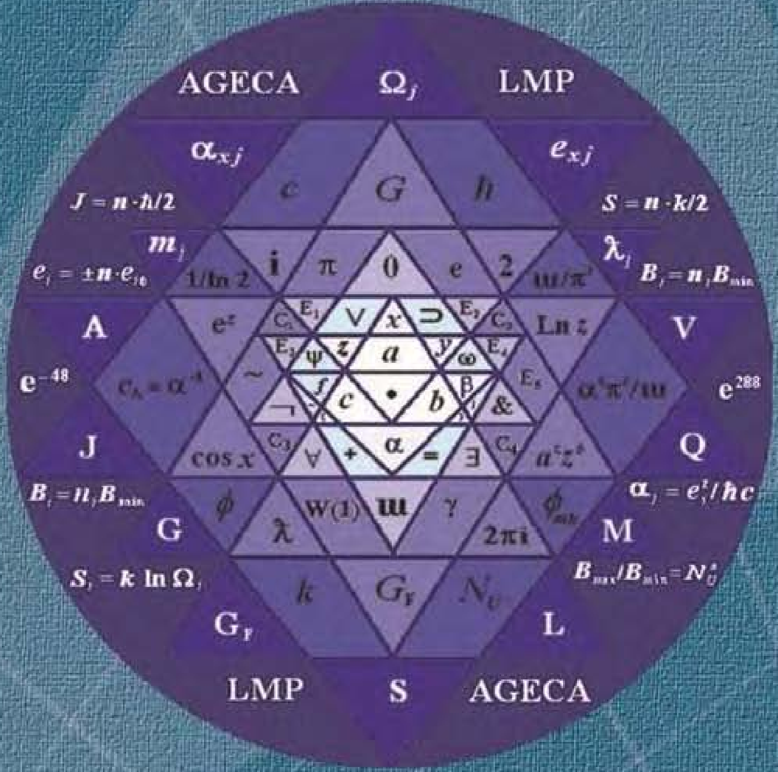
Hrant Arakelian

LMP
Fundamental Theory



Logic & Mathematics + Physics

Hrant Arakelian was born in 1947. He has Master of Science degree in physics and Doctor of Philosophy scientific degree. Dr. Arakelian currently works at the Institute of Philosophy, Sociology and Law in the structure of Armenian National Academy of Sciences. He holds the position of Leading Scientist and Head of a Group. The topic of his theses for Candidate of Science degree was "On Proof in Mathematics", gained in 1976 at the Yerevan State University.

His Doctor of Philosophy degree was awarded in 1992 at the St.-Petersburg State University, for a theses "Numbers and Quantities in Modern Physics". The sphere of interest includes foundations of mathematics and physics, philosophy and methodology of science physical theory, number theory,.

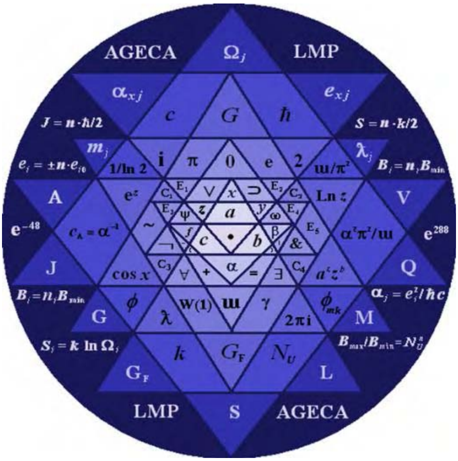
Dr. Arakelian is the author of numerous publications in academic area, including the following monographs:

- On Proof in Mathematics, 1979
- Fundamental Non-Dimensional Quantities, 1981
- Numbers and Quantities in Modern Physics, 1989
- Foundations of Physical Theory, 1997
- LMP-Theory, 2007
- From Logical Atoms to Physical Laws, 2007

Armenian National Academy of Sciences
Institute of Philosophy, Sociology and Law

LMP
Fundamental Theory

Hrant Arakelian



Publishing House
SARVARD HRAT LTD

Reviewers:

Hamlet Gevorkyan	Academic Philosopher
Gagik Karamyan	Ph.D. in Mathematical Sciences

UDC 510.6:53:1/14

ISBN 978-9939-824-03-01

LMP-THEORY

IS CONCEIVED as a **BASIC THEORY** of
PHYSICAL WORLD, GENERAL THEORY of
ALL PHYSICAL THEORIES GROWN on the **SOIL** of
MATHEMATICAL LOGIC and **PURE MATHEMATICS**

BASIC PRINCIPLES OF LMP-THEORY

The Concept of Triunity

Mathematical logic (L), formal numerical mathematics (M) and fundamental physical theory (P) constitute a unified trinomial system of knowledge.

Definition of Physics

Physics is a science of physical quantities.

Fundamental physical theory is a theory of fundamental physical quantities.

Basic Principles of Construction

Only constructions requiring no other logical-mathematical elements and means except the original are admissible in the LMP-Theory. On the other hand all the primary resources of the theory ought to be used in its construction.

Relationship Between the Components

In the LMP-Theory, the extension of logical deductive calculus represents the formal universal mathematics complemented by a system of physical equations – codes and a dimensionless measurement system. Transition from logic to mathematics is related with introduction of notion of number and initial numbers, such as the new mathematical constant ɯ. Transition from mathematical to physical components of the theory is primarily transition from mathematical quantities to fundamental physical quantities.

Basic Constructions and Main Results

Selection of axiomatic system **AG** having interpretation in the set of all numbers, as a universal logical-mathematical basis of the LMP-Theory. The **AG** system includes eighteen postulates of propositional and predicate calculus, seven mathematical axioms and all initial concepts, elements and principles required for further constructions.

Utilization of the main **AG** system resources for final construction of formal mathematics by means of a system **E** of five functional equations. The system **E** reduces multiplication and division operations to axiomatically specified operations of addition and subtraction, as well as extends the properties of axiomatic zero to functional analysis and uses the initial concept of superposition of functions.

Solution of **E** system of equations, unambiguously resulting in initial mathematical functions of logarithm Ln z and exponent e^z, as well as constants e, π, i, 2, W(1) – omega constant and ɯ – cosine superposition constant. The said six constants, jointly with 0 and Euler constant γ form the system of initial mathematical numbers. Functional equations **E** actually represent the only way for formally rigorous obtaining of truly functional mathematical constants (FMC), as a system of interrelated mathematical quantities.

Understanding of number ɯ = 0.73908 51332… as a missing link, a *hidden parameter* of mathematics. Using the fundamental constant ɯ makes possible solution of a number of physical theory problems, including the problem which appears to be unsolvable, namely obtaining the values of fundamental physical constants (FPC).

Exponential-logarithmic notation as a universal representation form of any number, except zero, and as a formal analytic basis of physical theory. Analysis of the simplest forms of said representation by means of FMCs and their physical interpretation. The procedure results in a system of fundamental physical equations (or codes) **C** in form of simple relations between the constant and variable physical quantities:

$$\alpha_{\mathrm{e}j} = e_j^2/\hbar c \qquad \alpha_{Gj} = Gm_j^2/\hbar c \qquad \alpha_{\mathrm{W}j} = (G_{\mathrm{F}}/\lambda_j^2)/\hbar c \qquad \Omega_j = \mathrm{e}^{S_j/k}$$

Detailed “decoding” of relations **C** by acceptable means, resulting in the major dimensions and fundamental physical laws of conservation (light velocity in vacuum, action, mass and generalized charges), variation (of entropy, number of microscopic states in the Universe and interaction constants), and quantization (of action, entropy, charges, Hall resistance and magnetic flux values).

Construction of dimensionless system for measurement of physical quantities (**A**-system) based on FPCs expressed through FMCs:

$$c_{\mathrm{A}} = \alpha^{-1} \qquad m_{\mathrm{eA}} = ш/\pi^2 \qquad k_{\mathrm{A}} = 1/\ln 2 \qquad \hbar_{\mathrm{A}} = \pi^2\alpha^2/ш$$

This system endows any physical quantity by its true mathematical expression, or to some or other accuracy by its true numerical value.

Transformation of various physical quantities into **A**-system revealing their mathematical features which cannot be found by any other method. Particularly, the role of family of number 137 in physical theory, the new formula for mean lifetime of muon, a general formula for masses of muon, τ-lepton and nucleons.

Transition to the **A**-system of Fermi coupling constant related with interaction probability of 48 fundamental particles – 24 leptons, quarks and their antiparticles, and 24 bosons from the $SU(5)$ group, giving the expression $G_{\mathrm{F}} \cong \mathrm{e}^{-48}$. This result represents absolutely precise “hitting” of the desired point in the infinite continuum of real numbers, unforeseen beforehand and obtained without any “aiming”. Randomness is eliminated here, even theoretically, while the revealed correspondence most clearly demonstrates the validity of **AGECA**-formalism, and thus of the entire LMP-Theory.

Boundaries of physical world determined by using known parameters of the Universe. Three independent methods are presented: dimensional analysis, consideration of one **C** system relation, and using the entropy formula for black hole. As a result, the minimal value for length, for example, has the order of magnitude 10^{-95} cm, while the ratio of maximal and minimal values for all physical quantities is expressed by integer or half-integer power values of the new cosmological constant $N_U \approx 10^{125}$. The number of microscopic states of the Universe is expressed through a tremendous number e^{N_U}.

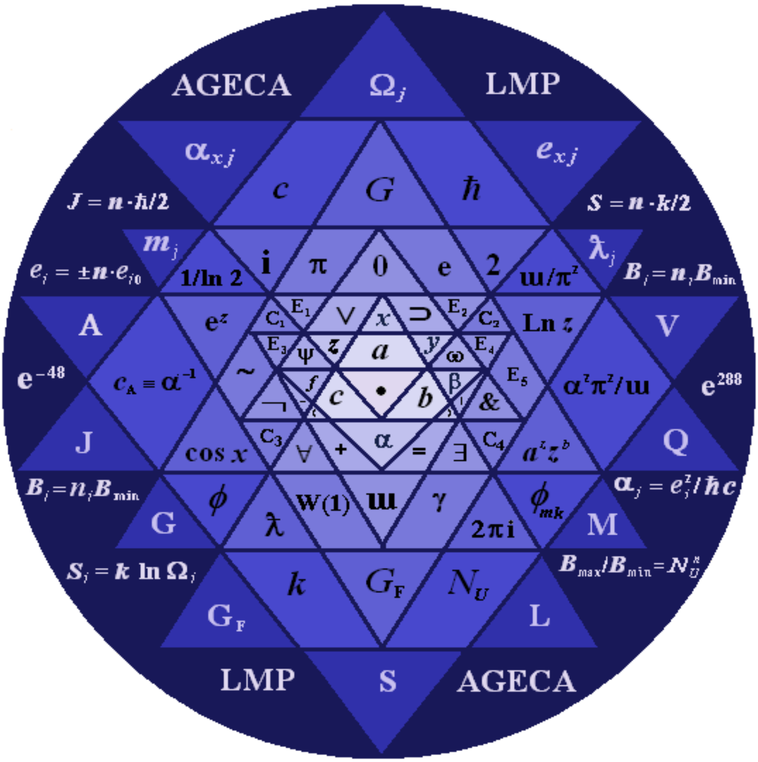
Generalization of fundamental physical laws by means of constant N_U. Such are the general law of conservation for numerical values of all FPCs, the general law of extreme values ratios for various physical quantities, and generalized law of conservation, variation and quantization.

Basic Numerical Predictions

Fine-structure constant	$\alpha^{-1} = 137.035\,999\,452\,021\ldots$	
Number of fundamental fermions and bosons	$48 = 24 + 24$	
Fermi coupling constant	$G_F = 1.166\,383\,14(6)\cdot 10^{-5}\ \text{GeV}^{-2}$	(0.05 ppm)
Muon mean lifetime	$\tau_\mu = 2.196\,975\,51(56)\cdot 10^{-6}\ \text{s}^*$	(0.25 ppm)
AMM of muon	$a_\mu = 1.165\,923\,55(7)\cdot 10^{-3}\ \mu_B$	(0.06 ppm)
Gravitational constant	$G = 6.673\,900(4)\cdot 10^{-8}\text{cm}^3\text{g}^{-1}\text{s}^{-2}$	(0.6 ppm)
Muon-electron mass ratio	$m_\mu/m_e = 206.768\,280\,26(5)$	(0.24 ppb)
Tau-electron mass ratio	$m_\tau/m_e = 3477.327\,024\,03(8)$	(0.023 ppb)
Proton-electron mass ratio	$m_p/m_e = 1836.152\,674\,94(20)$	(0,11 ppb)
Neutron-electron mass ratio	$m_n/m_e = 1838.683\,661\,82(15)$	(0.08 ppb)

* Updated version, depending on the latest value of radiation corrections in the formula for muon lifetime, of the previous theoretical prediction $\tau \approx 2.196\,973\cdot 10^{-6}$ s (and accordingly $G_F \approx 1.166383\,07\cdot 10^{-5}\ \text{GeV}^{-2}$) which recently has been fully confirmed (see Ch. 3).

Contents



The main logical, mathematical and physical elements of LMP theory inscribed in shri yantra

Chapter I. LMP-Theory: Logic and Mathematics

On attempts to construct the Fundamental Physical Theory (FPT) and reasons of failure

The *Fundamental Physical Theory* (FPT) is also called *Unified Theory*, or in ironic and pretentious way the *Theory of Everything* (TOE). It is generally known that after a number of great discoveries in the first thirty years of the last century the further development of physical theory ceased to be exclusively valuable for scientific cognition and philosophy of science. Thus far, all numerous attempts to construct a fundamental (or unified) theory called to comprehend the whole physical world were futile. Far from being complete, the list of such attempts includes the "Theory of matter" [Mie], "Fundamental theory" [Eddington], unified field theories [Einstein; Hilbert; Klein], unified theory of nonlinear spinor field [Heisenberg], various versions of axiomatic quantum field theory [Bogolyubov, Logunov and Todorov], supergravitation, superstrings [Freedman, van Nieuwenhuisen, and Ferrara; Golfand and Lichtman; Deser and Zumino; Schwarz; Green and Gross], and finally "An Exceptionally Simple Theory of Everything" [Lisi]. Retrospectively, from the height of contemporary physical knowledge and from the viewpoint of LMP concepts it is possible to point out some reasons, which may be considered now as insurmountable barriers in the path of success.

- Construction of FPT is possible only at a certain stage of physical theory and experiment development and only if some opportunities are at hand;
- FPT setup requires generation of fresh ideas, new understanding of physical theory foundations and novel methodology;
- The mathematical apparatus used in various attempts of building FPT is not sufficient for solving the problem totally, as far as it has serious gaps.

Definitions of physics and physical theory. Tree-diagram of FPT and its environment

All requirements and conditions necessary and sufficient for building a fundamental theory are presently available. It is reasonable to start from definition of physical science for presence of general ideas that underlie the fundamental LMP-Theory. As shown by [Arakelian 1997], any statement on the physical reality is inevitably a statement on some *physical quantity*, any equation, formula, or relation states an analytic connection between *physical quantities*, any physical measurement, experiment; empirical study comes to be specific information on *physical quantities*. According to this concept **physics is a science of physical quantities.** Hence:

Fundamental physical theory is a theory of fundamental physical quantities.

In any case, selection of primary objects, fundamental physical quantities is of paramount importance. There exist many alternative options, and the problem is how one should choose such primary objects which will fully meet the requirements of contemporary FPT. This problem is actually very hard, lying in any case beyond the scope of the physical theory itself. However, this key problem has a substantial solution. In order to understand how it should be solved, one must study at first the environment of FPT. It is convenient to represent the FPT environment by means of traditional tree-like diagram [Arakelian 1992, 11–12]:

- Atmosphere: Philosophy
- Soil: Methodology
- Roots: Logic (L)
- Trunk: Pure Mathematics (M)
- Branches: Fundamental Physics (P)
- Crone: The Rest of Physics
- Fruits: Application of Physics in Science and Technology

It should be noted that we shall not deal with applications of physics. Philosophical, epistemological and methodological issues, along with many specific physical, mathematical and logical problems have been discussed by the author in the monographs [Arakelian 1979; 1981; 1989; 1997; 2007; 2007*a*] and in a number of publication [Arakelian 1984; 1992; 1994; 1995]. Thus there are apparently all necessary prerequisites and conditions for construction of a consistent theory that would include formal logic as a part of unified trinomial logical-mathematical-physical monolith.

On integrity of logic, mathematics and physics

The suggested tree-diagram has the aim to visually demonstrate that, according to the proposed concept, the fundamental physical theory develops being supported by a trunk of pure mathematics stemming from logic. Thus, essential is the integrity of mathematics and logic (and not only mathematics) as a language of physical theory, method and means of description of the physical reality, etc. Substantive is integrity in a stronger sense, as a unity of a holistic system with rigorous natural relationship of the system components. The integrity of **logic**, **mathematics and fundamental physical theory** (conceived as a theory of fundamental physical quantities) is reflected in the name of the LMP-theory. Stated otherwise, *the LMP-Theory is conceived as a basic theory of physical world*, general theory of all physical theories, the physical theory of everything grown on the soil of mathematical logic and pure mathematics. LMP-Theory is treated in its extended form in the monograph **[**Arakelian 2007**]** and concisely in the book **[**Arakelian 2007*a***]**.

Note that each part of the LMP-system, especially the first (L) and to less extent the second (M) represent relatively self-contained structures, in conformity with general requirements of the concept. So each subsequent evaluation step is relied on the previous step which to some extent determines the structure and parameters of the whole theory. From the constructive viewpoint, the major objective of investigation is to reveal and utilize the “navel” which helps to find out the main characteristics of fundamental physical theory by connecting the core of nucleus of physical theory with its logical-mathematical basis. Stated otherwise, *we need such a logic and mathematics based on it, the natural development of which would result in transition from selected mathematical quantities to fundamental physical quantities, and then to the main physical principles and laws*.

The most rational, rigorous and logically reliable way of representing a natural scientific theory lies in axiomatization of that theory. The axiomatic method which proved to be excellently applicable in logic and mathematics (although, as established by Gödel, having limited capacity in mathematics) is limited also in other research areas, including physics. With account of this fact, we have limited the rigorous application of axiomatic method only by the **AG-**system, making the first two parts of LMP-Theory’s formalism and including the well-known sections of formal logic and less known mathematical axiomatics.

The main logical and mathematical functions and variables

As already mentioned above, the LMP-Theory is based on organic aggregate of logic, mathematics and fundamental physical theory conceived as the theory of fundamental physical quantities. One can see the roots of formalized mathematics just in logic, moreover in mathematical logic. That is to say, such theories of mathematics as (formal) arithmetic must start from logical atoms – propositions (statements) and other basic logical elements forming the propositional calculus. On this basis is constructed the predicate calculus with the initial notion of predicate, or logical function. Only after such logical-deductive formalism has been constructed, some or other system of mathematical axioms is added including new elements interpreted by means of some or other set of objects, not necessarily having numerical origin. Choice of adequate logical-mathematical basis of fundamental physical theory, called to provide the integrity of three components of the system, has practically no alternative relative to the logic. Classical predicate calculus (excluding equality), including as its part the propositional calculus, may serve, being duly modified, a formal basis of numerous different mathematical systems. Except the necessity to introduce the formal integrity of logic and mathematics, we are highly interested in obtaining a *complete list of principal, initial components* of the logical-mathematical system.

One of the most remarkable features of logic and mathematics is the possibility of reduction of all their forms to a minimal basis of initial elements and principles. Sequential exposition of general principles and construction of formal body of the LMP-Theory requires selection of major classes of logical, mathematical functions and variables:

- (*a*) logical *propositional functions*, or *predicates*, the limiting case of which are *individual propositions* (or *statements*)
- (*b*) *simple functions*, the limiting case of which are *constants*
- (*c*) *composite functions*, formed by means of *superposition*
- (*d*) *functionals*
- (*e*) *operators*

Respectively there are four potentially infinite sets of variables:

1) *objective (individual) variables*
2) *predicate* logical *variables*
3) *numerical variables*
4) operational *functions-arguments* of mathematics

Logical and mathematical operations, terms and formulas

The next step is selection of primary operations, or operators. In the LMP formalism their number is ten in total:

- logical connectives (or logical operators) $\sim$, $\supset$, $\&$, $\vee$, $\neg$
- universal quantifier $\forall$ and existential quantifier $\exists$ (inverted capital letters of English words “all” and “exist”)
- mathematical operations $=$, $+$, $-$

All operators have their assigned ranks. The operators are ordered in decreasing rank from left to right as follows:

$$\sim \quad \supset \quad \& \quad \vee \quad \neg \quad \forall \quad \exists \quad = \quad + \quad -$$

(the ranks of operators + and – may be conceived as the same, in which case their order is insignificant). The higher is the rank of operator, the larger is the area of its action, while the lower is the rank the stronger the operator binds its variable. This fact allows using minimal number of brackets in writing the logical-mathematical expressions; often no brackets are necessary at all. Only these ten logical and mathematical operators should be considered as independent. Only such operators and operations reduced to them are acceptable. Any other operation used in this study is just *a convenient construction* representable through the initial ten operations at any stage.

Having the alphabet of LMP system at hand we can now turn to study of the well-formed expressions called “terms” and “formulas” of the formal system. It is generally accepted that in the natural language grammar the analogs of term are “word”, “subject” and “object”; the analog of formula is “sentence”, or “judgment”, although, due to a certain ambiguity of the last word its correlation with formula seems somewhat weak. One must be always able to distinguish well-formed and *not* well-formed sequences of logical and mathematical symbols, as well as distinguish the words of formal language from terms, the sentences from formulas.

Definition of terms:

1. All logical objective variables, all mathematical numerical variables and functions-arguments are terms
2. All simple and composite mathematical functions, all constants and functionals are terms
3. If $\hat{\mathbf{A}}$ is an operator, and F is a function, then $\hat{\mathbf{A}}$F is a term
4. 0 is a term
5. If p is a term, –p is also a term
6. If p and q are terms, p + q, p – q are also terms
7. There are no terms except those defined in the items 1 to 6

Definition of formulas:

1. All propositions (zero-placed predicates) are formulas
2. All predicates $P(x_1, ..., x_n)$ and all predicate variables are formulas
3. If p and q are terms, p = q is a formula
4. If A and B are formulas, $A \sim B$, $A \supset B$, A & B, $A \vee B$, $\neg A$ are also formulas
5. If A is a formula, and x is a variable, then $\forall xA$, $\exists xA$ are formulas
6. There are no other formulas, besides those that are defined the items 1 to 5

All previously introduced logical and mathematical variables and functions are covered by these definitions, while all ten primary operations are used in *formation rules* of new terms and formulas. Any finite sequence of graphic signs obtained by application of these rules give well-formed terms and formulas of LMP-system.

Logical postulates of LMP-Theory

In the classical predicate calculus the simplest logical functions – propositions – can assume only two values, denoted as t (truth) and f (false). To values t and f of formula A correspond the values f and t of formula $\neg A$, i.e. $\neg A$ is true if and only if A is false and $\neg A$ is false if and only if A is true. Such is the natural interpretation of formula $\neg A$ in the model theory of two-valued formal logic. It is clear that the equivalence $A \sim B$ is true if and only if A and B are both true or both false; implication $A \supset B$ is false only if A is true and B is false; conjunction $A \,\&\, B$ is true only in the case when A and B are both true; alternation $A \vee B$ is false only if A and B are both false and is true in all other cases. Now, connecting the logical *atoms* A and B by means of implication and alternation to a logical formula $A \supset A \vee B$ and preparing a truth table of its values, it is easy to see that, independently of the values of sub-formulas A and B, the compound formula is true in all cases. Formula which is true at any arbitrary distribution of true values of sub-formulas A, B, C, ... is a tautology and such formulas are often called identically true, or universally significant. Similar reasoning is applicable to formulas, containing predicates and quantifiers. It is apparent that just from the set of identically true formulas must be chosen the logical axioms, or more precisely, the axiom schemes, which are transformed into certain axioms only when arbitrary A, B, C are substituted by concrete formulas. Fifteen axiom schemes together with three inference rules (transformation rules) form a system of postulates of classical predicate calculus, which are the logical postulates of LMP system at the same time.

