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Wolfgang Gaschütz: Life and Work

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Professor Dr. Dr. h. c. Wolfgang Gaschütz passed away in Kiel (Germany) on November 7, 2016. With him, the mathematical community lost one of its most prominent creative minds and last contemporary witnesses of the overwhelming development of the theory of groups throughout the decades after the Second World War.

Born on June 11, 1920, in the tiny village of Karlshof in the Oderbruch region near the eastern border of Germany, his adolescent years fell into the gloomiest chapter of recent German history. The vicissitudes at the end of the war made Gaschütz and his then young family of three end up in the province of Schleswig-Holstein in the very north of the country. In November of 1945, the University of Kiel was reopened. However, large parts of the town had been destroyed, and the academic life began in deplorable conditions. For one year, four ships in the harbour of Kiel were the only place to live for the students, and even lectures took place on board. The British authorities appointed the neuropathologist Hans Gerhard Creutzfeldt as Rector of the university, and students' enrolment took place in a bureau of the


Lichtbild des Inhabers
Eigenhändige Unterschrift des Inhabers:
(Vor- und Zuname)
Ort und Tag der Ausstellung:
Kiel, den 27. Nov. 1945
Der Rektor
der Christian-Albrechts-Universität

Die Ausweiskarte ist nur mit der angehängten ersten Hälfte und nur für die von der Hochschule bescheinigten Halbjahre gültig.
Gültigkeitsvermerke der Hochschulbehörden:
Universität Kiel
Winter-Semester 1945/46
Hochschulnummer: 1615
UNIVERSITÄT KIEL
SOMMER-SEMESTER 1946
Universität Kiel
Winter-Semester 1946/47
Universität Kiel
Winter-Semester 1947

Gaschütz: student identity document Foto: pur.pur, psychiatric clinic at Schleswig, about 60 km from Kiel. "Thus, after he insanity of the war, I was enrolled in the mental hospital"[1], said Gaschütz in 2005. This sounds humorous and ironical – the typical iridescent tone which he loved –, but was also meant as an omen of deeper relevance: "Our generation of those who had escaped the war longed for spiritual liberation through scientific virtues", he continued, and added: "As barely any aims of studying like a good job and a good salary were visible, we were happily able to choose our disciplines exclusively on the basis of our personal interests and to study them in academic freedom".[2] Scientific literature was rarely available so that the students would frequently exchange among each other the few books in their belongings. On one of these occasions, Gaschütz swapped a book on a geometric subject for Andreas Speiser's textbook [50]. The fascination of finite groups started to attract him irresistibly, and from then on never left him. For Kiel, this was to be the starting point of the growth of the branch of Algebra for which it since became widely known: Group Theory.

Group-theoretically, Gaschütz was essentially a self-made man. When he took up his studies, the only professor of mathematics at Kiel was Karl-Heinrich Weise, who, however, worked in non-algebraic research areas. Still Gaschütz owed him a great deal and was grateful throughout life for his teaching, encouragement and support. Many years later, Gaschütz dedicated papers [23],[43] to Weise on two special occasions, signs of his abiding gratitude. Gaschütz benefitted from the fact that Helmut Hasse and Ernst Witt were active in Hamburg. But there was no group theorist near Kiel in those days so that he had to shape his algebraic way for himself, without any local supervisor. In later years, he considered Helmut Wielandt at Tübingen as a transregional mentor of group theory. Under Weise's aegis, Gaschütz obtained his doctoral degree as early as 1949. Two years later and on a completely different subject, he submitted his first paper [13] for publication. Its famous main result, now known as the *Gaschütz splitting theorem*, has become an integral part of the textbooks on group theory. Gaschütz's first three publications [13],[14],[15] open up very different and wide areas of group

[1] "Ich wurde also nach dem Kriegsirrsinn in der Nervenheilanstalt immatrikuliert" [48]

