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COWLES FOUNDATION DISCUSSION PAPER NO. 725

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Trends, Random Walks, and Tests of the Permanent Income Hypothesis

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September 1984

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September 21, 1984

Abstract

Recent studies find that consumption is excessively sensitive to income. These studies assume that income is stationary around a deterministic trend. The data, however, do not reject the hypothesis that disposable income is a random walk with drift. If income is indeed a random walk, then the standard testing procedure is greatly biased toward finding excess sensitivity. Moreover, if income is borderline stationary, this procedure is also seriously biased.

We are grateful to Marjorie Flavin, Larry Katz, David Romer, and participants in the NBER Summer Institute on Consumption for helpful comments.

I. Introduction

Since the seminal work of Hall [1978] and Sargent [1978], much research has attempted to test the permanent income hypothesis under the assumption of rational expectations.¹ Some of this research produces evidence contradicting the theory. Most notably, Flavin [1981,1984] and Bernanke [1983] report that consumption is "excessively sensitive" to disposable income. To many economists, this excess sensitivity suggests that liquidity constraints are an important determinant of consumer spending.²

We argue that the standard test is biased toward finding excess sensitivity. The recent work of Nelson and Plosser [1982] suggests that unit roots are common in economic time series. Dickey and Fuller [1981] and Nelson and Kang [1981,1983] show that conventional test statistics are inadequate in the presence of unit roots. In particular, a researcher is likely to find a deterministic trend where none exists. Moreover, inappropriate detrending can produce spurious cycles even if the underlying data have no cyclical properties. These findings motivate our reexamination of tests of the permanent income hypothesis.

The intuition behind our analysis is as follows: Suppose income follows a random walk, so that permanent income equals current income. Suppose also that the permanent income hypothesis is true, and thus consumption equals income. Since the series contain unit roots, standard testing procedures are invalid. Moreover, if both consumption and income are

¹See King [1983] for a survey of this research.

²Dornbusch and Fischer [1984, pp. 186-187], for example, give Flavin's finding this interpretation in their textbook.

(inappropriately) detrended, then both series will exhibit spurious cycles. Since consumption tracks income perfectly over these seemingly transitory cycles, the econometrician will erroneously conclude that consumption is excessively sensitive to contemporaneous income.

In the second section of this note, we examine the stochastic properties of disposable income per capita over the post-war period. We conclude that disposable income is, or is very close to, a random walk. Thus, shocks to disposable income are essentially permanent.

In the third section, we perform Monte Carlo experiments to demonstrate the bias in the standard test procedure. We begin by assuming that income follows a random walk. We show how use of the conventional test, which assumes income is stationary around a deterministic trend, leads to incorrect inferences. We also generalize the Monte Carlo experiment by assuming that income is stationary but strongly autoregressive. We show that the conventional test is again biased toward rejection.

In our final section, we discuss the implications of our results for future research. Since shocks to aggregate income are almost completely permanent, it appears that aggregate data have little power to distinguish among alternative theories of consumption. More generally, our results provide a vivid and concrete example motivated by economic theory of the pitfalls inherent in the conventional and routine use of detrended data.

II. Disposable Income is a Random Walk

In this section, we show that quarterly disposable personal income per capita over the past thirty-five years is well approximated as a random

walk. In other words, shocks to disposable income are permanent. We use standard Box-Jenkins [1976] tools, which are essentially atheoretical, to identify the stochastic process.

In Table 1, we present the autocorrelations of the level of real disposable income per capita (Y_t). The autocorrelations begin at one and decline only slightly. The partial autocorrelations decline to zero after the first one. A random walk has these properties.

For comparison, we also present in Table 1 the autocorrelations of real GNP per capita. In contrast to disposable income, the second partial autocorrelation is significantly negative. This fact indicates that GNP and disposable income follow different stochastic processes.

In Table 2, we present the autocorrelations of the change in disposable income per capita. These autocorrelations are all small. The Q-statistic for the null hypothesis that the first 24 autocorrelations are zero is 36.1, which is significant at the ten percent level but not at the five percent level. Thus, the change in disposable income appears to be approximately white noise.

