# Optimal Machine Stopping Time and Ordering Cycle for Parts to Minimize the Total Cost of a Supply Chain

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**ABSTRACT** Many parts or components in a facility are sometime vulnerable to failure causing downtime and production loss. So, in order to decrease the downtime, parts of the same type that follow the same lifespan distribution need to be replaced at least once to minimize the cost of unscheduled labor and the damages to the products. A decision on replacing all such vulnerable parts decreases the total cost including penalty cost, purchasing cost and holding cost. Different preempt times of a machine result in different purchasing quantity, procurement cost and carrying costs. So, a rational decision of allowable stopping time of a machine and order cycle for vulnerable parts jointly minimizes the total cost of supply chain. This study proposes a novel model to minimize the total cost for the vulnerable parts which have joint influence on machine performance. An integrated nonlinear cost model developed here is optimally solved and tested with several different distributions. Strong results are propounded to support the proposed model in terms of manufacturing time and system cost.

**INDEX TERMS** Tool life, condition monitoring, preventive maintenance, allowable stopping time, procurement, operations planning and probabilistic models.

## I. INTRODUCTION

Nowadays, with the improvement of people's expectation of products and services, the market competition is more and more fierce. To enhance profits and reduce costs of enterprises, it is very necessary for companies to improve supply chain operational efficiency by various of methods. Although many effective supply chain strategies and models have been successfully applied in practice, there is still a need to develop new models or improve classical models to respond to the changing supply chain environment, considering some relevant trade-offs such as costs and benefits (Lina and Wang, 2011; Volling, *et al.* 2013; Yao and Askin, 2019) [1-3].

For a machine or equipment, there are many same type parts which need to be replaced over a period. These same type parts that need to be replaced regularly are called vulnerable parts. Failure of vulnerable parts could sometimes result in huge operating cost which could have be avoided by reasonable method. Therefore, it is necessary to explore this scenario and build appropriate model to make more accurate decision on how to choose the more suitable replacement time for the vulnerable parts of a machine to avoid loss. In general, vulnerable parts in a machine are purchased according to predetermined schedules. Based on different stopping times, we explore the impact of order cycle and order size of vulnerable parts on enterprise profitability and operating costs. In this study, we established a novel cost estimation model (CEM) where the jointly distributed lifespan of vulnerable parts which have same lifespan distribution function are introduced. According to the proposed CEM model, the optimal replacement time and optimal order cycle of the vulnerable can be determined and then total enterprise cost is minimized. There is still a lot of research on procurement and replacement strategy for vulnerable parts. But the existing scenarios are most often not in existence—failure of a part in operational mode has an impact on the parts.

### A. DISTRIBUTION OF LIFESPAN

Numerous researches have been done concentrating on the lifespan distribution of diverse parts, or products. The study of Moonseong *et al.* (1998) showed that power and the required sample size are dependent on the shape parameter and indicated power and sample size calculations under the Weibull (a typical continuous exponential distribution function with random variables) distribution [4]. To make the product reach its ideal lifespan, Chalkley *et al.* (2003) found that it is possible to calculate the point in the life of the product when replacement is most beneficial from information about new and existing machines [5]. Lamond and Sodhi (2006) investigated the processing time on a flexible machine with random tool lives [6]. Khan *et al.* (2018) argued that reduced product lifespan has a significant impact toward, both the environment and the economy and explored upgradability potential as a product lifespan extension strategy [7]. Combined with green and smart manufacturing, in order to solve production scheduling problems on CNC-machines having a set of cutting tools in a Flexible Manufacturing System (FMS), Setiawan *et al.* (2019) considered lifespan of cutting tool to maximize the cutting tools utilization [8]. De, Aghezzaf and Desmet (2019) studied two modelling approaches to the multi-echelon inventory optimization problem in a distribution network with stochastic demands and lead times [9]. Recently, Uzunoglu and Yalcin (2020) examined a continuous review inventory model for perishable items with two demand classes and demands for both classes occur according to Poisson process [10].

These contributions gave different examples on lifespan distribution in many manufacturing industries and provide some foundation for this paper. However, most papers discussed only one or several distributions separately and aimed at only one machine. This research studies the impact of lifespan joint distribution on part procurement policy.