$\mathbf{L}_1$ $\quad A \supset (B \supset A)$

$\mathbf{L}_2$ $\quad (A \supset B) \supset ((A \supset (B \supset C)) \supset (A \supset C))$

$\mathbf{L}_3$ $\quad \dfrac{A,\ A \supset B}{B}$ $\qquad$ modus ponens, or $\supset$-rule

$\mathbf{L}_4$ $\quad A \supset (B \supset A \,\&\, B)$

$\mathbf{L}_5$ $\quad A \,\&\, B \supset A$

$\mathbf{L}_6$ $\quad A \,\&\, B \supset B$

$\mathbf{L}_7$ $\quad A \supset A \vee B$

$\mathbf{L}_8$ $\quad B \supset A \vee B$

$\mathbf{L}_9$ $\quad (A \supset C) \supset ((B \supset C) \supset (A \vee B \supset C))$

$\mathbf{L}_{10}$ $(A \supset B) \supset ((A \supset \neg B) \supset \neg A)$

$\mathbf{L}_{11}$ $\neg\neg A \supset A$

$\mathbf{L}_{12}$ $(A \supset B) \supset ((B \supset A) \supset (A \sim B))$

$\mathbf{L}_{13}$ $(A \sim B) \supset (A \supset B)$

$\mathbf{L}_{14}$ $(A \sim B) \supset (B \supset A)$

$\mathbf{L}_{15}$ $\forall x A(x) \supset A(r)$ $\forall$-scheme

$\mathbf{L}_{16}$ $A(r) \supset \exists x A(x)$ $\exists$-scheme

$\mathbf{L}_{17}$ $\dfrac{C \supset A(x)}{C \supset \forall x\, A(x)}$ $\forall$-rule

$\mathbf{L}_{18}$ $\dfrac{A(x) \supset C}{\exists (x) A(x) \supset C}$ $\exists$-rule

The first fourteen postulates taken together constitute the axiomatics of propositional calculus; in conjunction with postulates $\mathbf{L}_{15}$–$\mathbf{L}_{18}$ they make up the predicate calculus. It is reasonable to state now that the first, logical part of LMP-system construction is fully executed.

From logic to mathematics: choice of axiomatic system. Formal G and AG systems

Having finished with logical roots we turn to the mathematical trunk of LMP-system. This is a key issue of construction complicated by existence of tens of mathematical axiomatic systems based on logical predicate calculus. The advantage of this calculus is that, in its various modifications it serves a natural, reliable and fairly simple basis for miscellaneous mathematical systems and therefore may be considered a universal logical-deductive foundation for the most part of formal mathematics. And now we face the problem, figuratively speaking, of finding a trunk of a unique tree, among the whole wood of trunks having almost identical roots, not knowing even if such a tree really exists. One may also state that thick and heavy branches of physical theory can hardly be supported by thin and under-grown trunk of arithmetic of natural numbers, and looking further, by any formal system having limited range of objects and capacities.

Thus, transition from universal logic to the yet unknown fundamental physics may be realized only by means of universal mathematics. This is not just a word-play but rather *modus vivendi* of triune LMP-system, which should be presently accepted by trust, in capacity of the working hypothesis. Although the system being sought is not so popular as the **N** system of natural numbers, it still is known and designated by a symbol **G**. The **G**-system includes the following formal symbols:

$$\sim\ \supset\ \&\ \vee\ \neg\ \forall\ \exists\ =\ +\ -\ 0\ a\ b\ c\ \ldots\ x\ y\ z\ \alpha\ \beta\ \ldots\ \psi\ \omega\ (\ )\ _{|}$$

The set consists of seven logical and three mathematical operations, decreasing in rank from left to right, with 0 (zero) individual object, 26 italic Latin letters, 24 small letters of Greek alphabet, left and right brackets, as well as the symbol $_{|}$. All other symbols of the present text, including punctuation marks, natural language words, such abbreviations as ≡, ≈, ψ, ≠, <, >, lim, Σ of corresponding logical-mathematical expressions refer to metalanguage, i.e. the language by means of which the objective language is tested.

Definition of terms and formulas were given above. One should keep in mind that in the case when the variables a, b, c, ..., x, y, z are conceived as numbers, then zero, all variables and constants, numerical functions (including composite functions), functionals, operator expressions, as well as any sequences of enumerated terms formed by operations + and – and application of rules $-p$, $p + q$, and $p - q$ also represent terms. Meanwhile, application of equality = gives a mathematical formula $p = q$, in addition with expressions formed by propositional connectives and quantifiers.

The following six axioms are mathematical axioms of the **G-**system:

$\mathbf{M}_1$ $\quad a = b \supset (a = c \supset b = c)$

$\mathbf{M}_2$ $\quad a = b \supset a + c = b + c$

$\mathbf{M}_3$ $\quad a = b \supset c + a = c + b$

$\mathbf{M}_4$ $\quad (a + b) + c = a + (b + c)$

$\mathbf{M}_5$ $\quad a + 0 = a$

$\mathbf{M}_6$ $\quad a - a = 0$

Axioms $\mathbf{M}_1$ to $\mathbf{M}_4$ define the properties of equality and addition, $\mathbf{M}_5$ establishes the unique properties of zero, and $\mathbf{M}_6$ defines the operation "–" and an object $-a$ opposite to a. Thus, eighteen postulates $\mathbf{L}_1$ to $\mathbf{L}_{18}$ of the predicate calculus, jointly with six mathematical axioms for operations of equality, addition and subtraction of objects a, b, c and zero constitute the logical-mathematical **G**-system of axioms.

One may ask what are the advantages of **G** system against other formal systems and what sense has the concept of infinite set of objects a, b, c, ...? In contrast to the **N** system with unique interpretation on the set of natural numbers, the formal **G** system admits a large number of interpretations, both of numerical and non-numerical group-theoretical character. However, significant is not the number and variety of interpretations but the remarkable fact that, along with other interpretations, there is one on the set of *all* numbers. It should be also noted that if we intend to have a formal system that would include all possible numerical sets, then the operations of addition, subtraction and constant 0 ought to be chosen, and not the operations of multiplication, division and number 1. It is also reasonable to include the commutative law for addition in the list of axioms. Thus the final system of mathematical axioms, denoted as **AG**, must additionally include the axiom

$\mathbf{M}_7 \quad a + b = b + a.$

Hence, we can state that a sufficiently universal logical-mathematical system is constructed on universal logical basis which axiomatically defines the mathematical number *in general* , i.e. a continuum of *all numbers* without any omission. It is also important that, along with a set of initial objects, the **AG** system specifies the complete set of primary logical and mathematical operations by means of which all the other operations may be expressed.

On necessity of introducing specific numbers and functions

It seems apparent that simple, "mechanical" extension of the system by addition of provable formulas is not enough for disclosure of internal potential of the **AG-**system. What numbers must follow zero in the formal hierarchy of mathematical quantities? What are the fundamental rules – laws establishing correspondence between various sets composed of variable and constant quantities? In other words, what are the initial, maternal functions needed in construction of other functions? By giving answers to these questions, we expect to obtain all necessary and sufficient tools and components for further construction of physical theory foundations. On the tree diagram, this step means transition from logical roots to mathematical trunk of the LMP-system.

Functional equations

The important issue is how specific numbers should be introduced in the **AG**-system if one stays in the framework of initial formal basis of the system. In such statement the problem seems unsolvable. Let us therefore formulate the problem in somewhat other way and ask ourselves what the **AG**-system misses in the first hand, what deep inner potentials of the system still remain undemanded and require disclosure? Clearly, in the absence of multiplication and division operations, as well as properties of 1 it is impossible to speak seriously about the theory of numbers and mathematics in general. Thus in any event these elements should be defined and introduced.

Let us first agree upon the terminology. We shall call a *relation* the equality including only constants. The equality including variables is called *equation*, while the equality where unknown quantity represents a function is called *functional equation*. Introduction of new mathematical realities by means of their reduction to initial elements, using functional equations, represents a powerful tool, a general method of formal system development. The method supplements the axiomatic properties of numbers by functional properties. The functional equations as we shall see later is the simplest and most reliable way of reducing the multiplication and division operations to addition and subtraction.

Thus, denoting the new operation of multiplication by a dot symbol which often may be omitted, we intend to determine and in some sense to reduce multiplication to addition, *using the simplest functional equations*. For two numerical expressions $x + y$, $x{\cdot}y$, and thus four functional expressions

$$f(x+y),\;\; f(x\cdot y),\;\; f(x)+f(y),\;\; f(x)\cdot f(y)$$

totally six equations are possible. Since the equations

$$f(x+y) = f(x\cdot y), \quad f(x)+f(y) = f(x)\cdot f(y)$$

simply mean identification of multiplication and addition, while the equations

$$f(x \cdot y) = f(x) \cdot f(y), \quad f(x+y) = f(x) + f(y)$$

do not reduce one operation to the other, only two so-called Cauchy functional equations are left. By denoting the unknown functions as $\psi(x)$ and $\alpha(x)$, we have:

$$\mathbf{E}_1 \quad \psi(x+y) = \psi(x) \cdot \psi(y)$$

$$\mathbf{E}_2 \quad \alpha(x) + \alpha(y) = \alpha(x \cdot y), \ (x \neq 0, y \neq 0).$$

It is easy to generalize the functional equations $\mathbf{E}_1$ and $\mathbf{E}_2$ for the case of multiple variables:

$$\mathbf{E}_{10} \quad \psi(x_1 + x_2 + \ldots + x_k) = \psi(x_1) \cdot \psi(x_2) \ldots \psi(x_k)$$

$$\mathbf{E}_{20} \quad \alpha(x_1) + \ldots + \alpha(x_k) = \alpha(x_1 \cdot x_2 \ldots x_k).$$

Time is now to introduce the constant λ, functional analog of the initial mathematical constant zero, i.e. to assign a functional character to main properties of equality $a \pm 0 = a$ fixed in the axioms $\mathbf{M}_5$ and $\mathbf{M}_6$. There is only one way of doing so – to replace the second variable in the functional equations $\mathbf{E}_1$ by expressions $\pm\lambda$, namely

$$\mathbf{E}_3 \quad \psi(x+\lambda) = \psi(x)$$

$$\mathbf{E}_4 \quad \psi(x-\lambda) = \psi(x).$$

In the more general case, where multiple application of the functional rule of zero (periodicity), these equations have the following form:

$$\mathbf{E}_{30} \quad \psi(x + \lambda + \ldots + \lambda) = \psi(x)$$

$$\mathbf{E}_{40} \quad \psi(x - \lambda - \ldots - \lambda) = \psi(x).$$

Using finally one more basic component of the **AG** formal system, namely the fundamental principle of superposition, we come to the following functional equation:

$$\mathbf{E}_5 \quad \lim_{n\to\infty} S(S(\ldots S(x)\ldots) = \text{const.}$$

Here the symbol S designates the yet unknown function whose infinite superposition must result in hypothetical constants, distinct from other constants; x denotes any arbitrary number.

Analysis of the functional equations **E** ($\mathbf{E}_1$ to $\mathbf{E}_5$) in the framework and strict limitations of the **AG** logical-mathematical system **[**Arakelian 2007, Ch. 2**]** is rather long and laborious. Ultimately we have a unique solution for the functions and numbers of the **E**-system.

Functions:

$\psi(z) \equiv \mathrm{e}^z \equiv \exp z$

$\alpha(z) \equiv \mathrm{Ln}\, z = \ln z \pm 2\pi n\mathrm{i}$

$S_1(x) = \psi(-x) \equiv \mathrm{e}^{-x}$

$$S_2(x) = \frac{\Psi(\mathrm{i}x) + \Psi(-\mathrm{i}x)}{2} \equiv \frac{\mathrm{e}^{\mathrm{i}x} + \mathrm{e}^{-\mathrm{i}x}}{2} \equiv \cos x.$$

Constants: e, π, i, 2, ɯ, W(1).

Hence, the well-known exponent e^z and logarithm Ln z are the initial, maternal functions of **AGE** formal system. $S(x)$ has two solutions: the inverse exponent and the arithmetic mean of exponent and inverse exponent. The number W(1) usually called omega-constant is the superposition constant of Lambert function. Incidentally, by analogy with ɯ, it can be obtained with some accuracy by consistently pressing the buttons e^x and $1/x$ of scientific calculator. It should also be borne in mind that the integral form of maternal functions (not presented in **E**) leads to the Euler–Mascheroni constant: $\gamma = \int_0^\infty -\frac{\ln(x)}{\mathrm{e}^x}dx$.

Now the equation $\mathbf{E}_5$ can be presented in explicit forms (z is an arbitrary real or imaginary number)

$$\mathbf{E}_{51} \quad \lim_{n\to\infty} \cos(\cos(\,...\,\cos(z)\,...\,)) = ɯ$$

$$\mathbf{E}_{52} \quad \lim_{n\to\infty} \psi^{-1}(\psi^{-1}(...\,\psi^{-1}(z)\,...\,)) = \mathrm{W}(1).$$

Thus the axiomatically given 0, the numbers π, e, i, 2, ɯ, W(1), together with γ represent the primary numbers of logical-mathematical system, formalizing the whole continuum and not only the sets of natural or real numbers. Therefore, as actually being the initial elements of continuum, these primary numbers should be considered as fundamental constants of mathematics. There exist numerous relations between *selected* constants, but to the best of our knowledge there is no other way of obtaining the really fundamental constants, as a compact group of deductively interrelated primary numbers.

Geometrically, the constants ɯ and W(1) represent triple points of intersection respectively of curves cos x, arccos x, x and e^{-x}, $-\ln(x)$, x.

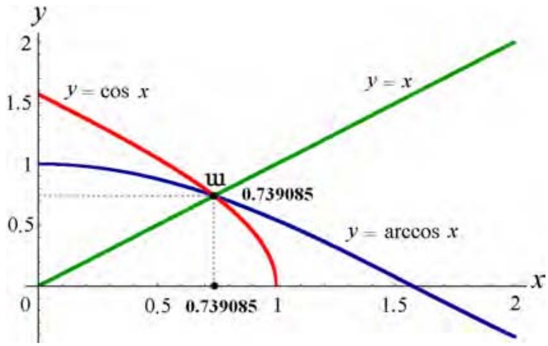


Fig. 1
The intersection point of functions $y = \cos x$, $y = \arccos x$, $y = x$

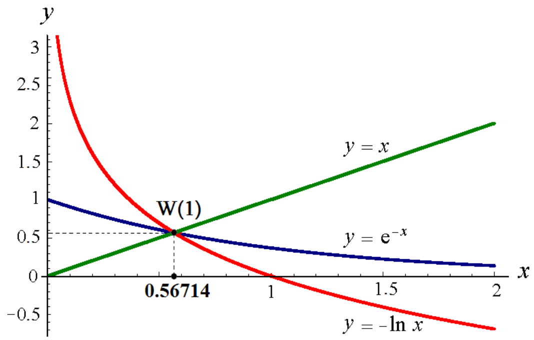


Fig. 2
The intersection point of functions $y = e^{-x}$, $y = -\ln(x)$, $y = x$

These points can be obtained by solution of transcendental equations

$\mathbf{E}_{53}$ $\cos x = \arccos x = x$

$\mathbf{E}_{54}$ $e^{-x} = -\ln(x) = x.$

Solving two of these six equations, for instance the cos $x = x$ and $e^{-x} = x$, we have the numbers

ɯ = 0.7390851332151606416553120876738734040134117589007574649656806357732846548835475945993761069317665318498
0124664398716302771490369130842031578044057462077868
8524903891539289438845095234801335631276772231580956
3537765724512043734199364335125384097800343406467004
7940214347808027180188377113613820420663163350372779
9169673122323006138865820362177081099789706268424058
8094898683261860600485898958548725736764015075227608
1803914595181016281591200964616460675440513264151710
6446628110936082584878371383955556175141494715939006
2775275632586349388697301408366515251152042678851530
2529417180365176420177086071899276016098743271545522
6756579824629761177553961669954931115856653483495383
8523159636025274995587252506666401313187401392538888
0552061869859213925252854154110791002998282929864052
1690465547366968714387356460065212254689149975920969
9758501364249508565047324972584248371554836483437275
8374675254533580066420047883971885848901453115506041
7812337047773953471710345119585460072656146472141978
7537388023680…

W(1) = 0.56714 32904 09783 87299 99686 62210 35554 97538 …

The constant ɯ value is given above with accuracy to thousand decimal places (the first 100000 digits may be found in **[**Arakelian 2007*b***]**, the first 6400000 digits in **[**Arakelian 2010**]**). Like other FMC, except 0, the constant ɯ represents the product of maternal function, namely of exponent. By its nature (superposition) ɯ is akin to W(1), by numerical value ɯ is closer to the constant $\gamma = 0.577215\ldots$, while by structure and relation to other FMC, the constant ɯ is similar to π. Compare:

$$(e^{i\pi} + e^{-i\pi})/2 = i\cdot i$$

$$(e^{iɯ} + e^{-iɯ})/2 = ɯ.$$

More about the constant ɯ

Thus we have a new mathematical quantity ɯ at hand, called to play a very important role in numerous constructions of the LMP-Theory, especially in its last component – fundamental physics. These constructions specially refer to some numerical problems of physical theory dealing with calculation of physical constants and often considered as "inaccessible", "unsolvable", etc. In order to better understand the significance of constant ɯ we need a brief overview of its intricate history and ways of its evaluation.

The simplest analytical way of obtaining the value of constant ɯ is solution of transcendental equation cos $x = x$, with accuracy to several decimal places, is pressing repeatedly the key COS of a calculator in radian mode. But the most apparent method is determining of cosine curve intersection point with diagonal line of the first quadrant.

The history of trigonometric functions including cosine counts probably four millennia [Joseph; Maor]. But it was a long way of understanding the main peculiarities of cosine function. The laws of cosine for acute and obtuse angles were presented in a rather specific form in the *Elements* of Euclid [Boyer]. In India, cosine as a function of circle arcs was discovered and studied in the VI–VII centuries. Some properties of cosine were known to medieval Chinese, Islamic and Jewish mathematicians [History of trigonometry; Espenshade; Simonson], as well as probably the scientists of other countries. Thus a lot of knowledge about the cosine function was gained in course of time. However the ancient and medieval mathematics was unable to reveal the constant ɯ even in its simplest geometrical form as a fixed point on the cosine graph. Plots of such functions appeared only when Descartes introduced his analytical geometry and after it became possible to reveal the intersection point of cosine curve with linear function graph. But even then this opportunity was not realized.

The next more sophisticated opportunity is related with Newton's iteration method making possible the obtaining the roots of transcendental equations with desired accuracy. Thus, it was not difficult in principle to evaluate the solution of equation cos $x = x$, however nobody has made that step at that time. The first appearance of number 0.739 085… probably took place in the second half of the 19th century [Bertrand; Briot; Heis; Miller]. Today one can find the cosine equation and, with some or other accuracy, its unique root in numerous studies, mostly in the textbooks explaining and illustrating Newton's method and other methods of functional iteration.

Computers have made easy evaluation of constants with desired accuracy, and the value of constant ɯ is known to 6 400 000 decimal points [Arakelian 2010]. However, everyone can find ɯ with accuracy 10 to 12 decimal places by a standard research calculator by repeatedly pressing the COS key (in radians). As already mentioned above, this is the "empirical" method of evaluating the constant ɯ. Mathematically, the following relation holds:

$$\cos(\cos(\ \ldots\ \cos(x)\ldots) = x.$$

This is the equation for cosine infinite superposition equivalent to three simple transcendental equations $\mathbf{E}_{53}$ for which, as we know, the number ɯ is a unique solution.

Actually, we have here an attractor, not just one of the numerous fixed points [Blanchard, Devaney, Hall], moreover a fundamental mathematical constant. Certainly, FMC is far beyond any attractor, but in its turn attractor is much more than an ordinary fixed point. Anyway, even in the rank of attractor the cosine superposition constant ought to be named. This is the story of how it was named as a *Dottie number*. "The story goes that Dottie, a professor of French, noticed that whenever she put a number in the calculator and hit the cos button over and over again, the number on the screen always went to the same value, about 0.739085… . She asked her math-professor husband why the calculator did this no matter what number she started with. He looked. He tried it. He said he had no idea, at least not that day. The next day he realized not only what was happening, but that his wife had found a beautiful, simple example of a global attractor." [Kaplan]. Thereafter the name "Dottie" has been used in some publications and mathematical forums, though it "is of no fundamental mathematical significance" [Weisstein].

However, the number 0.739085... as a universal attractor, and what is more important as a fundamental mathematical constant has appeared in the early 80s of the past century [Arakelian 1981]. It was named "cosine constant," or "cosine superposition constant" and designated by symbol ɯ (the first letter of Armenian alphabet pronounced as [a:]). The constant was used to solve some numerical problems of physical theory [Arakelian 1981; 1989; 1995; 1997; 2007; 2007*a*]. Taking into account all these facts we think that it would be better to speak about a *constant*, rather than simply a *number*. More specifically, about "Arakelian constant ɯ" or briefly about a "constant ɯ." Also, depending on the context, the constant ɯ may be called "cosine superposition constant," "cosine attractor," "cosine fixed point," "number ɯ," etc.

Back to the equation $\mathbf{E}_5$

Investigation and solution of the functional equations **E** although not technically difficult represents a long and tedious process fully described in [Arakelian 2007, Ch. 2]. In view of the great interest in the constant ɯ we shall go back and demonstrate a small fragment of this process, namely solution of the equation $\mathbf{E}_5$ jointly with equations $\mathbf{E}_1$ to $\mathbf{E}_4$. The process is directly related with examination of basic (non-composite) elementary functions.

First we have to consider particular real quantities specifying the domain of each function and allowed values of its parameters (if any). The general case is considered in the last part of the study.

Exponential and logarithmic functions:

a^x	$-\infty < x < +\infty$	$a > 0$	$a \neq 1$
$\log_a x$	$x > 0$	$a > 0$	$a \neq 1$

Power function:

x^μ	$-\infty < x < +\infty$	$\mu = k/m$ – irreducible fraction, m is odd
	$x > 0$	$\mu = k/m$ – irreducible fraction, m is even
	$x > 0$	μ is irrational, $x = 0$ only if $\mu > 0$

Trigonometric functions:

$\sin x$	$-\infty < x < +\infty$		
$\cos x$	$-\infty < x < +\infty$		
$\tan x$	$-\infty < x < +\infty$	except $x = (2n+1)\pi/2$	$n = 0, \pm1, \pm2\ldots$
$\cot x$	$-\infty < x < +\infty$	except $x = n\pi$	$n = 0, \pm1, \pm2\ldots$

$\sec x$	$-\infty < x < +\infty$	except $x = (2n+1)\pi/2$	$n = 0, \pm 1, \pm 2\ldots$
$\csc x$	$-\infty < x < +\infty$	except $x = n\pi$	$n = 0, \pm 1, \pm 2\ldots$

Inverse trigonometric functions:

$\operatorname{Arcsin} x$	$-1 \le x \le 1$
$\operatorname{Arccos} x$	$-1 \le x \le 1$
$\operatorname{Arctan} x$	$-\infty < x < +\infty$
$\operatorname{Arccot} x$	$-\infty < x < +\infty$
$\operatorname{Arcsec} x$	$\lvert x \rvert \ge 1$
$\operatorname{Arccsc} x$	$\lvert x \rvert \ge 1$

Hyperbolic functions:

$\sinh x$	$-\infty < x < +\infty$	
$\cosh x$	$-\infty < x < +\infty$	
$\tanh x$	$-\infty < x < +\infty$	
$\coth x$	$-\infty < x < +\infty$	except $x = 0$
$\operatorname{sech} x$	$-\infty < x < +\infty$	
$\operatorname{csch} x$	$-\infty < x < +\infty$	except $x = 0$

Inverse hyperbolic functions:

$\operatorname{Arsinh} x$	$-\infty < x < +\infty$	
Arcosh	$1 \le x < +\infty$	
$\operatorname{Artanh} x$	$-1 < x < 1$	
$\operatorname{Arcoth} x$	$\lvert x \rvert > 1$	
$\operatorname{Arsech} x$	$0 < x \le 1$	
$\operatorname{Arcsch} x$	$-\infty < x < +\infty$	except $x = 0$

Next we are going to successively eliminate the functions which do not satisfy the equation $\mathbf{E}_5$. With this purpose, we formulate in decreasing order of generality the list of necessary requirements that have to be imposed on the desired functions.

