

ON THE FORMATION OF HELIUM DOUBLE DEGENERATE STARS AND PRE-CATAclySMIC VARIABLES

Eric L. Sandquist^{1,2}, Ronald E. Taam¹, and Andreas Burkert³

¹Department of Physics & Astronomy, Northwestern University, Evanston, IL 60208;
erics@mintaka.sdsu.edu, taam@apollo.astro.nwu.edu

²present address: Department of Astronomy, San Diego State University, 5500 Campanile Drive,
San Diego, CA 92182

³Max Planck Institut for Astronomy, Koenigstuhl 17, D-69117 Heidelberg, Germany;
burkert@mpia-hd.mpg.de

ABSTRACT

The evolution of low mass ($M < 2.5 M_{\odot}$) binaries through the common envelope phase has been studied for systems in which one member is on its first ascent of the red giant branch. Three-dimensional hydrodynamical simulations have been carried out for a range of red giant masses ($1 - 2 M_{\odot}$) with degenerate helium cores ($0.28 - 0.45 M_{\odot}$) and companions ($0.1 - 0.45 M_{\odot}$) for initial orbital periods ranging from $\sim 15 - 1000$ days. The results suggest that these low mass binary systems can survive the common envelope phase provided that the helium degenerate core is more massive than about $0.2 - 0.25 M_{\odot}$ and that the mass of the red giant progenitor is $\lesssim 2 M_{\odot}$.

Specific applications are made to observed double helium degenerate systems, pre-cataclysmic variables, and subdwarf B stars in order to place constraints on progenitor systems evolving through the common envelope phase. For the observed short period double degenerate systems, it is found that evolutionary scenarios involving two phases of common envelope evolution are not likely and that a scenario involving an Algol-like phase of mass transfer followed by a common envelope phase is viable, suggesting that the first-formed white dwarf is often reheated by nuclear burning on its surface. A formation mechanism for two subdwarf B stars observed in eclipsing short period binaries with low mass main sequence stars is also described.

Subject headings: binaries: close — circumstellar matter — hydrodynamics — stars: interiors

1. Introduction

Short-period ($P \sim$ hours) binary systems involving low-mass white dwarf stars have been discovered in increasing numbers over the past five years (e.g. Marsh, Dhillon, & Duck 1995;

Marsh 1995). Because white dwarfs can only be created at the cores of giant stars, this indicates that the orbital separation of the components must have been much greater in the past. This situation is generally thought to be the result of common envelope (CE) evolution (Paczynski 1976), in which unstable mass transfer from a giant star to a companion causes the companion to be engulfed. The ejection of the giant’s envelope requires energy to be removed from the orbit, resulting in a short-period binary involving a white dwarf — what had been the core of the giant. For those white dwarfs with mass $M \lesssim 0.5 M_{\odot}$, the giant star is required to be on its first ascent up the red giant branch (Iben, Tutukov, & Yungelson 1997).

The efficiency at which the gas is ejected from the giant’s envelope (α_{CE}) is a critical parameter for determining the final orbital period of such systems. We use the definition

$$\alpha_{CE} = \frac{\Delta E_{bind}}{\Delta E_{orb}},$$

where ΔE_{orb} is the change in orbital energy of the binary [from (giant, companion) to (white dwarf, companion)], and ΔE_{bind} is the binding energy (as determined from the initial giant model) of mass unbound during the simulation. Because the evolution is rapid (making radiative transfer effects unimportant during the main mass ejection phase; Sandquist et al. 1998), the evolution is adiabatic. The only inefficiencies that affect calculations of α_{CE} are kinetic energy given to gas during the hydrodynamical interaction beyond what is necessary to unbind it, and kinetic energy given to center of mass motion of the remnant binary.

To determine α_{CE} for a real system, the final masses of the two components of the binary, the final orbital period, and the initial mass of the giant’s envelope should be known at least. The progenitor binary can be described by the initial mass of the giant M_g , its core mass M_c , its radius R , and the mass of the companion M_2 . The initial orbital separation a_i need not be specified since it can be obtained assuming the giant fills its Roche lobe. The giant’s radius can be estimated using a core mass-radius relation for a red giant if one component of the binary can be identified as the giant’s core. The initial mass of the giant envelope M_{env} is the greatest uncertainty since there are no obvious constraints except from measurement of nebular mass. However, a lower limit on the total mass of the giant can be obtained by requiring that it evolved within the age of the Galactic disk (although the giant mass could be even lower if there is an effective stellar wind prior to the common envelope event). So, at the very least we require observed binary systems with well-measured orbital periods and component masses in order to estimate α_{CE} .

We have attempted to model some of the best studied low-mass binary systems that meet these criteria. In this paper, we consider two types of systems containing helium white dwarfs. The first is the pre-cataclysmic variable stars, which are detached binary stars involving a white dwarf and a low-mass main sequence star. Subsequent angular momentum loss due to magnetic braking and gravitational radiation can reduce the orbital separation until the main sequence star exceeds its Roche radius and initiates mass transfer. The second class is double degenerate systems, in which both components are helium white dwarfs. The period of such systems determines whether the two white dwarfs can merge in less than a Hubble time through gravitational radiation.

Because of the low white dwarf masses, the merger of components will not typically form a Chandrasekhar mass object and supernova. Although such objects may lead to helium detonation, such systems can compose at most about 10% of observed Type I supernovae (Woosley, Taam, & Weaver 1986). This percentage follows from the overproduction of ^{44}Ca relative to iron (assuming that Type I supernovae account for the Galactic iron abundance), and is consistent with estimates by Iben & Tutukov (1984). The orbital parameters of the systems we have considered are given in Table 1.

In §2, we briefly describe our computational method and initial models. In §3, we discuss the general features of the hydrodynamical evolution of the system, and highlight features of individual simulations. In §4, we discuss observed binaries containing helium white dwarfs, and the constraints our models place on the process of common envelope evolution and on formation mechanisms for real binary systems.

2. Numerical Methods

The numerical techniques we have used in the simulations presented here are largely identical to those used in Sandquist et al. (1998) and discussed by Burkert & Bodenheimer (1993). We briefly summarize the method below, and enumerate the improvements made since the previous study.

The computational regime was composed of a 3-D grid of $64 \times 64 \times 64$ zones, with two subgrids of $64 \times 64 \times 32$ zones nested within, and centered on the main grid. The subgrids were a factor of 2 smaller in the x and y directions compared to the next coarser grid. Two point masses were used to represent the degenerate core of the red giant star and its companion (which can be identified as either a main-sequence star or helium white dwarf). Gravitational interactions between the collisionless particles and the gas were smoothed according to

$$\Phi_{PG} = \frac{-GM_P}{\sqrt{r^2 + \epsilon^2 \delta^2 \exp(-(r/\epsilon\delta)^2)}},$$

where δ is the width of a zone on the innermost subgrid, $\epsilon = 2.0$, and r is the distance between the gas and the particle of mass M_P . The gravitational potential of the gas was computed via fast Fourier transform solution of Poisson’s equation.

The orbit of the core particles decayed under the action of gravitational drag forces within the common envelope. The core positions were obtained using a four-point Runge-Kutta integration scheme each time the gas distribution (and thus, the gravitational force) was recalculated on the innermost subgrid. When the cores were at their closest approach, the gas timestep was subdivided to ensure that the core positions were computed at least 25 times during each orbit.

2.1. Initial Models

The initial gas distribution in the envelope of the red giant was taken from one-dimensional stellar models obtained from the code developed by Eggleton (1971, 1972) and updated by Pols et al. (1995). In order to stabilize the giant model once the core particle had been inserted, we adjusted the temperature of the 8 subgrid zones immediately surrounding the core to put them into hydrostatic equilibrium. The remainder of the grid was filled with a diffuse background gas in pressure equilibrium with the surface of the star.

