leqslant\rho(\mathcal{A}(S_{n,k,1}))$$

with the equality if and only if $G=S_{n,k,1}$, where $\rho(\mathcal{A}(G))$ is the spectral radius of the adjacency tensor $\mathcal{A}(G)$.

On the other hand, the spectral radius of the adjacency tensor has been widely investigated. For example, Fan et al. [15] determined the extremal spectral radii of several classes of k-uniform hypergraphs with a few edges. Yuan et al. [35] obtained several bounds for the spectral radius of uniform hypergraphs in terms of the degrees of vertices. Xiao et al. [33] determined the unique k-uniform supertrees with maximum spectral radii among all k-uniform supertrees with given degree sequences. Li et al. [27] determined the extremal spectral radii of k-uniform supertrees. Chen et al. [5] proved several good upper bounds for the adjacency spectral radius of uniform hypergraphs in terms of degree sequences. Bai and Lu [3] solved the problem of maximizing the spectral radius of k-uniform hypergraphs among all k-uniform hypergraphs with a given number of edges. Xiao and Wang [32] determined the unique hypergraphs with the maximum spectral radius among all the uniform supertrees and all the connected uniform unicyclic hypergraphs with a given number of pendant edges, respectively.

Motivated by the adjacency spectral version of the Erdős–Ko–Rado theorem and the results on hypergraph spectra, we continue to study other spectral versions of the Erdős–Ko–Rado theorem and make some contribution to the spectral hypergraph theory. This paper is organized as follows. In Sect. 2, we state some basic notations of tensors. In Sect. 3, some lemmas are presented regarding the $\mathcal{A}_\alpha$-spectral radius and incidence $\mathcal{Q}$-spectral radius of k-uniform hypergraphs on n vertices, including giving the exact value of $\mathcal{A}_\alpha$-spectral radius and incidence $\mathcal{Q}$-spectral radius of $S_{n,k,1}$. In Sect. 4, we state the main theorem of this paper and give the proof of the main theorem and some corollaries.

2 Preliminaries

In the sequel, we present some essential concepts of tensors which will be used later. A real symmetric tensor $\mathcal{T}$ of order k dimension n uniquely defines a kth-degree homogeneous polynomial function with the real coefficient by

$$F_{\mathcal{T}}(x) = \mathcal{T}x^k = \sum_{i_1,\cdots,i_k=1}^{n} t_{i_1\cdots i_k} x_{i_1} \cdots x_{i_k}.$$

$\mathcal{T}$ is called positive semi-definite if $F_{\mathcal{T}}(x) = \mathcal{T}x^k \geqslant 0$ for all $x \in \mathbb{R}^n$. Obviously, for the nontrivial case, k must be even. It is easy to see that $\mathcal{T}x^k$ is a real number. Remember that $\mathcal{T}x^{k-1}$ is a vector in $\mathbb{R}^n$, which its ith component is defined as

$$(\mathcal{T}x^{k-1})_i = \sum_{i_2,\cdots,i_k=1}^{n} t_{ii_2\cdots i_k} x_{i_2} \cdots x_{i_k}.$$

Definition 2.1 ([29]) Let $\mathcal{T}$ be a kth order n-dimensional real tensor and $\mathbb{C}$ be the set of all complex numbers. Then, λ is an eigenvalue of $\mathcal{T}$ and $x \in \mathbb{C}^n \setminus \{0\}$ is an eigenvector corresponding to λ if (λ, x) satisfies

$$\mathcal{T}x^{k-1} = \lambda x^{[k-1]},$$

where $x^{[k-1]} \in \mathbb{C}^n$ with $(x^{[k-1]})_i = (x_i)^{k-1}$.

If x is a real eigenvector of $\mathcal{T}$, then surely the corresponding eigenvalue λ is real. In this case, x is called an H-eigenvector and λ is called an H-eigenvalue. With more information on eigenvalues and eigenvectors of tensors, the readers are referred to the paper of Qi [29]. Moreover, it is easy to see that

$$(\mathcal{T}x^{k-1})_i = \lambda x_i^{k-1} \ \text{ for } \ i = 1, \cdots, n.$$

Shao [31] introduced the definition for tensor product, by the generalization of Bu et al. [4] for the tensor product, $\mathcal{T}x^{k-1}$, in Definition 2.1 can be simply written as $\mathcal{T}x$. Furthermore, if $x \in \mathbb{R}^n_+$, where $\mathbb{R}^n_+ = \{x \in \mathbb{R}^n : x \geqslant 0\}$, then λ is an H^+-eigenvalue of $\mathcal{T}$. If $x \in \mathbb{R}^n_{++}$, where $\mathbb{R}^n_{++} = \{x \in \mathbb{R}^n : x > 0\}$, then λ is said to be an H^{++}-eigenvalue of $\mathcal{T}$. The spectral radius of $\mathcal{T}$ is defined as

$$\rho(\mathcal{T}) = \max\{|\lambda| : \lambda \text{ is an eigenvalue of } \mathcal{T}\}.$$