(a) The equation $\mathbf{E}_5$ holds for all real numbers without any exceptions.

(b) The plots of functions $f(x), f^{-1}(x)$ and $y = x$ have a common point of intersection.

There exist ten functions meeting these requirements.

Table 1
Solutions of equations $f(x) = f^{-1}(x) = x$ for elementary functions

Function	**Solution of equation $\mathbf{E}_5$**
$\sin x$	0
$\sinh x$	0
$\tanh x$	0
$\operatorname{Arsinh} x$	0
$x^{k/m}$	0 and 1 – for all k/m
$\operatorname{Arctan} x$	0 – principal branch
	±4,4934094579…
	±7,7252518369…
	……………………
$\operatorname{Arccot} x$	±0,860333588… – principal branch
	±3,4256184556…
	±6,4372981764…
	……………………
a^x	$0 \div 1$ for $0 < a < 1$
	0,64118 57445 … for $a = 2^{-1}$
	0,56714 32904… for $a = \mathrm{e}^{-1}$
	0,39901 29782… for $a = 10^{-1}$
$\cos x$	0,73908 51332…
$\operatorname{sech} x$	0,76500 99545…

Zero has already been given axiomatically, and only a single solution is allowed for each function. Consequently, we have the third requirement:

(*c*) The functions $f(x)$, $f^{-1}(x)$ and $y = x$ must intersect in a single non-zero point.

The last condition is met by three functions: a^x, $\cos x$, and $\operatorname{sech} x$. In view of the fact that $\mathbf{E}_5$ is an integral part of system of equations $\mathbf{E}_1$ to $\mathbf{E}_5$, we conclude that inequality $0 < a < 1$ is satisfied by a value e^{-1}. Thus we come to the following functions and their respective constants:

$$e^{-x} = -\mathrm{Ln}(x) = x \qquad x_e = 0.56714\,32904\ldots$$

$$\cos x = \arccos x = x \qquad x_c = 0.73908\,51332\ldots$$

$$\mathrm{sech}\, x = \mathrm{arsech}\, x = x \qquad x_h = 0.76500\,99545\ldots$$

Since the system of formal mathematics has been constructed not only for real numbers, we have the last requirement:

(d) The equation $\mathbf{E}_5$ must hold for all imaginary numbers.

The function $\mathrm{sech}(\mathrm{i}x) = \sec(x) = 1/\cos x$ is not defined in the points $n\cdot\pi\mathrm{i}/2$ ($n = \pm1, \pm2, \ldots$), so in the long run we finally come to constants ɯ and W(1).

In the light of foregoing considerations we shall proceed from the assumption that the set of required fundamental mathematical constant is sufficiently full. From the heights of current knowledge the FMC have gained universal significance in the limits of mathematics and natural sciences using mathematics. Only in the case when the research arsenal has sufficiently full collection of basic, primary FMC, including superposition constants ɯ and W(1), the mystery of theoretical definition of physical constants can be converted from unsolvable category to solvable class of problems.

Trying to specify in a few words the characteristic feature of each FMC we give the following definitions:

0 absence of given quantity or property

π transition from rectilinear to curvilinear

e fast growth

i periodic processes

2 nonlinear relationship

γ transition to integral forms

ɯ, W(1) transition from multiple to single

Additional remarks on cosine and constant ɯ

In addition to preceding consideration there are two facts worth to be taken into account. The cosine function, as arithmetic mean of exponent and inverse exponent functions, may be generalized by means of relation

$$\cos_{apq} x = (a^{qix} + a^{-qix})/p$$

where a, p, q are positive numbers, $a \neq 1$. For example when $a = \phi$ (the number of golden mean), $p = \phi/2$ and $q = \gamma$, then the threefold point $x_{apq} = 0{,}87392\,11247\ldots$ When $p = 1$, while a and q *are* arbitrary positive numbers, the function $\cos_{apq} x$ assumes all values in the interval $-2 \le y \le 2$, and when p is arbitrary positive number, $-2/p \le y \le 2/p$. Particularly, when $p = 2$, we obtain the interval of values for ordinary cosine function: $-1 \le y \le 1$. The system of equations **E** uniquely determines the maternal functions e^x and Ln x, so in our case no alternative to numerator of cosine exists. But how about the denominator p? Isn't the expression $\cos_1 x = \mathrm{e}^{\mathrm{i}x} + \mathrm{e}^{-\mathrm{i}x}$ simpler than the arithmetic mean of the some exponents? It is worth noting that historically Euler established the true character of cosine by comparison of independently obtained expressions $\mathrm{e}^{\mathrm{i}x} + \mathrm{e}^{-\mathrm{i}x}$ and $2\cos x$.

Testing the function $\cos_1 x \equiv 2\cos x$ we arrive at a quite interesting result demonstrated by two plots in Fig. 4 compared with cos x superposition graph shown in Fig. 3.

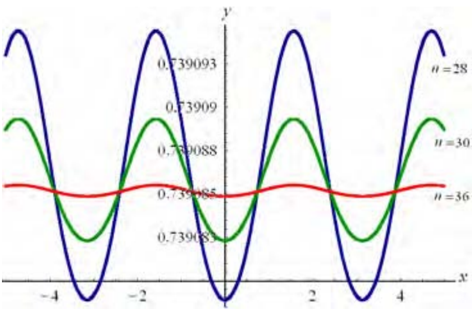


Fig. 3
Plot of superposition $\cos(\cos(\ldots\cos(x)\ldots))$ for $n = 28$, 30 and 36 steps

The superposition of cos x step by step leads, as it must, to the number 0.73908513… The picture is different for the function $\cos_1 x$.

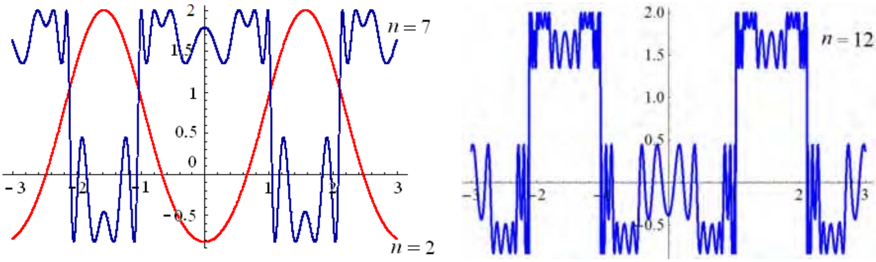


Fig. 4
Plots of superposition $\cos_1(\cos_1(\ldots\cos_1(x)\ldots)$ for $n = 2$, 7 and 12steps

Due to large number of peaks, only a short interval of abscissa may be considered. It is impossible to demonstrate the curves after $n \gg 12$ steps. But even here it is clear that the function $e^{ix} + e^{-ix}$ does not converge to some certain point. Therefore this combination does not satisfy the equation $\mathbf{E}_5$ and in that sense it does not represent an alternative or supplement to the ordinary cosine function.

The second fact results from comparison of the functions

$$\cos x = (e^{ix} + e^{-ix})/2 \quad \text{and} \quad \sin x = (e^{ix} - e^{-ix})/2i$$

from viewpoint of superposition. Geometrically, the cosine dependence is a sinusoid shifted by $\pi/2$ along the real line; analytically it is expressed by a relation $\sin(x \pm \pi/2) = \cos x$. This fact can create a false idea of proximity, almost identity of two functions. While parallel translation say of a linear function $y = ax + b$ along the x-axis does not affect its basic characteristics (it is actually the same function having other position in the coordinate plane), the cosine and sine functions have actually different structure, with essential difference being revealed by superposition procedure.

Consider for example the variable $z = 2i$. The infinite superposition of the cosine converts 2i, in the same way as real or imaginary number, into a constant ɯ. Meanwhile superposition sin(2i) results in a sequence of imaginary numbers 3.6i, 18.8i, 7.2·109i, 4.7·1031227831i ... with their absolute value growing to infinity. Among other things, this fact means that zero is superposition constant of sine function only in the range of *real* numbers (see the table given above). We can also state that the attractor in this case is of lower rank than the constants ɯ or W(1).

Thus, cosine is one of the most important functions in mathematics. It has numerous applications in engineering and is often used for mathematical description of various oscillatory processes. Whenever the solution of a problem contains the function cos φ, one can always simplify the result by substituting cos φ by the constant ɯ. Consider an example of logarithmic spiral. The length l of the arc starting from the pole and ending at arbitrary point of the spiral is given by the expression $l = r/\cos$, where the main parameter of spiral φ is the angle between the radius vector and the tangent to the curve at the selected point. It is easy to see that the simplest relation $l = r/$ɯ holds when the angle φ is equal to ɯ.

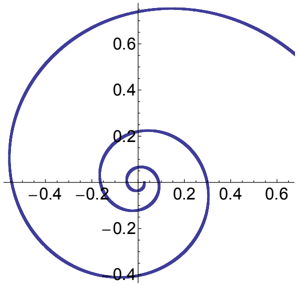


Fig. 5
Logarithmic spiral with angle φ = ɯ

In angular units ɯ·180°/π ≈ 42.3465°, or 42°20′47″.

Any new FMC represents a powerful tool for solving certain problems “unsolvable” before. Constant ɯ is something like a *hidden parameter* in mathematics. As we shall see later, the doors looking tightly closed are often easily opened by this constant. Like any other FMC, the constant ɯ may find numerous applications in other areas of science and technology except fundamental physics. The most promising fields today are oscillatory processes, the theory of chaos, fractals, dynamical systems, planetary orbits [Bojar; *AKiTi*].

Chapter II. LMP-Theory: Physics

System of physical codes

The last concept in triad of logic, mathematics, and physics is considered as extension of pure mathematics and not just a discipline supported by mathematics along with other research methods. In this connection, the commonly used term *mathematical physics* should be substituted by a more appropriate term *physical mathematics*. We assume that the main principles established or revealed for the formal logical-mathematical **AGE**-system serve at a same time the basis and constructive inception of the FPT, as a theory of fundamental physical quantities, at least in its unified semi-formal representation. In the **AGE**-system the embodiment of idea of primary quantities and corresponding primary laws is represented by a collection of eight FMC: 0, π, e, i, 2, γ, ɯ, W(1), jointly with two maternal functions ψ и α. From the viewpoint of LMP concepts, these constants lie in the natural origin of FPT. Since there are simply no alternatives, we can state that *only exponential-logarithmic representation and mathematical constants* are the single *adequate form for definition of fundamental physical laws and quantities, especially the constants*.

Turning to the tree-diagram we have to specify the role of branches, i.e. foundations of the physical theory, or extension of physical mathematics into the field of external world realities. Physical mathematics being a constituent part of LMP-system is called to solve a number of problems similar to the logical-mathematical parts of the system. Hence it is needed to select the principal components of fundamental physics, give a clear systematization and then re-expose and supply if necessary by corresponding material, presenting the system in unified and rigorous form. It is also desirable to reveal the system potential by obtaining some new results which could not be provided by other way.

Earlier, we have arrived at the ψ-α, or exponential-logarithmic presentation, a natural and only possible, from positions of **AGE**-system, formal-analytical basis of physical theory. In the general case, it is represented by the equation of **C**-type:

$$\mathbf{C} \quad z = \psi[\alpha(a)\cdot z + b\cdot\alpha(u)] \equiv \exp(\mathrm{Ln}\, a\cdot z + b\cdot \mathrm{Ln}\, u).$$

It should be also be equal to product of functions a^z ($a \neq 0$) and u^b ($u \neq 0$) depending on complex variables z and u and complex constants a and b; for the principal value of logarithm and real numbers

$$\mathbf{C}' \quad w = \psi[\alpha(a)\cdot x + b\cdot\alpha(y)] \equiv \exp(\ln a\cdot x + b\cdot \ln y) = a^x y^b.$$

The mathematical conservation law **C**, and its special case **C**′, are general forms of representation of all complex and real numbers, except 0, given earlier by axioms. By assigning different values to constants a and b one can obtain the whole collection of elementary functions or blocks required to construct, by mathematical operations, more complicated functions. Fixing therefore the values of variables one will come to definite relations between the mathematical quantities also designed to perform a real transition from pure mathematics to physical theory.

Using the requirement that all quantities should be single-valued and assuming that all (at least "independent") mathematical and physical constants (except i) must be real we shall take as a basis for construction the equation **C**′. Fixing a universal constant a_0 of exponential function, we identify b as a fundamental mathematical constant 2: $w = a_0 y^2$. Representing now the variable y^2 in all possible versions for the square-law form: u^2, $a_1 v^2$, and a_2/t^2 with new constants a_1, a_2 and variables u, v and t and denoting the new functions as w_1, w_2, and w_3, we obtain:

$$\mathbf{C}'_1 \quad w_1 = \exp(\ln a_0 + \ln u^2) \quad = a_0 u^2$$

$$\mathbf{C}'_2 \quad w_2 = \exp(\ln a_0 + \ln a_1 v^2) = a_0 a_1 v^2$$

$$\mathbf{C}'_3 \quad w_3 = \exp(\ln a_0 + \ln a_2/t^2) = a_0 a_2/t^2$$

Similarly, on symmetrical grounds, fixing the constant value of power-mode function a_3 and identifying a_0 with fundamental constant 2, we have:

$$\mathbf{C}'_4 \quad w_4 = \exp(x \ln 2 + a_3) = a_3\cdot 2^x$$

Note that the expression in parentheses is linear in variable x multiplied by the constant ln 2, so no other options are possible in this case. Using the index j for distinguishing the quantities-functions and arguments from constants, we introduce the final notations for the variables and constant mathematical quantities:

$a_0 \equiv 1/\hbar c \qquad a_1 \equiv G \qquad a_2 \equiv G_F \qquad \ln 2 \equiv 1/k$ (or $k \equiv 1/\ln 2$)

$w_1 \equiv \alpha_{ej} \qquad w_2 \equiv \alpha_{Gj} \qquad w_3 \equiv \alpha_{Wj} \qquad w_4 \equiv \Omega_j$

$u \equiv e_j \qquad v \equiv m_j \qquad t \equiv \lambda\!\!\!^{-}_j \qquad x \equiv S_j$

Assigning to symbols their usual physical interpretations:

- c is the light velocity in vacuum
- $\hbar$ is Planck constant
- G is gravitational constant
- G_F is Fermi coupling constant
- k is Boltzmann constant
- e_j is a family of charges (electric, weak, magnetic, strong)
- m_j is mass
- $\lambda\!\!\!^{-}_j$ is Compton length
- S_j is entropy
- Ω_j is the number of microscopic states of a macrosystem

we have a system of four equations for physical constants and variable quantities:

$$\mathbf{C}_1 \quad \alpha_{ej} = \exp\left(\ln\frac{1}{\hbar c} + \ln e_j^2\right) = \frac{e_j^2}{\hbar c}$$

$$\mathbf{C}_2 \quad \alpha_{Gj} = \exp\left(\ln\frac{1}{\hbar c} + \ln G m_j^2\right) = \frac{G m_j^2}{\hbar c}$$

$$\mathbf{C}_3 \quad \alpha_{Wj} = \exp\left(\ln\frac{1}{\hbar c} + \ln\frac{G_F}{\lambda\!\!\!^{-}_j{}^2}\right) = \frac{G_F/\lambda\!\!\!^{-}_j{}^2}{\hbar c}$$

$$\mathbf{C}_4 \quad \Omega_j = \exp\left(\frac{S_j}{k}\right)$$

We shall call the equations $\mathbf{C}_1$–$\mathbf{C}_4$ a *system of physical codes*. This is one of the most significant steps in construction of LMP system, which signifies transition from logic-mathematics to foundations of physical theory by adding the system of codes **C** to the system of logical postulates, mathematical axioms and initial functional equations. Complemented in this way the **AGE**-system becomes a **AGEC**-system. Integrating role of the code system **C** unifying the principal physical quantities and actually covering the whole theory is evident. Such equations and correlations could have appeared as a result of peculiar synthesis of the physical theory achievements and only at a certain stage of its development. Without these equations the concept of integrity and idea of logical-mathematical formalism transforming into the formalism of physical theory would look as a speculative chimera. On the other hand, the ideas lying in the basis of LMP-system act as a code of prohibitive laws and allowed constructions, as some selector correcting the research process, selecting and systemizing the phenomena that stay in accordance with the internal logic of their development. The logic imperatively requires transition from formal mathematics to fundamental physics to be realized by analytical laws and rules assumed to have basic character and by means of substantial elements.

In essence, the issue is how the systematic ψ-α-transition from mathematical to physical quantities, more specifically from mathematical quantities and FMC to physical quantities and constants should be performed by using the concept of dimensionality. The clue, as we believe, is given by the system **C** of four types of equations and correlations. This system contains universal codes of the physical theory related with main physical quantities, laws and dimensions. Dimensional analysis practically represents a complete theoretical product due to which the formal mathematical method is able to solve some general problems avoiding certain specific characteristics of physical theory and dealing only with physical quantities.

Physical quantities and dimensions

Dimensional analysis is almost a ready fragment of physical mathematics. All that is needed for dimensional analysis are physical quantities ready for use. And these quantities are given by code equations. All constants and variable quantities included in the initial system **C** of equation are naturally considered as fundamental. It is worth introducing now a number of notions and giving their definitions in order to establish relationship between the notions of *physical quantity* and *physical dimension*. We shall call dimensionless or zero-dimensional any physical quantity which actually represents mathematical quantity. Thus from the way the equations $\mathbf{C}_1 - \mathbf{C}_4$ are obtained it follows automatically that the parameters $\alpha_{\mathrm{e}j}$, α_{Gj}, $\alpha_{\mathrm{W}j}$, Ω_j are zero-dimensional physical quantities. And if the differences in the way of obtaining the first three quantities are taken into account, then one can speak about two main types of dimensionless parameters: the family of coupling constants α_{jk} and entropy quantity Ω_j. All other quantities entering the equations **C** are called dimensional. Thus we have four types of dimensionless equations relating dimensional physical quantities. The same denominator $\hbar c$ in first three **C** equations means that the expressions e_j^2, Gm_j^2, $G_F/\lambdabar_j^2$ have the same dimensionality, as well as their square roots e_j, $G^{1/2}m_j$, $G_F^{1/2}/\lambdabar_j$, which we shall call *charges*. Denoting the dimension of a physical quality by square brackets, we have five main dimensions in total, denoted by symbols A, V, J, S, Q:

A $\equiv [\alpha_{xj}] \equiv [\Omega_j]$ – dimension of α_{xj} or Ω_j, i.e zero-dimension

V $\equiv [c]$ – dimension of the velocity of light or simply of speed

J $\equiv [\hbar]$ – dimension of Planck constant or action

S $\equiv [k]$ – dimension of Boltzmann constant or entropy

Q $\equiv [e_j] \equiv [G^{1/2}m_j] \equiv [G_F^{1/2}/\lambdabar_j]$ – dimension of generalized charges

According to the general definition, *dimension* of physical quantity represents a simple analytical expression establishing formal relationship between the certain quantity and fundamental quantities. The same dimension of both parts of an equality is a universal requirement imposed on all physical sentences – formulas, equations, correlations, etc. Keep in mind that dimension is preserved when homogeneous quantities are added or subtracted. The dimension is altered when two quantities are multiplied and “vanishes” after their division. This fact actually means that one cannot add or subtract the quantities having different dimensionality, but only multiply and divide them. Such a strong limitation is absolutely inadmissible for mathematical quantities. Indeed, only division by zero is not allowed in mathematics while all other operations are quite “legal” for numbers. Note that the idea of mathematical and physical quantities unity demonstrates that dimensionality considerations are insufficient for establishing such unity.

It is important to continue disclosing the specific features of system **C** related with dimensionality. Even without formal analysis of the equations $\mathbf{C}_1$–$\mathbf{C}_3$ it is clear, at a glance, that one cannot obtain the dimensions of four fundamental quantities G, G_F, m_j, $\lambdabar_j$ in the system AVJSQ. It is evident in view of the fact that, along with the form including only the variable for the generalized charge e_j, the charge is also present in gravitational Gm_j^2 and weak interaction G_F/λ_j^2 options, with constants G and G_F and variables m_j and λ_j. Let us introduce the following notations:

$\mathrm{G} \equiv [G]$ dimension of gravitational constant

$\mathrm{G_F} \equiv [G_F]$ dimension of Fermi constant

$\mathrm{M} \equiv [m_j]$ dimension of mass

$\mathrm{L} \equiv [\lambda_j]$ dimension of Compton length

By substituting Q with any one of these four dimensions and having the dimension of charge $(\hbar c)^{1/2}$ in the equations $\mathbf{C}_1-\mathbf{C}_3$ we can obtain four other possible systems of principal dimensions in which the problem of construction of arbitrary physical dimension is already solvable in full extent.

In a word, any dimension taken from the initial equations may be considered as a principal dimension, at equal terms with other dimensions. This fact results in excessive number of dimensions, nine principal dimensions. Meanwhile their minimum amount, including A, must be equal to five, according to **C** system and physical theory data. This offers a considerable scope for varying the list of principal dimensions. Thus one can state that the method of dimensional analysis is unable to produce unambiguous results for the fundamental values of **C** system. Physical quantities are related by numerous expressions including different variables and constants. Moreover, the number of such “independent” relations significantly exceeds the number of quantities themselves. Thus, considerable freedom is given for choosing the system of initial dimensions.

On fundamental laws of physics. Conservation laws

Physics, as a science on physical quantities is called to reveal and order the families of interrelated quantities describing the physical reality. The essence and theoretical potential of a physical quantity may reveal themselves both from its internal characteristics and from its analytical relations with other quantities. All these features are encoded, in the most general form, in the system **C** of equations requiring sequential decoding and interpretation. In accordance with the initial concepts of variable and constant one should speak about three types of fundamental laws: ***conserva-***

tion, ***variation*** and ***quantization*** laws encoded in the **C** system in their integrity and formal unity.

It is known that any sentence of mathematics can be introduced by means of operation "=", propositional connectives and quantifiers. Also any equality represents a conservation law for analytic connection of mathematical quantities. But we are currently interested not in the "conservation law of laws" but in a more specific case of conservation laws of fundamental physical quantities, including constants, the initial collection of which is given by the **C** system of equations. Except the constants c, $\hbar$, k, G, G_F the fundamental physical constants also include singled out and marked values of functions α_j, α_{Gj}, $\alpha_{\mathrm{G}j}$, Ω_j, of the variables e_j, m_j, $\lambdabar_j$, S_j as well as all physically meaningful combinations of all listed variables. It is difficult to count the exact number of "independent" physical constants (in difference to mathematical constants), but one can speak of more than ten singled out special points of continuum – fundamental physical numbers which are preserved under all variations of physical reality.

One such number having the status of a great law of nature is unique being the single marked representative of its class of quantities. This is the constant c entering the equations $\mathbf{C}_1$–$\mathbf{C}_3$ representing not only a universal preserving quantity but also the only singled out velocity in the nature. Following the letter and spirit of the **AGEC**-system, one has only to warn that in correct and meaningful formulation of the law c = const it is inadmissible to use the expressions often encountered in the literature like *observer*, *in time* and *inertial frame of reference*. For example it is incorrect by many reasons to say that a physical quantity *is always* constant, or *always* has the same value. Stated otherwise, it is wrong to define a conservation law as invariance in time of some or other physical quantity. Clearly, the main reason of millennium-long worship of *Time* is due to the fact that we all are prisoners of limited and narrow character of our perception of the external world. According to this prejudice, any changes of physical characteristics are imagined and introduced as something taking place in space and time. Meanwhile, it has been reliably established that the "arrow of time" (Eddington) flies from past to future via the present only due to the law of increasing entropy (equation $\mathbf{C}_4$).

It is also meaningless to define the fundamental physical laws by means of the concepts like *inertial frame of reference*, *closed system*, *isolated system*, because such approach inevitably results in vicious circle of logic. Indeed, if one tries to disclose the content, for example, of an of inertial frame of reference (or coordinate system) then one immediately finds that it is a system where conservation laws are valid, which in their turn are true in the systems where conservation laws are applicable, etc. It is possible in principle to do without explicit indication of the physical system, by considering the corresponding equations. However the equations must refer to something, and this fact inevitably brings to the issue of existence of a certain isolated and privileged physical system allowing to overcome the mentioned difficulties.