The companion was typically placed on a circular orbit in the xy -plane with a radius about 30% larger than the surface of the giant as a compromise between computational time and matching the initial conditions for a real binary beginning mass transfer. The difference is unimportant because the deep structure of the giant (where most of the energy input occurs in our simulations) is unchanged by the difference in initial separation. The envelope of the giant star was given a rotation rate that would have made it synchronous with the companion if the companion was on an orbit that would cause the giant to just overflow its Roche lobe. As a result, the envelope was out of synchronicity with the companion. This was done because tidal forces are not able to maintain synchronism between the giant’s envelope and the companion once mass transfer begins accelerating.

3. Results

The initial parameters and results for each sequence are summarized in Tables 2 and 3 respectively. For all of the simulations that we have carried out in this study, the initial phases of the interaction between the giant star and its companion are similar to what was reported in Sandquist et al. (1998).

3.1. The Spiral-In and Rapid Infall Phases

Since these phases were discussed in more detail in Sandquist et al. (1998), we only summarize the general features here. In the earliest portion of the evolution, gas is pulled from the surface of the giant to the companion and is gravitationally torqued, removing angular momentum from the binary orbit until the companion and giant come into direct contact. Once this occurs, the orbital decay accelerates for a period of time. Large-scale spiral shock waves appear, but soon become tightly wound as the period of the binary orbit becomes smaller than that of the envelope gas. Once the two point masses begin to interact with higher density gas, the rate of energy transfer to the gas increases.

There are two distinct intervals during the evolution when gas is unbound in the system. The first occurs as the companion slingshots gas into a spiral wave during its descent through the giant

envelope. For a relatively long period after the initial infall, the amount of bound mass remains nearly constant, although there is still some expansion of the gas (as can be seen in Figures 5, 6, and 8 in the decrease in the amount of mass retained within the original volume of the star, M_{vol}). More substantial ejection of mass begins when the point masses are deep in the potential well of the binary, creating a more vigorous outflow in the orbital plane.

3.2. The Envelope Ejection Phase

The final separation of the binary is the most important quantity that can be derived from simulations. In our previous study, as well as for most of the sequences presented here, the resolution of the gas in the grid limited how long we could accurately compute interactions between point masses and the gas. For this reason most of the simulations had to be stopped at or before the start of the envelope ejection phase (defined by Sandquist et al. 1998 as the time at which the orbital decay timescale begins rising after the rapid infall). In sequence 3, however, this was not a factor until a much later stage in the computation — primarily because the binary had expelled most of the giant’s envelope before this point.

Previous papers in this series have indicated that the initial structure of the red giant plays an important role in the orbital decay. In an evolved giant branch star, there is an extremely small amount of mass in the radiative region between the hydrogen-burning shell and the base of the convection zone, even though that region encompasses a large volume. When the orbital separation becomes comparable to the size of this low mass region, the orbital decay will slow.

Simulation 3 shows that the orbital decay can be decelerated in a completely different way. From one-dimensional models of the initial giant, the base of the convection zone is 1.5×10^{12} cm from the core. However, when the orbital separation reaches this size, the orbital decay timescale has increased to 4500 days, indicating that the binary is essentially stable (although the giant’s envelope has not been unbound and ejected from the grid). This appears to show that with a relatively high-mass binary (compared to the mass of the giant’s gas envelope) gravitational torques can remove the gas and halt the orbital decay before it reaches what was the radiative zone of the giant.

In order to put all of our simulations in a theoretical context, we have computed the value of the ratio

$$\gamma_{CE} \equiv \frac{\tau_{spin-up}}{\tau_{decay}}$$

(Livio & Soker 1988) from the stellar models we used to initialize the hydrodynamics. γ_{CE} is a measure of the importance of the angular momentum transfer to the giant envelope. For $\gamma_{CE} < 1$, significant spin-up of the giant’s envelope is likely during the decay of the companion’s orbit. Assuming that the giant is not rotating, and that the companion is moving on a circular orbit at

its Keplerian speed, the ratio becomes

$$\gamma_{CE} \simeq \frac{4}{5} \left(\frac{\tilde{\rho}_a}{\bar{\rho}_a} \right) \left(\frac{M_1(a) + M_2}{M_2} \right)$$

where

$$\tilde{\rho}_a = \frac{5}{a^5} \int_{R_{core}}^a r^4 \rho(r) dr,$$

R_{core} is the radius of the core of the giant, $M_1(a)$ is the mass of the giant’s envelope within the orbit of the companion, and $\bar{\rho}_a$ is the average density of mass inside the orbit. We are in a position to compare the predictions of this simple model to the results of our hydrodynamical simulations.

Figure 1 shows plots of γ_{CE} as a function of radius in the envelope of the giant for each of our simulations. To check the validity of the simple spin-up model, in Figure 2 we plot slices for all of our simulations at the end of the rapid infall phase. The assumption of a Keplerian orbit for the companion is clearly violated at the times depicted, but the majority of the angular momentum transfer has occurred earlier. The simple model only predicts significant spin-up for simulations 2, 3, and 6, with simulation 3 being the most likely to show the effects. The more massive the companion and the lower the moment of inertia (whether due to low envelope mass or relatively small radius), the more likely spin-up will occur. In agreement with the prediction, simulation 3 shows dramatic effects of spin-up of the gas (including depletion of gas along the binary axis, and stable rotation), while simulations 2 and 6 show these features to a lesser extent.

3.3. Discussion of Individual Simulations

In Table 3, we list the separation and period of the point mass binaries at the end of our simulations. We emphasize that for all but simulation 3 the binary would undergo additional orbital decay. For that reason, the values listed for the efficiency of mass ejection α_{CE} should only be considered estimates. Our resolution prevents us from running our simulations far enough into the envelope ejection phase to accurately determine α_{CE} due to either rapid orbital decay or to slowly evolving gas on the grid.

Instead we will focus our discussion on whether the simulated binary is likely to survive or merge during the common envelope phase, and what conclusions can be drawn for real binary systems involving helium white dwarfs. Although we cannot make good estimates of α_{CE} , we can make a crude estimate of the final orbital separation for the binary using the energy needed to unbind the envelope mass remaining at the end of our simulations. Assuming values of $\alpha_{CE} = 0.4$ (Sandquist et al. 1998) and 1.0, we can compute how far the orbit would have to shrink to release the energy needed to complete the ejection.

3.3.1. *Simulation 1: 2 M_{\odot} Giant, 0.2 M_{\odot} Companion*

This simulation was run primarily to examine a case in which the binary was likely to merge because the low-mass remnant binary has insufficient energy to eject the relatively large envelope mass. Figure 3 compares the orbital decay timescale for the binary with the timescale for change in bound mass. The orbital decay timescale at the end of the simulation was several thousand times shorter than the timescale for unbinding and ejecting mass from the system, indicating that the binary is likely to merge.

As seen in Figure 4, very little mass had been unbound by the end of the simulation. The amount of energy necessary to unbind the remaining gas was over 10 times greater than what was removed during the simulation. Our estimates of the final orbital separation are also quite low — only a few tenths of a solar radius. We conclude that this system is indeed likely to merge.

3.3.2. *Simulation 2: 1 M_{\odot} Giant, 0.35 M_{\odot} Companion*

This binary was modeled as a plausible progenitor of the double degenerate system PG 1101+364, which has $q = 0.87 \pm 0.03$, a period of 3.47 h (Marsh 1995), and a mean mass of approximately $0.31 M_{\odot}$ (Bergeron et al. 1992). The temperatures of the two components of the binary leave some confusion as to the formation mechanism for this system (see § 4.2.2), but its short period indicates that it probably evolved through a common envelope phase. WD 0957-666 may be a better system for comparison — it also has a short period (1.46 h) and similar masses ($0.37 \pm 0.02 M_{\odot}$ and $0.32 \pm 0.03 M_{\odot}$; Moran et al. 1997).

A modest amount of energy is necessary to completely eject the envelope mass remaining at the end of this simulation, so the estimated final orbital separation for the system is not much different from the separation that was reached. Based on this, it is likely that this system will survive, particularly if both of the components are helium degenerates. Figure 5 corroborates this, showing that most of the gas has been removed from the original volume of the star, including the dense gas around the giant’s core. The estimated final periods for this simulation compare well with the observed double degenerate systems.