### B. LIFESPAN ESTIMATION

In the area of lifespan estimation, Masahiro *et al.* (2010) reviewed and categorized different types of lifespan distribution and methodologies for estimating the lifespan distribution of commodities and also examined the differences in actual lifespan between variant types of distribution, definition and methodology by comparing reported data [11]. To estimate the lifespan of agricultural tractor which can be quite useful for the market predictions, Munoz and Llanos (2012) established a diffusion aggregate adoption model by using a nonlinear estimation procedure [12]. Later Firoozi and Ariafar (2017) developed a model for network design of perishable items and proposed a Lagrangian relaxation-based heuristic algorithm to solve the model [13]. Then study of Li *et al.* (2019) reviewed data-driven battery health estimation methods and discussed these in view of their feasibility and cost-effectiveness in dealing with battery health in real-world applications [14]. From two different perspectives, Gu and Chen (2019) improved the similarity-based residual life prediction methods, which is an emerging technique and occupies a significant place in remaining useful life (RUL) prediction [15]. To better predict the lifespan of LEDs mounted on thermoplastic substrates, the so-called molded interconnect devices, Soltani *et al.* (2019) presented a novel approach for reliability investigation and lifespan estimation, based on simulation [16]. Later Huang *et al.* (2020) used the Weibull distribution model to calculate the lifetime of products based on survey data collected from selected formal recycling plants in various regions in China [17]. Later Wu *et al.* (2021) modeled a cellular network with repairable facilities characterized by a two-parameter Weibull distribution to minimize the expected life cycle cost of the cellular network [18].

Considering the maximum stopping time of a machine is jointly determined by multiple vulnerable parts, this paper proposes a novel lifespan estimation method and a cost assessment model (CEM) to get an optimal strategy through algorithm, after that it got the best lifespan for the part.

### C. TOTAL COST ASSESSMENT WITH THE CONSIDERATION OF LIFESPAN

In term of cost assessment study considering lifespan of parts or products, Weustink *et al.* (2000) developed a generic framework to control the product costs, to estimate the costs adequately and to store the cost data in a more generic way which would set a good example for the estimating of the total product cost [19]. Since quick and accurate information is extremely valuable to designers and manufacturing engineers. Then by considering multiple machine replacements under discounted costs, Safaei and Zuashkiani (2012) made progress on manufacturing system design [20]. According to an inventory control problem of aircraft spare parts during the end-of-life (EOL) phase of fleet operations, Hur, Keskin and Schmidt (2018) presented an algorithm that computes the optimal final order size of components under a budget constraint [21]. Ketzenberg, Gaukler and Salin (2018) addressed the problem of how to set expiration dates for perishable products to balance hazard costs and perishing costs in the context of a retailer that sells a random lifespan product under periodic review [22]. In order to select a final design representing the best trade-off between safety and economy, in a life-cycle perspective, Venanzi *et al.* (2019) proposed an automated procedure for the estimation of life-cycle repair costs of different bridge design solutions [23]. An inventory model has been developed by Kundu *et al.* (2019) under two levels of trade credit policy with customers' default risk consideration for a deteriorating item having a maximum lifespan [24]. By modeling the manufacturer-retailer relationship as a Stackelberg game where the retailer is the leader and decides the replenishment cycle that minimizes its mismatch cost between supply and uncertain demand. For a transition towards a circular built environment, Jansen *et al.* (2020) developed an economic assessment model in the form of a Circular Economy Life Cycle Cost (CE-LCC), which is based on existing Life Cycle Cost techniques and adapted to the requirements of CE products [25]. To optimize real-time maintenance decisions dynamically in a serial-parallel structure of a manufacturing system, Xia *et al.* (2021) proposed a capacity balancing-oriented leasing profit optimization (CB-LPO) policy by considering the constraints of the capacity balancing [26].

Different from the above studies, the stopping time of a part is taken into consideration and integrated into the total cost model which focuses on the vulnerable parts of a machine.