Actually the matter is not so complicated and hopeless as it appears at a first glance. The outcome has actually been found long ago [Clausius] and is reproduced often intuitively, by many researchers. The *Universe*, or integral physical world existing in a single copy with all its parameters, properties, relations and characteristics is the desired system, singled out by the very fact of its existence. Hence all fundamental physical laws must be attributed to the Universe.

Light velocity conservation law in vacuum

The constant c represents a permanent parameter of the Universe. The value of this quantity does not depend on any physical changes.

The more formal, mathematical statement of the same proposition, having no reference to physical system can be given as follows:

The number c is preserved in all physical equations and relations, as well as mathematical transformations having physical meaning.

And finally in the logical-mathematical terminology the statement has a form:

The individual term c represents an absolutely invariable constant of the LMP-system formalism.

Another great conservation laws refers to a quantity represented in codes by a constant $\hbar$ and called *action*, momentum, angular momentum, spin, etc. – depending on the physical domain and context where it appears. Only a single list of titles of this quantity speaks of plurality and broad spectrum of applications due to its wonderful characteristics. Here are some features of action given below.

(*a*) *Invariance* of quantum mechanical ψ-function to transformations $\psi \rightarrow \exp(-i\varphi J/\hbar)$ where φ is rotation angle. Physically this fact may be treated as isotropy, or equivalence of all directions in space making impossible selection of absolute direction;

(*b*) *Classification* of all elementary particles depending on their spin which results in different mathematical models and description methods of various particle groups;

(*c*) Heisenberg *uncertainty relations* for canonically conjugated quantities having the product of their dimensions equal to action dimension and lower limit equal either to $\hbar/2$ or $\hbar$;

(*d*) *Variational principle* related with the action integral, Lagrangian, Noether's theorem, as well as equality to zero of action variation (least action principle) in mechanics, quantum physics, field theory, elementary particle physics, i.e. almost everywhere;

e) Obtaining, by means of *Noether's* mathematical *theorems*, the whole family of secondary conservation laws, as well as unified method of obtaining different equations of the existing theory by variation of action.

Such are some major characteristics of the physical quantity as important as the *c* constant.

Conservation law of action

The action of the Universe preserves.

Clearly, the action is invariant to all physically meaningful transformations, so we won't give the definition of law either in mathematical or logical terms. With account of all external factors the action of course is conserved. This fact is proved by numerous experimental results concerning physical processes, although it actually is a consequence of the action value constancy in the Universe.

There are also charges left in the **C** system.

Generalized charges conservation law

Charges are conserved in the Universe.

The charges are known to conserve in all processes running in the Universe, but like in the case of action this statement is direct consequence of the general law. In difference to the action, several types of charges exist, therefore one has to consider a general law and plural number in its definition. The concept of generalized charge e_j conservation which is actually present in the $\mathbf{C}_1$ equation is specified by the equations $\mathbf{C}_2$ and $\mathbf{C}_3$. Today one can speak, with some or other certainty, about five types of fundamental charges: electrical e_e, magnetic e_m, strong interaction e_s, weak interaction e_W and gravitational charge e_G.

Quantization laws

Formally, some quantities entering the system **C** of equations represent numerical sequences constructed by a certain law which reflect the internal characteristics of physical quantities. The rules of composition of such mathematical sequences out of physical quantities form the second group of fundamental physical laws, namely ***the laws of quantization***. It looks like Nature prefers discrete series limited both from below and above to continuous, infinite continuums. The victorious march of quantum physics started from discovery of action quantum continues until now. Step by step, slowly but steadily the continuous quantities of classical physics are replaced in the theory by quantized quantities. It seems that the eternal question *continuous or discrete* is being resolved in physics in favor of the latter.

Thus, we have the following laws for setup of discrete numerical sequences forming the discrete spectra of fundamental physical quantities' values:

Quantization law of action: $J = \frac{\hbar}{2} \cdot n \quad n = 1, 2, ..., N_U$

Quantization law of entropy: $S = \frac{k}{2} \cdot n \quad n = 1, 2, ..., N_U$

Quantization law of charges: $Q = \pm e_{j0} \cdot n \quad n = 1, 2, ...$

In the last law, quantization of all types of charges goes in elementary charge which in the case of electrical charge is equal to $\pm e$ for leptons and $\pm e/3$ for quarks.

Presenting the equation $\mathbf{C}_1$ in form

$1/\alpha_j = e_j/c^2 \cdot \hbar$

is easy to guess what other quantization laws may be obtained for the secondary quantities containing the multiplier $\hbar$:

Integer quantization law of Hall resistance: $R_j = \dfrac{2\pi\hbar}{e^2} \cdot \dfrac{1}{n^2}$

Fractional quantization law of Hall resistance: $R_j = \dfrac{2\pi\hbar}{e^2} \cdot \dfrac{n}{2k+1}$

Quantization law of magnetic flow: $\Phi = \Phi_0 n = \dfrac{\pi c}{e} \cdot hn$

Finally we observe that discreteness is a universal characteristic of the physical world, so that quantization as the secondary law is possible for other physical quantities as well. For example quantum of circulation as a combination $\pi\hbar/m_e$ from the equation $\mathbf{C}_2$ (with defined value of the π).

Variation laws

Any mathematical equation containing variable physical quantities may be considered as a variation law of these quantities. Usually the statement on conservation of a quantity is executed as an equation including except the constants and transformation invariants also the variable quantities. Actually, conservation law of some quantities is presented as a variation law of other quantities provided that the first remain constant. One may think therefore that no essential difference exists between the two types of physical laws. However, their differences being hidden when secondary quantities are used in the laws become entirely clear when laws are formulated in general form. All fundamental variation laws (with account of assumption on necessary and sufficient character of the **C** system for solution of such problems and due to formal and meaningful characteristics of quantities entering the system) refer to the functions/variables α_j, α_{Gj}, α_{mj}, α_{Gj}, α_{sj} and S_j. In accordance with this fact one has a group of variation laws for five coupling constants of five fundamental interactions with five types of charges, as independent variables as well as somewhat isolated law for entropy. Formulation of the latter is very simple.

Entropy variation law

The entropy of the Universe increases.

Mathematically, this is the equation $\mathbf{C}_4$: $S_j = k \ln \Omega_j$. Combining it with the entropy quantization law: $S_j = j{\cdot}k/2$ we obtain the Universal microscopic states quantity variation law

$$\Omega_j = \mathrm{e}^{j/2}, \quad j = 1, 2, 3, ..., N_U$$

which generates a fast growing exponential series $\mathrm{e}^{1/2}$, e, $\mathrm{e}^{3/2}$, e^{2}, $\mathrm{e}^{5/2}$...

Concerning the α-functions, discovery of these dimensionless physical quantities belongs to the most remarkable events in the history of science. In the unified exponential-logarithmic form **C** the quantities α_{xj} are varying by the laws which together with law $\mathbf{C}_4$ belong to fundamental variation laws of the physical quantities. Generally, all variables singled out to some or other extent in the system $\mathbf{C}_1$–$\mathbf{C}_3$ correspond to selected values of functions α_{xj}. Their combinations form discrete ordered sets of zero-dimensional functions establishing the numerical framework of the physical world. On the other hand all values of functions α_{xj} correspond to certain values of e_{xj}, m_j, $\lambda\!\!\!^{-}_j$ on basis of equalities

$$e_j = \sqrt{\alpha_j \hbar c}, \quad m_j = \sqrt{\frac{\alpha_{Gj}\hbar c}{G}}, \quad \lambda\!\!\!^{-}_j = \sqrt{\frac{G_\mathrm{F}}{\alpha_{\mathrm{W}j}\hbar c}}, \text{ and also } \lambda\!\!\!^{-}_j = \hbar/m_j c.$$

Such inverse dependence between the function and argument is particularly important for the gravitational charge and mass having no quantization law. At least if such a law exists, it is not simple as for the charges e_e, e_m, e_s, e_W of four fundamental interactions.

A-system: absolute dimensionless system of physical quantities measurement

The **A**-system is defined as a final merger of physical and mathematical quantities, which was intended from the very beginning and partially realized in the system of codes **C** for constant $k = 1/\ln 2$ and then for $c_\mathrm{A} \equiv \alpha^{-1}$, however not resolved yet in the general form. The idea is to reduce physical quantities to a form of mathematical number. With this purpose, one has to build first the system of physical quantities of measurement which instead of gram, centimeter, second, etc. would be based on fundamental physical constants (M. Planck) and then to reduce all physical constants to mathematical constants (D. Hilbert). Only the first part of this program has been realized in the dimensionless systems of measurement: Planck system ($c = \hbar = G = k = 1$), Hurtree system ($\hbar = m_e = e = 1$), relativistic quantum theory ($c = \hbar = m_e = 1$) and others. Meanwhile the idea of D. Hilbert proposed in form of manifesto (although on other reason) "to reduce all physical constants to mathematical constants, as a result of which physics may well become a science like geometry" **[**Hilbert**]** has not been realized by these systems. Indeed, the PCs are taken equal to unity in these systems, rather than FMC or their simplest combinations. Therefore all such dimensionless systems determine only the values of physical quantities in the *arbitrarily chosen unity scales*, rather than true values of the physical quantities. In the Planck system, for example, all velocities $\upsilon_P \leq 1$, while all values of action, with account of law of its quantization, are expressed by integers or half-integers.

The dogma on primacy of natural numbers as lying in the foundations of mathematics was extended to principles of physics. The remarkable idea of Planck which was certainly revolutionary for its time turned out to be largely devaluated by unreasonable decision to set the FPC equal to unity. Practical advantages of this procedure, particularly a simple form of physical equations free of unity scale constants, do not compensate the losses in understanding the sense, the physical meaning of equation. They are able even to be misleading. Unjustified introduction of unitary constants is distinctly seen in the background of zero-dimensional quantities whose values are independent of measurement system. Such are the dimensionless functions α_{xj} and Ω_j in the **C**-system of physical codes representing just numbers often far from unity.

For the FMC, and moreover for their combinations, it is natural to distinguish graphic symbols π, e, ɯ and true expressions made of these symbols, e.g. e^π, $\pi/2$, $ɯ/\pi^2$ from the numerical values of these expressions. Hence, "the true expressions of physical constant" ought to be understood as the formula of its connection with mathematical and other physical quantities. Meanwhile the numerical value of the physical constant is the usual representation of this constant in some or other (most often decimal) number system. For example, the mathematical term 1/ln 2 is the true expression of Boltzmann constant k, while the number 1.44279604… is decimal (infinite) representation of the same constant.

For completeness, we need another three true expressions which will reduce any dimensional physical constant, and generally any physical quantity to a dimensionless form of mathematical number. Four, as minimal number of initial physical constants expressed mathematically corresponds to the number of independent equations entering the **C**-system, as well as the number of fundamental dimensions, except the zero-dimension. Such coincidence is clearly not occasional, since the **C**-system has been composed to include the number of basic equations necessary and sufficient for solution of cardinal issues of fundamental physical theory, including the aspects of dimensionality. We shall call **A-**system the measurement system based on true mathematical expressions for the initial physical constants. All quantities attributed to the **A-**system except the initially dimensionless like α_{xj} will be marked by subscript A.

Truth of the expression $k_A = 1/\ln 2$, besides the considerations of obtaining the physical codes by means of ψ-α functional representation, is supported by purely physical considerations. They are related with distinct recognition of the physical meaning of constant k and comparison of the Boltzmann formula $\mathbf{C}_4$ with Shannon's expression for the entropy of minimal code. According to the well-known equipartition of energy, the quantity $kT/2$ represents the average energy per one degree of freedom of a system existing in the state of thermodynamic equilibrium. Proceeding from the equipartition law, one should interpret the Boltzmann constant k, or more correctly its half-value $k/2$, as a quantum of entropy per one degree of freedom. For additional confirmation of this fact we turn to the third law of thermodynamics also called Nernst theorem. Classical interpretation of the third law states that the entropy of any system tends to zero as the temperature goes down to zero. In the more rigorous formulation, when nuclear spins of the cooled body are taken into account, the entropy tends to its minimal positive value S_0 when temperature goes down to absolute zero $T_{\min}$. Existence of the minimal finite entropy S_0 may be interpreted as a statement that absolute order is unattainable in the nature. The minimal value of entropy is easily obtained through the Boltzmann relation for $\Omega = 2$:

$$S_{\min} = k \ln 2.$$

Thus $S_{\min}$ is proportional to the constant k with coefficient ln 2 ≈ 0.693. Hence we come to a conclusion that, with the coefficient having the order of unity, the constant k represents elementary portion of quantum of entropy. The conclusion is based on the third law of thermodynamics, Boltzmann formula and law of energy equipartition. This is the principal physical meaning of the constant k, making it similar to quants of action $\hbar$ and electromagnetic charge e. It remains to find the multiplier of order 1, for which we continue the search in the field of information theory and cybernetics. Here entropy is defined as a measure of information uncertainty in the structure of a system, disorder, etc. In all these cases the entropy is measured in the same dimensionless units, the bits, serving at the same time the measure of information quantity. The entropy of two elementary events having the same probability is one bit. Stated otherwise, the entropy expressed in bits determined the number of binary symbols required for writing the given information. The entropy (in bits) of a physical system having Ω_j microscopic states is known to be given by the expression due to Shannon:

$$I = \log_2 \Omega_j$$

where $\log_2$ is the symbol of logarithm to the base 2. Comparison with Boltzmann formula gives a simple relation:

$$k = 1 \text{ bit}/\ln 2$$

transforming the constant k into bits and vice versa. Proceeding from minimal character of binary code for recording the information and assuming *bit* as an absolute unit of information amount, we obtain the true value of constant k equal to 1/ln 2 = 1.44269… in decimal form, while the Boltzmann formula assumes in **A**-system the following form:

$$S_A = \ln \Omega / \ln 2.$$

Thus the physical consideration complemented by Shannon formula confirms the earlier obtained "code" value of the constant k. The issue may be thus finally resolved.

Construction of the **A**-system requires three more expressions for corresponding physical constants. The most suitable quantities are light velocity in vacuum, Planck's constant and a mass of a truly fundamental particle measured besides with high accuracy. In the publication of the author **[**Arakelian 1981, 139–144**]** the expressions for m_{eA} and $\hbar_A$ were obtained, including the ш constant. Thus the entrance of constant ш in mathematical expressions for many physical constants was provided. The final list **A** of initial relations for the absolute system of measurement of physical quantities has the following form:

$\mathbf{A}_1 \quad k_A \equiv 1/\ln 2$

$\mathbf{A}_2 \quad c_A \equiv \alpha^{-1}$

$\mathbf{A}_3 \quad m_{eA} \equiv ш/\pi^2$

$\mathbf{A}_4 \quad \hbar_A \equiv \pi^2\alpha^2/ш$

Clearly, the complete empirical confirmation of basic relations and **A**-system as a whole is possible only by analysis of numerous consequences resulting from this choice and admitting direct comparison with experimental data. The ultimate list of initial equation and true expressions is given below in form of a table, where expressions for constants and their decimal values as given.

Constants	**The equation and true expressions**	**Decimal values**
c_A	$\cos x + \frac{e^{x-44\pi}}{x^2} - e^{-\sqrt{x}} - \frac{1}{e} = 0$	137.035 999 452 0214...
k_A	$\frac{1}{\ln 2}$	1.442 695 040 888 963...
m_{eA}	$\frac{ш}{\pi^2}$	0.074 884 980 509 814...
$\hbar_A$	$\frac{\pi^2\alpha^2}{ш}$	0.000 711 108 607 804...

Any physical constant, as a combination of initial constants, can be now expressed through mathematical constants. Thus, for the charges e and e_{m0}, or for the Compton time $\tau_{Ce} = \hbar/m_e c^2$ and Bohr magneton $\mu_B = e\hbar/2m_e c$ we obtain the following expressions:

$$e_{\mathrm{A}} = \pm\frac{\pi\alpha}{\sqrt{ш}},\ e_{\mathrm{m0A}} = \frac{\pi}{\sqrt{ш}},\ \tau_{\mathrm{CeA}} = \frac{\pi^4\alpha^4}{ш^2},\ \mu_{\mathrm{eA}} = \frac{\pi^5\alpha^4}{2ш^2\sqrt{ш}}.$$

Hence we have a number of curious relations

$$e_{\mathrm{A}} = \pm\hbar_{\mathrm{A}}^{1/2},\quad e_{\mathrm{m0A}} = \pm 1/\sqrt{m_{\mathrm{eA}}} = \pm c_{\mathrm{A}}\hbar_{\mathrm{A}}^{1/2},\quad \tau_{\mathrm{CeA}} = \hbar_{\mathrm{A}}^{2},\quad \mu_{\mathrm{BA}} = c_{\mathrm{A}}\hbar_{\mathrm{A}}^{5/2}/2$$

reducing these constants to c_{A} and $\hbar_{\mathrm{A}}$.

In regard to arbitrary physical quantities one can speak only on their decimal values calculated by rules general for all dimensionless systems. If, for example, some physical quantity has in the LMTΘ system (L – length, M – mass, T – time and Θ – temperature in Kelvins) a value $B_{\mathrm{LMT\Theta}}$ and dimensionality $\mathrm{L}^p\mathrm{M}^q\mathrm{T}^r\Theta^s$ then in the **A**-system its value B_{A} is determined from the general expression

$$B_{\mathrm{LMT\Theta}} = B_{\mathrm{A}}\, l_{\mathrm{A}}^p m_{\mathrm{A}}^q t_{\mathrm{A}}^r \Theta_{\mathrm{A}}^t$$

where l_{A}, m_{A}, t_{A}, Θ_{A} are uniquely determined transition coefficients from LMTΘ to **A**-system, or vice versa. They are calculated by the same rules as Planck coefficients, Hartree system coefficients, or any other transition coefficients between dimensional and dimensionless measurement systems. We use the recommended values for constants α, k, m_{e} and Rydberg constant R_∞. The following expressions for coefficients are given below, where relative errors are indicated in parentheses:

$$l_{\mathrm{A}} = \frac{ш^2}{4\pi^5\alpha R_\infty} = 5.572626246(19)\cdot 10^{-7}\ \mathrm{cm} \qquad 3.4\ \mathrm{ppb}$$

$$t_{\mathrm{A}} = \frac{ш^2}{4\pi^5\alpha^2 R_\infty} = 2.547263565(17)\cdot 10^{-15}\ \mathrm{s} \qquad 6.7\ \mathrm{ppb}$$

$$m_{\mathrm{A}} = \frac{m_{\mathrm{e}}}{m_{\mathrm{eA}}} = \frac{m_{\mathrm{e}}\pi^2}{ш} = 1.21644989(21)\cdot 10^{-26}\ \mathrm{g} \qquad 170\ \mathrm{ppb}$$

$$\theta_{\mathrm{A}} = \frac{\pi^2\alpha^2}{ш\ln 2}\cdot\frac{m_{\mathrm{e}}c^2}{k} = 6.083550(12)\cdot 10^{6}\ \mathrm{K} \qquad 2.0\ \mathrm{ppm}$$

Calculation of these coefficients for a number of other dimensional measurement systems used in physics, e.g. CGSK (centimeter, gram, second, and Kelvin) is an easy task. One has also add the coefficient

$$m_{\mathrm{A0}} = 6.823\,783\,61(58)\cdot 10^{-3}\ \mathrm{GeV}\ \ (85\ \mathrm{ppb})$$

translating the A-value of mass into GeVs used in elementary particle physics.

Thus, we have finished construction of the initial basis for **A**-system, supplied by its relations with other measurement systems of physical quantities, such as CGSK. From now on, all physical quantities can be represented by ordered collections of zero-dimensional numbers distinguishing from each-other only by formal features and ontological content. There are no more dimensions as such and all mathematical operations (not only multiplication and division) are admissible for the physical numbers.

Chapter III. Applications of the LMP-Theory

Fermi constant in A-system

We shall start from one special relation valuable in many aspects which will turn out to have key character and remarkably require no other assumption except those made earlier. We have established that in accordance with formal unification of all mathematical and physical quantities any dimensional physical quantity is reduced to a number in **A**-system. Particularly, the fundamental physical constants c, $\hbar$, k, m_e, as well as physically meaningful combinations like $\hbar/m_e c$, $e\hbar/2m_e c$ are directly expressed through mathematical constants by means of relations $m_{eA} = ш/\pi^2$ and $k_A = 1/\ln 2$, or by ψ-α equations.

These relations and equations reveal the true mathematical origin of physical constants often containing correction multiplies or summands, which require special and mostly tedious calculations having approximate character. One cannot neglect serious and often insurmountable limitations imposed on the accuracy by collective effect of corrections. Meanwhile, having been represented by dimensionless multiplies of value close to unity the corrections are separated from the constant itself. In view of this fact we expect all FPC to be determined by the relations and equations of the said type, i.e. often including multipliers close to 1.

Out of five code constants c, $\hbar$, k, G, G_F we have expressions (although unconfirmed empirically) for the first three: c, $\hbar$ and k. Let us see now the state of things with Fermi constant G_F. The exceptional role played by G_F in physical theory is well-known. It is sufficient to note that G_F, often in powers of 1/2, 2, 3, ... characterizes all 156 four-fermion interactions between 24 fundamental particles (12 leptons and antileptons and 12 quarks and antiquarks). No specific considerations exist on how the Fermi constant should be expressed through mathematical constants. One can only suggest that by its physical content G_F must be an exponential quantity. All that can be done now is to calculate *formally* and with highest accuracy the A-system value of G_F. In the CGS natural system the value of G_F is determined from the famous formula for mean lifetime of a muon. It is convenient to give this formula in form:

$$G_{\mathrm{F}} = \left(\frac{192\pi^3 \hbar^2 c\, \lambda\!\!\!^{-}{}^5_{C\mu}}{R_\mu \tau_\mu} \right)^{\frac{1}{2}}. \tag{1}$$

The factor $192\pi^3$ was obtained from diagram calculations; R_μ, often presented in the form $1 + \Delta q$, is the radiative corrections evaluated since 1950s. [Berman; Sirlin; Green, Veltman; Marciano; Ritbergen and Stuart]. Using the consistent values for α, m_μ, m_e/m_μ, the latest experimental result [Webber et al.]

$$\tau_{+}(\mathrm{M\mu Lan}) = 2.196\,980.3(2.2)\cdot 10^{-6}\ \mathrm{s} \tag{2}$$

which was actually predicted in [Arakelian 2007*a,* 180–181] and the new theoretical value [Pak and Czarnecki; Lynch, 15]

$$R_\mu = 0.995\,6104(2.2) \tag{3}$$

we obtain the values

$$G_{\mathrm{F}} = 1.435\,854\,41(75)\cdot 10^{-49}\ \mathrm{cm^5 g{\cdot}s^{-2}} \quad (0.5\ \mathrm{ppm}) \tag{4}$$

$$G_{\mathrm{F}} = 1.166\,381\,87(63)\cdot 10^{-5}\ \mathrm{GeV^{-2}} \quad (0.5\ \mathrm{ppm}) \tag{5}$$

Transition to **A**-system by general formula (1) brings about the expression

$$G_{\mathrm{FA}} = G_{\mathrm{F}} l_{\mathrm{A}}^{-5} m_{\mathrm{A}}^{-1} t_{\mathrm{A}}^{2}. \tag{6}$$

In explicit form we have:

$$G_{\mathrm{FA}} = \frac{64\pi^{13}}{\text{ш}^5} \cdot \frac{R_\infty^3 \alpha}{m_e c^2} G_{\mathrm{F}} = 1.425\,162\,53(74)\cdot 10^{-21}\ (0.5\ \mathrm{ppm}) \tag{7}$$

In decimal notation this number is negligible, but as soon as it is presented in the initial exponential form, $\psi(x) \equiv \exp(x)$, we come to absolutely amazing result

$$G_{\mathrm{FA}} = \mathrm{e}^{-48.000\,001(0.5)} \tag{8}$$

which in the limits of experimental error is practically equal to

$$\mathrm{e}^{-48} \equiv \exp(-48) \equiv \psi(-48).$$

It should be specially emphasized that the last relation was obtained automatically, just as a simple consequence of identity $c_{\mathrm{A}} \equiv \alpha^{-1}$ and initial relations $\mathbf{A}_3$ and $\mathbf{A}_4$ for $m_{e\mathrm{A}}$ and $\hbar_{\mathrm{A}}$ set up absolutely with no regard of the Fermi constant. Hardly such a number could be due to "game of chance", since nothing incidental is present in foundations of physics. Note that from the positions of the physical world simplicity and harmony the numerical term $\psi(-48)$ is not only surprisingly simple but also highly convenient for those calculations where Fermi constant is invariably present. It enters in various relations, as already observed, in powers of $\pm 1/2$, ± 1, $\pm 3/2$, ± 2, ... which corresponds to simple terms

$$\psi(\pm 24),\ \psi(\pm 48),\ \psi(\pm 72),\ \psi(\pm 96), \ldots$$

It is generally very convenient to deal with a number 24, as well as its homological numbers 6, 12 and 48 since all they are champions of the natural series by the number of devisors. As for the power of exponent, it is quite possible that its value is exactly -48, especially if we assume that all small corrections are actually taken into account in the factor R_μ. Even independently of this fact the approximation accuracy (0.2 ppm) is so high that a question naturally arises: why namely 48 (or 24, 96, etc.), how this number, significant for mathematics in the above mentioned sense, appeared in the physical theory? What aspects of the physical reality are hidden behind this number? As we have stated, 48 is practically the most suitable number. Now we wish to understand the physical meaning of this constant. In the mathematical expressions for FPC *yet unexplained* but not *incidental* numbers can be present.