Let x be a column vector of dimension n. A k-uniform hypergraph $G = (V(G), E(G))$ on n vertices consists of a vertex set $V(G) = [n]$ and an edge set $E(G) = \{e_1, \cdots, e_m\} \subseteq P_k(V(G))$, where $P_k(V(G))$ is the set of all k-subsets of $V(G)$. It is easy to see that

$$x^{\mathrm{T}}(\mathcal{A}(G)x) = \sum_{e \in E(G)} kx^e,$$

where $x^e = x_{i_1} \cdots x_{i_k}$ for $e = \{i_1, \cdots, i_k\} \in E(G)$, and

$$(\mathcal{A}(G)x)_i = \sum_{e \in E_i(G)} x^{e\setminus\{i\}}.$$

Hu and Qi [21] defined the signless Laplacian tensor as $\mathcal{Q} = \mathcal{D} + \mathcal{A}$, where $\mathcal{D}$ is a kth-order n-dimensional tensor with its diagonal element $d_{i\cdots i}$ being d_i, the degree of vertex i, for all $i \in [n]$. It is easy to see that

$$x^{\mathrm{T}}(\mathcal{Q}(G)x) = \sum_{e\in E(G)} (x^{[k]}(e) + kx^e),$$

where $x^{[k]}(e) = x_{j_1}^k + \cdots + x_{j_k}^k, x^e = x_{j_1} \cdots x_{j_k}$ for $e = \{j_1, \cdots, j_k\} \in E(G)$. Furthermore,

$$x^{\mathrm{T}}(\mathcal{Q}(G)x) = \sum_{i\in V(G)} d_i x_i^k + \sum_{e\in E(G)} kx^e$$

and

$$(\mathcal{Q}(G)x)_i = d_i x_i^{k-1} + \sum_{e\in E_i(G)} x^{e\backslash\{i\}}.$$

Inspired by the innovating work of Nikiforov [28], Lin et al. [25] proposed corresponding notation of the convex linear combination $\mathcal{A}_\alpha(G)$ of $\mathcal{D}(G)$ and $\mathcal{A}(G)$, which is defined as

$$\mathcal{A}_\alpha(G) = \alpha\mathcal{D}(G) + (1-\alpha)\mathcal{A}(G),$$

where $0 \leqslant \alpha < 1$. The spectral radius of $\mathcal{A}_\alpha(G)$ is called the $\mathcal{A}_\alpha$-spectral radius of G and denoted by $\rho_\alpha(G)$. Then, $\rho_0(G)$ is the spectral radius of $\mathcal{A}(G)$, which is called the adjacency spectral radius of G and denoted by $\rho(\mathcal{A}(G))$. Moreover, $2\rho_{1/2}(G)$ is the spectral radius of $\mathcal{Q}(G)$, which is called the signless Laplacian spectral radius of G; some bounds for the signless Laplacian spectral radius for uniform hypergraphs can be found in [5, 22, 26, 30]. And the $\mathcal{A}_\alpha$-spectral radius of uniform hypergraphs has been studied in [17, 25].

For $k \geqslant 2$, let G be a k-uniform hypergraph with $V(G) = [n]$, and x be an n-dimensional column vector. Clearly,

$$\begin{aligned} x^{\mathrm{T}}(\mathcal{A}_\alpha(G)x) =& \alpha \sum_{i\in V(G)} d_i x_i^k + (1-\alpha) \sum_{e\in E(G)} kx^e, \\ x^{\mathrm{T}}(\mathcal{A}_\alpha(G)x) =& \sum_{e\in E(G)} \left(\alpha \sum_{i\in e} x_i^k + (1-\alpha)kx^e \right), \end{aligned}$$

and

$$(\mathcal{A}_\alpha(G)x)_i = \alpha d_i x_i^{k-1} + (1-\alpha) \sum_{e\in E_i(G)} x^{e\backslash\{i\}}.$$

The incidence matrix [2] of a hypergraph G is defined as a matrix $R = (r_{ij})$ whose rows and columns are indexed by the vertices and edges of G, respectively. The (i, j)-entry of R is

$$r_{ij} = \begin{cases} 1 & \text{if } v_i \in e_j, \\ 0 & \text{otherwise.} \end{cases}$$

Li et al. [27] introduced the concept of the incidence Q-tensor of a k-uniform hypergraph G, which is defined as $Q^* \equiv Q^*(G) = R\mathcal{I}R^{\mathrm{T}}$, where R is the incidence matrix of G and $\mathcal{I}$ is the identity tensor, i.e., $\mathcal{I}_{i_1\cdots i_k} = 1$ if $i_1 = \cdots = i_k \in [m]$, and zero otherwise when the dimension is m. It is easy to see that $R\mathcal{I}R^{\mathrm{T}}$ is a symmetric tensor of order k and dimension n. Clearly,

$$x^{\mathrm{T}}(\mathcal{Q}^*(G)x) = \sum_{\{i_1,\cdots,i_k\}\in E(G)} (x_{i_1} + \cdots + x_{i_k})^k = \sum_{e\in E(G)} x(e)^k,$$

where $x(e) = x_{i_1} + \cdots + x_{i_k}$ for $e = \{i_1, \cdots, i_k\}$. Moreover, the $(i_1, \cdots, i_k)$-entry of $\mathcal{Q}^*(G)$ is

$$(\mathcal{Q}^*(G))_{i_1,i_2,\cdots,i_k} = \sum_{j=1}^{m} r_{i_1 j} r_{i_2 j} \cdots r_{i_k j}$$

and

$$(\mathcal{Q}^*(G)x)_i = \sum_{e\in E_i(G)} x(e)^{k-1},$$

where $x(e) = x_{j_1} + \cdots + x_{j_k}$ for $e = \{j_1, \cdots, j_k\} \in E(G)$. Furthermore, $\mathcal{Q}^*(G)$ is positive semi-definite for even k.

A kth-order n-dimensional tensor $\mathcal{T} = (t_{i_1 i_2 \cdots i_k})$ is called reducible (see [7]) if there exists a nonempty proper index subset $I \subset [n]$, such that

$$t_{i_1 i_2 \cdots i_k} = 0, \qquad \forall i_1 \in I,\ \forall i_2, \cdots, i_k \notin I.$$

$\mathcal{T}$ is called weakly reducible (see [11]), if there exists a nonempty proper index subset $I \subset [n]$, such that

$$t_{i_1 i_2 \cdots i_k} = 0, \quad \forall i_1 \in I, \text{ and at least one of } i_2, \cdots, i_k \notin I.$$

If $\mathcal{T}$ is not reducible, then $\mathcal{T}$ is called irreducible. Analogously, if $\mathcal{T}$ is not weakly reducible, then $\mathcal{T}$ is called weakly irreducible. It is easy to see that irreducibility implies weak irreducibility.