### D. SUPPLY CHAIN UNDER JOINTLY DISTRIBUTED PRODUCTION

Blackhurst *et al.* (2005) developed a decision support modeling methodology for supply chain, product and process design decisions, which integrate the production process into a supply chain system, but they rarely considered the impact of product lifespan on supply chain cost [27]. In order to maintenance spare parts planning and

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control, a framework on how to plan and control a spare parts supply chain had been provided (Cavalieri *et al.*, 2008) [28]. Later a cold standby repairable system consisting of two dissimilar components and one repairman is studied by Wang and Zhang (2016) [29]. In this research, they proposed a replacement policy for a two-dissimilar-component cold standby system with different repair actions. Then Khan *et al.* (2019) proposed two integrated models to incorporate quintessential and omnipresent supply chain practices as repairing or replacing non-conforming items supplied by the vendor [30]. Taking production and cycle time as decision variables, Iqbal and Sarkar (2019) design a forward and reverse supply chain system that produces two different types of products, which are subject to deterioration [31]. Bacchetti *et al.* (2020) formulated a mixed integer linear programming model to ensure an optimal replenishment of the regional warehouses and an optimal choice of the distribution strategies in a supply chain composed of set of production plants and a set of regional warehouses [32]. Later Shen, Hu and Ma (2020) studied systems with a critical subsystem and a protective auxiliary subsystem subject to degradation and economic dependence. Based on such systems, they presented two preventive replacement models and discussed their application conditions [33].

Previous research on supply chain management focused more on optimizing the management process. But an impact of product lifespan assessment on total cost was mentioned in few studies. This study regards the lifespan of vulnerable parts as a part of the CEM, which makes the cost evaluation more reasonable.

### *E. STATUS OF THE PREVIOUS RELATED RESEARCH, VULNERABLE PARTS AND THE PRESENT STUDY*

In many scenarios of the manufacturing system, a machine or equipment usually may have many vulnerable parts that play important roles in the regular running process. The machine with these parts has also uptime/downtime distribution that is also dependent on the part failure and other affecting parameters that control the machine performance. In order to decrease the machine down-time, all parts of the same type that follow the same lifespan distribution, are changed almost simultaneously at a time, resulting in minimization of the cost of unscheduled labor and damages to the product in case of random failure. For example, assume a test bench for testing flow leak that has a few seals made of silicone; if one of these silicone seals fails, the test bench leaks, and it will not pass the test. So, a reasonable decision on the preemption of all the vulnerable parts (silicone seal, in this case) and their replacement decreases the total cost of failure penalty, purchasing, and holding of parts. At the same time, the different preempt stopping times of a machine result in the requirements of variant quantity of purchasing parts resulting in different procurement and carrying costs. Consequently, a scientific decision of allowable downtime of a machine and ordering cycle for replenishing vulnerable parts jointly minimizes the total cost of penalty, purchasing and holding costs. Thus, this study proposes a novel model to minimize the total cost (holding, penalty, purchasing and fixed replenishing cost) for the vulnerable parts which have joint influence on machine performance. So, it is a tradeoff situation on letting the parts to fail or when to stop the machine to change the parts instead of parts to fail—which is the main issue of the current research directed to an economic decision-making process.

Based on determining appropriate inspection intervals and a maintenance threshold, Ahmadi (2019) proposed a new approach to minimize the long-run average maintenance cost per unit time [34]. Later Seif *et al.* (2020) developed a mixed-integer linear programming model for optimally allocating maintenance items to campaigns so that total shutdown cost is minimized. The model incorporates constraints on maintenance deadlines, campaign times, maintenance item suppression and labor hours per campaign [35]. In order to solve parts procurement and inventory management problems during the product life cycle after obsolescence, Shi and Liu (2020) formulated an optimal stopping model and established the optimality of a threshold policy for the design refresh choice [36]. According to analyzing status data collected from the sensors, Chien and Chen (2020) developed a data-driven framework to prolong the maintenance cycles for enhancing capacity utilization and productivity, and thus reduce the cost [37].

In this study, we explore the influence of the joint life distribution of multiple vulnerable parts on the maintenance interval distribution of machine. Then, through the relatively dependent maintenance interval distribution, an appropriate maintenance interval is found, and the corresponding procurement strategy is determined. In other words, according to the research of the predetermined distribution of multiple vulnerable parts, this manuscript established a procurement cost evaluation model to minimize the total cost.

## II. OPTIMAL COST ESTIMATION MODELING

Since many vulnerable parts for a machine are usually expensive and essential to the regular running of the machine, rationally allowing preemption for their usage and logically justifiable procurement have significant importance in manufacturing industry. In this study, a cost evaluation model which considers a variety of operating costs of enterprises is established and it is optimized for determining the optimal stopping usage time and procurement policy of parts.

### *A. ASSUMPTIONS AND NOTATIONS*

In the process of building the CEM model, there are some very complex derivations involved. Given the rigor of the research, some necessary assumptions are considered and listed as below.