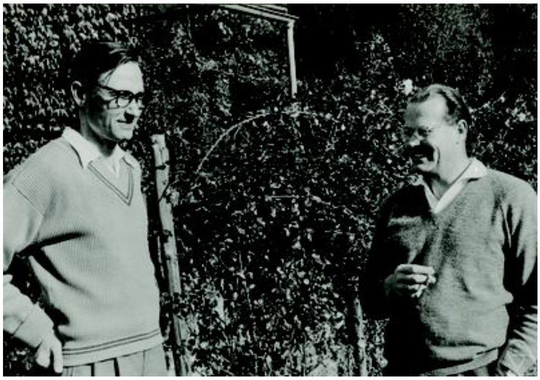
[2] "Unsere Generation der dem Krieg Entronnenen sehnte sich nach geistiger Befreiung durch die Wissenschaft. Da sich kaum Berufs- und Verdienstmöglichkeiten als Studienziel anboten, war man in der glücklichen Lage, die Studienfächer allein nach eigenem Interesse zu wählen und in akademischer Freiheit zu studieren" [48]

theory. In a broader sense, they may stand for the main topics of his long-term research:

1) *Cohomology of groups*,
2) *Representation theory of groups*,
3) *Soluble groups.*

The results in his papers [13],[20],[22] (the latter being based on [19, Satz 1]) are approached by a brilliant usage of a repertoire which constitutes a preliminary stage of Gaschütz's later cohomological thinking. For Gaschütz it was not just the outcome that was important. He was never satisfied before the proof had not reached the level of elegance which he desired in his deep sense of algebraic beauty. He did not like lengthy case analyses, and he always tried to find some general reason when a proof initially seemed to hinge upon complicated calculations. In this sense, he felt that the contents of [13] deserved a more satisfactory approach. He found the desired general view for it in cohomology theory which he mainly treated as an instrument for tackling structural problems on groups. His introductory notes [31] from a course for students at Padova (Italy) are distinctively written in this spirit. For example, part of the contents of [22] is included here in a cohomological environment. Gaschütz loved to generalise – not, however, for the sake of generalising, but when it offered a gain in insight. The cohomological presentation of his splitting theorem in [47, I, 17.4] is therefore an example of his algebraic handwriting of the 1960s. Gaschütz saw the usefulness of cohomology theory for the pursuit of group-theoretical problems underlined by a further discovery due to this approach: He showed that every finite p-group of order $> p$ has an outer p-automorphism ([27],[28],[47, III, §19]). His highly original and short cohomological proof was qualified by Karl W. Gruenberg as "one of the most ingenious applications of cohomology to a purely group theoretical problem" (see [46, 7.5, p. 110]).

In his second paper [14], Gaschütz treats the connection between Maschke's idea of "averaging" and the concepts of projectivity and injectivity of modules, proving their equivalence for group algebras of finite groups over fields. Ten years later, it was presented as the starting point of the important development called *Gaschütz-Ikeda-Higman theory* in the now classical textbook [7]. In a further contribution [18] to representation theory he obtained an elegant solution of a problem which may be traced back to Burnside, characterising the finite groups which have a faithful irreducible complex representation. Modular representations and cohomology, both being topics of powerful developments of substantial theories in their own right, were in the first line instruments for the structure theory of groups in Gaschütz's hands. Furthermore, a highly original usage of concepts from the *theory of free groups*, some in surprising combination with those other tools, is prominent in a number of his papers (see [17],[19],[20],[26],[43]). His own words from the introduction of [17] shed light on his general view as he mentions "the ever-growing significance of the connection finite group – free group on the one hand and the modular representation theory on the other hand for the study of finite groups"[3]. A very natural link of this kind consists in the basic fact that the abelian chief factors of a finite group are irreducible modules over prime fields. Considerably simplifying a first and less general approach in [12], it is proved in [6] that all these modules belong to the 1-block of the group ring over the respective prime field. In Gaschütz's forthcoming investigations on finite soluble groups, the corresponding irreducible representations played an important role, first to be seen in [24].