In contrast, the change in GNP per capita appears autoregressive. The Q-statistic is 52.2, which is significant at the one percent level. These autocorrelations confirm that GNP follows a higher order process than disposable income.³

Finally, we present a simple univariate forecasting equation. When we regress $Y_t - Y_{t-1}$ on Y_{t-1} , $Y_{t-1} - Y_{t-2}$, and a time trend, we obtain:

³The pattern of autocorrelations suggests that GNP is ARIMA(1,1,0), which is not "cyclical". Alternatively, GNP may be a time-aggregated random walk.

$$(1) \quad Y_t - Y_{t-1} = 84. - 0.04 Y_{t-1} + 0.04 (Y_{t-1} - Y_{t-2}) + 0.8 \text{ Time}$$

(41.) (0.02) (0.08) (0.4)

$$R^2 = 0.03 \quad \bar{R}^2 = 0.01 \quad D.W. = 1.96 \quad s.e. = 36.$$

Standard errors are in parentheses.

Although some of the coefficients appear significant using conventional significance levels, one should not draw this conclusion. As Dickey and Fuller [1981] make clear, if Y_t in fact follows a random walk, the conventional tests are inadequate. In particular, higher t-statistics are required to reject a null hypothesis for a test of a given size. The adjusted R-squared suggests, though, that the forecastability of disposable income changes from lagged values and from trend is negligible.⁴

Thus, from this examination of the data, we conclude that disposable income is approximately a random walk with drift:

$$(2) \quad Y_t = 18. + Y_{t-1}$$

(3.)

$$s.e. = 36.$$

Comparing the standard error of estimate of equations (1) and (2), it is evident that one cannot reject the hypothesis that disposable income is a random walk with drift against the alternative in equation (1), even with the conventional critical value.

⁴The same equation estimated for GNP per capita yields an adjusted R-squared of 0.16.

III. Testing the Permanent Income Hypothesis

In this section, we examine tests of the permanent income hypothesis. We first consider an economy in which income follows a random walk. We then show that the problems in the non-stationary case also arise when income follows a borderline stationary process.

A. Non-stationarity

We begin by supposing that income follows exactly the process in equation (2). We then examine the standard tests of the permanent income hypothesis performed on detrended data. Even when the permanent income hypothesis is true, these tests often yield rejections of the theory that appear both statistically and economically significant.

If income follows a random walk, then permanent income equals current income.⁵ If, in addition, the permanent income hypothesis is true, then consumption (C_t) also equals current income. Consider the now standard test of the theory, derived first by Hall [1978] and examined more fully by Flavin [1981], that changes in consumption are not forecastable. We might regress the change in consumption on lagged income:

$$(3) \quad C_t - C_{t-1} = \mu + \pi Y_{t-1} + v_t$$

Since $C_t = Y_t$ in our fictitious economy, the test that $\pi = 0$ is just the test that consumption follows a random walk.

Suppose that, in an economy in which consumption and income are generated by equation (2), an econometrician first detrended the data on both

⁵If income follows a random walk with drift, then permanent income equals current income plus a constant. Since we always include a constant term in our equations, this constant is inessential and thus we do not

consumption and income and then performed the regression test (3).⁶ What would he find? We answer this question by a Monte Carlo experiment assuming 25 years of quarterly data. Based on 1000 replications, we find the median value of his t-test would be -2.20. Using the conventional "five percent" critical value of 1.96, he would reject the null hypothesis 61 percent of the time. Using the conventional "one percent" critical value of 2.58, he would reject 30 percent of the time.

Would the econometrician conclude that the rejection was economically significant? Consider Flavin's [1981] measure of excess sensitivity. In the first-order case, the just-identified system of equations is:

$$(4) \quad Y_t = \delta + \rho Y_{t-1} + \epsilon_t$$

$$(5) \quad C_t - C_{t-1} = \alpha + \beta (E_{t-1} Y_t - Y_{t-1}) + v_t$$

where β is the measure of the excess sensitivity. According to the theory, predictable changes in income should not be related to changes in consumption; that is, $\beta = 0$. Flavin estimates β by noting that

$$E_{t-1} Y_t = \delta + \rho Y_{t-1}.$$

Thus, equations (3) and (4) are the reduced form system and

$$\pi = \beta(\rho - 1).$$

The test for excess sensitivity is thus just the test that $\pi = 0$ in equation (3). The excess sensitivity parameter, β , is recovered from the estimates of π and ρ .

discuss it.