3.3.3. *Simulation 3: Evolved 1 M_{\odot} Giant, 0.35 M_{\odot} Companion*

This simulation was intended for comparison with the pre-CV system PG 1224+309, which has an orbital period of 6.21 h, a white dwarf with a mass of approximately $0.45 M_{\odot}$, and an M dwarf companion of $0.28 \pm 0.05 M_{\odot}$ (Orosz et al. 1999). At present, PG 1124+309 is the pre-CV system with main sequence companion of largest measured mass, although CV companion masses can be somewhat larger. The mass of the giant is approximately the minimum for a star that could have evolved to the giant branch in the age of the Galactic disk. Together with the highly

evolved state of the giant (low mass and binding energy of the giant envelope) and the relatively massive remnant binary, this represents a system that is likely to survive the common envelope phase.

Because the orbital decay of the binary slowed substantially before reaching the point where grid resolution became important, we were able to follow this simulation much farther in its evolution than any other. Several features distinguish this simulation from the previous ones. As can be seen from a cut through the yz -plane in Figure 2 (panel c), there is a very low density region oriented along an axis perpendicular to the orbital plane of the binary. In addition, we find that the circulation pattern that is often seen in common envelope simulations (e.g. Sandquist et al. 1998) is almost completely absent in this simulation because of the support given to the gas by centrifugal forces. During a portion of the simulation, a large and stable differentially rotating disk-like structure occupies much of the computational domain.

As can be seen from a plot of mass tracers in Figure 6, negligible mass is unbound after the initial spiral-in phase. Towards the end of our simulation, the mass distribution appears to stabilize for a time. However, beginning at approximately 900 d, convective plumes appear and begin to disrupt the disk. Stability of the material against local axisymmetric adiabatic perturbations was tested using the Høiland criterion (Tassoul 1978; Igumenshchev, Chen, & Abramowicz 1996):

$$\frac{1}{r^3} \frac{\partial l^2}{\partial r} - \left(\frac{\partial T}{\partial P} \right)_S \nabla P \cdot \nabla s > 0$$

and

$$-\frac{1}{\rho} \frac{\partial P}{\partial z} \left(\frac{\partial l^2}{\partial r} \frac{\partial s}{\partial z} - \frac{\partial l^2}{\partial z} \frac{\partial s}{\partial r} \right) > 0$$

where l is the specific angular momentum, and s is the specific entropy. By the Høiland criterion, the plumes we see in our simulation are indeed unstable. The unstable regions for one timestep are plotted in Figure 7.

At the end of the simulation the total bound mass (the sum of what remained on the grid and what moved off the grid) had decreased only by about $0.05 M_\odot$ down to $0.5 M_\odot$. However, the gas contained within the original volume of the star had decreased to about $0.1 M_\odot$, so that most of the gas had expanded to relatively large distances from the point masses. The final orbital period of the point masses was 13 days, indicating that it may be possible to create binaries with periods of several days in the common envelope scenario. The orbital decay timescale of the binary had also risen to about 4500 days. Given the low binding energy of the gas remaining in the simulation, the binary is likely to remain with an orbital period on the order of several days. A period on the order of days is considerably larger than that of PG 1224+309, which suggests that the mass of the giant’s envelope must have been larger. This possibility is examined in simulation 5. However, to make a definitive statement about the final state of the binary, we need to understand the role of the gas that remained on the grid. This gas influences the evolution on timescales longer than we are able to follow.

Although this simulation is best suited for a useful computation of α_{CE} , the calculation yields a rather low value (0.09). In this case, the efficiency is low because the gas has been moved far from the cores without being unbound. This is a consequence of our definition of α_{CE} , and is not necessarily a reflection of the efficiency of mass ejection for common envelope systems.

3.3.4. *Simulation 4: Evolved 1 M_{\odot} Giant, 0.1 M_{\odot} Companion*

This run is to be compared with the pre-CV system GD 448, which has an orbital period of 2.47 h, a white dwarf with a mass of approximately $0.44 \pm 0.03 M_{\odot}$, and an M dwarf companion of mass $0.09 \pm 0.01 M_{\odot}$ (Maxted et al. 1998). It may also simulate the formation of the subdwarf B/main sequence star binaries HW Vir (Wood, Zhang, & Robinson 1993) and PG 1336–018 (Kilkenny et al. 1998), which have periods of 2.8 and 2.4 hours respectively. The sdB/MS binaries will be discussed in more detail in § 4.2.3, but sdB stars are generally believed to have a mass of about $0.5 M_{\odot}$ (Saffer et al. 1994), and mass ratios $q \sim 0.3$ are inferred in the cited observational studies. The companion mass and initial orbital separation are the only differences between the initial parameters of this simulation and simulation 3. The companion mass is near the hydrogen burning limit for stars, and is at the low end of the companion mass distribution for well-observed pre-CV systems.

The orbital decay timescale for the binary at the end of the simulation is about 200 days, while the mass unbinding timescale is considerably longer (and is not well-determined due to the low rate of change of the bound mass). The mass tracers for this simulation are shown in Figure 8. Very little mass ($\sim 0.02 M_{\odot}$) was made unbound by the time the simulation was terminated, and the majority of the gas originally within the volume of the giant remained there. As a result, the binary is certain to undergo substantial further orbital decay.

The estimated final orbital periods range from 0.8 to 2.6 days, which are again substantially larger than those of the observed systems. A common envelope binary involving a higher mass giant in the same evolutionary phase could potentially produce the observed systems. However, we must be careful: if the α_{CE} values we have used are overestimates, the final periods would also be smaller. Because this simulation shows less evidence of mass ejection than simulations 2 and 3, and because the low mass of the companion makes it more likely that the binary will merge, it is not certain that α_{CE} will be in the range we used to calculate the periods. Better resolved computations should be undertaken.

3.3.5. *Simulation 5: Evolved 2 M_{\odot} Giant, 0.35 M_{\odot} Companion*

In order to examine the effects of giant mass on the binary, we reran simulation 3, doubling the giant mass while leaving the mass of the giant core and companion the same. This places the giant mass near the maximum that can produce a helium degenerate object.

The additional gas mass drastically changes the outcome. The timescale for orbital decay at the end was approximately 100 days, while the mass-loss timescale was around 10^4 days. Energetic considerations indicate that the binary is likely to shrink to a separation of a few solar radii. However, the caveats from simulation 4 can be applied here also — unless the value of α_{CE} is proven to be between 0.4 and 1.0 (what we consider to be the most likely range), we cannot be certain that the mass ejection is less efficient than in previous simulations. As a result, we believe that the estimates of the final orbital separation in Table 3 are upper limits. Those upper limits are still greater than is found for the observed systems, so it is still possible that a giant of this mass could produce them.

3.3.6. *Simulation 6: 1 M_{\odot} Giant, 0.45 M_{\odot} Companion*

This simulation was run as one test of the validity of the double CE mechanism for forming double helium white dwarf systems. It is intended to model a binary after it has undergone the first common envelope phase, which shrank the orbit by a large factor. As a result, the secondary would not have been able to evolve as far before mass transfer began. Here we assume the giant has been able to form a $0.28 M_{\odot}$ core, putting it on the lower half of the red giant branch.

This binary is likely to survive the common envelope interaction — by the end of the simulation only $0.25 M_{\odot}$ of the original $0.72 M_{\odot}$ of gas was left within the original volume of the giant. Because of the evidence that the system will eject much of its mass, we can be more confident in believing the range of final periods in Table 3. These estimates of the final period fall on the order of a few hours, which is comparable to the short period double degenerates in Table 1.