We have a sufficiently convincing explanation of the number 48 and its homologies appearance in physics. The point is that the total amount of leptons and quarks (including antiparticles) characterized by the Fermi constant is equal to 24. It is also known that G_F refers to bosons having spin $\hbar$, and in accordance with Grand Unification Theory, the famous $SU(5)$ contains exactly 24 generators. And each generator has a corresponding vector boson, $W^\pm$ or Z^0 bosons, photon γ, 8 gluons and 12 X- and Y-particles. It should be also mentioned that the expression $G_{FA} \cong e^{-48}$ was first obtained in the authors paper **[**Arakelian 1995], much later than the **A-**system **[**Arakelian 1981**]**. The number 24 for α^{-1} appeared only recently (see later). Designating the number of fundamental fermions by n_F and bosons by n_B and assuming that the power index 48 is exact, we can write

$$G_{FA} = e^{-(n_F + n_B)}. \qquad (9)$$

Immediately analogy may be seen with the entropy relation $\Omega_j = e^{j/2}$ for the number of microstates in the Universe. The formal similarity of Ω_j

and G_{FA} is by no means incidental. It results from analogous content of these quantities. By its physical meaning Ω_j varies inversely proportional to the probability of state. However, the probability characteristics are also applicable to the Fermi constant. It is sufficient to turn to the formula (1), where $G_F^2 \sim 1/\tau_\mu$, and since the constant τ_μ is inversely proportional to decay probability, the relationship of Fermi constant with probability is clear. The exponential character of dependence is initially laid down in the concept of lifetime for a quantum mechanical system. Particularly, for a system unstable with respect to any decay of free particles. Indeed, the quantity τ is defined as a time interval during which the probability of finding the particle in the given state is reduced in e times. This fact is certainly not a result of some arbitrary convention, but rather a reflection of exponential character ultimately due to characteristics of maternal ψ-function encoded in the system **E** of functional equations. In general, the mean lifetime of any particle can be represented in universal form

$$\tau = \tau_C \cdot a = \tau_C \cdot a_1 e^{a_2} \quad (a_1 > 0,\ a_2 > 0)$$

where $\tau_C = \lambda\!\!\!^{-}_C/c$ is Compton time of the particle, a_1 and a_2 are positive dimensionless coefficients. With account of available A-expressions,

$$\tau_\mu = \tau_{C\mu} \frac{192\pi^3 \hbar_A^{10}}{R\ d^4 \alpha^6} e^{\boldsymbol{96}} = 2.196\,975\,51(56) \cdot 10^{-6}\ \text{s} \quad (0{,}25\ \text{ppm}) \qquad (10)$$

where $d_\mu = m_\mu/m_e$, while bold italic type conditionally denotes the relative empirical error (10 ppb) of the exponent power index in the expression for G_{FA}. This value is four times more precise than the latest experimental result τ_{+}(MμLan) = $2.196\,980.3(2.2) \cdot 10^{-6}$ **[**Webber et al.**]**, thus it is a forecast for the further experimental value. Accordingly

$$G_F = 1.166\,383\,14(6) \cdot 10^{-5}\ \text{GeV}^{-2} \quad (0.05\ \text{ppm}) \qquad (10')$$

In the **A-**system

$$\tau_{\mu A} = \frac{3\pi^3 d_\mu}{R_\mu} \left(\frac{2\pi^4 \alpha^3}{ш^2 d_\mu} e^{\boldsymbol{16}} \right)^6. \qquad (11)$$

We assume this relation as a true mathematical expression for the mean lifetime of muon, i. e. a “prototype” and in vast number of cases also a constituent part of similar relations.

Returning to the Fermi constant we summarize our results. The **A**-system, experimental data, Fermi constant and the entropy, mean lifetime and probability expressions, the numbers of fundamental fermions, bosons and the number of microscopic states, great synthesis and $SU(5)$ symmetry group, as well as the fine-structure constant (as we shall see it later) all these components of the physical theory excellently converge in a simple and elegant expression $G_{\mathrm{FA}} = \mathrm{e}^{-48}$. We have actually hit one of the significant points of the infinite numerical continuum without any "aiming" or even any expectation of this account. The possibility of incidental coincidence of numbers seems to be highly unlikely here. One can therefore assert the following. Somewhat unexpected and unprovoked *testing by mathematical harmony* turned out so successful for the system that there are all reasons to consider it beyond the reach of any serious refutation. Truly, such a strong statement on "invulnerable" character of the system strictly speaking refers to its part (constant ш, initial relations of the **A-**systems) whose immediate consequence is the expression (7). But because the LMP-Theory and its formal core **AGECA** form indivisible organic integrity of all its parts one can speak here of a powerful factor confirming the whole theory.

The second expression of Fermi constant

In connection with the result for the Fermi constant obtained in the previous section the question arises whether other fundamental physical constants can be determined theoretically, in the framework of LMP-Theory. However, even the correct statement of formal construction of such constants is beyond capabilities of any existing canonical physical theory. It is not incidental that the problems of this kind have gained the reputation of deadlock problems. Meanwhile, the LMP-Theory possesses all necessary tools for their correct statement and solution. With some problems the theory it is able to cope quite easily, i.e. actually in form of deductive inference from the initial elements and principles. Other problems require more detailed analysis and application of non-trivial solution methods taken from formal **AGECA** arsenal. Amazing is the completely automatic result $G_{\mathrm{FA}} \cong \mathrm{e}^{-48}$, actually representing a direct, although peculiar verification of **A-**system validity, and thus indirect verification of the whole LMP-Theory as well. In the framework of this theory one can distinguish four levels of definition for the fundamental physical numbers: *deduction*, *almost canonical construction* (confirmed by indirect data), *semi-intuitive* *construction* (having no such confirmation) and arbitrary *play with numbers*. The first ideal level can be reached only in rare cases, the second fairly high level is most prospective and desirable, the third level is insufficient and shall be given no credit, and finally the fourth should be fully rejected. An efficient way to increase the reliability of theoretical construction is the method of unified determination of physical constants in the framework of some system of inter-correlated quantities. The uniformity in calculation of more than one constants related by physical meaning can be a key to solution of many "unsolvable" problems in the physical theory. And certainly a factor of paramount importance is representation of all dimensional quantities in the **A**-system. Without this step solution of some problems is practically impossible.

Here we continue the discussion of Fermi constant. Since relations between the physical quantities are numerous and diverse, the formula (1) for G_F is not single. The equation $\mathbf{C}_3$ prompts the relation of G_F with other quantities. Fermi constant has the dimension of expression $(e_j ƛ_{Cj})^2$ or $(e_j \cdot \hbar / m_j c)^2$. The same dimensionality has the square of a fundamental quantity, Bohr magneton μ_B, i.e. magnetic moment of electron in its "pure" form, without corrections. Its value should be naturally supplemented by its quantum-relativistic corrections a_e and a_μ, the anomalous magnetic moments of electron and muon. One can distinctly see the relationship between G_F and μ_B^2. With account of the said correction the relationship is seen between the expressions $G_F R_\mu^{1/2}$, μ_B^2 and $(a_e/a_\mu)^2$. In order to characterize the intensity of various interactions we introduce an exponential quantity $e^{-9\theta_\mu/4}$, where the fundamental parameter θ_μ (by analogy with tangential expression for Cabibbo angle) is determined from the following equation:

$$\mathrm{tg}\,\theta_\mu = 2ɯ - 1. \tag{12}$$

A sufficiently simple relation connecting all said quantities has a form

$$G'_F = \left(\frac{a_e}{a_\mu}\right)^2 \frac{\mu_B^2}{\sqrt{R_\mu}}\, e^{-9\theta_\mu/4} \tag{13}$$

where $\theta_\mu = 3\pi - 2\theta_C = 8.978\,746\,151\,439...,$

$$\theta_C = \frac{\mathrm{arctg}\,(2ɯ - 1)}{2} = 0.223\,015\,904\,665\ldots \tag{14}$$

Inserting the values of θ_μ and R_μ, as well as consistent values of a_e, μ_B and the last value for a_μ **[**Bennett**]** we come to the number

$$G'_{FA} = e^{-47,999\ 995(2)} \qquad (15)$$

which is also close to e^{-48}.

It looks like everything is in its place in the expression (13): the exponent, the correction associated with G_F present in combination $G_F R_\mu^{1/2}$, the quantities a_e, a_μ and μ_B directly related with G_F – a constant of four-fermion interaction related with electron and muon. However, the main criterion of validity should be the correspondence of expression to experimental data. Although the experiment is of primary importance it is unable, due to poor precision, to verify the truth of expression. We can only state that the dependence (13) gives a number practically undistinguishable from "deductive" expression $\cong e^{-48}$.

Acceptance or rejection of this formula depends mainly on empirical verification of the forecast (given by a formula in the **A**-system)

$$a_\mu = \frac{a_e \pi^5 \alpha^4}{R_\mu^{1/4} 2ш^{5/2}}\ e^{24\ -\ 9\theta_\mu/2} = 1.165\,923\,55(7)\cdot 10^{-3}\ \ (0.06\ \text{ppm}) \qquad (16)$$

concerning the value of AMM of muon in the units of μ_B.

A brief review of golden section generalized theory

The LMP-Theory not only gives the necessary tools for determination of any known physical constant and for ascribing with limited or absolute accuracy the true value to any physical quantity. In the light of LMP-Theory, a number of mathematical quantities studied far and wide reveal some new and unknown features. In the first head this observation applies to *proportion* in geometry (Euclides), *golden section* (Leonardo), *golden ratio*, *golden mean*, or just a *golden number* in arithmetic. The traditional theory of golden mean, as well as the theory of Fibonacci and Lucas numbers with all their various applications in mathematics, different fields of science and techniques, nature, architecture, art, music and so on, has been detailed and rather fully considered in **[**Arakelian 2007, Ch. 4 and 5**]**. Generalized theory of golden proportion (GTGP) was constructed in **[**Arakelian 2007, Ch. 6**]** as application of the LMP-Theory. The main relations of GTGP are based on the generalized exponential form of golden number:

$$\phi_{mk} = \mathrm{e}^{\pm \operatorname{arsh}(m/k)}$$

or in more general case

$$\phi_{mk} = \mathrm{e}^{\pi \mathrm{i} n/2 \pm \operatorname{arsh}(m/k)} \quad (n = 1, 2, 3, 4)$$

where m and k are real numbers.

In the same book we have also considered the generalization of the Benford's law (first-digit distribution law), generalization of the silver sequence, Fibonacci and Lucas numbers, Lévy's formula, as well as golden logarithmic spiral, Platonic solids, da Vinci constant and a number of other applications of the LMP-Theory.

General principles of physical constants construction. Mass formula

Before turning to consideration of physical constants (PC) whose theoretical definition, especially of the dimensional constants, is possible only in the framework of **A**-system, we shall make a general observation. The logical-mathematical basis of all PCs defined and analyzed earlier is associated with some special relations and connections existing between them. Besides the correlation, expressing one quantity in terms of other quantities, the analytical relationship between constants may be also implicit. It can be, for example, represented by means of transcendental equations with roots including mathematical constants (MC), as well as some unknown values of physical quantities. Sufficiently full comparison of theory and experiment requires the list of results obtained above to be supplied by new systematic data. In doing so, we ought to carefully observe the construction canons and methodological rules of the **AGECA**-system, briefly listed below:

(*a*) All PC are expressed by means of mathematical and/or other physical constants in either explicit form, or through transcendental e-i-2-equations;

(*b*) The genuine mathematical expression of any dimensional PC can be obtained only in the **A**-system;

(*c*) Except the MCs, such expressions can contain only simple coefficients, as well as correction coefficients and summands;

(*d*) In the PC system the number of various relations is much larger than the quantity of PCs, which allows mutual corrections to be made and comparison of results obtained by different calculation methods;

(*e*) While determining a PC, should be taken into account the physical meaning of constant, its entrance into some or other family of quantities, etc.

One should also keep in mind the higher the PC measurement accuracy, the narrower is its error interval found experimentally, the less are the chances to hit this interval incidentally. But even the highest accuracy fitness in the narrow interval of experimental data is not enough to guarantee the full success. Moreover, even ideal agreement of mathematical form and experimental results is not enough to consider this form as a real candidate for genuine expression of the empirically found value.

In light of this approach, let us consider a group of constants theoretical determination of which is possible only by means of **A**-system. It should be observed that the whole burden of LMP-Theory verification lies on the most accurately measured experimental parameters admitting direct comparison with theory. Particularly, such are the mass ratios m_μ/m_e, m_p/m_e, m_n/m_e measured with accuracy of order 10^{-8}–10^{-10}. The general formula proposed for these ratios, as well as one for tau lepton can be written in form:

$$m_{jA} = n_j\pi - f^{-1}(n_{1j}/n_{2j})[1 - \sigma_j(\Delta m_{jA}) - k_j\varepsilon)] \qquad (17)$$

Here f_j is one of the trigonometric (e-i-2) functions; n_{1j} and n_{2j} are "quantum integers", θ_C = 0.223 015… is universal coupling constant (14) and Δm_{jA} are mass differences for nucleons and leptons; n = 1 for barions and n = 2 for leptons; φ is function of isospin determined by the expression

$$\varphi(I_j) = \gamma_j[I_j(I_j + 1) - Q^2] \qquad (18)$$

where γ_j = 2 is for leptons and γ_j = 4 for nucleons. The values of function φ for these particles are shown in the table below.

	I	γ	*Q*	φ
μ	1/2	2	–1	**–1/2**
τ	1/2	2	–1	**–1/2**
p	1/2	4	1	**–1**
n	1/2	4	0	**3**

Note that the values of φ_μ and φ_τ strongly differ by modulo from φ_p and φ_n and have the same opposite sign.

There is also an expression

$$\tau_\mu = \tau_{C\mu} e^{\theta_\mu (9/2 - \varepsilon)} \tag{19}$$

different from (16). Correlation of these two formulas gives the relation

$$\varepsilon = -\frac{1}{\theta_\mu} \ln\left(\frac{192\pi^3}{\alpha^2} \left(2 \frac{a_\mu / a_e}{m_\mu / m_e} \right)^4 \right) = 6.720(28) \cdot 10^{-6} \tag{20}$$

having the order $(\alpha/\pi)^2 \sim 5 \cdot 10^{-6}$. In the general formula (17), the groups of coefficients n_{1j}/n_{2j} and k_j are related and the latter have values: $k_\mu = k_\tau = 2/3$, $k_p = -2/3$, $k_n = 1$.

Transforming masses in the **A-**system and using notations

$$m_{nA} - m_{pA} \equiv \Delta m_{npA}, \quad m_{\mu A} - m_{eA} \equiv \Delta m_{\mu eA}, \quad m_{\tau A} - m_{\mu A} \equiv \Delta m_{\tau\mu A}$$

we ultimately obtain the following dimensionless A-expressions for masses of two leptons and two nucleons:

$$m_{\mu A} = 5\pi - \arcsin\frac{2}{9}\left[1 + \frac{1}{2}\theta_C \left(\Delta m_{eA} \cdot \frac{\alpha}{\pi} \right)^2 - \frac{2}{3}\varepsilon \right] =$$
$$= 15.483\,838\,637(4) \qquad (0.26 \text{ ppb}) \tag{21}$$

$$m_{\tau A} = 83\pi - \arcsin\frac{1}{3}\left[1 + \frac{1}{2}\theta_C \left(\Delta m_{\tau\mu A} \cdot \frac{\alpha}{\pi} \right)^2 - \frac{2}{3}\varepsilon \right] =$$
$$= 260.399\,566\,421(6) \qquad (0.023 \text{ ppb}) \tag{22}$$

$$m_{pA} = 44\pi - \arcsin\frac{2}{3}\left[1 + \theta_C \left(\Delta m_{npA} \cdot \frac{\alpha}{\pi} \right) + \frac{2}{3}\varepsilon \right] =$$
$$= 137.500\,257\,276(15) \quad (0{,}11 \text{ppb}) \tag{23}$$

$$m_{nA} = 44\pi - \operatorname{arctg}\frac{3}{5}\left[1 - 3\theta_C \left(\Delta m_{npA} \cdot \frac{\alpha}{\pi} \right) - \varepsilon \right] =$$
$$= 137.689\,790\,179(11) \quad (0.08 \text{ ppb}) \tag{24}$$

It is easily seen that in all four relations the "super-fine structure" is three orders of magnitude less that the "fine- structure", which in its turn is three orders of magnitude smaller than the "principal member". The higher approximations, typically for such expressions, are related with tedious and never-ending calculation of various corrections. Now the numerous futile attempts to determine and calculate the physical numbers m_μ/m_e, m_p/m_e and m_n/m_e are explained by wrong search methods. We have all reasons to state here that the true expressions for masses should be first determined in the **A**-system by means of relation (6). Then division by $m_{eA} = ɯ/\pi^2$ results in the experimentally obtained ratios:

$$m_\mu/m_e = 206.768\,280\,26(5) \qquad (0.24 \text{ ppb}) \tag{25}$$

$$m_\tau/m_e = 3477.327\,024\,03(8) \qquad (0.023 \text{ ppb}) \tag{26}$$

$$m_p/m_e = 1836.152\,674\,94(20) \qquad (0,11 \text{ ppb}) \tag{27}$$

$$m_n/m_e = 1838.683\,661\,82(15) \qquad (0.08 \text{ ppb}) \tag{28}$$

which actually are coincident with the recommended experimental data with precision up to 9–11 significant figures. Accordingly, one may state that the mass ratios

(*a*) can be obtained only through the **A**-system;

(*b*) can be universally constructed by means of expressions (17)–(24).

All this analysis has shown that with high degree of confidence we have obtained the expressions of at least second level accuracy.

Fine-structure constant equation

Recall that the Sommerfeld constant α^{-1} and light velocity value c in vacuum are identical. This fact is confirmed by analysis of the equations $\mathbf{C}_1$ for a certain value e of variable e_j: $e^2/\hbar c = \alpha$ and comparison of main characteristics of constants α^{-1} and c, α and $1/c$, as well as magnetic charge e_{m0} versus electric charge e. In short, the identity $c_A \equiv \alpha^{-1}$ is completely proved. We define α^{-1} as a mathematical quantity satisfying the equation [see Arakelian 1981, 136, 146; also Arakelian 1989, 46–50]

$$\cos x \equiv \frac{e^{ix} + e^{-ix}}{2} = \frac{1}{e} \tag{29}$$

which among others has a solution

$$x = 2\pi \cdot 22 - \arccos(1/e) = 137.036\,007\,939\,214... \qquad (30)$$

Interestingly, that after two decades this relation giving only the first crude approximation to the recommended value $\alpha^{-1}(2002)$ has appeared in the Internet as exact mathematical value for α^{-1}. Meanwhile, the expression (29) assumed later as a basic equation for obtaining the value of α^{-1} needs further improvement carried out in **[**Arakelian 2007, Ch. 3**]**.

The empirical data related with α^{-1} contains numerous direct and indirect measurements which were performed during the last eight decades. The value

$$\alpha^{-1}(2002) = 137.035\,999\,11(46) \quad (3.3 \text{ ppb}) \qquad (31)$$

may be taken as a reliable benchmark for theoretical search of the constant's true value, since it is certainly better than the last CODATA recommendation **[**FPC−Extensive Listing**]**

$$\alpha^{-1}(2006) = 137.035\,999\,679(94) \quad (0.68 \text{ ppb}). \qquad (32)$$

The latter value almost coincides with the value $\alpha^{-1} = 137.035\,999\,710(96)$ **[**Gabrielse et al.**]** obtained, in units of μ_B, from the expression for the electron AMM

$$a_e = \frac{1}{2}\left(\frac{\alpha}{\pi}\right) - 0.328\,478\,444\,003\left(\frac{\alpha}{\pi}\right)^2 + 1.181\,234\,017\left(\frac{\alpha}{\pi}\right)^3 -$$

$$1.7283(35)\left(\frac{\alpha}{\pi}\right)^4 + 1.70(3)\cdot 10^{-12} \qquad (33)$$

where $a_e = 1.159\,652\,180\,85(76)\cdot 10^{-3}$. In the *Erratum* to the same paper the multiplier $-1.7283(35)$ is "improved" to the value $A_8 = -1.9144(35)$ giving

$$\alpha^{-1} = 137.035\,999\,070(98) \quad (0.71 \text{ ppb}) \qquad (34)$$

which has a deviation from the preceding value $\sigma = -6.5$. This deviation is amazing, although not the only, in the case of QED results for α. The "improved" A_8 in the expression for a_e and the new measurement

$$a_e = 1.159\,652\,180\,73(28)\cdot 10^{-3}$$

provide the number **[**Hanneke et al.**]**

$$\alpha^{-1} = 137.035\,999\,084(51) \quad (0.37 \text{ ppb}). \qquad (35)$$

Anyhow, the recommended value $\alpha^{-1}(2006)$ was confirmed in 2008 **[**Mohr et al.**]**. Therefore, until the situation is not fully resolved, we have to keep in mind the result of previous recommendation.

Deviation $\delta \approx 19$ of the value (30) from $\alpha^{-1}(2002)$ may be assumed as evidence of small correction summand to the basic equation $\cos x = 1/\mathrm{e}$ the total contribution of which does not exceed α^2:

$$\cos x = 1/\mathrm{e} - \varepsilon_\alpha \quad (0 < \varepsilon_\alpha < \alpha^2). \tag{36}$$

It is a common form of numerous physical quantities: the principal member complemented by small and super-small corrections. In the QED, the structures of many quantities include additions proportional to α, α^2, α^3… But how should the value of α itself be found? Being guided by the ideas which lie in the basis of LMP-Theory, we accept the main principle of mathematical determination, namely that the equation for α^{-1} must not contain any constants except FMC and most significant PCs. Hence the desired equation constituted on basis of principle: "principal member + fine structure + superfine structure" should have the form:

$$\frac{\mathrm{e}^{\mathrm{i}x} + \mathrm{e}^{-\mathrm{i}x}}{2} + \frac{\mathrm{e}^{x-2\pi n}}{x^2} - \mathrm{e}^{-\sqrt{x}} = \frac{1}{\mathrm{e}}. \tag{37}$$

The variable x is directly related with FMC through the mathematical forms $\mathrm{e}^{\pm \mathrm{i}x}$, $\mathrm{e}^{x-2\pi n}$, $\mathrm{e}^{-\sqrt{x}}$, x^2 containing yet unknown integer n which determines the period of function

$$f(x) = \cos(x) + \frac{\mathrm{e}^{x-2\pi n}}{x^2} - \mathrm{e}^{-\sqrt{x}} - \mathrm{e}^{-1}. \tag{38}$$

It is natural to seek the solution of this transcendental equation in the set of real numbers. The variable x^2 in denominator and $x^{1/2}$ in the exponent power eliminate negative solutions; it is also easy to see that no real roots exist if $n = -1, -2, -3,\ldots$ Hence n can be only positive integer, in which case all roots of the equation are positive. Analysis of function $f(x)$ shows that starting from $n = 10$ the number n_λ of function periods is equal to

$$n_\lambda = n + 2.$$

Also the roots close to 137 are obtained starting from $n = 21$.

n	Value	n_λ
10	71,1030850…	12
……	………………	…..
15	103,8368208…	17
……	………………	…..
20	135,6695885…	22
21	137,0268256…	23
22	137,0359994…	24
23	137,0360167…	25
24	137,0360168…	26
……	………………	…..
500	137,0360168…	502

Thus, for any $n > 22$ all roots of equation (37) slightly differ from each-other and only the solution at $n = 22$ stays in full agreement with the accepted experimental value. The intersection point of the curve (38) with x-axis corresponding to the constant α^{-1} is shown below.