***Assumptions:***

1. The machine is deemed to work on a model of mass production which means a continuous requirement of the vulnerable parts with insignificant variation in demand quantity during working time.
2. Runtime of each vulnerable part of the machine is independent of another part, that is, the lifespan of one vulnerable part has no impact on the same type of other parts.

3. If one of the specific vulnerable parts is damaged, the machine will shut down because of the irreplaceable role of these parts.
4. Only one type of vulnerable part in a single machine is considered, and there exists more than one of the same vulnerable parts in a machine, or equipment.
5. A failure of one vulnerable part may cause potential damage to a work piece, thus resulting in a penalty cost.
6. Procurement lead time is not listed and discussed as a variable. The procurement lead time is constant and included in the purchasing cycle. In order to highlight the focus of the study, the lead time is not described respectively.

***Notations and definition:***
For better understanding of all definitions of parameters, variables and components, descriptive definitions are provided here:
*(a) Parameters*

$c_f$ : Fixed cost per order ($/order or cycle),

$c_h$ : Unit holding cost ($/part/ year),

$c_u$ : Unit purchase (variable) cost ($/part),

$c_p$ : Unit penalty cost of part failure ($/part),

$T_w$ : Total working time for a machine per year (time-units/year).

*(b) Intermediate variables*

$C_f$ : Fixed cost of order replenishment per year ($/year),

$C_h$ : Annual holding cost ($),

$C_{pc}$ : Annual purchasing cost of parts ($),

$C_{pt}$ : Annual penalty cost ($),

$D_p$ : Demand of vulnerable parts required per year,

$Q$ : Order size of parts (parts/order),

$T$ : Lifespan of vulnerable parts (in time-unit),

$T_M$ : Working time for a machine (in time-unit),

$TC$ : Total cost ($/year).

*(c) Decision variables*

$T_c$ : Interval between two consecutive orders (year/cycle or order),

$T_M^{\max}$ : Maximum allowable working time of the machine (in time-unit).

*(d) Measure of Performance*

$TC$ : Total cost ($/year).

### B. PROBLEM DEFINITION

In the manufacturing industry, machines are important basis for processing or detecting and play an indispensable role in the operation of an enterprise. In general, this machine is not a separate part but a combination of many parts where there are often several vulnerable key parts performing the same function. The illustration of a sample machine is shown in Figure 1. For example, there are three vulnerable parts, named as *Part 2*, lie in different levels of the machine, and their lifespans may differ depending on the usage and operations at different levels and locations in the machine.

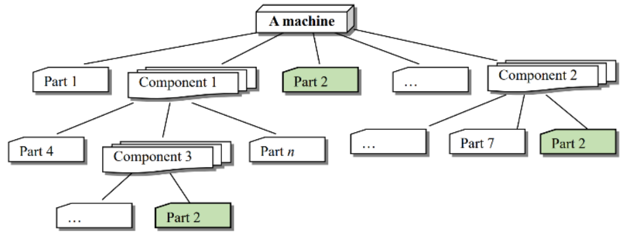


**FIGURE 1.** **Illustration of vulnerable parts in a machine**

It is assumed that the lifespan of the vulnerable parts, $T$, follows a general distribution, $T \sim f(t)$. If a vulnerable part is damaged, it must be replaced immediately. Otherwise, it will result in a penalty of $c_p$ for the damage caused by the failed part to the product. On the one hand, if potentially damaged parts in probability are replaced before failure, there will be no additional penalty cost. On the other hand, replacing parts too early will cause waste of resource, which means additional purchase and maintenance cost. Thus, that is a kind of tradeoff between usage time and inventory procurement due to early preemption of the vulnerable parts.

For the stated condition, variable cost of every part is $c_u$ dollars. The annual maintenance cost of a part is $c_h$ dollars. In addition, every order incurs a fixed expense, symbolized as $c_f$ dollars per order. According to optimizing the machine maximum allowable time ($T_M^{\max}$) and ordering cycle ($T_c$), minimization of the total cost ($TC$) is the ultimate goal of the study where $T_M^{\max}$ and $T_c$ are the two decision variables that will eventually control the part's optimal usage time ($T$) and the order quantity ($Q$).