4. Discussion

4.1. Constraints on CE Evolution

The common envelope interaction of binary stars with a low-mass ($M < 2.5 M_{\odot}$) red giant has been investigated for the formation of double degenerate systems and pre-cataclysmic variable systems. The results of three-dimensional hydrodynamical simulations suggest that common envelope evolution is a viable method of producing these systems. For red giants with helium white dwarf cores $M_c \lesssim 0.2 - 0.25 M_{\odot}$, the absence of an extensive region characterized by a flat mass-radius profile inhibits the survival of the system as a binary (see Figure 9). Although an extensive set of calculations has not yet demonstrated that all red giants of low core mass merge with their companions, it is likely in this case that gravitational torques remain effective in bringing the two cores together, which may lead to the dissolution of the companion and to the formation of a rapidly rotating subgiant star. If so, then the two stars would continue to spiral

together *even* if sufficient energy is available to unbind the envelope since the timescale for orbital decay is expected to be shorter than the timescale for mass loss from the common envelope. In this interpretation, the observational detection of a system containing a white dwarf less massive than $\sim 0.2 - 0.25 M_{\odot}$ in double degenerate and pre-cataclysmic systems would imply the existence of an alternative evolutionary path. Such a path might involve substantial mass and angular momentum loss via an Algol-type evolution for the double degenerate systems (Sarna, Marks, & Smith 1996).

When the red giant star is in an advanced evolutionary state, a larger fraction of the star’s mass is stored at larger radii. As shown in this paper (simulation 3, for example), the envelope can be pushed farther outward by the interaction between the envelope and the two cores, provided that the cores are sufficiently massive. In that case, gravitational torques are less effective in forcing the two stars together, and energy considerations become the important determinant of the survival of the binary. Our simulations indicate that for a system to survive as a double degenerate, the giant’s envelope mass at a given evolutionary stage is constrained by the mass of the companion white dwarf.

The energy release associated with hydrogen recombination has previously been suggested as an additional source of energy for ejection of the common envelope. For planetary nebulae, detailed calculations have failed to demonstrate that hydrogen recombination can cause mass ejection (see Harpaz 1998). The reduction in opacity associated with the recombination increases radiative energy losses, thereby reducing the efficiency of the mass ejection process. Although recombination may play a minor role in helping eject the outer surface layers of the giant, it is likely to be ineffective in unbinding matter at higher temperatures in the deep interior of the common envelope.

4.2. Constraints on Formation Mechanisms

The success in ejecting the common envelope during the red giant stage provides theoretical support to the hypothesis that the common envelope phase can produce low-mass double degenerate and pre-cataclysmic systems. The primary difference between double degenerates and the pre-cataclysmics is the evolutionary state of the companion — for pre-CVs, the companion is a main sequence star, which is considerably larger than a white dwarf. Thus, for pre-CVs, orbital separations must be such that the tidal lobe is not smaller than the thermal equilibrium radius of the main sequence star.

4.2.1. Pre-Cataclysmic Variable Stars

For pre-cataclysmic variable systems containing a white dwarf and a detached low-mass main sequence companion, we are assured that only one CE phase can have occurred. There are two

well-studied systems of this type that we can compare with our simulations. Maxted et al. (1998) observed GD448 (WD 0710+741), a system containing a white dwarf and an M dwarf companion orbiting with a period of 2.47 h. Although the period places it in the CV period gap, making it possible that GD 448 is in a temporary detached state, Marsh & Duck (1996) argue that it was born into this state following a common envelope phase. Their arguments are based on the short cooling time for the white dwarf (5×10^7 yr) compared to the timescale for angular momentum loss via gravitational radiation (1.8×10^9 yr) necessary to reduce the period to the present value from the 3 h upper edge of the period gap. (Note that some of the orbital evolution may have taken place via magnetic braking.) Orosz et al. (1999) studied the system PG 1224+309, which orbits every 6.21 h.

The masses of the white dwarfs in both systems indicate that the giants were approaching helium flash when they formed a common envelope. This fact ($M_c \gtrsim 0.4 M_\odot$) implies that the giant’s envelope had relatively low binding energy (radius of $\sim 10^{13}$ cm), enabling a low-mass main sequence companion to eject the envelope. In order for the giant to have evolved to this state, its main-sequence evolutionary timescale must be less than the age of the Galactic disk (≈ 10 Gyr), which provides the constraint that $M \gtrsim 1.0 M_\odot$. Our numerical results indicate that a binary containing a $0.35 M_\odot$ companion and an evolved $1 M_\odot$ giant at a period of several days is highly likely to survive. The fact that the final orbital period found in simulation 3 is considerably longer than that of PG 1224+309 indicates either that the mass of the giant star was larger than $1 M_\odot$, or the gas remaining on the grid will interact with the binary to further reduce the orbital separation. Our knowledge of the mass of the white dwarf in this system fixes the evolutionary state of the giant.

With regards to the formation of pre-CVs from an initial binary composed of a $1 M_\odot$ giant (with a $0.45 M_\odot$ core) and a $0.1 M_\odot$ companion, our results are uncertain. The existence of GD 448 indicates that such systems probably do survive. It is possible though that mass loss on the giant branch (e.g., Reimers 1975, 1977) may have reduced the envelope mass prior to the onset of the common envelope phase. The energy required to eject the envelope would be reduced, and survival of the binary could become possible. The importance of mass loss for single red giant branch stars has long been realized because it appears to be needed to explain the masses of horizontal branch stars in globular clusters (Rood 1973). Other evidence provided by the width of the color distributions in these clusters suggests that there are star-to-star differences in the amount of mass loss (Rood 1973). In addition, a metallicity dependence might be expected (Renzini 1981), which would imply that mass loss among stars in the Galactic plane is likely to be larger than for stars in globular clusters on average. The existence of a companion in close proximity to the giant could further enhance the mass loss (Tout & Eggleton 1988) and facilitate the successful ejection of the common envelope.

4.2.2. Double Degenerate Systems

In the following, we discuss three possible mechanisms for forming binary helium white dwarfs, which we shall refer to as the Algol/CE, CE/CE, and CE/Algol mechanisms.

In the Algol/CE scenario, the primary begins mass transfer as a subgiant (with an initial separation $a_i \lesssim 20 R_\odot$). Provided that the system does not undergo a dynamical instability (which can occur if the mass ratio exceeds a critical value that depends on the adiabatic response of the star), further evolution results in mass transfer on a nuclear timescale as an Algol-like system. The donor’s envelope is eventually depleted to the point that it contracts within its Roche lobe and is seen as a white dwarf. By the end of this stage, the orbital separation has increased beyond its initial value. The system remains detached until the secondary evolves onto the first-ascent giant branch and fills its Roche lobe. If the convective envelope of the (now more massive) donor is sufficiently massive, mass transfer is unstable and the system enters into the common envelope phase. Envelope ejection takes place, and the final binary separation is determined by the evolutionary state of the giant and the amount of energy released from the binary orbit.

The mass ratio of a system can serve as an important clue to the formation channel. For the Algol/CE scenario, the mass ratio of the fainter to the brighter white dwarf is constrained very tightly to $q = 0.88 \pm 0.03$ (Tutukov & Yungelson 1988). In the following, we briefly outline this argument, and update the derivation.

The radius of a giant branch star depends sensitively on the mass of the degenerate core, almost independent of envelope mass:

$$R/ R_\odot \approx 9500(M_c/ M_\odot)^{4.8}.$$

In contrast, the size of the Roche lobe depends in part on the *total* mass M of the star through the mass ratio q :

$$R_L/ R_\odot = \frac{0.49q^{2/3}}{0.6q^{2/3} + \log(1 + q^{1/3})}(a/ R_\odot)$$

(Eggleton 1983). At the very end of the Algol stage, the mass of the primary is nearly stripped to its helium core and the mass of the secondary has been supplemented by transfer from the original primary. When the secondary fills its Roche lobe and enters into the common envelope phase, we can make use of the fact that the binary separation was the same in both cases to derive:

$$q_f \equiv M_{1,R}/M_{2,R} = \left(\frac{1 + \frac{5}{3}q_m^{2/3} \log(1 + q_m^{-1/3})}{1 + \frac{5}{3}q_m^{-2/3} \log(1 + q_m^{1/3})} \right)^{1/4.8},$$

where $M_{1,R}$ and $M_{2,R}$ are the masses of the remnants of the original primary and secondary respectively, and $q_m = M_{1,R}/(M_2 + M_1 - M_{1,R})$ is the mass ratio just before the onset of the common envelope phase, assuming conservative mass transfer during the Algol stage. Because the Roche lobe of the (more massive) secondary is larger than that of the remnant

of the primary, the secondary is able to evolve slightly farther than the primary. Thus, the core of the secondary becomes more massive than the core of the primary. The range allowed in q_f for helium double degenerates can be illustrated by two cases. q_m is maximized when $M_1 \approx M_2 = 1.0 M_\odot$ and $M_{1,R} = 0.5 M_\odot$, which gives $q_m = 1/3$ and $q_f = 0.90$. q_m is minimized when $M_2 + M_1 - M_{1,R} = 2.25 M_\odot$ (the maximum secondary mass that would produce a helium white dwarf), and $M_{1,R} \approx 0.25 M_\odot$, which gives $q_m = 0.11$ and $q_f = 0.81$. So, our derived range ($q_f = 0.85 \pm 0.05$) is slightly larger than the range derived by Tutukov & Yungelson (1988).