If G is a connected k-uniform hypergraph with $k \geqslant 2$, it is easy to see that both $\mathcal{A}_\alpha(G)$ and $Q^*(G)$ are weakly irreducible (see [17] and [27]).

Lemma 2.2 ([20]) *Let $\mathcal{T}$ be a symmetric nonnegative tensor of order k and dimension n. Then,*

$$\rho(\mathcal{T}) = \max\left\{ x^{\mathrm{T}}(\mathcal{T}x) \middle| x \in \mathbb{R}^n_+, \sum_{i=1}^{n} x_i^k = 1 \right\}.$$

Furthermore, $x \in \mathbb{R}^n_+$ with $\sum_{i=1}^{n} x_i^k = 1$ is an eigenvector of $\mathcal{T}$ corresponding to $\rho(\mathcal{T})$ if and only if it is an optimal solution of the above maximization problem.

Lemma 2.3

i) [34] *If $\mathcal{T}$ is a nonnegative tensor of order k and dimension n, then $\rho(\mathcal{T})$ is an H^+-eigenvalue of $\mathcal{T}$.*
ii) [11] *If, furthermore, $\mathcal{T}$ is weakly irreducible, then $\rho(\mathcal{T})$ is the unique H^{++}-eigenvalue of $\mathcal{T}$, with the unique eigenvector $x \in \mathbb{R}^n_{++}$, up to a positive scaling coefficient.*
iii) [7] *If, moreover, $\mathcal{T}$ is irreducible, then $\rho(\mathcal{T})$ is the unique H^+-eigenvalue of $\mathcal{T}$, with the unique eigenvector $x \in \mathbb{R}^n_+$, up to a positive scaling coefficient.*

For more details on the Perron–Frobenius theorem of nonnegative tensors, one can refer to a survey [8]. From Lemma 2.2, it is easy to derive that $\rho(\mathcal{T})$ can also be rewritten as follows:

$$\rho(\mathcal{T}) = \max\left\{ \frac{x^{\mathrm{T}}(\mathcal{T}x)}{x^{\mathrm{T}}(\mathcal{I}x)} \Big| x \in \mathbb{R}^n_+, x \neq 0 \right\},$$

where $\mathcal{T}$ and $\mathcal{I}$ have the same order and dimension. Here, $x^{\mathrm{T}}(\mathcal{I}x) = x_1^k + x_2^k + \cdots + x_n^k = \|x\|_k^k$. By Lemma 2.3, for a symmetric weakly irreducible nonnegative tensor $\mathcal{T}$, there exists a unique positive eigenvector x with $\|x\|_k^k = 1$ corresponding to $\rho(\mathcal{T})$ which is called the principal eigenvector of $\mathcal{T}$.

3 Some Lemmas

In this section, we present some results which will be used in the proof of the main results.

Lemma 3.1 ([6]) *Let $G = (V(G), E(G))$ be a k-uniform hypergraph that is the disjoint union of k-uniform hypergraphs $G_1 = (V(G_1), E(G_1))$ and $G_2 = (V(G_2), E(G_2))$. Then, as sets, $spec(\mathcal{A}(G)) = spec(\mathcal{A}(G_1)) \cup spec(\mathcal{A}(G_2))$. Considered as multisets, an eigenvalue λ with multiplicity m in $spec(\mathcal{A}(G_1))$ contributes λ to $spec(\mathcal{A}(G))$ with multiplicity $m(k-1)^{|V(G_2)|}$.*

Similarly, we have the analogous results for $\mathcal{A}_\alpha$-tensors and incidence $\mathcal{Q}$-tensors. Let $G = (V(G), E(G))$ and $G' = (V(G'), E(G'))$ be two k-uniform hypergraphs. If $V(G') \subset V(G)$ and $E(G') \subset E(G)$, then G' is called a subhypergraph of G. If G' is a subhypergraph of G and $G' \neq G$, then G' is called a proper subhypergraph of G.

Lemma 3.2

(i) *Let $G = (V(G), E(G))$ and $G' = (V(G'), E(G'))$ be two k-uniform hypergraphs. If G' is a subhypergraph of G, then $\rho(\mathcal{A}_\alpha(G')) \leqslant \rho(\mathcal{A}_\alpha(G))$.*

(ii) *Let $G = (V(G), E(G))$ and $G' = (V(G'), E(G'))$ be two k-uniform hypergraphs. If G' is a subhypergraph of G, then $\rho(\mathcal{Q}^*(G')) \leqslant \rho(\mathcal{Q}^*(G))$.*

Proof

(i) By Lemma 3.1, without loss of generality, we assume that both G and G' are connected. Let x be a nonnegative H-eigenvector corresponding to $\rho(\mathcal{A}_\alpha(G'))$ with $\|x\|_k^k = 1$. Let y be the vector with the ith component $y_i = x_i$ for $i \in V(G')$, $y_i = 0$ for $i \in V(G) \setminus V(G')$. Obviously, $\|y\|_k^k = 1$. Then, by Lemmas 2.2 and 2.3,

$$
\begin{aligned}
\rho(\mathcal{A}_\alpha(G')) &= x^{\mathrm{T}}(\mathcal{A}_\alpha(G')x) \\
&= \alpha \sum_{i\in V(G')} d'_i x_i^k + (1-\alpha)k \sum_{e\in E(G')} x^e \\
&= \alpha \sum_{i\in V(G')} d'_i y_i^k + (1-\alpha)k \sum_{e\in E(G')} y^e \\
&\leqslant \alpha \sum_{i\in V(G)} d_i y_i^k + (1-\alpha)k \sum_{e\in E(G)} y^e \\
&= y^{\mathrm{T}}(\mathcal{A}_\alpha(G)y) \\
&\leqslant \rho(\mathcal{A}_\alpha(G)).
\end{aligned}
$$

Furthermore, it is easy to see that the equality holds if and only if $G = G'$ by Lemma 2.2.