### C. MACHINE LIFESPAN EXPECTATION CONSIDERING VULNERABLE PARTS

Although a few vulnerable parts may be different from each other because of individual minor differences, they may be assumed to be the same type. As indicated earlier, their lifespans may differ depending on the usage, performed operations, and their locations at different levels in the machine; so, it may be fair to assume that the parts follow a generic distribution $T \sim f(t)$. Under this circumstance, the maximum working time of the machine depends on the shortest lifespan of all the vulnerable parts. The relationship between the working time of a machine and its vulnerable parts is illustrated in Figure 2. The vulnerable part with the shortest life is the bottleneck of the machine, just like well-known buckets effect.

The total number of vulnerable parts in a machine is expressed by $n$ and $T_i$ ($i = 1,2,\ldots n$) representing the life of vulnerable part $i$ if they were allowed to be used until

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failure. So, the maximum working time of the machine, $T_M$ can be expressed easily as

$$T_M = \min\{T_i, i = 1,2,\ldots n\}, \qquad (1)$$

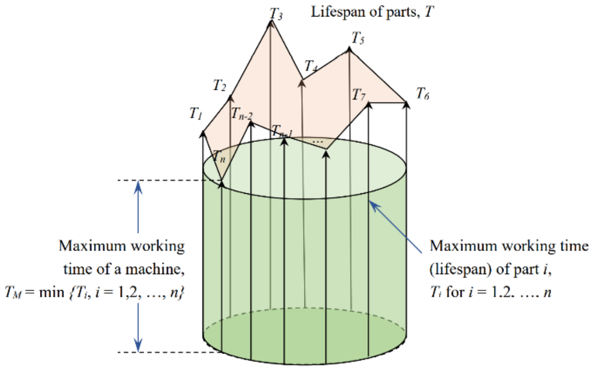


**FIGURE 2. Relationship between machine working time and vulnerable part's lifespan**

Since under this assumption of cask theory, each part's lifespan follows the same identical distribution $T_i \sim f(t)$. One machine or equipment includes *n* vulnerable parts. As shown in Equation (1), the distribution of the machine working time follows a distribution that is jointly dependent on all these *n* vulnerable parts. Maximum working time of the machine under each setup, $T_M$, is also a random variable. Thus, the probability density function (PDF) of the maximum working time $T_M$ for a machine is defined as $g(t)$, that is, $T_M \sim g(t)$ which needs to be evaluated. The corresponding cumulative distribution function (CDF) is defined as $G(T_M^{\max})$ where $T_M^{\max}$ is the maximum allowable working time of the machine while *n* parts were loaded. Since the lifespan of each part of the machine is independent variable, by using set theory, $g(t)$, the probability density function of the machine uptime can be deduced.

Since $T_M = \min\{T_i, i = 1,2,\ldots,n\}$, the cumulative probability, $G(T_M^{\max})$, can be written as $G(T_M^{\max}) = P\{\min(T_i, i = 1,2,\ldots,n) < T_M^{\max}\}$, where $T_M^{\max}$ indicates the maximum allowable working time (uptime) for a machine . This means that the parts which do not fail by time $T_M^{\max}$ are preemptively withdrawn/unloaded from the machine to safeguard the damage by the parts or due to its failure. Based on the above analysis, the derivation of $G(T_M^{\max})$ as shown in Figure 3 (also see Appendix A.5) is described as

$$G(T_M^{\max}) = 1 - [1 - F(T_M^{\max})]^n \qquad (2)$$

Where, $F(T_M^{\max}) = P\{T_M < T_M^{\max}\} = P\{\min(T) < T_M^{\max}\} = P\{\min(T_i, i = 1,2,\ldots,n) < T_M^{\max}\}$.

Taking derivative of equation (2) with respect to time *t*, the probability density function of maximum working time for a machine (i.e., uptime), $g(t)$, is given by