In this discussion we have referred to the initially most massive star as the primary. However, the star actually observed to be brighter in the double degenerate systems is most often referred to as the primary in the literature. Since the secondary according to mass (initially) forms the second white dwarf, it will be brighter in observed systems, and so would be called the primary. Thus, for observed systems, the mass ratio signature of this process would be $q'_f = M_{bright}/M_{faint} = 1.17 \pm 0.06$ with the most recently formed white dwarf being more massive and brighter than its companion.

There are four helium double degenerate systems that have adequately measured mass ratios. The system L101-26 (WD 0957-666) is probably the best measured of the four. It has a mass ratio $q'_f = 1.15 \pm 0.10$ (Moran, Marsh, & Bragaglia 1997), providing evidence that it has been formed by the Algol/CE mechanism. The mass ratio for the system WD 0136+768 is somewhat higher than the theoretical predictions assuming conservative mass transfer. Although a better estimate of the errors will be necessary to determine the degree to which theory and observation are in disagreement, the final mass ratio q' is expected to be less than about 1.11 for a double degenerate created by the Algol/CE mechanism if mass is not conserved during the Algol phase.

The remaining two systems have mass ratios that are the inverse of the value expected for Algol/CE evolution. The components of the double-lined spectroscopic binary PG 1101+364 have approximately the same surface temperatures and a mass ratio $q'_f = 0.87 \pm 0.03$ (Marsh 1995). Because less massive white dwarfs are expected to cool faster than more massive ones due to their larger surface areas, this indicates that PG 1101+364 may not have formed by the Algol/CE scenario (since the more massive and younger white dwarf would not have been able to cool to the same temperature as the older, less massive white dwarf) unless the temperature of the less massive white dwarf was elevated during the CE phase.

The system L870-2 (WD 0135-052) also has a measured mass ratio $q'_f = 0.86 \pm 0.02$, although some uncertainty exists as to whether both components are helium white dwarfs (Saffer, Liebert, & Olszewski 1988). If it can be confirmed that the components are both helium white dwarfs, then this binary would also appear to have the component temperatures opposite the predictions of the Algol/CE mechanism.

The most recent observations of L870-2 and PG 1101+364 suggest that these two systems did not form as a result of a CE/CE or a CE/Algol mechanism (see below), where the lower mass white dwarf is most recently formed and therefore brighter. The Algol/CE scenario is untenable

unless rejuvenation of the first-born white dwarf has taken place (Iben et al. 1997). In this case, it is unlikely that such heating would be a consequence of the high entropy of the accreted matter or gravitational compression of the white dwarf envelope since energy would only be conducted into the white dwarf interior to a depth set by the lifetime of the common envelope. This timescale ($< 10^2 - 10^3$ yrs) is orders of magnitude shorter than the cooling timescale ($\sim 10^7$ yrs, see Mazzitelli & D’Antona 1986) for the brightest white dwarf (Feige 36, with $T_{eff} < 30,000$ K) in the sample of double degenerate systems, therefore ruling out this mechanism. However, heating from the nuclear burning of hydrogen-rich matter at the base of the non-degenerate envelope could take place. Since the helium white dwarfs in the systems under consideration are of low mass ($\lesssim 0.5 M_{\odot}$), this hypothesis would suggest that such white dwarfs may have hydrogen-rich envelopes of $\gtrsim 10^{-3} M_{\odot}$ (Benvenuto & Althaus 1998). Since the first-born white dwarf may have residual hydrogen from its formation, this hydrogen mass need not be accreted during its interaction with the common envelope. The white dwarf rejuvenation hypothesis could be tested if the white dwarfs were found to exhibit pulsations characteristic of ZZ Ceti variables. It is generally accepted that this phenomenon is due to the excitation of gravity modes in the envelopes of white dwarf stars (see Winget et al. 1982). Their detection could provide the means of probing the composition structure of white dwarf envelopes to look for signs of rejuvenation. Thus, this kind of independent measurement of the mass of the hydrogen-rich layer of a white dwarf (as well as the white dwarf itself) can provide important links to white dwarf asteroseismology and may help illuminate important processes taking place during the common envelope interaction.

The eclipsing double-line binary system HD 185510 is an example of a system which may be on the Algol/CE evolutionary path. This system is a long period ($P = 20.7$ d; Frasca, Marilli, & Catalano 1998) binary containing a K0 giant star and an underluminous B-type star. Although there is still some uncertainty as to whether the B star companion ($T = 31000$ K, $\log g \sim 7$; Jeffery & Simon 1997) is a helium white dwarf or a subdwarf B star, either possibility requires that the progenitor must have had its evolution truncated while on its first ascent of the giant branch. The long period of the system and the relatively large mass of the K giant ($M \approx 2.3 M_{\odot}$) are expected after an Algol-like phase of conservative (or nearly so) mass transfer.

We can also consider the possibility that binary systems with larger initial orbital separations ($a_i \gtrsim 20 R_{\odot}$) evolve through two common envelope phases as the envelopes of the primary and secondary are successively ejected. The final mass ratio of the white dwarf remnants is much smaller in this scenario because the binary separation decreases by a large factor after the first common envelope stage. As a result, the secondary is not able to evolve as far up the giant branch before filling its Roche lobe, and so will leave the lower mass remnant. In contrast to the Algol/CE evolutionary path, the secondary (in terms of initial mass) would also be hotter and more luminous (the primary in terms of brightness) in the absence of rejuvenation. However, the constraint that the secondary is able to evolve to a radius where it can undergo unstable mass transfer and survive a common envelope phase is a serious one (see below).

To determine the viability of the CE/CE formation mechanism, we estimate the maximum

binary separation possible after the first common envelope stage. For the binding energy of the envelope of a giant star, we use an expression given by de Kool (1990):

$$E_{bind} = \frac{G(M_c + M_e)M_e}{\lambda R}$$

where M_c and M_e are the core and envelope masses of the giant, R is the giant’s radius, and λ is a constant with value ~ 0.5 . Assuming that the secondary must be able to form a helium core of $0.25 M_\odot$ to be able to survive a common envelope phase, a lower limit for the orbital separation after the first common envelope phase can be obtained. As a consequence, the amount of orbital energy available for ejecting the envelope of the primary can be determined. Using the equation

$$E_{bind} = \alpha_{CE} E_{orb},$$

we can then derive a lower limit for the core mass of the *primary* star. Because the radius of the giant has a relatively large dependence on core mass, this lower limit has a relatively low dependence on the initial primary and secondary masses, α_{CE} , and λ . From our assumptions we derive the equation (to be solved numerically):

$$q_f^{4.8} = \frac{9.8}{q_i} \left(\frac{0.25}{\lambda \alpha_{CE}} \right) \left(\frac{q_m}{q_i} - 1 \right) \left(1 + 2.04 \frac{\log(1 + q_m^{1/3})}{q_m^{2/3}} \right)$$

where $q_m = M_2/M_{1,R}$ and $q_i = M_2/M_1$. With the constraints that $M_{2,R} > 0.25 M_\odot$ and $M_2 < M_1$, we find that the parameter space can be limited to $M_{1,R} > 0.47 M_\odot$, $M_1, M_2 < 1.2 M_\odot$. If we use the less strict condition $M_{2,R} > 0.2 M_\odot$, the limits are relaxed substantially: the only useful mass constraint is that $M_{1,R} > 0.4 M_\odot$. However, the mass ratio is constrained to be $q_f = q'_f < 0.53$ while the remnant of the secondary remains hotter, beyond which $q'_f > 1.89$. None of the observed double degenerates are consistent with the mass ratio constraint, although there are few helium double degenerate systems known and selection biases could be important.