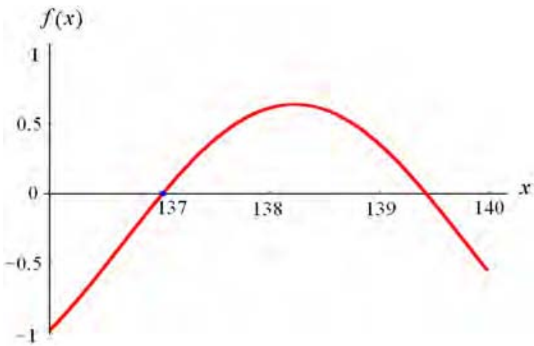


Fig 6.
The point on abscissa corresponding to α^{-1}

Geometrical meaning of exponential power index $2\pi n - x$ is quite clear. The real projection length of the curve $f(x)$ on the x-axis measured from the origin to the point x_n is equal to x; the same distance for the period 2π is equal to $2\pi n$. The difference of these lengths is equal to $\Delta x_n = x - 2\pi n$. The ratio $\Delta x_n/x$ is the measure of function non-periodicity, since for periodic function this ratio is zero.

Having finally available all necessary results of the $f(x)$ function analysis we turn to solution of the problem. The main characteristics of such partly-periodic function having limited number n_λ of periods is the (limiting) value of the period length λ and the corresponding integer n_λ. Since for the considered function the length λ tends to 2π and this value is present in the equation for α^{-1}, it remains only to explain the meaning of number n_λ which unambiguously determines the value of parameter n. Provided that the number of period n_N including the solution $n_N = n$ (clearly, the number of roots is $n_N + 1$), the only parameter requiring explanation is the integer n_λ equal to 24. In the long run the problem reduces to integer 24 as fundamental physical quantity. This fact may be explained as follows. The Sommerfeld constant, or in other words the light velocity in vacuum, is closely related with the quantum of electromagnetic field, photon, having spin $\hbar$ and representing a fundamental boson. But the $SU(5)$ symmetry group includes exactly 24 generators, as we have seen earlier when interpretation of power index 48 = 24·2 was given, appearing in A-expression of Fermi constant. So there are all reasons to think that these 24 generators (or 24 elementary particles) lie in the basis of Sommerfeld constant value. It is plausible to observe that we have found the key to solution of a century-long mystery of number 137 in modern physics. Indeed, oughtn't we to take as fundamental physical constant the number of fundamental bosons or fermions? The solution of equation for periodic function requires an integer and what can be here better than this constant? If that's how matters stand, the equation for α^{-1} may be written in a form containing no constants other than e, π, i, 2 plus PC $n_\text{B} = 24$:

$$\cos(x) + \frac{\mathrm{e}^{x-2\pi n(n_\text{B})}}{x^2} - \mathrm{e}^{-\sqrt{x}} - \mathrm{e}^{-1} = 0 \tag{39}$$

with additional condition: $n_N = n = 22$. The single solution

$$\alpha^{-1} = 137.035\,999\,452\,021\ldots \tag{40}$$

well matches ($\delta = 0.74\sigma$) the value $\alpha^{-1}(2002)$.

It is also worthy of notice that the number (40) obtained theoretically stays in excellent agreement with the most accurate non-QED value **[**Cadoret et al.**]**

$$\alpha^{-1} = 137.035\,999\,45(62) \qquad (41)$$

as well as with the world average **[**Arakelian 2007, Ch.7**]**

$$\alpha^{-1} = 137{,}035\,999\,42(40) \quad (2{,}9 \text{ ppb}) \qquad (42)$$

of more than forty, non-QED values. At last, the world average without (41) is

$$\alpha^{-1} = 137.035\,999\,39(53) \quad (3.9 \text{ ppb}). \qquad (43)$$

It only remains to add that direct empirical verification of the value (40) requires the increase of α^{-1} reliable measurement by one or better two orders of accuracy.

Chapter IV. Boundaries and Generalized Laws of Physical World

On the extreme values of physical quantities

We have carried out the construction of the **AGECA-**system principal components, namely: the logical postulates and mathematical axioms **AG**, functional equations **E**, physical codes **C** and dimensionless physical quantities' measurement system **A**. Now we pass to consideration of some problems immediately related with singular points of physical world – the physical constants. Different aspect of physical constants were considered by M. Planck, P. Dirac, A. Eddington, W. Heisenberg, A. Einstein, D. Hilbert, H. Weyl, B. Russel, M. Born and many others. They are multi-functional and often extreme values of the physical quantities, the milestones by which Nature outlines the boundaries of physical reality. Here we shall consider the *upper and lower boundaries*, the ***initial conditions*** and corresponding *generalized laws* of the physical world. These laws are directly related with the concepts of atoms, discreteness and quantization of the world, its reflection in form of various physical quantities. The discreteness of all physical quantities has become apparent after discovery of atoms, charges, minimal mass of charged particles, quantum of action, etc. Including the most "stubborn" space and time, although the concept of their atomicity is one of the first natural scientific hypotheses. The sources of this concept can be clearly seen in the atomism of Leucippus and Democritus, later developed by Epicurus. However, the return to space discreteness concept occurred only in the 19th century.

In contemporary physics the hypothetical atom of space l_f is called *fundamental length* and jointly with the time quantum t_f called *chronon* forms a relation $l_f = ct_f$. These atoms are considered as universal constants determining the applicability limits of relativity theory, quantum theory, and causality. At the same time the fundamental length and chronon represent indivisible elements of space and time. The fundamental length is usually expressed through the FPC by dimensional analysis. The main idea of fundamental length persists, although the candidates change from time to time. The early pretenders were Compton lengths of electron ($ƛ_e \sim 10^{-11}$ cm, electromagnetic interaction), π-meson ($ƛ_\pi \sim 10^{-13}$ cm, strong interaction) and nucleon ($ƛ_N \sim 10^{-14}$ cm, strong interaction). Later the characteristic length of weak interaction $(G_F/\hbar c)^{1/2} \sim 10^{-16}$ cm. The last candidate is the Planck length $l_P \sim (G\hbar/c^3)^{1/2} \sim 10^{-33}$ cm. All these lengths except the last one were rejected successively by the experiment. In some sense l_P may be indeed as an extremely important limit of intermediate character, a singular point to a new weakly explored domain of physical phenomena. Although the classical notions on continuity of space-time are not applicable for lengths less than $l_P \sim 10^{-33}$ cm, one cannot state that smaller lengths are impossible. The Planck length occupies the last position in the hierarchy of fundamental interactions lengths decreasing. But it is easy to indicate many significant lengths shorter than l_P. For example, such a physically significant quantity as gravitational radius of electron defined by the relation $R = 2Gm/c^2$ can be obtained by dimensional analysis with accuracy up to coefficient 2 produced by gravitation theory. The matter is that $R_e \approx 1.4{\cdot}10^{-55}$ cm, i.e. is 22 orders of magnitude smaller than l_P. Gravitational radius of hadrons lies in the limits $3.6{\cdot}10^{-53}-3{\cdot}10^{-51}$ cm and is also are much smaller than the Planck length.

Taking this into account we shall try to solve the problem of minimal or fundamental length and time by means of dimensional analysis. Thus we assume first of all that the matter is discrete in all its manifestations and there exist non-zero $l_f = l_{\min}$ and $t_f = t_{\min}$; secondly, the fundamental length and chronon, as well as other significant values, can be expressed through FPC; third, the relation between constants can be determined by dimensional analysis. All these natural assumptions should be complemented by one more, most constructive prerequisite: all extreme values of fundamental physical quantities must constitute a closed system of consistent and profoundly interrelated fundamental parameters. This requirement immediately implies that once the extreme values of some quantities are known, the remaining values can be expressed through the known parameters. Thus we can state the problem as follows: reveal the list of extreme values and establish analytical connections between them. From the viewpoint of LMP-Theory the task is to select a system of physical quantities of the required type which can be expressed through the system of FPC.

One should also keep in mind that the selected set of physical extremes must refer not only to fundamental quantities but to secondary physical quantities as well. For the present, we have to determine the extreme values of length and time interval. At the first step only a rough approximation will be found by means of dimensional analysis. Then applying fine methods we shall try to obtain more accurate values. Not touching yet the most specific equation $\mathbf{C}_4$ we recall that the initial list of quantities given by the equations $\mathbf{C}_1 - \mathbf{C}_3$ includes dimensionless coupling constants α_{xj}, dimensional constants c, $\hbar$, G, G_F and variables e_j, m_j, $\lambdabar_j$. In view of the goal to be reached, we shall take as a constant value of m_j the mass of Universe having the estimated order of magnitude $m_j \sim 10^{57}$ g. Dimensional analysis provides a whole number of possibilities for constitution of fundamental length out of seven dimensional quantities. Without loosing generality of consideration we shall not consider yet any other dimensional quantity except the length. Thus any available combination having L dimension contains from three to seven quantities and a number of l values much smaller than l_P. We shall, however, restrict our consideration by three quantities. Along with m_U, we shall take as initial extreme quantities the quantum of action $\hbar$, the maximal velocity c and elementary electrical charge e. Dimensional analysis gives the following expressions:

$$\lambdabar_U = \frac{\hbar}{m_U c} \sim 10^{-95}\ \text{cm} \tag{44}$$

$$l_U = \frac{e^2}{m_U c^2} \sim 10^{-97}\ \text{cm} \tag{45}$$

$$a_{U0} = \frac{\hbar^2}{m_U e^2} \sim 10^{-93}\ \text{cm} \tag{46}$$

related by a constant α:

$$\lambdabar_U = l_U/\alpha = \alpha a_{U0}. \tag{47}$$

These simple results obtained purely by dimensional analysis prove that there are no reasons to consider the Planck constant l_P as a "fundamental length." Moreover, these relations give the coarse value of minimal length $l_{\min}$ having approximately the order of magnitude $\sim 10^{-95}$ cm (or $\sim 10^{-89}$ in the **A**-system) i.e. 60 orders of magnitude less than the Planck length. The latter value itself, however, is 60 orders of magnitude smaller than the assumed value of *maximal* length usually called "radius of the Universe." Here we have a good fit not only in numerical magnitude but more importantly in physical sense: by accepting the two values as physical extremes staying at equal logarithmic "distance" from the Planck length. One could also define the fundamental length as the Compton length $\lambda\!\!\!^{-}_U$ of the Universe. The intermediate character of Planck length means that this parameter represents geometric mean of extreme values. Denoting the dimensionless coefficient of any Planck quantity by k_{P} we have:

$$l_{\mathrm{P}} = \sqrt{l_{\min} \cdot l_{\max}} \ , \quad \sqrt{k_{\mathrm{P}} \frac{G\hbar}{c^3}} = \sqrt{\frac{\hbar}{m_U c} \cdot R_U} \tag{48}$$

whence follows that $R_U = \dfrac{k_{\mathrm{P}} G m_U}{c^2}$. In the theory of gravitation the expression for gravitational radius has coefficient 2, so finally $k_{\mathrm{P}} = 2$. This means that among several treatments of the Planck length the preferable definition should be based on equality of Compton and gravitational lengths: $\hbar/mc = 2Gm/c^2$. The extreme values of length $l_{\min}$ and $l_{\max}$ should be respectively taken as Compton length and gravitational radius of Universe. The last quantity, $2Gm_U/c^2$, should be assumed as the upper limit of L dimension.

It is necessary to observe that the special status of Compton, gravitational and Planck quantities is actually embedded in the initial physical equations. Particularly the equation $\mathbf{C}_2$ includes all those quantities which are present in combinations $c\hbar m_j$, cGm_j, $c\hbar G$, the first two containing the variable mass m_j while the last only code constants. The equation $\mathbf{C}_2$ itself is easily seen to be a relation between Compton and gravitational quantities:

$$\alpha_{Gj} = \frac{Gm_j^2}{\hbar c} = \frac{1}{2}\frac{2Gm_j}{c^2} : \frac{\hbar}{m_j c} = \frac{1}{2}\frac{l_{Gj}}{\lambda\!\!\!^{-}_{Cj}} \tag{49}$$

One can easily see that the same relation holds for other dimensional parameters as well:

$$\alpha_{Gj}/\alpha_{\mathrm{P}} = B_{Gj}/B_{Cj} \tag{50}$$

In a very simple formula, three physical essences of highest significance are encoded by means of functional variable α_{Gj}, independent variable m_j and FPCs G, $\hbar$ and c. Special interest represents the particular case $m_j = m_U$.

In the general case of extreme values we have the relation

$$B_{\mathrm{P}} = \sqrt{B_{\min} \cdot B_{\max}} \tag{51}$$

where obvious notations are used: B_{P}, $B_{\min}$ and $B_{\max}$. This relation actually gives a simple way to express one extreme quantity through the other. For example inserting the quantities m_{P} and $m_{\max} = m_U$ we obtain the minimal mass value $m_{\min} = \frac{m_{\mathrm{P}}^2}{m_{\max}} \sim 10^{-68}$ g which is forty orders of magnitude smaller than electron mass. In the **A-**system the values of these masses are

$$m_{\min} \sim 10^{-42}, \quad m_{\mathrm{P}} \approx 1{,}3 \cdot 10^{21}, \quad m_{\max} \sim 10^{83}$$

while the ratio of masses $m_{\max}/m_{\min}$ is the same in all systems and is expressed by a vast number

$$N_U \sim 10^{125}. \tag{52}$$

Comprehensive consideration of number N_U, being highly significant problem here, requires additional judgement about the code equation $\mathbf{C}_4$.

Entropy and number N_U

Speculative character of constructions related with the fundamental length is based on the extremality concept and dimensional analysis. This approach caused serious difficulties in determining the accurate value of fundamental parameter N_U. Therefore we have to turn to independent source expecting to confirm the previous result by new data. The only such source is the equation $\mathbf{C}_4$ for entropy and quantity Ω_j. To appreciate the potential of $\mathbf{C}_4$ we shall pull back a little for discussion of the entropy concept.

The issue of constants and variables was discussed in the general form while considering the laws of conservation and variation. Speaking of the Universe parameters it is clear that some physical quantities are constant, such as mass, total energy, action, electrical charge, etc., while others quantities, such as radius, lifetime, temperature, density, volume, etc., tend to their extreme values. Special role in the last list has entropy, the principal source of changes in the physical world. Entropy explains the universal character of various physical processes.

In statistical mechanics the entropy is defined by means of Boltzmann formula $S_j = k \ln \Omega_j$ which was taken above as one of four initial equations of the physical theory. The dependence of spatial and temporal quantities of entropy was obtained by Hawking for black holes:

$$S = \frac{k}{4}\frac{A}{l_{\mathrm{P}}^2} = \frac{k}{4}\frac{c^3}{G\hbar}A. \tag{53}$$

This formula was deduced rigorously in the relativistic astrophysics, establishing a simple relation between the black hole entropy S, area of its horizon A, the Boltzmann constant k and Planck length l_{P}. When the quantum of entropy is assumed to be equal to $k/2$, the last formula is represented in form

$$\frac{S_{\max}}{S_{\min}} \equiv \frac{S_{\max}}{k/2} = \frac{A_{\max}}{2G\hbar/c^3}. \tag{54}$$

The coefficient 2 standing before G is a clear evidence in favour of equality $\alpha_G = 1/2$. In other words we get a confirmation of the statement that Planck quantities are obtained from the equality of Compton and gravitational values, particularly of Compton length and gravitational radius. Both these lengths depend on mass and starting from the Planck length, i.e. the intersection point $\lambda\!\!\!^{-}_{\mathrm{C}} = l_G$ the Compton length decreases with m_j up to the value $l_{\min}$, while gravitational radius grows up to $l_{\max}$ (and vice versa). Since the maximal length by definition is equal to radius of Universe, it is easy to evaluate the ratio (54). Indeed, the horizon area A is proportional to the square of black hole radius $R_G = 2Gm/c^2$ and in coarse approximation is given by the expression

$$A = 4\pi R^2 = 4\pi(2Gm/c^2)^2$$

valid for the area of a 3-dimensional sphere. However a finer analysis gives the coefficient of unity order (i.e. smaller than $4\pi \sim 12.6$). If our Universe is a black hole, so that $A_{\max}$ corresponds to a mass $m_U \sim 10^{57}$ g, then we obtain an estimate

$$\frac{S_{\max}}{S_{\min}} = \frac{R_U^2}{2G\hbar/c^3} \sim 10^{125}. \tag{55}$$

The ratio of extreme values for entropy again resulted in a tremendous number which by its order of magnitude is equal to the earlier obtained value N_U. Moreover, the second appearance of N_U, ignoring fine internal bonds, is in no way related with the first.

The following conclusions may be drawn from the preceding consideration:

(*a*) confirmation is obtained of the earlier made assumption that Planck length is intersection points of Compton and gravitational quantities;

(*b*) the existence of fundamental constant $N_U \sim 10^{125}$ is proved independently and its value is confirmed;

(*c*) in exponential form this number is close to $e^{288} = e^{48\cdot 6}$, therefore it may be interpreted as a member of $\psi(48\cdot n)$ numbers family.

Recall that the first appearance of constant N_U was related with the space quantization concept and is due to Compton and gravitational lengths and radius of Universe R_U. As for the second appearance of N_U, we observe that the formula (55) already contains all necessary information, so all one has to do is only apply this formula to the Universe. But there also exists the third, most immediate method of obtaining the number N_U by substituting the Universe mass in the initial equation $\mathbf{C}_2$ in which case N_U is obtained with accuracy up to coefficient 1/2. Thus we can state with certainty: existence of the fundamental constant N_U is a firmly established scientific fact.

Boundaries of physical reality

The extensive subject of extreme physical quantities considered earlier requires logical development and termination. The initial provisions have been already stated, so what we need now is to briefly list these provisions for the length, although they are applicable to any physical quantity.

- We have shown the high significance of Compton and gravitational lengths and their equality in the intersection point known as Planck length.
- Remind that gravitational and Compton lengths respectively are proportional and inverse proportional to the mass. Therefore the divergence between these two lengths, and generally between the quantities of $c\hbar m_j$ and cGm_j families, is growing with distance of cor-

responding points from the intersection point. Meanwhile, as we have shown the Compton and gravitational lengths are interchangeable in the limitary points of the physical world, i.e. one quantity is transformed into the other. Namely, the Compton length of extremely small mass $m_{\min}$ is *exactly* equal to gravitational radius R_U of the Universe, while the Compton length $\lambda\!\!\!^{-}_U$ of the Universe is equal to gravitational radius of mass $m_{\min}$.

Expressing our statements in mathematical language we come to the following system of three equalities:

$$\hbar/m_{\mathrm{P}}c \quad = 2Gm_{\mathrm{P}}/c^2 \tag{56}$$

$$\hbar/m_U c \quad = 2Gm_{\min}/c^2 \tag{57}$$

$$m_U/m_{\min} = N_U \tag{58}$$

Solving this system for unknown quantitis $m_{\min}$ and m_U we obtain

$$m_U \quad = \sqrt{\frac{\hbar c}{2G} N_U} \;=\; l_{\mathrm{P}}\sqrt{N_U} \tag{59}$$

$$m_{\min} = \sqrt{\frac{\hbar c}{2GN_U}} = \frac{l_{\mathrm{P}}}{\sqrt{N_U}} \,. \tag{60}$$

Inserting these relations into the equation $\mathbf{C}_2$ we come to expressions

$$N_U = \frac{\alpha_{\mathrm{P}}}{\alpha_{G\min}} \tag{61}$$

$$N_U = \frac{\alpha_{GU}}{\alpha_{\mathrm{P}}} \,. \tag{62}$$

Thus the assumption made earlier brings to the simple relation between the constant N_U and initial function α_{GU}. Namely N_U is equal to the value of α_{Gj} at the point $m_j = m_U$. This simple result completely agrees with the results obtained earlier by other methods. The expressions for extreme values of major physical quantities are given in the table.

Table 2
Extreme values of physical quantities

Quantity	Notation	Expression	Decimal Values	
			A-system	CGS or other
Initial function	$\alpha_{j\min}$	$\frac{1}{2N_U} = \frac{\alpha_P}{N_U}$	$5 \cdot 10^{-126}$	$5 \cdot 10^{-126}$
	$\alpha_{j\max}$	$\frac{N_U}{2} = \alpha_P N_U$	$5 \cdot 10^{124}$	$5 \cdot 10^{124}$
Entropy	$S_{\min}$	$k/2$	0,7	$7 \cdot 10^{-17}$ erg/K
	$S_{\max}$	$N_U \cdot k/2$	$7 \cdot 10^{124}$	$7 \cdot 10^{108}$ erg/K
Action	$J_{\min}$	$\hbar/2$	$4 \cdot 10^{-4}$	$5 \cdot 10^{-28}$ erg·s
	$J_{\max}$	$N_U \cdot \hbar/2$	$4 \cdot 10^{121}$	$5 \cdot 10^{97}$ erg·s
Mass	$m_{\min}$	$\sqrt{\frac{\hbar c}{2GN_U}} = \frac{m_P}{\sqrt{N_U}}$	$4 \cdot 10^{-42}$	$5 \cdot 10^{-68}$ g
	$m_{\max}$	$\sqrt{\frac{\hbar c}{2G} N_U} = m_P \sqrt{N_U}$	$4 \cdot 10^{83}$	$5 \cdot 10^{57}$ g
Fill energy	$E_{\min}$	$\sqrt{\frac{\hbar c^5}{2GN_U}} = \frac{E_P}{\sqrt{N_U}}$	$8 \cdot 10^{-38}$	$4 \cdot 10^{-47}$ erg
	$E_{\max}$	$\sqrt{\frac{\hbar c^5}{2G} N_U} = E_P \sqrt{N_U}$	$8 \cdot 10^{87}$	$4 \cdot 10^{78}$ erg
Temperature	$T_{\min}$	$\sqrt{\frac{\hbar c^5}{2Gk^2 N_U}} = \frac{T_P}{\sqrt{N_U}}$	$5 \cdot 10^{-38}$	$3 \cdot 10^{-31}$ K
	$T_{\max}$	$\sqrt{\frac{\hbar c^5 N_U}{2Gk^2}} = T_P \sqrt{N_U}$	$5 \cdot 10^{87}$	$3 \cdot 10^{94}$ K
Length	$\lambda\!\!\!^{-}_{C\min} = l_{G\max}$	$\sqrt{\frac{2G\hbar}{c^3 N_U}} = \frac{l_P}{\sqrt{N_U}}$	$1 \cdot 10^{-89}$	$7 \cdot 10^{-96}$ cm
	$\lambda\!\!\!^{-}_{C\max} = l_{G\min}$	$\sqrt{\frac{2G\hbar}{c^3} N_U} = l_P \sqrt{N_U}$	$1 \cdot 10^{36}$	$7 \cdot 10^{29}$ cm

Time	$\tau_{\text{Cmin}} = t_{\text{Gmax}}$ $\tau_{\text{Cmax}} = t_{G\text{min}}$	$\sqrt{\dfrac{2G\hbar}{c^5 N_U}} = \dfrac{t_\text{P}}{\sqrt{N_U}}$ $\sqrt{\dfrac{2G\hbar}{c^5} N_U} = t_\text{P}\sqrt{N_U}$	$1\cdot10^{-91}$ $1\cdot10^{34}$	$2\cdot10^{-106}$ s $2\cdot10^{19}$ s
Critical density	ρ_C	$\dfrac{3c^5}{16\pi G^2\hbar N_U} = \dfrac{3\rho_\text{P}}{4\pi N_U}$	$4\cdot10^{-26}$	$3\cdot10^{-33}\ \text{g}\cdot\text{s}^{-3}$
Universe volume	V_U	$\dfrac{4\pi R_U^3}{3} = \dfrac{4\pi l_\text{P}^3 N_U^{3/2}}{3}$	$9\cdot10^{108}$	$2\cdot10^{90}\ \text{cm}^3$
Number of microscopic states	Ω_{min} Ω_{max}	$\text{e}^{1/2}$ e^{N_U}	1,6 $10^{0.43\cdot10^{125}}$	1,6 $10^{0.43\cdot10^{125}}$

Remarkably the constant N_U is present, in various degrees, in all expressions given in the table. The intermediate character of Planck parameters as geometric means between the maxima and minima of physical quantities makes the presentation of extreme values particularly simple. Introducing a notation $B_{j\text{ext}}$ we have a general relation

$$B_{j\text{ext}} = B_{j\text{P}} N_U^n,\ \ n = \pm 1/2,\ \pm 1,\ \pm 3/2,\ \pm 2,\ \pm 3. \tag{63}$$

It should be also mentioned once more that extremality problem is identical to the task of finding the highest and smallest values of physical quantities.