$$g(t) = n\left[1 - F(T_M^{\max})\right]^{n-1} f(t). \qquad (3)$$

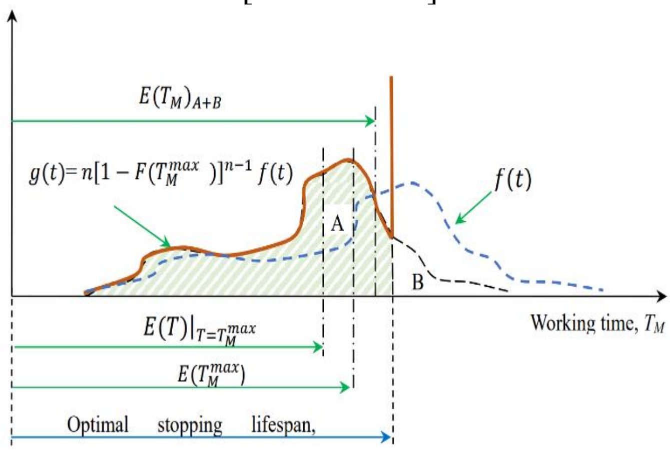


**FIGURE 3. Expected effective working time for a machine**

Under the distribution described in equation (3), if the stopping time for changing parts in a machine is assumed to be $T = T_M^{max}$, there is also some probability of machine failure *before* the stopping point, $T_M^{max}$, due to the parts whose lifespan is $T < T_M^{max}$, and also a probability of regular running at the stopping time $T = T_M^{max}$, where the cumulative distribution function is $F(T_M^{max}) = \int_{-\infty}^{T_M^{max}} f(t)dt$. As illustrated in Figure 3, the mathematical expectation of maintenance interval expectation for one normal machine includes a couple of areas, *A* (non-preemptive failure) and *B* (preemptive failure).

*(a) Non-preemptive failure*

For the area A enveloped by shaded line in Figure 3, which means the region $T < T_M^{max}$, the expected effective working time in the range $T < T_M^{max}$, can be expressed as

$$E(T)|_{t=T_M^{max}} = \int_0^{T_M^{max}} t g(t) dt \qquad (4)$$

Replacing $g(t)$ in equation (4) with equation (3), the effective expected working time in area *A* is given by

$$E(T)|_{t=T_M^{max}} = \int_0^{T_M^{max}} t\left\{n\left[1 - F(T_M^{\max})\right]^{n-1} f(t)\right\} dt \qquad (5)$$

*(b) Preemptive failure*

For the region B in figure 3 [white area under $g(t)$], which means the region $T > T_M^{max}$, the mathematical expectation of maintenance interval of the machine exceeds the maximum allowable usage time and its actual working time extends to the time $T_M$. Thus, the value of expectation time for the preempted region (i.e., B) can be calculated and expressed as formula (6).

$$E(T_M^{max}) = T_M^{\max} \int_{T_M^{\max}}^{\infty} g(t)dt = T_M^{\max}\left[1-G(T_M^{\max})\right] \quad (6)$$

where $g(t)$ follows a distribution described in equation (3).

Upon combining equation (6) with equation (2), with the portion marked with *B* area, of which expectation time can be calculated and expressed as formula (7).

$$E(T_M^{\max}) = T_M^{\max}[1-F(T_M^{\max})]^n . \quad (7)$$

Because the probability distribution of machine working time consists of a couple of parts, area A and B in figure 3, the two parts of the probability distribution of working time have different mathematical expectations. So, the total mathematical expectation of machine working time ( $T_M < T_M^{\max}$ ) should be expressed as

$$E(T_M)_{A+B} = E(T)\big|_{T=T_M^{\max}} + E(T_M^{\max}) \quad (8)$$

In order to have an insight of the properties with the jointly distributed maximum working time of a machine, Equation (8) also reveals several attributes that are expressed in the following theorems:

***Theorem 1***: $E(T_M)_{A+B} \le E(T_M)$.

**Proof:** From the definition of expectation for distribution $E(T_M) = \int_{-\infty}^{T_M} tg(t)dt + \int_{T_M}^{\infty} tg(t)dt$. Equations (5), (6) and (7) yields $E(T_M)_{A+B}$ = $E(T)\big|_{T=T_M^{\max}} + E(T_M^{\max})$ = $\int_{-\infty}^{T_M^{\max}} tg(t)dt + \int_{T_M^{\max}}^{\infty} T_M^{\max} g(t)dt$.

Let $\Theta = E(T_M) - E(T_M)_{A+B}$.

Thus, $\Theta$ = $\int_{-\infty}^{T_M^{\max}} tg(t)dt + \int_{T_M^{\max}}^{\infty} tg(t)dt - \int_{-\infty}^{T_M^{\max}} tg(t)dt - \int_{T_M^{\max}}^{\infty} T_M^{\max} g(t)dt$ $=\int_{T_M^{max}}^{\infty} tf(t)dt - \int_{T_M^{max}}^{\infty} T_M^{max} f(t)dt$.