Ritter (1999) indicates that stable mass transfer from a giant star can occur for mass ratios up to $q = M_{donor}/M_{accretor} = 0.83$ in the conservative case, and up to $q = 1.2$ if mass can be lost from the accretor in an isotropic wind having the accretor’s specific angular momentum. If the latter holds true (since Algol systems are known to transfer mass non-conservatively), then no systems having $M_{2,R} > 0.25 M_\odot$ could survive the CE/CE mechanism since $M_{1,R} \sim 0.5 M_\odot$ is a constraint on helium white dwarfs. Such a constraint on q would also cut into the parameter space allowed if $M_{2,R} > 0.2 M_\odot$, but would not entirely eliminate it. Mass loss from the primary before it fills its Roche lobe would tend to make CE/CE less likely by making stable mass transfer possible. The volume of parameter space available for the CE/CE mechanism can be visualized using Figure 10.

According to these estimates, we conclude that binary scenarios involving two common envelope phases ending with a detached binary are possible only in systems with very rare combinations of initial orbital parameters. Our hydrodynamical simulations indicate that orbital shrinkage of at least a factor of 20 occurs as a result of the common envelope evolution for even

our most favorable case. Provided that the binary system survives the first common envelope phase, the second stage of mass transfer involving the expansion of the secondary component is most likely to occur on the subgiant branch or low on the giant branch (with $M_c \lesssim 0.25 M_\odot$). As the mass ratio of the system is likely to be greater than 2 (since the first-born white dwarf is less massive than $0.5 M_\odot$ and the secondary component is greater than about $1 M_\odot$ to ensure nuclear evolutionary expansion), mass transfer leads to a second common envelope phase with the merger of the system a likely outcome.

If the mass transfer from the secondary begins in the Hertzsprung gap ($P \sim 1$ day), the binary may escape a second common envelope phase, but the unstable mass transfer is likely to lead to substantial mass and angular momentum non-conservation. This third mechanism (CE/Algol) for the formation of double degenerates has been studied by Sarna et al. (1996) under the assumption that the mass transfer process is non-conservative. By also including angular momentum losses associated with magnetic braking, Sarna et al. (1996) found that the evolution of an Algol system following the common envelope phase depended on the period after the common envelope phase. For orbital periods less than about 1 day, the system evolved to shorter orbital periods (\sim hours), whereas longer period systems evolved to periods greater than 1 day. This bifurcation in evolution is similar to that discussed by Pylyser & Savonije (1988) and Ergma, Sarna, & Antipova (1998) for low mass X-ray sources. Although the bifurcation period is a function of orbital parameters of the system, the results of these studies, taken as an aggregate suggest that it is less than 1.5 days. We note that for this mechanism the most recently formed white dwarf (and hence the brighter member of the binary) is predicted to be lower in mass than the first born white dwarf. In addition, since mass transfer takes place in the Hertzsprung gap or on the subgiant branch, the mass of the secondary's remnant will be low ($\sim 0.15 - 0.2 M_\odot$ for the systems studied by Sarna et al. 1996). If we assume that the remnant mass of the primary is at least $0.25 M_\odot$ for the binary to have undergone common envelope evolution and survived, then the mass ratio is expected to be less than 0.8. This upper limit would decrease further if the primary gained mass as a result of the stable mass transfer (Shara, Prialnik, & Kovetz 1993). As with the CE/CE mechanism, none of the observed systems matches the mass and mass ratio constraints, although again selection effects might be important.

The formation of double degenerate stars in the common envelope scenario is also tested by the range of a factor of ~ 100 in orbital periods — the shortest period known is 1.4 hr for L 101-26, while the longest period (6.3 days) is for WD 1824+040. Because it is unlikely that ionization energy can be tapped efficiently during the common envelope phase, and because sufficient nuclear energy is not likely to be available on the timescale over which matter is ejected ($\lesssim 10$ yrs), the conversion of orbital energy into binding energy is almost certainly the prime factor in determining the final periods for common envelope systems. The main issue is whether this range of periods can be explained by differences in the initial masses and evolutionary states of the components in a single mechanism, or whether different formation mechanisms must be invoked. The theoretical evidence indicates that an Algol/CE scenario is a viable formation mechanism for

both long period and short period double degenerates. For this evolutionary channel, we expect a correlation between final orbital period and mass of the double degenerates since long period systems should have component masses close to $0.5 M_{\odot}$, while short period systems could have a range of masses resulting from differences in the mass of the secondary before common envelope. An investigation of the CE/CE evolutionary scenario indicates that it is a rare contributor to the double degenerate systems and could only be of importance to the short period population at best. The fact that only one of the observed systems has a mass ratio consistent with formation by the Algol/CE mechanism indicates that the evolutionary theory is incomplete or other formation mechanisms can operate. The possibility of the formation of double degenerates in a manner similar to that suggested for the evolution of Algol systems where mass and angular momentum are lost without invoking a common envelope phase (see Sarna et al. 1996) may be fruitful. Observational determinations of the masses and mass ratios for long period ($\gtrsim 1$ day) helium double degenerates would help in determining their origin in terms of the CE/Algol or Algol/CE evolutionary channels more securely.

4.2.3. *Subdwarf B Stars*

Systems containing subdwarf B (sdB) stars appear to share a number of common characteristics (e.g., orbital period) with both pre-cataclysmic variables and double degenerate stars. The current observational evidence indicates that sdB stars are core helium burning stars with masses of approximately $0.5 M_{\odot}$ (Saffer et al. 1994). This mass is sufficiently close to the core mass of low-mass giants at helium flash that we should consider formation mechanisms similar to those for systems involving helium degenerates.

Several studies (Allard et al. 1994; Theissen et al. 1995) have indicated that the fraction of sdB stars in binaries is fairly large, although there is still debate on the exact number. The companions in sdB binaries are inferred to be subgiants in many cases, although there are at least two short-period binaries known to have main sequence companions. The two systems, HW Vir (Wood et al. 1993) and PG 1336–018 (Kilkenny et al. 1998) are both eclipsing, have periods (2.8 h and 2.4 h, respectively) in the period gap for cataclysmic variables, and have mass ratios $q \sim 0.3$ if the sdB star has the canonical mass of $0.5 M_{\odot}$. Intermediate-mass giants with masses of $4.0 \pm 0.3 M_{\odot}$ also form hydrogen-exhausted cores that could become sdB stars. While it is possible to create helium burning stars of $0.5 M_{\odot}$ by stripping the envelope from an intermediate mass giant, it is very difficult to explain the abundance of sdB stars in the field because of the relatively small number of stars expected from the initial mass function (Saffer et al. 1994), and because sdB kinematics are indicative of old, low-mass disk stars (Saffer 1991). In addition, the ages of Galactic open (Liebert, Saffer, & Green 1994) and globular clusters (e.g. Moehler, Heber, & Rupprecht 1996) containing sdB stars rule out intermediate mass stars as their progenitors.

Low-mass stars provide a more natural way of creating helium burning stars of $0.5 M_{\odot}$, although it is difficult to understand how a star that evolves to the tip of the giant branch could

lose nearly all of its envelope mass, and yet still ignite helium in its degenerate core. If too much mass is lost, the envelope of the giant will collapse, the hydrogen burning shell will be extinguished, and the core of the star will not be sufficiently massive to ignite helium in a flash. On the other hand, if too little mass is lost, the giant will evolve through the helium flash to the core helium burning phase (where it would most likely be identified as a red clump star), and eventually to the asymptotic giant branch to become a carbon-oxygen white dwarf. Mengel, Norris, & Gross (1976) proposed that binary mass transfer (which they assumed to be conservative) can create an sdB star, although only for a small range of orbital separations.