(ii) Also by Lemma 3.1, we assume that G and G' are connected. Let x be a nonnegative H-eigenvector corresponding to $\rho(\mathcal{Q}^*(G'))$ with $\|x\|_k^k = 1$. Let y be the vector with the ith component $y_i = x_i$ for $i \in V(G')$, $y_i = 0$ for $i \in V(G) \setminus V(G')$. Obviously, $\|y\|_k^k = 1$. Then, by Lemmas 2.2 and 2.3,

$$
\begin{aligned}
\rho(\mathcal{Q}^*(G')) &= x^{\mathrm{T}}(\mathcal{Q}^*(G')x) \\
&= \sum_{e\in E(G')} x(e)^k \\
&= \sum_{\{i_1,\cdots,i_k\}\in E(G')} (x_{i_1} + \cdots + x_{i_k})^k \\
&= \sum_{\{i_1,\cdots,i_k\}\in E(G')} (y_{i_1} + \cdots + y_{i_k})^k \\
&\leqslant \sum_{\{i_1,\cdots,i_k\}\in E(G)} (y_{i_1} + \cdots + y_{i_k})^k \\
&= \sum_{e\in E(G)} y(e)^k \\
&= y^{\mathrm{T}}(\mathcal{Q}^*(G)y) \\
&\leqslant \rho(\mathcal{Q}^*(G)).
\end{aligned}
$$

Furthermore, it is easy to see that the equality holds if and only if $G = G'$ by Lemma 2.2.

Li et al. [27] introduced an operation of moving edges on hypergraphs. Let G be a hypergraph with $v \in V(G)$ and $e_1, \cdots, e_r \in E(G)$, such that $v \notin e_i$ for $i = 1, \cdots, r$. If $e'_i = (e_i \setminus \{v_i\}) \cup \{v\} \notin E(G)$ are distinct for $v_i \in e_i$ with $i = 1, \cdots, r$, then we can obtain a hypergraph G' from G by deleting edges $\{e_1, \cdots, e_r\}$ and adding edges $\{e'_1, \cdots, e'_r\}$. Moreover, this operation is called the edge-shifting operation and G' is said to be the hypergraph obtained from G by the edge-shifting operation with moving edges $(e_1, \cdots, e_r)$ from $(v_1, \cdots, v_r)$ to v. Notice that $v_1, \cdots, v_r$ need not be distinct. Roughly speaking, the edge-shifting operation can be regarded as the shifting method (see [16]) in extremal set theory. According to the property of the shifting method, one can easily deduce that the resulting k-uniform hypergraph of an intersecting k-uniform hypergraph after the edge-shifting operation is still intersecting. Furthermore, the following two lemmas present how the $\mathcal{A}_\alpha$-spectral radius and the incidence $\mathcal{Q}$-spectral radius of a hypergraph change after the edge-shifting operation.

Lemma 3.3 ([17]) *Let G be a connected hypergraph and G' be the hypergraph obtained from G by the edge-shifting operation with moving edges $(e_1, \cdots, e_r)$ from $(v_1, \cdots, v_r)$ to v. If x is a principal eigenvector of $\mathcal{A}_\alpha(G)$ corresponding to $\rho(\mathcal{A}_\alpha(G))$ and $x_v \geqslant \max\limits_{1\leqslant i\leqslant r}\{x_{v_i}\}$, then $\rho(\mathcal{A}_\alpha(G')) > \rho(\mathcal{A}_\alpha(G))$.*

Lemma 3.4 ([27]) *Let G be a connected hypergraph and G' be the hypergraph obtained from G by the edge-shifting operation with moving edges $(e_1, \cdots, e_r)$ from $(v_1, \cdots, v_r)$ to v. If x is a principal eigenvector of $\mathcal{Q}^*(G)$ corresponding to $\rho(\mathcal{Q}^*(G))$ and $x_v \geqslant \max\limits_{1\leqslant i\leqslant r}\{x_{v_i}\}$, then $\rho(\mathcal{Q}^*(G')) > \rho(\mathcal{Q}^*(G))$.*

Furthermore, we can use the above lemmas to prove the following assertions.

Corollary 3.5

(i) *If G is a hypergraph with the maximum $\mathcal{A}_\alpha$-spectral radius among all the connected hypergraphs with a fixed number of vertices and edges, then G contains a vertex v adjacent to all other vertices.*

(ii) *If G is a hypergraph with the maximum incidence $\mathcal{Q}$-spectral radius among all the connected hypergraphs with a fixed number of vertices and edges, then G contains a vertex v adjacent to all other vertices.*

Proof We only prove (i). By Lemma 2.3, let x be the principal eigenvector of $\mathcal{A}_\alpha(G)$ corresponding to $\rho(\mathcal{A}_\alpha(G))$ and let $x_{u_0} = \max\{x_v : v \in V(G)\}$. Suppose that there exists a vertex not adjacent to u_0, say w. As G is connected, there exists a path connecting u_0 and w, say $u_0e_1u_1 \cdots u_{t-1}e_tu_t$, where $t \geqslant 2$ and $u_t = w$. Let $e_t' := (e_t \setminus \{u_{t-1}\}) \cup \{u_0\}$. Then, $e_t' \notin E(G)$, otherwise w would be adjacent to u_0. We obtain a connected hypergraph G' from G by the edge-shifting operation with moving the edge e_t from u_{t-1} to u_0. Moreover, the edge-shifting operation does not change the number of vertices and edges. Since $x_{u_0} \geqslant x_{u_{t-1}}$, by Lemma 3.3, we obtain $\rho(\mathcal{A}_\alpha(G')) > \rho(\mathcal{A}_\alpha(G))$, which is a contradiction. Hence, we complete the proof.