⁶By the Frisch-Waugh [1933] theorem, first detrending the data is numerically equivalent to estimating the trend simultaneously with the other parameters.

As already noted, the econometrician would reject the hypothesis that $\pi = 0$ much too frequently. That is, the hypothesis of no excess sensitivity would be rejected 61 percent of the time using a test with a nominal size of five percent. The estimate of β would also be large. Since $C_t = Y_t$, equations (3) and (4) are in fact the same, except for normalization. In particular, the estimate of ρ is always one plus the estimate of π . Therefore, the econometrician would always infer that $\beta = 1$, that is, complete excess sensitivity. Hence, he would always conclude that the extent of excess sensitivity is economically significant.

B. Borderline Stationarity

While our example above concerns an income series that follows exactly a random walk, the same issues arise for any non-stationary series. Moreover, if the income series is barely stationary (that is, has a root close to the unit circle), the asymptotic theory justifying standard test procedures is misleading for samples of typical size. To illustrate the problems of conventional inference with barely stationary series, we examine a slight modification of the above example.

Suppose income follows a first-order autoregressive process with autoregressive parameter ρ :

$$(6) \quad Y_t = 18. + \rho Y_{t-1}$$

$$\text{s.e.} = 36.$$

Suppose consumption is set according to the permanent income hypothesis:

$$(7) \quad C_t = r(W_t + H_t)$$

where r = the real interest rate,

W = non-human wealth,

H = human wealth, and

$$(8) \quad H_t = \sum_{k=0}^{\infty} \left(\frac{1}{1+r}\right)^k E_t Y_{t+k}$$

Non-human wealth evolves as:

$$(9) \quad W_{t+1} = (1+r)[W_t + Y_t - C_t]$$

Finally, we set initial non-human wealth equal to zero, initial income equal to its unconditional mean of $18/(1-\rho)$, and the quarterly real interest rate to 1.25 percent.

Suppose our econometrician obtained data generated by this economy, detrended it, and then performed regression (3).⁷ Table 3 presents the results of this Monte Carlo experiment for various values of ρ . The bias is largest for values of ρ close to one. Even at $\rho = 0.95$, the median value of the test statistic is -1.37. In 27 percent of the cases he would obtain a test statistic exceeding 1.96 in absolute value. In other words, while he would think his significance level is 5 percent, it would actually be 27 percent. Moreover, the median estimate of the excess sensitivity parameter, β , is 0.11. Thus, while the bias in the standard procedures is smaller under the assumption that income is stationary, the bias is nonetheless large.

⁷While there is no true trend in income in this example, our econometrician does not know this. Moreover, the same issues arise if there is a trend term in equation (6) that must be estimated by the econometrician.

V. Conclusions

Tests of the permanent income hypothesis that assume income is stationary around a deterministic trend are biased toward rejection if income follows a non-stationary, or barely stationary, process. We believe this finding casts doubt on the conclusion that consumption is excessively sensitive to income.⁸

Our finding that disposable income is approximately a random walk indicates that measuring the excess sensitivity of consumption is not possible using aggregate post-war data.⁹ Such a test entails measuring the response of consumption to transitory income; but shocks to aggregate income appear completely permanent.¹⁰ Tests for the importance of liquidity constraints

⁸Flavin [1981, pp. 1005-1006] reports that her rejection of the theory is decisive only with detrended data. Bernanke [1983] reports that using non-detrended data typically leads to non-convergence of his estimation procedure or to nonsensical results. As Plosser and Schwert [1978] discuss, using first-differenced data is often preferable to using detrended data. It is not generally valid simply to use the raw non-stationary series, because the asymptotic distribution theory justifying the hypothesis testing assumes stationarity.