The short periods of HW Vir and PG 1336–018 imply that a common envelope phase has indeed occurred. A common envelope event may appear to be incompatible with helium ignition since a giant immediately contracts after the flash, but D’Cruz et al. (1996) find that the giant’s remnant can ignite helium provided that the core mass lies within $\sim 0.018 M_{\odot}$ of the value at helium flash for normal evolution. The continuing gravitational contraction of the helium core results in an eventual flash, even if the star has reached the white dwarf cooling curve. Upon ignition, the star evolves to the helium burning main sequence. Although D’Cruz et al. (1996) examined stars undergoing enhanced mass loss, we suggest that common envelope evolution can create the same outcome with nearly complete ejection of the envelope. In this way, a giant could produce an sdB star provided that a common envelope phase occurs near the red giant tip. If so, an sdB star could be formed with masses as low as about $0.45 M_{\odot}$ for a metal-rich composition (D’Cruz et al 1996).

As is the case with the Mengel et al. mechanism, an important question is whether this process could produce a significant number of sdB systems. To try to answer this question, we have used a Monte Carlo method using assumptions about binary mass and separation given in a population synthesis study by Han (1998). We assumed a sdB binary system could be formed if the primary star had a mass less than $2.25 M_{\odot}$, the core mass of the primary was in a range of $0.018 M_{\odot}$ above $M_c = 0.45 M_{\odot}$. Using the core mass – radius relation for giants, the range in core masses corresponds to a 25% increase in radius. If the accretor can drive an isotropic stellar wind, then unstable mass transfer can only occur for initial mass ratios $q = M_1/M_2 > 1.2$ (Ritter 1999). In that case, we find that the birthrate is approximately 8 systems per 1000 years, which is more than half the birthrate of helium double degenerate systems (from values given in Han 1998). If there is not a stellar wind from the accretor, the birthrate increases to 10 systems per 1000 years. Selection effects would tend to make sdB systems appear to be more populous than double degenerates due to their $\sim 10^8 \text{yr}^{-1}$ lifetimes. This value appears to make this mechanism viable. Very precise mass measurements for the helium degenerates and sdB stars in short period systems and continued searches for binaries involving sdB stars can provide an important test of this formation mechanism.

We would like to acknowledge helpful conversations with A. Cool, E. Green, and R. Wade. We would also like to thank the anonymous referee for a number of useful comments. This work

has been supported by NSF grants AST-9415423 and AST-9727875.

REFERENCES

- Allard, F., Wesemael, F., Fontaine, G., Bergeron, P., & Lamontagne, R. 1994, *AJ*, 107, 1565
- Benvenuto, O. G. & Althaus, L. G. 1998, *MNRAS*, 293, 177
- Bergeron, P., Saffer, R. A., & Liebert, J. 1992, *ApJ*, 394, 228
- Bergeron, P., Wesemael, F., Liebert, J., & Fontaine, G. 1989, *ApJ*, 345, L91
- Bragaglia, A., Renzini, A., & Bergeron, P. 1995, *ApJ*, 443, 735
- Burkert, A. & Bodenheimer, P. 1993, *MNRAS*, 264, 798
- de Kool, M. 1990, *ApJ*, 358, 189
- D’Cruz, N., Dorman, B., Rood, R. T., & O’Connell, R. W. 1996, *ApJ*, 466, 359
- Edmonds, P. D., Grindlay, J. E., Cool, A. M., Cohn, H. M., Lugger, P. N., Bailyn, C. D. 1998, *ApJ*, 516, 250
- Eggleton, P. P. 1971, *MNRAS*, 151, 351
- Eggleton, P. P. 1972, *MNRAS*, 156, 361
- Eggleton, P. P. 1983, *ApJ*, 268, 368
- Ergma, E., Sarna, M. J., & Antipova, J. 1998, *MNRAS*, 300, 352
- Frasca, A., Marilli, E., & Catalano, S. 1998, *A&A*, 333, 205
- Han, Z. 1998, *MNRAS*, 296, 1019
- Harpaz, A. 1998, *ApJ*, 498, 293
- Iben, I. Jr. & Tutukov, A. V. 1984, *ApJS*, 54, 335
- Iben, I. Jr., Tutukov, A. V., & Yungelson, L. R. 1997, *ApJ*, 475, 291
- Igumenshchev, I. V., Chen, X., & Abramowicz, M. A. 1996, *MNRAS*, 278, 236
- Jeffery, C. S. & Simon, T. 1997, *MNRAS*, 286, 487
- Kilkenny, D., O’Donoghue, D., Koen, C., Lynas-Gray, A. E., & van Wyk, F. 1998, *MNRAS*, 296, 329
- Liebert, J., Saffer, R. A., & Green, E. M. 1994, *AJ*, 107, 1408
- Livio, M. & Soker, N. 1988, *ApJ*, 329, 764
- Marsh, T. R. 1995, *MNRAS*, 275, L1
- Marsh, T. R., Dhillon, V. S., & Duck, S. R. 1995, *MNRAS*, 275, 828
- Marsh, T. R. & Duck, S. R. 1996, *MNRAS*, 278, 565
- Maxted, P. F. L., Marsh, T. R., Moran, C., Dhillon, V. S., & Hilditch, R. W. 1998, *MNRAS*, 300, 1225.
- Mazzitelli, I., & D’Antona, F. 1986, *ApJ*, 311, 762

- Mengel, J. G., Norris, J., & Gross, P. G. 1976, *ApJ*, 204, 488
- Moehler, S., Heber, U., & Rupprecht, G. 1996, *A&A*, 319, 109
- Moran, C. 1999, Ph.D. Thesis, University of Southampton, in preparation
- Moran, C., Marsh, T. R., & Bragaglia, A. 1997, *MNRAS*, 288, 538
- Orosz, J. A., Wade, R. A., Harlow, J. J. B., Thorstensen, J. R., Taylor, C. J., & Eracleous, M. 1999, *AJ*, 117, 1598
- Paczynski, B. 1976, in *IAU Symposium No. 73, The Structure and Evolution of Close Binary Systems*, ed. P. Eggleton, S. Mitton, & J. Whelan, (Dordrecht: Reidel), 75
- Pols, O. R., Tout, C. A., Eggleton, P. P., & Han, Z. 1995, *MNRAS*, 274, 964.
- Pylyser, E. & Savonije, G. E. 1988, *A&A*, 191, 57
- Reimers, D. 1975, *Mem. Soc. R. Sci. Liege*, 6 ser., 8, 369
- Reimers, D. 1977, *A&A*, 57, 395
- Renzini, A. 1981, *Mass Loss and Stellar Evolution*, ed. C. Chiosi & R. Stalio (Dordrecht: Reidel), p. 319
- Ritter, H. 1999, *MNRAS*, 309, 360
- Rood, R. T. 1973, *ApJ*, 184, 815
- Saffer, R. A. 1991, Ph.D. Thesis, University of Arizona
- Saffer, R. A., Bergeron, P., Koester, D., & Liebert, J. 1994, *ApJ*, 432, 351
- Saffer, R. A., Liebert, J., & Olszewski, E. W. 1988, *ApJ*, 334, 947
502, 394
- Saffer, R. A., Wade, R. A., Liebert, J., Green, R. F., Sion, E. M., Bechtold, J., Foss, D., & Kidder, K. 1993, *AJ*, 105, 1945
- Sandquist, E. L., Taam, R. E., Chen, X., Bodenheimer, P., & Burkert, A. 1998, *ApJ*, 500, 909
- Sarna, M. J., Marks, P. B., & Smith, R. C. 1996, *MNRAS*, 279, 88
- Schmidt, G. D., Smith, P. S., Harvey, D. A., & Grauer, A. D. 1995, *AJ*, 110, 398
- Shara, M. M., Prialnik, D. & Kovetz, A. 1993, *ApJ*, 406, 220
- Tassoul, J.-L. 1978, *Theory of Rotating Stars* (Princeton: Princeton)
- Theissen, A., Moehler, S., Heber, U., Schmidt, J. H. K., & de Boer, K. S. 1995, *A&A*, 298, 577
- Tout, C. A., & Eggleton, P. P. 1988, *MNRAS*, 231, 823
- Tutukov, A. V. & Yungelson, L. R. 1988, *Sov. Ast. Lett.*, 14, 265
- Winget, D. E., Van Horn, H. M., Tassoul, M., Fontaine, G., Hansen, C. J., & Carroll, B. W. 1982, *ApJ*, 252, L65
- Wood, J. H., Zhang, E.-H., & Robinson, E. L. 1993, *MNRAS*, 261, 103

Woosley, S. E., Taam, R. E., & Weaver, T. A. 1986, ApJ, 301, 601

Fig. 1.— γ_{CE} as a function of stellar radius for the giants used in our common envelope simulations. The curves are labeled by simulation number.