Recall that an automorphism of a k-uniform hypergraph G is a permutation σ of $V(G)$, such that $\{i_1, i_2, \cdots, i_k\} \in E(G)$ if and only if $\{\sigma(i_1), \sigma(i_2), \cdots, \sigma(i_k)\} \in E(G)$, for any $i_j \in V(G), j = 1, \cdots, k$. The group of all automorphisms of G is denoted by $\mathrm{Aut}(G)$. Shao [31] introduced the concept of permutational similarity for tensors as follows: for two order k and dimension n tensors $\mathcal{A}$ and $\mathcal{B}$, if there exists a permutation matrix $P = P_\sigma$ (corresponding to a permutation $\sigma \in S_n$), such that $\mathcal{B} = P\mathcal{A}P^{\mathrm{T}}$, then $\mathcal{A}$ and $\mathcal{B}$ are called permutational similar. Furthermore, $\mathcal{A}$ and $\mathcal{B}$ are permutational similar, and they have the same characteristic polynomials and the same spectra. Using the theory of automorphism of a k-uniform hypergraph G. Li et al. ([27]) proved the following result.

Lemma 3.6 ([27]) *Let G be a connected k-uniform hypergraph and $\mathcal{Q}^* = \mathcal{Q}^*(G)$ be its (irreducible) incidence $\mathcal{Q}$-tensor. If x is the principal eigenvector of $\mathcal{Q}^*$ corresponding to $\rho(\mathcal{Q}^*)$, then*

(i) *$P_\sigma x = x$ for each automorphism of G;*
(ii) *for any orbit Ω of $Aut(G)$ and each pair of vertices $i, j \in \Omega$, the corresponding components x_i, x_j of x are equal.*

Lemma 3.7 *Let $S_{n,k,1}$ be the intersecting k-uniform hypergraph on n vertices as defined above. Then,*

$$\rho(\mathcal{A}_\alpha(S_{n,k,1})) = \binom{n-2}{k-2}(\alpha + (1-\alpha)\eta^*),$$

where $\eta^* \in \left(\frac{\alpha}{1-\alpha}(\frac{n-1}{k-1}-1), \frac{\alpha}{1-\alpha}(\frac{n-1}{k-1}-1)+(\frac{n-1}{k-1})^{\frac{1}{k-1}}\right)$ is the largest real root of the equation $x^k + \frac{\alpha}{1-\alpha}\left(1-\frac{n-1}{k-1}\right)x^{k-1} - \frac{n-1}{k-1} = 0$.

Proof Clearly, $S_{n,k,1}$ is connected, and the degree sequence of $S_{n,k,1}$ is $\left(\binom{n-1}{k-1}, \binom{n-2}{k-2}, \cdots, \binom{n-2}{k-2}\right)$. Let $x \in \mathbb{R}^n$ be a positive H-eigenvector of $\mathcal{A}_\alpha(S_{n,k,1})$ corresponding to $\rho(\mathcal{A}_\alpha(S_{n,k,1}))$. We may assume that $x_1 = \eta \in \mathbb{R}$ and $x_2 = \cdots = x_n = 1$. Hence,

$$(\mathcal{A}_\alpha(S_{n,k,1})x)_1 = \alpha\binom{n-1}{k-1}\eta^{k-1} + (1-\alpha)\binom{n-1}{k-1},$$

and for $i \in \{2, \cdots, n\}$,

$$(\mathcal{A}_\alpha(S_{n,k,1})x)_i = \alpha\binom{n-2}{k-2} + (1-\alpha)\binom{n-2}{k-2}\eta.$$

Then, by the eigenvalue equation $\mathcal{A}_\alpha(S_{n,k,1})x = \rho x^{[k-1]}$, where ρ denotes $\rho(\mathcal{A}_\alpha(S_{n,k,1}))$ for convenience, we have

$$\rho\eta^{k-1} = \alpha\binom{n-1}{k-1}\eta^{k-1} + (1-\alpha)\binom{n-1}{k-1}, \tag{1}$$

$$\rho = \alpha\binom{n-2}{k-2} + (1-\alpha)\binom{n-2}{k-2}\eta. \tag{2}$$

Combining the above two equations (Eqs. (1) and (2)), η is a real root of the following equation:

$$(1-\alpha)\eta^k + \alpha\left(1-\frac{n-1}{k-1}\right)\eta^{k-1} - (1-\alpha)\frac{n-1}{k-1} = 0. \tag{3}$$

Let

$$g(x) := (1-\alpha)x^k + \alpha\left(1-\frac{n-1}{k-1}\right)x^{k-1} - (1-\alpha)\frac{n-1}{k-1},$$

and

$$h(x) := x^k + \frac{\alpha}{1-\alpha}\left(1-\frac{n-1}{k-1}\right)x^{k-1} - \frac{n-1}{k-1}.$$

We have that $h\left(\frac{\alpha}{1-\alpha}\left(\frac{n-1}{k-1}-1\right)\right)=-\frac{n-1}{k-1}<0$, and $h\left(\frac{\alpha}{1-\alpha}\left(\frac{n-1}{k-1}-1\right)+\left(\frac{n-1}{k-1}\right)^{\frac{1}{k-1}}\right)>0$.