Huppert (left) and Gaschütz (right), 1961
Foto: Konrad Jacobs

The Frattini subgroup had essentially had a wallflower life for almost 70 years until Gaschütz realised its potential as a group-theoretical concept and devoted an entire and vital publication [15] to this topic, on the basis of his doctoral dissertation of 1949. One of the basic results of high impact in this paper is the observation that a normal subgroup of a finite group G is nilpotent if and only if its epimorphic image modulo the Frattini subgroup $\Phi(G)$ is nilpotent. The class of finite nilpotent groups is therefore *saturated*, as introduced by Gaschütz later for an arbitrary class $\mathfrak{F}$ of groups by the condition

[3] Original text: "[die] Bedeutung, die der Zusammenhang endliche Gruppe – freie Gruppe einerseits und die modulare Darstellungstheorie andererseits für das Studium der endlichen Gruppen immer mehr erlangen"

that $G/N \in \mathfrak{F}$ implies $G \in \mathfrak{F}$ whenever G is a finite group and $N \trianglelefteq G$ such that $N \leqslant \Phi(G)$. He called a class $\mathfrak{F}$ a *formation* if it is closed under isomorphisms and satisfies the following closure conditions:

1. $N \trianglelefteq G,\ G \in \mathfrak{F} \Rightarrow G/N \in \mathfrak{F}$
2. $M, N \trianglelefteq G, G/M, G/N \in \mathfrak{F} \Rightarrow G/(M \cap N) \in \mathfrak{F}$

These definitions were triggered by the key objective of finding a common root for Hall's classical theorem and the analogous result on Carter subgroups, then still a rather young discovery [5]. Both the class of finite π-groups for a set of primes π and the class of finite nilpotent groups are saturated formations. The two theorems thus turned out to be special cases of Gaschütz's main result in [25] about non-empty saturated formations $\mathfrak{F}$: In every *finite soluble* group there exists a unique conjugacy class of so-called *$\mathfrak{F}$-projectors* of G, i. e., subgroups H such that $H\sigma$ is a maximal $\mathfrak{F}$-subgroup of $G\sigma$ for every homomorphism σ of G. The natural example of the Hall π-subgroups, being the projectors for the class of π-groups, shows, by Ph. Hall's characterisation of solubility, that the limitation to soluble groups in Gaschütz's result is indispensable.

For the universe of all finite soluble groups, however, a rapidly growing general theory arose from Gaschütz's discoveries. His principal intention was to define subgroup systems in finite soluble groups for which general existence and conjugacy theorems could be proved, in the spirit of the results on Hall subgroups and on Carter subgroups – a research programme aiming at *a general Sylow theory for finite soluble groups* as he and others would sometimes call it. Gaschütz and Ulrike Lubeseder, one of his research students at that time, obtained a most satisfactory description of all saturated formations (see [25],[41] and, for a first comprehensive account, [47, VI, § 7]) where the abovementioned action of a finite soluble group on its chief factors plays a vital role (see [47, VI, 7.25]). The general question for which classes $\mathfrak{F}$ there exist $\mathfrak{F}$-projectors in every finite soluble group led to the more general notion of a *Schunck class* [49], still being saturated but not necessarily a formation. They were investigated by and named after Hermann Schunck, again one of Gaschütz's research students.