Fig. 2.— yz density slices near the end of each of our simulations. Density contours are 5 per decade. The velocity fields in each panel have the same scale, with the maximum speed corresponding to about 60 km s^{-1} . The times pictured are: a) 70 d for simulation 1, b) 21 d for simulation 2, c) 900 d for simulation 3, d) 1053 d for simulation 4, e) 373 d for simulation 5, and f) 16 d for simulation 6.

Fig. 3.— The bound mass change ($-M_{bound}/\dot{M}_{bound}$) and orbital decay ($-a/\dot{a}$) timescales as a function of time for simulation 1.

Fig. 4.— Tracers of gas mass for simulation 1. The curve labels are as follows: M_{tot} , total mass; M_{bound} , total mass remaining bound (including bound mass lost off of the grid); $M_{3*orbit}$, the mass within a circular orbit three times the current binary separation; and M_{lost} , mass lost from the grid.

Fig. 5.— Tracers of gas mass for simulation 2. The curve labels are as follows: M_{tot} , total mass; M_{bound} , total mass remaining bound (including bound mass lost off of the grid); M_{vol} , mass in the original volume of the giant; $M_{3*orbit}$, the mass within a circular orbit three times the current binary separation; and $M_{lost,xy}$, $M_{lost,z}$, mass lost from the grid in the (cylindrical) radial direction and in the z -direction.

Fig. 6.— Tracers of gas mass for simulation 3. The curve labels are the same as in Figure 5, with the addition of $M_{R/10}$, the mass within a distance of 10% of the original giant’s radius of the core.

Fig. 7.— A snapshot of the yz -plane at 1000 days into simulation 3. The solid lines enclose areas that are unstable according to the Høiland criterion. The dotted lines are contours of specific entropy.

Fig. 8.— Tracers of gas mass for simulation 4. The curve labels are the same as in Figures 4 and 6.

Fig. 9.— The spatial variation of the mass interior to a radius R is illustrated for a $1 M_{\odot}$ star at different evolutionary phases at the base of the red giant branch. The solid, dashed dot, and dashed curves correspond to degenerate helium core masses of 0.19, 0.23, and 0.27 M_{\odot} respectively. Note that the range of radii for which the mass radius profile is flat increases with larger core masses. The more rapid increase in mass with radius ($\gtrsim 3 \times 10^9 \text{ cm}$) for a star with a core mass of 0.19 M_{\odot} hinders the successful ejection of the common envelope.

Fig. 10.— The bounding box for the parameter space available to binaries that can survive two common envelope phases without merging. The four panels are the edges of the space that is available to real binaries that can create helium degenerates: $M_1 = 2.25 M_{\odot}$, the maximum primary mass that has a degenerate helium core on the giant branch; $M_2 = 1.0 M_{\odot}$, the minimum mass for

a secondary that can evolve to the giant branch in the age of the disk of the Galaxy without having accreted mass; $M_1 = M_2$, the condition that the first evolving star must be at least as massive as the secondary; and $M_{1,R}$, the maximum mass for a helium degenerate core before helium flash. The panels are configured in such a way that the paper can be folded to create a wedge-shaped box with the axes being $(M_1, M_2, M_{1,R})$, and the letters a, b, and c showing points of attachment. The condition that the secondary must reach a minimum core mass $M_{2,R}$ for the binary to survive the second common envelope phase is represented by solid lines showing cuts through the boundaries of parameter space. The heavy solid line assumes $M_{2,R} = 0.25 M_\odot$, while the thin solid line assumes $M_{2,R} = 0.20 M_\odot$. Binaries with parameters which fall above the solid lines would survive. The dashed line represents a mass ratio $q = 1$, while the dotted line shows where the $q = 1.2$ condition cuts the volume. If $q > 1.2$ is necessary for unstable mass transfer (and hence common envelope), only binaries with parameters to the right of the dotted line would survive.

Table 1. Orbital Parameters for Helium White Dwarf Binary Systems

Name	P (d)	M_1 (M_\odot)	M_2 (M_\odot)	q^a	T_1 (K)	T_2 (K)	References ^c	
Double Helium Degenerate Systems								
0135–052	L870-2	1.56	0.47 ± 0.05^b	0.52 ± 0.05^b	0.86 ± 0.02	7500	6900	1
0136+768		1.41	0.34	0.26	1.31			2
0957–666	L101-26	0.06	0.37 ± 0.02	0.32 ± 0.03	1.15 ± 0.10	27000		3, 4
1101+364		0.15	0.27	0.31	0.87 ± 0.03	13600	13600	5, 6
Pre-Cataclysmic Variable Systems								
0308+096		0.29	$0.39^{+0.13}_{-0.10}$	0.18 ± 0.05		26200		7
0710+741	GD448	0.10	0.44 ± 0.03	0.09 ± 0.01	0.22 ± 0.03	19000		8
1224+309		0.26	0.45 ± 0.05	0.28 ± 0.05	0.62 ± 0.08	29300		9
2256+249	GD245	0.17	0.48 ± 0.03	0.22 ± 0.02	0.47 ± 0.03	22170		10

^aFor the double degenerates, the mass ratio q is defined to be the ratio of the mass of the brighter to the mass of the fainter component. For the pre-CV systems, it is defined to be the mass of the (less massive and fainter) main-sequence companion to that of the white dwarf.

^bIt is still possible that L870-2 may be a double degenerate system composed of carbon-oxygen white dwarfs.

^c1. Bergeron et al. 1989 2. Moran 1999 3. Moran et al. 1997 4. Bragaglia et al. 1995 5. Marsh 1995 6. Bergeron et al. 1992 7. Saffer et al. 1993 8. Maxted et al. 1998 9. Orosz et al. 1999 10. Schmidt et al. 1995

Table 2. Initial Parameters of Common Envelope Sequences

Sequence	M_1 (M_\odot)	M_c (M_\odot)	M_2 (M_\odot)	a_i (10^{12} cm)	P_i (d)
1	2	0.355	0.2	4.0	34.1
2	1	0.28	0.35	2.0	15.4
3	1	0.45	0.35	22.0	560.7
4	1	0.45	0.1	20.0	989.2
5	2	0.45	0.35	16.0	263.6
6	1	0.28	0.45	1.6	12.8

Table 3. Final Parameters of Common Envelope Sequences

Sequence	a_f (R_\odot)	P_f (days)	α_{CE}^a	$M_{bound,f} / M_\odot$	$\alpha_{CE} = 0.4$		$\alpha_{CE} = 1.0$	
					$a_{f,c}$ (R_\odot) ^b	$P_{f,c}$ (days)	$a_{f,c}$ (R_\odot) ^b	$P_{f,c}$ (days)
1	2.1	0.48	0.26	1.59	0.3	0.02	0.6	0.06
2	1.8	0.35	0.21	0.65	0.8	0.10	1.2	0.18
3	21.3	12.7	0.09	0.49	14.9	7.5	18.2	10.0
4	33.2	29.8	0.14	0.53	3.0	0.79	6.5	2.60
5	18.9	10.6	0.56	1.45	2.0	0.37	4.3	1.16
6	2.4	0.49	0.34	0.62	0.8	0.10	1.3	0.21

^a α_{CE} as tabulated here is the efficiency of mass ejection as computed from the simulation when the run was terminated.

^b $a_{f,c}$ is an estimate of the final orbital separation assuming that orbital energy is removed to unbind the remainder of the gas with the efficiency α_{CE} given.