From above, we know $h'(x):=x^{k-2}\left(kx+\frac{\alpha}{1-\alpha}(k-n)\right)$, and when $h'(x)=0$, we have $x_1=\frac{\alpha}{1-\alpha}(\frac{n-k}{k}), x_2=0$ (with multiplicity $k-2$). It is easy to see that $h'(x)>0$ when $x\in\left(\frac{\alpha}{1-\alpha}\left(\frac{n-k}{k}\right),+\infty\right)$. Hence, $h(x)$ is increasing in the interval $\left(\frac{\alpha}{1-\alpha}\left(\frac{n-1}{k-1}-1\right),+\infty\right)$, since $\frac{\alpha}{1-\alpha}\left(\frac{n-1}{k-1}-1\right)>\frac{\alpha}{1-\alpha}\left(\frac{n-k}{k}\right)$. Moreover, $h\left(\frac{\alpha}{1-\alpha}\left(\frac{n-1}{k-1}-1\right)+\left(\frac{n-1}{k-1}\right)^{\frac{1}{k-1}}\right)>0$. Therefore, $h(x)=0$ has a unique real root in the interval $\left(\frac{\alpha}{1-\alpha}\left(\frac{n-1}{k-1}-1\right),\frac{\alpha}{1-\alpha}\left(\frac{n-1}{k-1}-1\right)+\left(\frac{n-1}{k-1}\right)^{\frac{1}{k-1}}\right)$. Since $1-\alpha>0$, from above discussion, $g(x)=0$ has the largest real root in the interval $\left(\frac{\alpha}{1-\alpha}\left(\frac{n-1}{k-1}-1\right),\frac{\alpha}{1-\alpha}\left(\frac{n-1}{k-1}-1\right)+\left(\frac{n-1}{k-1}\right)^{\frac{1}{k-1}}\right)$. Let η^* be the largest root of Eq.(3). By Eq.(3), we have

$$\rho=\binom{n-2}{k-2}(\alpha+(1-\alpha)\eta^*)=\binom{n-1}{k-1}\left(\alpha+(1-\alpha)\frac{1}{\eta^{*k-1}}\right).$$

Therefore, the assertion holds.

When $\alpha=0$, it follows from Lemma 3.7 that $\rho(\mathcal{A}(S_{n,k,1}))=\binom{n-2}{k-2}\left(\frac{n-1}{k-1}\right)^{\frac{1}{k}}$. When $\alpha=\frac{1}{2}$, it follows from Lemma 3.7 that

$$\rho(\mathcal{Q}(\mathcal{S}_{n,k,1}))=\binom{n-2}{k-2}(1+\eta^*),$$

where $\eta^*\in\left(\frac{n-1}{k-1}-1,\frac{n-1}{k-1}\right]$ is the largest real root of the equation $x^k+\left(1-\frac{n-1}{k-1}\right)x^{k-1}-\frac{n-1}{k-1}=0$.

Lemma 3.8 *Let $S_{n,k,1}$ be the intersecting k-uniform hypergraph on n vertices as defined above. Then,*

$$\rho(\mathcal{Q}^*(S_{n,k,1}))=\binom{n-2}{k-2}\left(\left(\frac{n-1}{k-1}\right)^{\frac{1}{k-1}}+k-1\right)^{k-1}.$$

Proof It follows from the definition of $S_{n,k,1}$ that $S_{n,k,1}$ has the edge number $m=\binom{n-1}{k-1}$. Let $V_0\cup V_1\cup\cdots\cup V_m$ be the disjoint partition of $V(S_{n,k,1})$, such that $|V_0|=1,|V_1|=\cdots=|V_m|=k-1$ and $E=\{V_0\cup V_i|i=1,\cdots,m\}$. Note that V_0 and $V_1\cup\cdots\cup V_m$ are two orbits of automorphism group $\mathrm{Aut}(S_{n,k,1})$. Let x be the principal eigenvector of $\mathcal{Q}^*(S_{n,k,1})$ corresponding to $\rho(\mathcal{Q}^*(S_{n,k,1}))$. Since $S_{n,k,1}$ is connected, by Lemma 3.6, we have that the components of x corresponding to vertices in V_0 and $V\setminus V_0$ are constant, respectively, and let a and b be these common values, respectively. Hence,

$$(\mathcal{Q}^*(S_{n,k,1})x)_1=\binom{n-1}{k-1}(a+(k-1)b)^{k-1},$$

and for $i \in \{2, \cdots, n\}$,

$$(\mathcal{Q}^*(S_{n,k,1})x)_i = \binom{n-2}{k-2}(a+(k-1)b)^{k-1}.$$

Then, by the eigenvalue equation $\mathcal{Q}^*(S_{n,k,1})x = \rho x^{[k-1]}$, where ρ denotes $\rho(\mathcal{Q}^*(S_{n,k,1}))$ for short, we have

$$\rho a^{k-1} = \binom{n-1}{k-1}(a+(k-1)b)^{k-1}, \tag{4}$$

$$\rho b^{k-1} = \binom{n-2}{k-2}(a+(k-1)b)^{k-1}. \tag{5}$$

Dividing Eq. (4) by Eq. (5), we obtain $(\frac{a}{b})^{k-1} = \frac{n-1}{k-1}$ which implies that $\frac{a}{b} = (\frac{n-1}{k-1})^{\frac{1}{k-1}}$. Therefore, by Eq. (5), we have

$$\rho = \binom{n-2}{k-2}\left(\frac{a}{b}+k-1\right)^{k-1} = \binom{n-2}{k-2}\left(\left(\frac{n-1}{k-1}\right)^{\frac{1}{k-1}}+k-1\right)^{k-1}.$$

Therefore, the assertion holds.