The consideration would not be complete without discussing the question whether the constant 0 (zero), axiomatically given in LMP-Theory, should be attributed to the extreme values. Long time, particularly in the 18th and 19th centuries, the concepts of continuum and infinity were dominant in science. The physical world was thought as infinite and boundless both in space and time, with possibility of physical quantities to be infinitely large or small. Meanwhile, in reality the fundamental physical quantities turned out to be discrete and finite, at least where clarity could be reached. We don’t know any natural scientific fact which is able to prove the opposite assertion. The mathematical symbol ∞ is, strictly speaking, inadmissible in physics, in difference to number zero reflecting for example the absence of some or other characteristic, say electrical charge in particular object.

Considering the mass, one can ask a question far from being rhetoric: if a particle can have zero charge or spin, then why some particles cannot have zero mass, say photon, graviton, or gluon? In the quantum field theory, QFT, zero mass is attributed to strong interaction carriers, gluons, while the Goldstone theorem proves the existence of massless particles, the so-called "Goldstone bosons" created in cases of violated *continuous* (not discrete) symmetries. As for the carriers of electromagnetic and gravitational interactions, the arguments in favor of their zero mass are related with very large, but limited, interaction radius for such carriers. In view of the fact that in situation where experimental database for theoretical considerations is very scanty or generally absent, there are three efficient methods to overcome such "empirical vacuum." First is requirement of systematic interrelationship, second is systematic consistency of results, and third is possibility of obtaining the same results at least by two independent methods. Of course, even the existence of three said requirements cannot serve a full guarantee of correctness of the ultimate results. However the chances for success are sufficiently higher in this case.

Returning to the issue of extremal quantities and admitting the existence of massless particles with small action radius, of order of nuclear size, one may try to explain the Compton and gravitational length limits. Particularly, mass and length minimal values and length maxima. A number of factors should be taken into account, namely the fundamental character of electromagnetic and gravitational forces and their agents, photons and gravitons, the relation between mass and action radius of fundamental interactions carriers, as well as some other factors. For example the Compton length of minimal mass $m_{\min}$ is equal to gravitational radius R_U of the Universe; also that Compton length λ_U of the Universe is equal to gravitational radius of $m_{\min}$. One may therefore state that the characteristics of smallest particles define the parameters of Universe and vice versa. It should be also added that geometrical mean values of the largest and smallest physical quantities are called "Plankeons." For example the Planck mass is geometric mean of photon or graviton mass and the mass of Universe. In view of this consideration one can state that zero constant means the absence of some or other physical quantity, rather than its minimal value. The conclusion is clear: minimal values or "quanta" of various physical quantities are expressed by finite numbers which are not always small (take for example the quantum of entropy, $k/2 \approx 1.44$).

Consideration of physical extreme values has resulted, in the case of length and entropy, to the same gigantic natural number N_U encoded in the initial equation $\mathbf{C}_2$. The chain of steps bringing to number N_U is following:

1. Physical codes $\mathbf{C}_1$–$\mathbf{C}_3$ and selection of initial FPCs → dimensional analysis aimed at obtaining the length dimension → application to Universe parameters → account of intermediate value of Planck length with respect to extreme values → signification of Compton length → N_U as a ratio of extreme values.
2. Black hole entropy expression → application to Universe parameters, with account of quantum of entropy → N_U as a ratio of extreme values of entropy.
3. Substitution of $m_j = m_U$ value in the equation $\mathbf{C}_2$.

Each chain of consideration step, long and tedious in the first case, much shorter and immediate in the second, and shortest in the last chain is finished by the number N_U as a fundamental constant defining *from and to* of the physical reality. One may generally conclude that all variable parameters of the Universe can be expressed through the entropy with fixed lower limit $S_{\min} = k_A/2 = 1/2 \cdot \ln 2$ and upper limit $S_{\max} = N_U \cdot k/2 = N_U/2 \cdot \ln 2$. Then, in view of the limits $S \to S_{\min}$ and $S \to S_{\max}$, we obtain the chain of equalities for the ratios of type $B_{\max}/B_{\min}$:

$$\lim\frac{l_{\max}}{l_{\min}} = \lim\frac{\lambda_{\max}}{\lambda_{\min}} = \lim\frac{t_{\max}}{t_{\min}} = \lim\frac{\tau_{C\max}}{\tau_{C\min}} = \lim\frac{T_{\max}}{T_{\min}} = \lim\frac{V_{\max}^{1/3}}{V_{\min}^{1/3}} = \\ = \lim\frac{\rho_{\max}^{1/2}}{\rho_{\min}^{1/2}} = \lim\ln\frac{\Omega_{\max}}{\Omega_{\min}} \ldots = N_U \tag{64}$$

For conserving fundamental quantities the ratios of extreme values are obvious:

$$\frac{J_{\max}}{J_{\min}} = \frac{m_{\max}}{m_{\min}} = \frac{Q_{\max}}{Q_{\min}} = \ldots = N_U \tag{65}$$

The issue on exact value of the Universal constant N_U still remains open, due to absence of any reliable milestones. However the N_U order of magnitude is known and it can be verified that N_U is expressed by a number $\exp(288 \pm \varepsilon)$, where ε is of order 10^{-1} or even 10^{-2}. Anyway, since $288 = 24 \cdot 12$, there are all reasons to attribute this number to the family $\psi(24 \cdot n)$.

Such numbers are also very elegant and convenient for multiplication, division, rising to a power and taking the root, differentiation and integration, since in all these cases the operations with $\psi(24n)$ are extremely simple being reduced to transformation of argument $24n$. Thus because of exponent characteristics and the fact that $288 = 2^5 \cdot 3^2$, all numbers N_U^n belong to this family, for all physically possible values of n. Thereby the constant N_U becomes the central member of the said family of numbers including the Fermi constant G_{FA} and particular values of the initial physical quantity Ω. Hence the constant N_U by its content and formally is incorporated with the whole system of physical problems dealing with supersymmetry and Grand Unification, the number of fundamental fermions and bosons mentioned earlier with the key constant $G_{\mathrm{FA}} \cong \psi(24 + 24)$.

Generalized physical laws

The list of cosmic constant merits, however, is not yet exhausted. The cycle of discussions focused on the aconstant N_U as a principal physical-mathematical quantity defining the limits of physical reality returns us to the fundamental physical laws underlying the **AGECA** system. It is time to remember now that correct statement of fundamental physical laws of conservation and variation is possible only for the whole Universe. Action, mass and electrical charge of Universe are conserved, while the entropy is growing. Any attempt to substitute Universe by inertial frame of reference, or by a closed system inevitably creates controversies and does not withstand any serious criticism. On the other hand, consideration of boundaries of physical reality, revealing the number N_U gives an opportunity to make more definite in values the physical laws, as well as pass to generalized laws of conservation, variation and quantization. It should also be borne in mind the speculative character of all judgments regarding Universe as the only one integral system.

We shall begin with formulation of the law which is not directly related with the origin of constant N_U.

Generalized conservation law of fundamental physical constants

Numerical values of FPCs are invariable.

First of all we observe that only those physical constants that are elements of the integrated system of interrelated physical numbers should be included in the FPC system. Such are first the code constants c, $\hbar$, k, G, G_{F}, a number of other most significant quantities like m_e, e, N_U, as well as numerous combinations of physically meaningful constants including parameters of the Universe. The invariance postulate of FPCs is based on dual character of their origin, rather than absence of any serious empirical data proving the opposite view. The dualism of FPCs means that they are natural physical quantities on one hand and a definite mathematical numbers on the other.

Generalized law on extreme values ratio

The ratios of extreme values of a physical quantity are expressed through the constant N_U by means of relations

$$\frac{B_{j\max}}{B_{j\min}} = N_U^n \quad (n = 1/3,\ 1/2,\ 2/3,\ 1,\ 3/2,\ 2,\ 5/2,\ 3) \tag{66}$$

$$\ln\frac{\Omega_{\max}}{\Omega_{\min}} = N_U. \tag{67}$$

The arguments in support of these relations as generalized laws are as follows: for a broad class of fundamental and secondary constants, as well as variable physical quantities the ratios of extreme values are expressed through integer or fractional degrees of cosmic number N_U.

Generalized law of conservation, variation and quantization

For integer-quantized physical quantity holds the following relation:

$$B_j = n_j B_{\min}, \quad n_j = 1, 2, ..., c. \tag{68}$$

In light of the previous consideration it seems clear that the upper limit of integer series should be taken equal to N_U, e.g. in fundamental laws of action and entropy quantization. In this case the laws of conservation, variation and quantification for such physical quantities may be expressed by a simple relation symbolizing the internal and formal unity of three types of physical laws. Specifying the quantities $B_{\min}$ or fixing constant values of $n_j = N_j$ we obtain the corresponding laws for action, entropy, etc.

A question originates in connection with generalized laws, related with space-time properties and characteristics. Judging by all its manifestation, the physical world is discrete. Theoretical reflection of this fact is that physical quantities are quantized. But only some quantities, like entropy, action, electrical charge, etc. are quantized according to the integer-values law, why for others, e.g. mass, length and time, the quantization law is unclear. Generalized laws like no exceptions. Any such law, if correct, tends to extend its application area to a maximally broad domain of physical reality. Why one code variable, the entropy (or say the action), for which the extreme values ratio is expressed by a magic number N_U, is quantified in accordance with integer-value law, while the other code variable, mass (or length, time, etc.) having the same extreme values ratio should not obey the same law? Is there any interface, either meaningful or formal, or some selection principle to distinguish integer-quantized from non-quantized physical quantity (if any), or from the one subject to non-integer law?

One may set forth at least three mutually exclusive hypotheses.

(a) This fact is purely incidental;

(b) All physical quantities according to integer or fractional value law are expressed by numbers close to unity in their order of magnitude;

(c) The world is discrete and all quantities are subject to universal quantization law. However some physical quantities are too small to be empirically revealed and justified.

The hypothesis (a) seems most unlikely to intuition. Indeed, the gap between two mentioned groups of quantities is too broad (thirty orders of magnitude and higher) in order to see a simple coincidence. So we shall abandon the (a) and consider hypotheses (b) and (c) in more detail.

It is admissible that in the numerical domain close to unity includes not only most of mathematical constants, but also quants of all integer and fractionally quantized physical quantities in the **A**-system. Other quantities are quantized by other rules, or form a discrete values set not subject to the general law. The assumption (b) fixes the state-of-the-art; its weak point is clear discrimination of numerous physical realities, including the important quantities like mass, Compton, gravitational and Planck quantities.

Finally, the third hypothesis (c) is most preferable from viewpoint of generalized laws. If the general law of extremes ratio really has universal character then the general law of conservation, variation and quantization can hardly have selective effect being applicable only to some particular physical quantities. We think that no serious reasons for such conclusion exist. Thus all admissible values of physical quantities must be multiples of corresponding minimal values of these quantities. Exceptions represent the quantities like Ω quantized according to exponential law. It is extremely difficult to deal with physical values lying far beyond the domain accessible to empirical investigation. However in order to bring our analysis to logical end we shall formulate the forth law, which may be less justified and reliable than the previous three laws:

Generalized law of conservation, variation and quantization – II

Numerical values of many quantities are multiples of their minimal values:

$$B_j = n_j B_{\min}, \ \ n_j = 1, 2, ..., N_U. \tag{69}$$

The difference from the similar law (68) is in that the relation (69) covers a significantly broader class of physical quantities, instead of integer-quantized ones. Such generalization of the previous law is related with a great risk.

We close this chapter the main subject of which was the N_U constant, along with entropy, extreme values of different quantities and generalized laws. Regretfully, we do not know exactly the value of N_U. Should we be able to significantly increase the accuracy of N_U measurement in experiments, we would be able to calculate its exact mathematical value. However such a breakthrough requires, for example, higher accuracy of the mass m_U evaluation. One can hardly expect this step to be made in near future. Anyhow, the number N_U may be considered as a *Number of Nature*, symbolizing its integrity and mathematical harmony.

Conclusion. LMP-Theory and its Applications
(in a thesis form)

I. BASIC PRINCIPLES

A. FFT IN FORM OF A TREE-DIAGRAM

- Atmosphere: Philosophy
- Soil: Methodology
- Roots: Logic (L)
- Trunk: Pure Mathematics (M)
- Branches: Fundamental Physics (P)
- Crone: The Rest of Physics
- Fruits: Application of Physics in Science and Technology

B. CONCEPT OF TRIUNITY

Mathematical logic (L), formal numerical mathematics (M) and fundamental physical theory (P) constitute a unified trinomial system of knowledge.

C. DEFINITION OF PHYSICS

Physics is a science of physical quantities.

Fundamental physical theory is a theory of fundamental physical quantities.

D. BASIC PRINCIPLES OF CONSTRUCTION

Only constructions requiring no other logical-mathematical elements and means except the original are admissible in the LMP-Theory. On the other hand all the primary resources of the theory ought to be used in its construction.

E. RELATIONSHIP BETWEEN THE COMPONENTS

In the LMP-Theory, the extension of logical deductive calculus represents the formal universal mathematics complemented by a system of physical equations – codes and a dimensionless measurement system. Transition from logic to mathematics is related with introduction of notion of number and initial numbers, such as the new mathematical constant ɯ; transition from mathematical to physical components of the theory is primarily transition from mathematical quantities to fundamental physical quantities.

II. FORMALISM OF LMP-THEORY

A. GENERAL NOTIONS

VALUES: objective (individual) and predicate logical variables; numerical variables (particularly constants), functions-arguments

OPERATIONS: propositional connectives, quantifiers, mathematical operations

FUNCTIONS: predicates (particularly unit propositions), elementary mathematical functions, composite functions, functionals, operators

TERMS, FORMULAS, FORMATION RULES

B. THE ALPHABET OF AGECA SYSTEM

$\sim \; \supset \; \& \; \vee \; \neg \; \forall \; \exists \; = \; + \; - \; a \; b \; c \; \ldots \; x \; y \; z \; \alpha \; \beta \; \gamma \; \ldots \; \psi \; \omega \; (\;) \; |$

C. LOGICAL POSTULATES

$\mathbf{L}_1$ $\quad A \supset (B \supset A)$

$\mathbf{L}_2$ $\quad (A \supset B) \supset ((A \supset (B \supset C)) \supset (A \supset C))$

$\mathbf{L}_3$ $\quad \dfrac{A,\, A \supset B}{B}$ modus ponens, or $\supset$-rule

$\mathbf{L}_4$ $\quad A \supset (B \supset A \,\&\, B)$

$\mathbf{L}_5$ $\quad A \,\&\, B \supset A$

$\mathbf{L}_6$ $\quad A \,\&\, B \supset B$

L$_7$ $A \supset A \vee B$

L$_8$ $B \supset A \vee B$

L$_9$ $(A \supset C) \supset ((B \supset C) \supset (A \vee B \supset C))$

L$_{10}$ $(A \supset B) \supset ((A \supset \neg B) \supset \neg A)$

L$_{11}$ $\neg\neg A \supset A$

L$_{12}$ $(A \supset B) \supset ((B \supset A) \supset (A \sim B))$

L$_{13}$ $(A \sim B) \supset (A \supset B)$

L$_{14}$ $(A \sim B) \supset (B \supset A)$

L$_{15}$ $\forall x A(x) \supset A(r)$ $\forall$-scheme

L$_{16}$ $A(r) \supset \exists x A(x)$ $\exists$-scheme

L$_{17}$ $\dfrac{C \supset A(x)}{C \supset \forall x\, A(x)}$ $\forall$-rule

L$_{18}$ $\dfrac{A(x) \supset C}{\exists (x)\, A(x) \supset C}$ $\exists$-rule

D. MATHEMATICAL AXIOMS

M$_1$ $a = b \supset (a = c \supset b = c)$

M$_2$ $a = b \supset a + \mathrm{c} = b + c$

M$_3$ $a = b \supset c + a = c + b$

M$_4$ $(a + b) + c = a + (b + c)$

M$_5$ $a + 0 = a$

M$_6$ $a - a = 0$

M$_7$ $a + b = b + a$

E. FUNCTIONAL EQUATIONS E

E$_{10}$ $\psi(x + y + \ldots + z) = \psi(x)\cdot\psi(y)\cdot \ldots \cdot\psi(z)$

E$_{20}$ $\alpha(x) + \alpha(y) + \ldots + \alpha(z) = \alpha(x\cdot y\cdot \ldots \cdot z)$

$\mathbf{E}_{30}$ $\psi(x+\lambda+\ldots+\lambda)=\psi(x)$

$\mathbf{E}_{40}$ $\psi(x-\lambda-\ldots\lambda)=\psi(x)$

$\mathbf{E}_{50}$ $\lim S(S(\ldots S(x)\ldots)=\text{const}$

Initial (maternal) functions as a solution of equations E

$$\psi(z)\equiv \mathrm{e}^{z}\equiv \exp z$$

$$\alpha(z)\equiv \mathrm{Ln}\, z=\ln z\pm 2\pi n\mathrm{i}$$

$$\frac{\psi(\mathrm{i}x)+\psi(-\mathrm{i}x)}{2}\equiv\frac{\mathrm{e}^{\mathrm{i}x}+\mathrm{e}^{-\mathrm{i}x}}{2}\equiv\cos x$$

$$\psi(-z)\equiv \mathrm{e}^{-z}$$

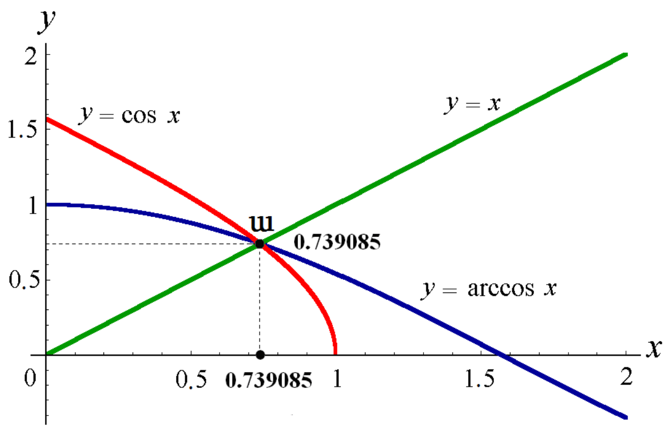
$$\psi(-\mathrm{W}(z))\equiv \mathrm{W}(z)$$

$$\frac{\psi(\mathrm{i}\pi)+\psi(-\mathrm{i}\pi)}{2}=\mathrm{i}\cdot\mathrm{i}$$

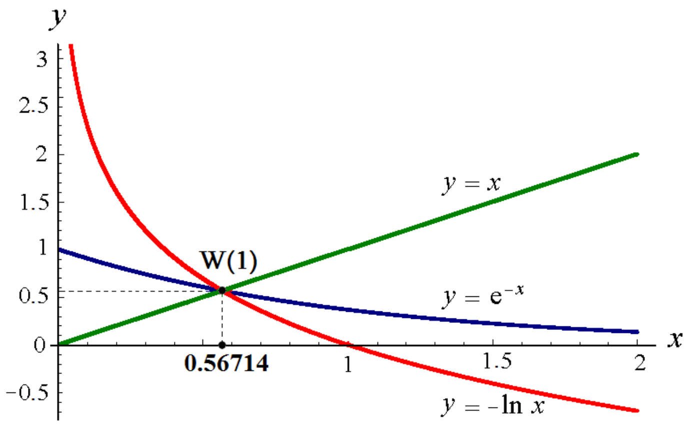
$$\frac{\psi(\mathrm{i}ш)+\psi(-\mathrm{i}ш)}{2}=ш$$

The constants ш and W(1) as triple points of intersection of function, inverse function and argument

$\mathbf{E}_{53}$ $\cos x=\arccos x=x,\qquad x=ш$



$\mathbf{E}_{55}$ $\quad e^{-x} = -\mathrm{Ln}(x) = x \qquad x = \mathrm{W}(1)$



Fundamental mathematical constants and their decimal values

0	
2	
i	$\sqrt{-1} \quad i^2 = -1$
π	3.14159 26535 89793 23846 26433 83279 50288 41972...
e	2.71828 18284 59045 23536 02874 71352 66249 77571...
ɯ	0.73908 51332 15160 64165 53120 87673 87340 40134...
W(1)	0.56714 32904 09783 87299 99686 62210 35554 97538…
γ	0.57721 56649 01532 860606 51209 00824 02431 04215...

F. PHYSICAL CODES C

General mathematical form

$$\boxed{\mathbf{C} \quad z = a^{z_1} \cdot z_2^{b} \equiv \psi(\alpha(a) z_1 + b\alpha(z_2)) \equiv \exp(\mathrm{Ln}\, a \cdot z_1 + b \cdot \mathrm{Ln}\, z_2)}$$

(z function, z_1, $z_2 \neq 0$ variables, a, b constants)

Physical interpretations (variables with subscript j)

$\mathbf{C}_1$	$\alpha_{ej} = \psi[\alpha(1/\hbar c) + \alpha(e_j^2)] \equiv \dfrac{e_j^2}{\hbar c}$
$\mathbf{C}_2$	$\alpha_{Gj} = \psi[\alpha(1/\hbar c) + \alpha(Gm_j^2)] \equiv \dfrac{Gm_j^2}{\hbar c}$
$\mathbf{C}_3$	$\alpha_{Wj} = \psi[\alpha(1/\hbar c) + \alpha(G_F/\lambda\!\!\!^{-}{}_j^2)] \equiv \dfrac{G_F/\lambda\!\!\!^{-}{}_j^2}{\hbar c}$
$\mathbf{C}_4$	$\Omega_j = e^{S_j/k}$

Fundamental physical quantities and dimensions

Quantities		*Dimensions*
$\alpha_{ej}, \alpha_{Gj}, \alpha_{Wj}, \alpha_{sj}, \alpha_{mj}, \Omega_j$	non-dimensional functions	$[\alpha_{xj}] \equiv \ldots \equiv \dim \alpha \equiv \mathrm{A}$
c	velocity of light in the vacuum	$[c] \equiv \dim c \equiv \mathrm{V}$
J	action	$[\hbar] \equiv \dim \hbar \equiv \mathrm{J}$
S	entropy	$[k] \equiv \dim k \equiv \mathrm{S}$
Q	charge	$[e_j] \equiv [G^{1/2}m_j] \equiv [G_F^{1/2}/\lambda\!\!\!^{-}{}_j] \equiv \dim e \equiv \mathrm{Q}$
G	gravitational constant	$[G] \equiv \dim G \equiv \mathrm{G}$
G_F	Fermi constant	$[G_F] \equiv \dim G_F \equiv \mathrm{G_F}$
m_j	mass	$[m_j] \equiv \dim m \equiv \mathrm{M}$
$\lambda\!\!\!^{-}{}_j$	Compton length	$[\lambda\!\!\!^{-}{}_j] \equiv \dim \lambda\!\!\!^{-} \equiv \mathrm{L}$

G. GENERAL PHYSICAL LAWS

CONSERVATION LAWS

- **General mathematical form**

  $Y = BF(X, A) + C.$

- **Velocity of light in the vacuum**
  The *number c* ($c_A = \alpha^{-1} \approx 137.036$) *is preserved in all physical equations and relations*, *as well as mathematical transformations having physical meaning.*

- **Action**
  The action of the Universe preserves.

- **Mass**
  The mass of the Universe preserves.