⁹It is still possible, however, that modeling income as a multivariate process might permit such measurement.

¹⁰This research is not meant to imply that measured consumption should equal measured income. Even given that the univariate process of income is a random walk, there are a variety of other explanations beyond liquidity constraints for the failure of consumption exactly to equal income. The real interest rate may not be constant (Mankiw [1981], Michener [1984]), the utility function may not be quadratic (Zeldes [1983]), consumption goods may be durable (Hayashi [1983]), the utility function may not be additively separable over time and between the measure of consumption, leisure and other goods (Mankiw, Rotemberg and Summers [1984]), or taste shifts may be important (Garber and King [1983], Hall [1984]). Our point in this note is that even if these factors are irrelevant, as is often assumed, the standard testing procedure is invalid.

may require data on individuals.¹¹ Such studies of micro-data, however, reach mixed conclusions.

¹¹See, for example, Bernanke [1984], Hall and Mishkin [1982], Hayashi [1983], Runkle [1984], Shapiro [1984], and Zeldes [1984].

Table 1: Autocorrelations of Levels

Sample: 1948:1 - 1983:4

| | <u>Disposable Income</u> <u>Per Capita</u> | <u>Gross National Product</u> <u>Per Capita</u> |
|----------------|---|--|
| <u>Simple</u> | | |
| First | 1.00 | 1.00 |
| Second | 1.00 | 1.00 |
| Third | 1.00 | 1.00 |
| Fourth | 0.99 | 0.99 |
| Fifth | 0.99 | 0.99 |
| <u>Partial</u> | | |
| First | 1.00 | 1.00 |
| Second | -0.03 | -0.39 |
| Third | 0.02 | -0.06 |
| Fourth | -0.14 | 0.15 |
| Fifth | 0.12 | 0.01 |

Table 2: Autocorrelations of Changes

Sample: 1948:1 - 1983:4

| | <u>Disposable Income Per Capita</u> | <u>Gross National Product Per Capita</u> |
|----------------|---|--|
| <u>Simple</u> | | |
| First | 0.04 | 0.37 |
| Second | 0.00 | 0.18 |
| Third | 0.12 | -0.02 |
| Fourth | -0.12 | -0.09 |
| Fifth | -0.20 | -0.14 |
| <u>Partial</u> | | |
| First | 0.04 | 0.37 |
| Second | 0.00 | 0.05 |
| Third | 0.12 | -0.12 |
| Fourth | -0.14 | -0.07 |
| Fifth | -0.19 | -0.08 |

Table 3: Results of Monte Carlo Experiment

Y_t is generated by equation (6) and C_t is generated by equation (7). Then equations (3) and (4) are estimated on detrended data. Each sample is 100 periods long. Distribution based on 1000 replications. Detrended data is generated by taking residuals from regression on linear trend. Percent Rejections for the null hypothesis that $\pi = 0$ is based on conventional "five percent" critical value of t-test of 1.96.

| <u>True value of ρ</u> | Median Estimate of: | | | | <u>Percent Rejections</u> |
|--|--------------------------|-------------------------|---------------------------|---------------|---------------------------|
| | <u>ρ</u> | <u>π</u> | <u>β</u> | <u>t-test</u> | |
| 1.0 | 0.91 | -0.09 | 1.00 | -2.20 | 61 |
| 0.995 | 0.91 | -0.06 | 0.68 | -2.08 | 54 |
| 0.99 | 0.91 | -0.05 | 0.50 | -1.95 | 50 |
| 0.98 | 0.91 | -0.03 | 0.31 | -1.76 | 41 |
| 0.95 | 0.89 | -0.01 | 0.11 | -1.37 | 27 |
| 0.90 | 0.84 | -0.01 | 0.04 | -1.02 | 17 |
| 0.80 | 0.75 | 0.00 | 0.01 | -0.71 | 12 |
| 0.50 | 0.46 | 0.00 | 0.00 | -0.41 | 7 |
| 0.00 | -0.02 | 0.00 | 0.00 | -0.21 | 5 |

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