HADAMARD'S COURS D'ANALYSE

Cours d'Analyse. By J. Hadamard. Volume I. Paris, J. Hermann, 1925, 1927. xxxii+624 pp.

The title *Cours d'Analyse* calls to mind a long array of prominent names, Jordan, Picard, Goursat, de la Vallée-Poussin, whose books on this subject have long been standard works of reference and sources of inspiration. The name of Hadamard joined to "Cours d'Analyse" would lead one to expect something in the same category, a treatment of some standard subjects, but with some decidedly interesting additional original contributions. If one brings such expectations to the volume under consideration, one is doomed to disappointment. Hadamard is careful to warn the reader in the introduction that he does not contemplate the type of product which his predecessors have made famous, by including topics and developments not considered in lectures, but to limit himself to the material which he has been presenting in the course in analysis at the École Polytechnique. As a consequence, the first volume of this work is much of the character of an advanced calculus with a development of the fundamentals of differential geometry thrown in for good measure.

After an introduction devoted mainly to a brief resumé of the essentials of infinite series and indefinite integration, there are two sections on what might be termed additional material in differentiation and integration. The former includes such topics as differentials of functions of more than one variable, functional determinants, change of variables (with material on contact transformations) and maxima and minima, especially of functions of two variables. The latter includes in addition to the Riemann integral, reference to the Stieltjes integral, elliptic integrals, material on definite integrals with some emphasis on the approximate calculation of the latter by different methods and comparison of these methods as regards accuracy, improper integrals and multiple integrals as well as continuity properties of these.

The application of integrals is limited mainly to a discussion of

$$\int_0^\infty e^{-x^2}dx,$$

and its relatives, and the Euler integrals, together with an elegant introduction to the theory of Fourier series. The applications of differentiation are in the field of differential geometry, beginning with a consideration of orders of contact and obtaining the principal results with respect to space curves and surfaces, including the applicability of surfaces on each other.

Naturally, there is a chapter devoted to the fundamental formulas of mathematical physics, called by Hadamard the formulas of Ostrogradsky, Riemann, and Green, respectively, to which is joined a brief introduction to vector analysis in three space, in which a forward look into higher dimensional vector analysis plays a role. The latter chapter closes with a tabular comparison of the notations in common vogue by different schools of vector analysis. It would be helpful if vector analysts and mathematical physicists could agree on the names of operators and formulas.

The concluding section of the volume is concerned with differential equations, and gives the elementary results in the subject relating to differential equations of the first order, and of special types of higher order, with main emphasis on linear differential equations with constant coefficients and systems of such.

The volume closes with notes on (a) comparison of the Euler-Maclaurin and trapezoid formulas for computing definite integrals and a suggestion of a combination giving in some cases a better approximation than either, (b) summation by means as applied to Fourier series, leading naturally to (c) the Weierstrass Theorem on approximation to continuous functions by polynomials; and (d) a brief introduction to affine vectorial geometry. There is appended an incomplete table of errata.

As outstanding features of the book might be mentioned the introduction and treatment of the Stieltjes integrals on a par with the Riemann integral; further, the derivation of some of the characterizing magnitudes of space curves and surfaces by the use of kinematic considerations, making these seem perhaps more natural to a student preparing for a technical career. He bows to the modern trend, especially where technical students are concerned, in devoting considerable attention to approximation theorems and calculations.

The presentation of the material is lucidly and interestingly done, remarks and subremarks being frequently added in the hope of adding further points of interest to the main material. In matters of rigorous treatment, involving so-called ϵ, δ considerations and existence theorems, he prefers to pass over the finer points and refer to consideration of these in the next volume, so as not to divert the attention too much from the material expounded,—a method in common practice and perhaps justifiable on pedagogical grounds. As a careful and clear exposition of topics in advanced calculus this volume is worthy of a high place, but it is not at all remarkable in presenting material in a novel way, or being revolutionary in its point of view.

T. H. Hildebrandt