BOOK REVIEWS

La Théorie des Spineurs. By Élie Cartan. (Actualités Scientifiques et Industrielles, nos. 643 and 701.) Paris, Hermann, 1938. 95 pp. and 91 pp.

The theory of spinors has been treated in the literature from three points of view: the infinitesimal, the algebraic and the geometric. Cartan originated the theory in its infinitesimal aspect in 1913 and he has now written a full length account in which it has been his intention "de développer systematiquement la théorie des spineurs en donnant de ces êtres mathématiques une définition purement géométrique."

Cartan's definition of a spinor (§§93 and 109) is based upon the one-to-two correspondence between the ν-vectors which are coordinates of linear spaces of ν dimensions on the fundamental quadric cone in 2ν (or $2\nu+1$) dimensions and certain vectors with 2^ν components, called "simple" spinors. Despite this geometric introduction to the concept of a spinor, knowledge of the algebraic theory of the representation of linear groups will be of more use to the reader than any acquaintance with the classical ideas of geometry.

The first two chapters of the first volume do not contain the word "spinor." Instead, they include an elementary discussion of the following topics: euclidean space (including the complex and the real definite and indefinite cases), rotations and reflections, multivectors, definition of tensors, tensor algebra, irreducible and reducible tensors, matrices (their algebra and a brief account of unitary, orthogonal and hermitian matrices), and the irreducibility of multivectors. Any reader already acquainted with the material in these forty-seven pages will delight in the concise elegance of the exposition while others will benefit from its lucidity and accuracy.

In the third and fourth chapters the spinor theory is developed in detail for a three dimensional space. The components of a spinor are first introduced as specific functions of the components of an isotropic vector, affected however by an ambiguous sign. After showing that a spinor is a "euclidean tensor," the author introduces in a natural fashion the relation $(x_1)^2+(x_2)^2+(x_3)^2=XX$, where X is a matrix with linear forms in x_1, x_2 and x_3 as elements and the left member is a scalar matrix. This association of vectors and matrices is used to obtain the double-valued representation of the rotation group by linear transformations on spinors. Chapter III concludes with a discussion of reality conditions in the euclidean and pseudo-euclidean cases. In Chapter IV several theorems on the linear representations of the various rotation groups are proved by making use of the infinitesimal generators of the groups.

The first chapter of the second volume extends the spinor theory to spaces of any odd number of dimensions while the following chapter does the same for spaces of an even number of dimensions. In each of these cases the chapter closes with a brief discussion of reality conditions.

The last three chapters (totalling thirty pages) are entitled: Spinors in the Space of Special Relativity; Linear Representations of the Lorentz Group; and Spinors and Dirac Equations in Riemannian Geometry.

Comparison of this exposition of the spinor theory with the work of Veblen and his students[1] reveals a surprisingly small amount of common material. Thus Cartan does not employ Veblen's useful concept of a linear family of geometric transformations, which is a generalization of the well known linear family of involutions on the projective line. The avoidance of any use of the index notation is also interesting since the use of dotted indices has sometimes been regarded (quite erroneously) as a characteristic feature of the theory. These monographs do contain a full and excellent account of the group representation aspect of the theory and are an invaluable contribution to the literature of the subject.

Aside from a few misprints, the only error noted was on page 18 of the first volume, where the first lemma is false in case the fundamental quadratic form in n variables is of signature $\pm(n-2)$. Even in this case a weakened form of the lemma suffices for the following proofs.

WALLACE GIVENS

Gesammelte Werke. By Johannes Kepler. Vol. 1. *Mysterium Cosmographicum. De Stella Nova.* Edited by Max Caspar. 1938. 15+493 pp. Vol. 2. *Astronomiae Pars Optica.* Edited by F. Hammer. 1939. 467 pp. Vol. 3. *Astronomia Nova.* Edited by Max Caspar. 1937. 488 pp. Munich, Beck.

In 1936 Max Caspar published the *Bibliographia Kepleriana* as a part of this series.

The three imposing volumes of Kepler's collected works represent

[1] In his introduction Cartan refers to Veblen's work as unpublished, although in his bibliography he lists Veblen's paper in the Comptes Rendus Congrès Oslo and the reviewer's dissertation, which was done under Veblen's direction. The most complete publication of the work at Princeton was in the form of mimeographed notes on lectures by Veblen and Givens under the title "Geometry of Complex Domains."