4 Main Results

Now, we are ready to state the main theorem.

Theorem 4.1 *Assume that $k \geqslant 2$ and $n \geqslant 2k$. Let G be an intersecting k-uniform hypergraph on n vertices. Then,*

$$\rho(\mathcal{A}_\alpha(G)) \leqslant \rho(\mathcal{A}_\alpha(S_{n,k,1})),$$

and

$$\rho(\mathcal{Q}^*(G)) \leqslant \rho(\mathcal{Q}^*(S_{n,k,1})).$$

Moreover, when $n > 2k$, either one of the above equalities holds if and only if $G = S_{n,k,1}$.

Proof First, we give the proof of the first assertion. Let G_0 be a hypergraph having the maximum $\mathcal{A}_\alpha$-spectral radius among all intersecting k-uniform hypergraphs on n vertices. Furthermore, there are no isolated vertices in G_0 by Lemma 3.2. In other words, for every vertex $v \in V(G_0)$, there exists one edge $e \in E(G_0)$ containing v. For any two vertices $u, v \in V(G_0)$, if there exists an edge e containing u, v, then u and v are adjacent; if there exist two edges, such that $u \in e_1$ and $v \in e_2$, then $e_1 \cap e_2 \neq \emptyset$ by G_0 being an intersecting hypergraph, so there is a path ue_1we_2v, where $w \in e_1 \cap e_2$. Hence, G_0 is connected. Let x be the principal eigenvector corresponding to $\rho(\mathcal{A}_\alpha(G_0))$ with $x_{u_0} = \max\{x_v : v \in V(G_0)\}$. Now, we prove that G_0 contains a vertex adjacent to all other vertices. If not, suppose that there exists one vertex w which is not adjacent to u_0. Since G_0 is connected, there must exist one path connecting u_0 and w, say $u_0e_1u_1e_2u_2\cdots e_tu_t$, where $t \geqslant 2$, and $u_t = w$. Let $e'_t = (e_t \setminus \{u_{t-1}\}) \cup \{u_0\}$. Then, $e'_t \notin E(G_0)$; otherwise, u_0 would be adjacent to w. Let G'_0 be the hypergraph obtained from G_0 through the edge-shifting operation with moving the edge e_t from u_{t-1} to u_0. Worth to say, G'_0 is still an intersecting k-uniform hypergraph by the property of the edge-shifting operation. Since $x_{u_0} \geqslant x_{u_{t-1}}$, by Lemma 3.3, we have $\rho(\mathcal{A}_\alpha(G'_0)) > \rho(\mathcal{A}_\alpha(G_0))$, which is a contradiction. Furthermore, it follows from the definition of $S_{n,k,1}$ that G_0 must be a subhypergraph of $S_{n,k,1}$. By Lemma 3.2, we have $\rho(\mathcal{A}_\alpha(G_0)) \leqslant \rho(\mathcal{A}_\alpha(S_{n,k,1}))$. Note that $S_{n,k,1}$ is an intersecting k-uniform hypergraph on n vertices. Then, $\rho(\mathcal{A}_\alpha(G_0)) \geqslant \rho(\mathcal{A}_\alpha(S_{n,k,1}))$. Hence, $\rho(\mathcal{A}_\alpha(G_0)) = \rho(\mathcal{A}_\alpha(S_{n,k,1}))$. By Lemma 3.2 and G_0 is a subhypergraph of $S_{n,k,1}$, we have $G_0 = S_{n,k,1}$.

The proof of the second assertion is very similar to that of the first assertion except for Lemma 3.4 and omitted.

Corollary 4.2 *Let G be an intersecting k-uniform hypergraph on n vertices with $k \geqslant 2$, $n \geqslant 2k$. Then,*

$$\rho(\mathcal{A}_\alpha(G)) \leqslant \binom{n-2}{k-2}(\alpha + (1-\alpha)\eta^*),$$

where $\eta^* \in \left(\frac{\alpha}{1-\alpha}\left(\frac{n-1}{k-1}-1\right), \frac{\alpha}{1-\alpha}\left(\frac{n-1}{k-1}-1\right)+\left(\frac{n-1}{k-1}\right)^{\frac{1}{k-1}}\right)$ *is the largest real root of the equation* $x^k + \frac{\alpha}{1-\alpha}\left(1-\frac{n-1}{k-1}\right)x^{k-1} - \frac{n-1}{k-1} = 0$, *and*

$$\rho(\mathcal{Q}^*(G)) \leqslant \binom{n-2}{k-2}\left(\left(\frac{n-1}{k-1}\right)^{\frac{1}{k-1}} + k - 1\right)^{k-1}.$$

Moreover, when $n > 2k$, either one of the above equalities holds if and only if $G = S_{n,k,1}$.

Proof The assertions follow directly from Lemmas 3.7, 3.8, and Theorem 4.1.

Acknowledgements The authors would like to thank the anonymous referees for their careful reading and providing helpful suggestions and comments on an earlier version of this paper, which lead to a great improvement of the presentation.