A major step in this flourishing theory was the idea of dualising its basic notions. An *$\mathfrak{F}$-injector* of a group G for a class of groups $\mathfrak{F}$ (closed under isomorphisms) is defined to be a subgroup H of G such that $H \cap N$ is a maximal $\mathfrak{F}$-subgroup of N for all $N \trianglelefteq\trianglelefteq G$. In a remarkable analogy with the behaviour of projectors, it was proved in [11] (interestingly, making use of [5]) that there exists a unique conjugacy class of $\mathfrak{F}$-injectors in every finite soluble group if and only if the (non-empty) class $\mathfrak{F}$ of finite soluble groups is a *Fitting class* – the definition of which is the following exact dualisation of that of a formation:

1. $N \trianglelefteq G,\ G \in \mathfrak{F} \Rightarrow N \in \mathfrak{F}$

2. $M, N \trianglelefteq G,\ M, N \in \mathfrak{F} \Rightarrow MN \in \mathfrak{F}$

Noboru Ito's attempt of dualising the definition of the Frattini subgroup, by considering the factor group modulo the join $\Psi(G)$ of all minimal subgroups of a group G, led to an elegant and unexpected result. "Ingeniously applying Schreier's theorems on free groups" (N. Ito in MR178058), Gaschütz proves that every finite group occurs in the form $G/\Psi(G)$ [26].

Projectors of Schunck classes (in particular, of saturated formations) and injectors of Fitting classes provide a rich source of canonical conjugacy classes of subgroups in finite soluble groups. The "untypical" case that these consist of a single subgroup in every finite soluble group is examined in [4] where the classes with this property are called *normal*. While the normal Schunck classes (of finite soluble groups) were easily identified as the classes of all soluble π-perfect groups for some set of primes π, the description of all normal Fitting classes turned out to be an intricate problem. By the main result in [4] (a deepening of which may be found in [32]), a non-trivial Fitting class $\mathfrak{F}$ of finite soluble groups is normal if and only if the factor group modulo the maximal normal $\mathfrak{F}$-subgroup of a finite soluble group is always abelian. Hence there exists a unique smallest non-trivial normal Fitting class, the Fitting class generated by all commutator subgroups of finite soluble groups. Its nature remained mysterious until T.R. Berger, in his extensive investigation [2], gave a description, instrumentalising a type of construction of normal Fitting classes which is based on the transfer. Recently, further progress has been made on this topic [3].

Gaschütz's pioneering work had an enormous impact on worldwide group-theoretic research activities (some comprehensive publications on these developments are [1],[8],[9],[10],[51],[52]). But during the 1970s, he observed that most of these went into directions which were no longer his algebraic home. Gaschütz never aimed at a "theory of group classes". He loved *finite* (and, in particular,

Gaschütz 2005 Foto: pur.pur, Copyright: CAU 2005

soluble) *groups* and devoted all his creative energy to them. He knew that his concepts would give rise to many generalisations and to theory formation on a "meta level". But he was not willing to enter these areas. At the end of the 1970s he found that the development had reached a conclusive stage for the aims which he had had in mind. Otherwise always refusing to write books or long survey articles, he made an exception in this case. In 1979 he published a complete account [37] of the theory in the best organised way in his view, on the basis of a preliminary version [29]. It was his final word on the topic. It is no coincidence that we find the following paragraph of reminding character in its introduction:

> "*Intentionally, 'formations' and 'Fitting classes' are not mentioned in the title, as is the case with most publications on our subject. This expresses that, here, for us, classes of groups are interesting only if they yield universal constructions like characteristic subgroups or classes of subgroups which can effectively serve the structural analysis for all soluble groups. It seems this aspect for the foundation of the theory was sometimes lost later on.*"

Gaschütz's early paper [24] exhibits a specific conjugacy class of subgroups the intersection of which is the Frattini subgroup, in every finite soluble group. Unlike the Frattini subgroup, these pre-Frattini subgroups "behave well" with respect to homomorphisms, as is proved in [24, Satz 6.4]. The paper was a contribution to the objective recalled in the above quotation, from the time before the idea of group classes was born, and certainly a close forerunner. But also later Gaschütz pursued his original aim occasionally without referring to any group classes: One result of [30] is that, for every finite soluble group G and $R \trianglelefteq P \in Syl_p(G)$, all subgroups H which are maximal with respect to the property that $R \in Syl_p(H)$ are conjugate.