- **Generalized law of charges**
  All charges (electrical, magnetic, strong, weak and gravitational) *are conserved in the Universe.*

QUANTIZATION LAWS

- **Action**
  $J = n \cdot \hbar/2 \qquad n = 1, 2, \ldots, N_U$

- **Entropy**
  $S = n \cdot k/2 \qquad n = 1, 2, \ldots, N_U$

- **Charges e_j (e_e, e_m, e_W, e_s)**
  $E_j = \pm n \cdot e_{j0} \qquad n = 1, 2, \ldots$ (e_{j0} elementary charge)

- **Integer quantization law of Hall resistance**
  $$R_j = \frac{2\pi\hbar}{e^2} \cdot \frac{1}{n^2}$$

- **Fractional quantization law of Hall resistance**
  $$R_j = \frac{2\pi\hbar}{e^2} \cdot \frac{n}{2k+1}$$

- **Quantization law of magnetic flow**
  $$\Phi = \Phi_0 n = \frac{\pi c}{e} \cdot hn$$

VARIATION LAWS

- **Entropy**
  The entropy of the Universe increases: $S_j = k \ln \Omega_j, \quad j = 1, 2, 3, \ldots$

▲ **Number of microscopic states of the Universe**

$\Omega_j = e^{j/2}$, $j = 1, 2, 3, \ldots, N_U$.

GENERALIZED LAWS

⬟ **Conservation of FPC**

Numerical values of FPCs are invariable.

⬟ **Extreme values ratio**

The ratios of extreme values of a physical quantity are expressed through the constant N_U by means of relations $\dfrac{B_{j\max}}{B_{j\min}} = N_U^n$ ($n = 1/3$, $1/2$, $2/3$, 1, $3/2$, 2, $5/2$, 3)

$$\ln \frac{\Omega_{\max}}{\Omega_{\min}} = N_U.$$

⬟ **Conservation, variation and quantization**

For integer-quantized physical quantity holds the following relation:

$B_j = n_j B_{\min}$, $n_j = 1, 2, \ldots, N_U$ $N_U \sim 10^{125}$ – the new cosmic FPC

H. A-SYSTEM (of physical values' measurement)

Initial relations

$$\boxed{c_A = \alpha^{-1} \qquad m_{eA} = \frac{ш}{\pi^2} \qquad k_A = \frac{1}{\ln 2} \qquad \hbar_A = \frac{\pi^2\alpha^2}{ш}}$$

Genaral form and transition coefficients to CGSK and Gev/c^2

$$B_{LMT\Theta} = B_A l_A^p m_A^q t_A^r \theta_A^t$$

$$l_A = \frac{ш^2}{4\pi^5\alpha R_\infty} = 5.572626246(19)\cdot 10^{-7}\ \text{cm} \qquad 3.4\ \text{ppb}$$

$$t_A = \frac{ш^2}{4\pi^5\alpha^2 R_\infty c} = 2.547263565(17)\cdot 10^{-15}\ \text{s} \qquad 6.7\ \text{ppb}$$

$$m_A = \frac{m_e}{m_{eA}} = \frac{m_e\pi^2}{ш} = 1.21644989(21)\cdot 10^{-26}\ \text{g} \qquad 170\ \text{ppb}$$

$m_{A0} = 6.82378382(58)\cdot 10^{-3}$ GeV 85 ppb

$$\theta_A = \frac{\pi^2\alpha^2}{ш \ln 2}\cdot\frac{m_e c^2}{k} = 6.083550(12)\cdot 10^6 \text{ K} \qquad 2.0 \text{ ppm}$$

Fermi constant in the A-system

$$\boxed{G_{FA} = \psi(-48 \pm 0{,}00001) \cong e^{-48} = e^{-(n_F + n_B)}}$$

$n_F = 24$ – number of fundamental fermions (leptons and quarks)

$n_B = 24$ – number of bosons in $SU(5)$ group

III. PHYSICAL CONSTANTS

A. SOMMERFELD CONSTANT ERUATION

$$\boxed{\cos(x) + \frac{e^{x-2\pi n(n_B)}}{x^2} - e^{-\sqrt{x}} - e^{-1} = 0} \qquad n(n_B) = n(24) = 22$$

$\alpha^{-1} = 137.035\,999\,452\,021\ldots$

B. Mass Formula for of leptons and nucleons

The general formula for them may be written in a form

$$m_{jA} = n_j\pi - f^{-1}(n_{1j}/n_{2j})[1 - \sigma_j(\Delta m_{jA}) - k_j\varepsilon)].$$

Here f_j is one of the trigonometric functions; n_{1j} and n_{2j} are “quantum integers”, $\theta_C = 0.223\,015\ldots$ is universal coupling constant and Δm_{jA} are mass differences for nucleons and leptons; $n = 1$ for barions and $n = 2$ for leptons; φ is function of isospin determined by the expression

$$\varphi(I_j) = \gamma_j[I_j(I_j + 1) - Q^2]$$

where $\gamma_j = 2$ is for leptons and $\gamma_j = 4$ for nucleons,

$$\varepsilon = -\frac{1}{\theta_\mu}\ln\left(\frac{192\pi^3}{\alpha^2}\left(2\frac{a_\mu/a_e}{m_\mu/m_e}\right)^4\right) = 6.720(28)\cdot 10^{-6}.$$

Thus

$$m_{\mu A} = 5\pi \quad - \arcsin\frac{2}{9}\left[1 + \frac{1}{2}\theta_C\left(\Delta m_{eA}\cdot\frac{\alpha}{\pi}\right)^2 - \frac{2}{3}\varepsilon\right] =$$

$$= 15.483\,838\,637(4) \qquad (0.26 \text{ ppb})$$

$$m_{\tau A} = 83\pi - \arcsin\frac{1}{3}\left[1 + \frac{1}{2}\theta_C\left(\Delta m_{\tau\mu A}\cdot\frac{\alpha}{\pi}\right)^2 - \frac{2}{3}\varepsilon\right] =$$

$$= 260.399\,566\,421(6) \qquad (0.023 \text{ ppb})$$

$$m_{pA} = 44\pi - \arcsin\frac{2}{3}\left[1 + \theta_C\left(\Delta m_{npA}\cdot\frac{\alpha}{\pi}\right) + \frac{2}{3}\varepsilon\right] =$$

$$= 137.500\,257\,276(15) \quad (0{,}11\text{ppb})$$

$$m_{nA} = 44\pi - \operatorname{arctg}\frac{3}{5}\left[1 - 3\theta_C\left(\Delta m_{npA}\cdot\frac{\alpha}{\pi}\right) - \varepsilon\right] =$$

$$= 137.689\,790\,179(11) \quad (0.08 \text{ ppb})$$

Dividing by $m_{eA} = ш/\pi^2$ we come to the experimentally given relation:

$$m_\mu/m_e = 206.768\,280\,26(5) \qquad (0.24 \text{ ppb})$$

$$m_\tau/m_e = 3477.327\,024\,03(8) \qquad (0.023 \text{ ppb})$$

$$m_p/m_e = 1836.152\,674\,94(20) \qquad (0{,}11 \text{ ppb})$$

$$m_n/m_e = 1838.683\,661\,82(15) \qquad (0.08 \text{ ppb})$$

Muon mean lifetime in the A-system and in CGS

$$\tau_{\mu A} = \frac{3\pi^3 d_\mu}{R_\mu}\left(\frac{2\pi^4\alpha^3}{ш^2 d_\mu}e^{16}\right)^6$$

$$\tau_\mu = 2.196\,975\,51(56)\cdot 10^{-6}\ \text{s}^* \qquad (0.25 \text{ ppm})$$

The second formula for the Fermi constant

$$G'_F = \left(\frac{a_e}{a_\mu}\right)^2 \frac{\mu_B^2}{\sqrt{R_\mu}} e^{-9\theta\ /4}$$

$\theta_\mu = 3\pi - 2\theta_C = 8.978746151439...$, thence

$$\tau_\mu = \tau_{C\mu}\cdot 192\pi^3\alpha^{-2}\left(2\frac{a_\mu/a_e}{m_\mu/m_e}\right)^4 e^{9\theta_\mu/2}$$

$$G'_{FA} = \frac{a_e^2}{a_\mu^2}\cdot\frac{\pi^{10}\alpha^5}{4ш^5\sqrt{R_\mu}}\, e^{-[3\pi - \mathrm{arctg}(2ш - 1)]9/4} \quad \text{in } \mathbf{A}\text{-system}$$

Fundamental Hall resistance: $R_{fA} = 2\pi$

Some other formulas

$$e_A = \pm\frac{\pi\alpha}{\sqrt{ш}} \qquad e_{m0A} = \pm\frac{\pi}{\sqrt{ш}} \qquad e_{PA} = \frac{\pi\alpha}{\sqrt{2ш}} \qquad e_A/m_{eA} = \frac{\pi^3\alpha}{ш^{3/2}}$$

$$ƛ_{mA} = \frac{1}{2\alpha}\sqrt{\frac{\pi}{2ш}}\, e^{-\eta/2} \qquad ƛ_{PA} = \frac{1}{2}\sqrt{\frac{\pi\alpha}{ш}}\, e^{-\eta/2} \qquad ƛ_{eA} = \frac{\pi^4\alpha^3}{ш^2} \qquad r_{eA} = \frac{\pi^4\alpha^4}{ш^2}$$

$$a_{0A} = \frac{\pi^4\alpha^2}{ш^2} \qquad R_{\infty A}c_A = \frac{ш^2}{4\pi^5\alpha^2} \qquad R_{\infty A} = \frac{ш^2}{4\pi^5\alpha} \qquad R_{\infty A}\hbar_A c_A = \frac{ш}{4\pi^3}$$

$$\tau_{CeA} = \frac{\pi^4\alpha^2}{ш^2} \qquad \tau_{PA} = \frac{\alpha}{2}\sqrt{\frac{\pi\alpha}{ш}}\, e^{-\eta/2} \qquad \mu_{BA} = \frac{\pi^5\alpha^4}{2\,ш^{3/2}}$$

$$h_A/m_{eA} = \frac{2\pi^5\alpha^2}{ш^2} \qquad m_{mA} = 2\pi\alpha^2\sqrt{\frac{2\pi}{ш}}\cdot e^{\eta/2} \qquad m_{eA}c_A^2 = \frac{ш}{\pi^2\alpha^2}$$

$$E_{0A} = \frac{ш}{2\pi^2} m_{PA} = 2\pi\alpha^2\sqrt{\frac{\pi\alpha}{ш}}\cdot e^{\eta/2}$$

C. NUMBERS OF $\psi(24n)$ FAMILY

$G_{FA} \cong \psi(-48) \qquad n = 2$

$N_U \cong \psi(288) \qquad n = 12$

$\Omega_j = \psi(24n) \qquad n = 1, 2, \ldots, N_U/48$

$$\lim_{S\to S_{\max}} \frac{B_{j\max}}{B_{j\min}} = N_U^n \cong \psi(288n) \quad n = 1, 2, 3$$

$\frac{B_{j\,\max}}{B_{j\,\min}} = N_U^n \cong \psi(288\, n)$ for conserving quantities

$\Omega_{\max} \cong \psi[\psi(288)] \approx e^{N_U} \approx e^{10^{125}}$ the number of microscopic states of the Universe, the largest number in nature

D. BOUNDARIES OF THE PHYSICAL WORLD

The initial relations

$$B_{\mathrm{P}} = \sqrt{B_{\min} \cdot B_{\max}}$$

$$\hbar/m_{\mathrm{P}}c = 2Gm_{\mathrm{P}}/c^2 \qquad \hbar/m_U c = 2Gm_{\min}/c^2 \qquad m_U/m_{\min} = N_U$$

$$\alpha_{G\min} = 1/2N_U \qquad \alpha_{GU} = N_U/2$$

E. NUMERICAL PREDICTIONS OF LMP-THEORY

Fine-structure constant	$\alpha^{-1} = 137.035\,999\,452\,021\ldots$	
Number of fundamental fermions and bosons	$48 = 24 + 24$	
Fermi coupling constant	$G_{\mathrm{F}} = 1.166\,383\,14(6)\cdot 10^{-5}\ \mathrm{GeV}^{-2}$	(0.05 ppm)
Muon mean lifetime	$\tau_{\mu} = 2.196\,975\,51(56)\cdot 10^{-6}\ \mathrm{s}^{*}$	(0.25 ppm)
AMM of muon	$a_{\mu} = 1.165\,923\,55(7)\cdot 10^{-3}\ \mu_{\mathrm{B}}$	(0.06 ppm)
Gravitational constant	$G = 6.673\,900(4)\cdot 10^{-8}\mathrm{cm}^3\mathrm{g}^{-1}\mathrm{s}^{-2}$	(0.6 ppm)
Muon-electron mass ratio	$m_{\mu}/m_{\mathrm{e}} = 206.768\,280\,26(5)$	(0.24 ppb)
Tau-electron mass ratio	$m_{\tau}/m_{\mathrm{e}} = 3477.327\,024\,03(8)$	(0.023 ppb)
Proton-electron mass ratio	$m_{\mathrm{p}}/m_{\mathrm{e}} = 1836.152\,674\,94(20)$	(0,11 ppb)
Neutron-electron mass ratio	$m_{\mathrm{n}}/m_{\mathrm{e}} = 1838.683\,661\,82(15)$	(0.08 ppb)

References

AKiTi Kepler's Equation of Elliptical Motion, 2004 http://www.akiti.ca/KeplerEquation.html

Arakelian H. *On Proof in Mathematics (Methodological Analysis)*. Yerevan: Acad. Sci., 1981 (in Russian).

– *Fundamental Non-Dimensional Quantities*. *(Their role and importance for the methodology of science)*. Yerevan: Acad. Sci., 1981 (in Rus.).

– *Physical Numbers in the Absolute System*. In: Methodological Analysis of Scientific Cognition. Ann. of Arm. Philos. Soc., 1983. Yerevan: Acad. Sci. of Armenia Press, 1984, p. 34–41 (in Armenian).

– *Numbers and Quantities in Modern Physics*. Yerevan: Acad. Sci. of Armenia Press, 1989 (in Rus.).

– *Numbers and Quantities in Contemporary Physics*. Abstract of Thesis for the Degree of Doctor, S.-Petersburg, 1992 (in Rus.).

– *Fundamental Theory in Physics: Dream and Reality*. Synopsis, Yerevan, **2**, 80–94 (1993).

– *On the Dominant of Intra-Scientific Evolution of the Physical Knowledge: Physical Values in the Evolution of Physics*. Synopsys, Yerevan, **2**, 39–52 (1994).

– *The New Fundamental Constant of Mathematics*. Pan-Armenian Scientific Rev., London, **3**, 18–21 (1995).

– *Foundations of Physical Theory*. Yerevan: David, 1997 (in Rus.).

– *LMP Fundamental Theory*. Yerevan, 2007 (in Rus.) http://www.hrantara.com/Monograph.pdf

– *From Logical Atoms to Physical Laws*. Yerevan: Lusabatz, 2007*a* (in Rus.) http://www.hrantara.com/Book.pdf

– *24 – Number of Nature*, 2009 (in Rus.) http://www.info-mica.com

– *The New Fundamental Constant* ɯ, 2007*b* http://www.hrantara.com./NewConstant.pdf

– *The New Fundamental Constant* ɯ, 2010 http://www.hrantara.com./NewConstant2.pdf

Bennett G. et al. *Muon (g−2) Collaboration*. Phys. Rev. Lett. **92**, 161802 (2004).

Berman S.M. Phys. Rev. **112**, 267 (1958).

Bertrand J. *Exercise III in Traité d'algèbre, Vols. 1-2, 4th ed*. Paris: Librairie de L. Hachette et Cie, p. 285, 1865.

Blanchard P., Devaney R.L., Hall G.R. *Differential equations, 3rd ed*. Thomson Brooks/Cole Publishing Co., pp. 677–691, 2006.

Bojar O. *Chaos přehledně* http://chaospace.hyperlinx.cz/index.php?pgid=230

Bogolyubov N.N., Logunov A.A. and **Todorov I.T.** *Fundamentals of the axiomatic approach in quantum field theory*. Moscow: Nauka, 1969 (in Rus.)

Boyer C.B. *Greek Trigonometry and Mensuration*, *A History of Mathematics*, 2nd ed. John Wiley & Sons, 1991.

Cadoret M. et al. Phys. Rev. Lett. **101**, 230801 (2008).

Chitwood D.B. et al. *Improved Measurement of the Positive Muon Lifetime and Determination of the Fermi Constant*
http://www.npl.uiuc.edu/exp/mulan/TalksAndPresentations/MuLan2004_PRL_Submit.pdf

Clausius R. Ann. Phys. **125**, 353 (1865).

Deser S. and **Zumino B.** Phys. Lett. **B**62, 335 (1976).

Eddington A.S. *Fundamental Theory*. Cambridge: Camb. Univ. Press, 1946.

Einstein A. *Autubiographisches (Autobiographical Notes)*. In: Albert Einstein – Philosopher-Scientist, ed. by P.A. Schilpp, Evanston (Illinois), p. 1–95 (1945).

Espenshade P.H. *A text on trigonometry by Levi ben Gerson (1288-1344)*. Math. Teacher **60**, 628−637 (1967).

Freedman D.Z., van Nieuwenhuisen P., and **Ferrara S.** Phys. Rev. **D**13, 3214 (1976).

Fundamental Physical Constants−Extensive Listing http://physics.nist.gov/constants

Gabrielse G., **Hanneke D.**, **Kinoshita T.**, **Nio M.**, and **Odom B.** *New Determination of the Fine Structure Constant from the Electron g Value and QED*. Phys. Rev. Lett. **97**, 030802 http://hussle.harvard.edu/~gabrielse/gabrielse/papers/2006/HarvardFineStructureConstant2006.pdf (2006)

Golfand A. and **Lichtman E.P.** Pisma JETP **13** (1971), p. 452 (in Rus.).

Green M.B. and **Gross D.J.** *Unified String Theories*. Singapore: World Scientific, 1985.

Green M. and **Veltman M.** Nucl. Phys. **B169**, 137 (1980).

Hanneke D., **Fogwell S.**, and **Gabrielse G.** *New Measurement of the Electron Magnetic Moment and the Fine Structure Constant*. Phys. Rev. Lett. **100**, 120801 (2008) http://hussle.harvard.edu/~gabrielse/gabrielse/papers/2008/HarvardElectronMagneticMoment2008.pdf

Heis E. *Schlüssel zur Sammlung von Beispielen und Aufgaben aus der allgemeinen Arithmetik und Algebra, Vol. 2, 3rd ed*. Cologne, Germany: Verlag der M. DuMont-Schauberg'schen Buchhandlung, p. 468, 1886.

Heisenberg W. *Introduction to the Unified Field Theory of Elementary Particles*. Interscience Publishers, London, New York, Sidney, 1966.

Hilbert D. *Die Grundlagen der Physik*. Nachrichten K.Gesellschaft Wiss. Götingen, Math.-phys. Klasse, 1915, Heft 3, S. 395.

History of Trigonometry. Wikipedia http://en.wikipedia.org/wiki/History_of_trigonometry

Joseph G.G. *The Crest of the Peacock: Non-European Roots of Mathematics* (2nd ed.). London: Penguin Books, 2000, pp. 383−384.

Kaluza Th. *Zum Unitätsproblem in der Physik*. Sitzungsber. Preuss. Akad. Wiss. Berlin (Math. Phys.), 1921, S. 966–972.

Kaplan S.R. *The Dottie Number*. Math. Mag. **80**, 73−74, 2007.

Klein O. *Quantentheorie und fünfdimensionale Relativitätstheorie*. Zeitschrift für Physik a Hadrons and Nuclei **37** (12), (1926), S. 895−906.

Lisi A.G. *An Exceptionally Simple Theory of Everything,* (2007) http://arxiv.org/PS_cache/arxiv/pdf/0711/0711.0770v1.pdf [hep-th].

Lynch K. *Extracting G_F from* τ : *Obtaining the Central Value and Propagated Errors*. Version 3: March 3, 2010.

Maor Eli. *Trigonometric Delights*, Princeton Univ. Press, 1998, p. 20.

Marciano W.J. *The Muon: A Laboratory for "New Physics"*. Journ. Phys. G: Nucl. Part. Phys. **29**, 23−29 (2003).

Mie G. *Die Grundlagen einer Theorie der Materie*. Ann. Phys. **37**, 511−534 (1912); Ibid. **39**, 1−40 (1912); Ibid. **40**, 1−66 (1913).

Miller T.H. *On the Numerical Values of the Roots of the Equation* $\cos x = x$. Proc. Edinburgh Math. Soc. **9**, 80–83, 1890.

Mohr P.J., **Taylor B.**N. and **Newell D.B.** *CODATA Recommended Values of the Fundamental Physical Constants: 2006. Rev. Mod. Phys.* **80**, 633 (2008) http://physics.nist.gov/cuu/Constants/RevModPhys_80_000633acc.pdf

Pak A. and **Czarnecki A.** Phys. Rev. Lett. **100**, 241807 (2008);

Ritbergen T. van and **Stuart R.G.** *On the Precise Determination of the Fermi Coupling Constant from the Muon Lifetime*. arXiv:hep-ph/9904240, v. 1, 6 Apr (1999).

Schwarz J.H. *Superstrings*. Singapore: World Scientific, 1985.

Simonson S. *The Mathematics of Levi ben Gershon, the Ralbag* http://u.cs.biu.ac.il/~tsaban/Pdf/MathofLevi.pdf

Sirlin A. Rev. Mod. Phys. **50**, 573 (1978)

D.M.Webber et al. *Measurement of the Positive Muon Lifetime and Determination of the Fermi Constant to Part-per-Million Precision*, arXiv:1010.0991v1 [hep-ex], 5 Oct. 2010.

Weisstein E.W. *Dottie Number*. From *MathWorld*. A Wolfram Web Resource http://mathworld.wolfram.com/DottieNumber.html

Remark*

A message appeared recently in the Internet, when the present book was already in process of printing. The report said that an essentially new result was obtained in measurement of the muon lifetime[1]:

$$\tau_{\mu+}(\text{M}\mu\text{Lan}) = 2.196\,980.3(2.2)\cdot 10^{-6}\ \text{s}$$

Hence, using the new theoretical result $R = 0.995\,6104(2.2)$ for radiative correction[2], the value for the Fermi constant is obtained:

$$G_{\text{F}} = 1.166\,3788(7)\cdot 10^{-5}\ \text{GeV}^{-2}.$$

The new τ_μ value is an order of magnitude more precise and notably smaller than the world average value $\tau_\mu = 2.197\,019(21)\cdot 10^{-6}$ s (standard deviation $\delta = -1.8$).

Such variation was clearly predicted in the framework of the LMP-Theory, as early as in publication[3]. Members of the sited, as well as other experimental groups were informed of the fact. Automatic use of the τ_μ(MμLan) value in dimensionless **A**-system brings to even more accurate, phenomenal result (five zeros instead of four, error of order 10^{-8}) of hitting the desired point of infinite continuum:

$$G_{\text{FA}} = \text{e}^{-48.000\,0011\,(6)}.$$

The author can only consider this news as another very important confirmation of the LMP-Theory presented in this book. Now we have the following LMP-forecast for the muon lifetime, Fermi coupling constant and anomalous magnetic moment of muon:

$$\tau_\mu = 2.196\,975\,51(56)\cdot 10^{-6}\ \text{s} \qquad (0.25\ \text{ppm})$$

$$G_{\text{F}} = 1.166\,383\,14(6)\cdot 10^{-5}\ \text{GeV}^{-2} \quad (0.05\ \text{ppm})$$

$$a_\mu = 1.165\,923\,55(7)\cdot 10^{-3}\ \mu_{\text{B}} \qquad (0.06\ \text{ppm})$$

---

[1] **D. M. Webber** et al. *Measurement of the Positive Muon Lifetime and Determination of the Fermi Constant to Part-per-Million Precision*, arXiv:1010.0991v1 [hep-ex], 5 Oct. 2010.

[2] **A. Pak** and **A. Czarnecki.** Phys. Rev. Lett. **100**, 241807 (2008); **K. Lynch.** *Extracting* G_{F} *from* τ *: Obtaining the Central Value and Propagated Errors*. Version 3: March 3, 2010.

[3] **H. Arakelian.** *From Logical Atoms to Physical Laws*. Yerevan: Lusabatz, 2007 (in Rus.); (see also pp. 46 and 84 of this book).

*Added to the published text of the book. The latest experimental values for τ_μ, G_{F} and R_μ are taken into account in this Internet version (see *Preface*, *Ch. 3* and *Conclusion*).