References

1. Berge, C.: Graphs and Hypergraphs, vol. 6, 2nd edn. North-Holland Mathematical Library, North-Holland Publishing Co., Amsterdam; London (1976)
2. Bretto, A.: Hypergraph Theory: an Introduction. Springer, Berlin (2013)
3. Bai, S.L., Lu, L.Y.: A bound on the spectral radius of hypergraphs with *e* edges. Linear Algebra Appl. **549**, 203–218 (2018)
4. Bu, C.J., Zhang, X., Zhou, J., Wang, W.Z., Wei, Y.M.: The inverse, rank and product of tensors. Linear Algebra Appl. **446**, 269–280 (2014)
5. Chen, D.M., Chen, Z.B., Zhang, X.D.: Spectral radius of uniform hypergraphs and degree sequences. Front. Math. China **12**, 1279–1288 (2017)
6. Cooper, J., Dutle, A.: Spectra of uniform hypergraphs. Linear Algebra Appl. **436**, 3268–3292 (2012)
7. Chang, K.C., Pearson, K., Zhang, T.: Perron–Frobenius theorem for nonnegative tensors. Commun. Math. Sci. **6**, 507–520 (2008)

8. Chang, K.C., Qi, L.Q., Zhang, T.: A survey on the spectral theory of nonnegative tensors. Numer. Linear Algebra Appl. **20**, 891–912 (2013)
9. Erdős, P., Ko, C., Rado, R.: Intersection theorems for systems of finite sets. Quart. J. Math. Oxford Ser. **12**(2), 313–320 (1961)
10. Frankl, P.: Erdős–Ko–Rado theorem with conditions on the maximal degree. J. Combin. Theory Ser. A **46**, 252–263 (1987)
11. Friedland, S., Gaubert, S., Han, L.: Perron–Frobenius theorem for nonnegative multilinear forms and extensions. Linear Algebra Appl. **438**, 738–749 (2013)
12. Frankl, P., Han, J., Huang, H., Zhao, Y.: A degree version of the Hilton–Milner theorem. J. Combin. Theory Ser. A **155**, 493–502 (2018)
13. Frankl, P., Tokushige, N.: A note on Huang–Zhao theorem on intersecting families with large minimum degree. Discrete Math. **340**, 1098–1103 (2017)
14. Frankl, P., Tokushige, N.: Invitation to intersection problems for finite sets. J. Combin. Theory Ser. A **144**, 157–211 (2016)
15. Fan, Y.Z., Tan, Y.Y., Peng, X.X., Liu, A.H.: Maximizing spectral radii of uniform hypergraphs with few edges. Discuss. Math. Graph Theory **36**, 845–856 (2016)
16. Godsil, C., Meagher, K.: Erdős–Ko–Rado theorems: algebraic approaches. Cambridge Studies in Advanced Mathematics, vol. 149. Cambridge University Press, Cambridge (2016)
17. Guo, H.Y., Zhou, B.: On the α-spectral radius of uniform hypergraphs. Discuss. Math. Graph Theory **40**, 559–575 (2020)
18. Han, J., Kohayakawa, Y.: The maximum size of a non-trivial intersecting uniform family that is not a subfamily of the Hilton–Milner family. Proc. Am. Math. Soc. **145**, 73–87 (2017)
19. Hilton, A.J.W., Milner, E.C.: Some intersection theorems for systems of finite sets. Quart. J. Math. Oxford Ser. **2**(18), 369–384 (1967)
20. Hu, S.L., Qi, L.Q.: The Laplacian of a uniform hypergraph. J. Comb. Optim. **29**, 331–366 (2015)
21. Hu, S.L., Qi, L.Q.: The eigenvectors associated with the zero eigenvalues of the Laplacian and signless Laplacian tensors of a uniform hypergraph. Discrete Appl. Math. **169**, 140–151 (2014)
22. Hu, S.L., Qi, L.Q., Xie, J.S.: The largest Laplacian and signless Laplacian H-eigenvalues of a uniform hypergraph. Linear Algebra Appl. **469**, 1–27 (2015)
23. Huang, H., Zhao, Y.N.: Degree versions of the Erdős–Ko–Rado theorem and Erdős hypergraph matching conjecture. J. Combin. Theory Ser. A **150**, 233–247 (2017)
24. Keevash, P., Lenz, J., Mubayi, D.: Spectral extremal problems for hypergraphs. SIAM J. Discrete Math. **28**, 1838–1854 (2014)
25. Lin, H.Y., Guo, H.Y., Zhou, B.: On the α-spectral radius of irregular uniform hypergraphs. Linear Multilinear Algebra **68**, 265–277 (2020)
26. Lin, H.Y., Mo, B., Zhou, B., Weng, W.M.: Sharp bounds for ordinary and signless Laplacian spectral radii of uniform hypergraphs. Appl. Math. Comput. **285**, 217–227 (2016)
27. Li, H.H., Shao, J.Y., Qi, L.Q.: The extremal spectral radii of k-uniform supertrees. J. Comb. Optim. **32**, 741–764 (2016)
28. Nikiforov, V.: Merging the A- and Q-spectral theories. Appl. Anal. Discrete Math. **11**, 81–107 (2017)
29. Qi, L.Q.: Eigenvalues of a real supersymmetric tensor. J. Symbolic Comput. **40**, 1302–1324 (2005)
30. Qi, L.Q., Shao, J.Y., Wang, Q.: Regular uniform hypergraphs, s-cycles, s-paths and their largest Laplacian H-eigenvalues. Linear Algebra Appl. **443**, 215–227 (2014)
31. Shao, J.Y.: A general product of tensors with applications. Linear Algebra Appl. **439**, 2350–2366 (2013)
32. Xiao, P., Wang, L.G.: The maximum spectral radius of uniform hypergraphs with given number of pendant edges. Linear Multilinear Algebra **67**, 1392–1403 (2019)
33. Xiao, P., Wang, L.G., Lu, Y.: The maximum spectral radii of uniform supertrees with given degree sequences. Linear Algebra Appl. **523**, 33–45 (2017)
34. Yang, Y.N., Yang, Q.Z.: Further results for Perron–Frobenius theorem for nonnegative tensors. SIAM J. Matrix Anal. Appl. **31**, 2517–2530 (2010)
35. Yuan, X.Y., Zhang, M., Lu, M.: Some upper bounds on the eigenvalues of uniform hypergraphs. Linear Algebra Appl. **484**, 540–549 (2015)