A further result to be mentioned here is a fine generalisation of Hall's theorem, an existence and conjugacy theorem for subgroups which are maximal with respect to certain index conditions (see [38] which considerably improves upon [35]).

Clearly, the main lines of Gaschütz's research, which have been sketched here, are accompanied by numerous further publications on miscellaneous topics which aroused his never fading interest during all the years (see References). The typical feature of his papers is the unexpected trick, the spontaneous and original idea in a proof, being the salient point in the solution of an attractive problem of immediate general group-theoretic interest. Accordingly, his preferred form of publication was the "note" of very few pages, which in numerous cases contained an ingenious stroke. Gaschütz was extremely interested in the following problems from his main areas of research, which are still open:

1. How can the conjugacy part of the Schur-Zassenhaus theorem be proved without involving the odd order theorem by Feit and Thompson?

2. Which are the irreducible factor modules of the maximal submodule of a principal indecomposable module (in the 1-block) of a finite group?

3. Which groups belong to the smallest Fitting class containing the non-abelian group of order 6?

After receiving lecturer's rights by his habilitation in 1953, Gaschütz was appointed "Diätendozent" (a position remotely similar to that of a reader in Britain) in 1956, and full professor in 1963. Preferring a chair at Kiel for his activity, he declined calls to Karlsruhe and Mainz. From 1963, Gaschütz was visiting professor at several universities in various parts of the world (Queen Mary College London (twice), Warwick (3 times), Padova,

Gaschütz 1973
Foto: Konrad Jacobs.

Florence, Naples, Chicago, Canberra). He loved the vivid discussions with his colleagues in these places and on international meetings. They were the origin of several joint publications on various group-theoretic subjects, on top of some which developed occasionally in Kiel: [4],[6],[11], [12],[41],[42],[43],[44],[45]. Gaschütz's captivating and frequently amusing manner of lecturing attracted many students. Those who wrote their thesis with him will never forget his completely informal discussion time, a generous hour offered daily in a small lecture room close to his study and always ending at noon. Whoever came and had a modest question, a doubtful idea or a sparkling proposal would certainly provoke some irony but, much more importantly, more spontaneous ideas than could be carried home. Gaschütz had many descendants as a glance at the impressive list of the Mathematics Genealogy Project shows.[4] The first was Joachim Neubüser who completed his dissertation in 1957. During the particularly fruitful period between 1962 and 1976, almost each year a research student or even more than one received their doctorate with a thesis under Gaschütz's supervision. Moreover, eight of Gaschütz's former research students took up postdoctorate studies and received their habilitation; in Kiel D. Blessenohl, K. Johnsen, O.-U. Kramer, H. Laue, J. Neubüser, K.-U. Schaller (in alphabetical order); furthermore R. Laue in Aachen and D. Voigt in Bonn.

During the preparation of his monumental work [47] and its two subsequent further volumes (with co-author Norman Blackburn (Manchester)), Bertram Huppert (Mainz) would make extensive visits to Kiel on a regular annual basis, to discuss in detail the draft of the planned book with Gaschütz and a growing number of younger group theorists. For many years, Gaschütz, Huppert and Gruenberg (London)

4 https://www.genealogy.math.ndsu.nodak.edu/id.php?id=21552%

were the organisers of an annual international conference on group theory at the Oberwolfach Research Institute. In 1988, Gaschütz became professor emeritus. In 2000, the Francisk Scorina Gomel State University, Republic of Belarus, conferred an honorary doctorate on Gaschütz.

Gaschütz's farewell paper [40] appeared in 2006, in the typical style of so many of his publications: original, relevant, short. For a last time he rose to speak on a topic in representation theory, here on group rings over the integers. For ten more years, he was regularly seen on the campus on Tuesdays when he would meet colleagues from his active time for discussions.

Gaschütz 2005 Foto: pur.pur, Copyright: CAU 2005

Gaschütz has been married since 1943 and is survived by his wife, his two daughters and his son.

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