Since $T_M^{max} \le t \le \infty$, so $\int_{T_M^{max}}^{\infty} tf(t)dt - \int_{T_M^{max}}^{\infty} T_M^{max} f(t)dt \ge 0$, because $T_M^{max} < t$. Thus, $\Theta = E(T_M) - E(T_M)|_{T=T_M^{max}} - E(T_M^{max}) \ge 0$, and which leads to the conclusion, $E(T_M) \ge$ $E(T)\big|_{T=T_M^{\max}} + E(T_M^{\max})$.

***Corollary 1.1***: As $T_M^{\max} \to \infty$, $E(T_M)_{A+B} \to E(T_M)$.

**Proof:** $\lim_{T_M^{\max}\to\infty} \left[E(T)\big|_{T=T_M^{\max}} + E(T_M^{\max})\right]$ = $\lim_{T_M^{\max}\to\infty} \int_{-\infty}^{T_M^{\max}} tg(t)dt + \lim_{T_M^{\max}\to\infty}\left[T_M^{\max}\int_{T_M^{\max}}^{\infty} g(t)dt\right]$.

Since $\lim_{T_M^{\max}\to\infty} \int_{-\infty}^{T_M^{\max}} tg(t)dt \to \int_{-\infty}^{\infty} tg(t)dt = E(T_M)$ and $\lim_{T_M^{\max}\to\infty}\left[T_M^{\max}\int_{T_M^{\max}}^{\infty} g(t)dt\right] = 0$, so $E(T)\big|_{T=T_m} + E(T_M) \to E(T_M)$. From equation (8), Thus, $E(T_M)_{A+B} \to E(T_M)$, for $T_M^{\max} \to \infty$.

Corollary 1 shows that if there is no preemption for any of the vulnerable parts, the population in group *B* will be less and the expected maximum working time of the machine will tend to be the same with the expectation of the machine without preemption. It also indicates that the maximum expectation of the preemption with the machine is always less than the expectation of the population without preemption.

***Theorem 2***: $E(T)|_{T\sim g(t)} \le E(T)|_{T\sim f(t)}$.

**Proof:** Assume there are $k$ groups of vulnerable parts, and there are $n$ parts needed for the changing each time with a machine (or a group). $E_i(T)$ is defined to be the expectation of group $i$, where $i = 1,2,\ldots,k$. Since $T_M^i = \min\,(T_{Mi}^j)$, where $T_M^i$ represents the value of the $i$ th changing of parts, $T_{Mi}^j$ represents the lifespan of the $j$th part in the $i$ th group, where $j = 1,2,\ldots,n$. So, $T_M^i \le \sum_{j=1}^{n} T_{Mi}^j \Big/ n$. For all the groups, $i = 1,2,\ldots,k$, $\sum_{i=1}^{k} T_M^i \Big/ k \le \sum_{i=1} k \sum_{j=1}^{n} T_{Mi}^j \Big/ (nk)$ where $\sum_{i=1}^{k} T_M^i \Big/ k$ is approximately equals to $E(T)|_{T\sim g(t)}$, and $\sum_{i=1} k \sum_{j=1}^{n} T_{Mi}^j \Big/ (nk)$ approximately tends to be $E(T)|_{T\sim f(t)}$. As a result, $E(T)|_{T\sim g(t)} \le E(T)|_{T\sim f(t)}$.

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***Corollary 2.1***: $E(T_M)_{A+B} \leq E(T)|_{T\sim f(t)}$.

**Proof:** Theorem 1 indicates that $E(T_M)_{A+B} \leq E(T)|_{T\sim g(t)}$, and Theorem 2 gives the relation of $E(T)|_{T\sim g(t)} \leq E(T)|_{T\sim f(t)}$. Combining these two theorems, the corollary is given by $E(T_M)_{A+B} \leq E(T)|_{T\sim f(t)}$.

Corollary 2.1 indicates that the expectation of the maximum working time for a machine is usually less than the expected lifespan of the vulnerable parts. As the number of vulnerable parts installed in one machine increases, the expected maximum working time will decrease.

### D. MINIMUM TOTAL COST

In this study, for a machine or equipment, working time in a year can be predicted and determined in advance. Because there have been many related prediction studies, this study takes a fixed value of working time in a year. When $T_M^{\max}$ is given by equation (8), $E(T_M)_{A+B}$, the expected allowable working time period can be computed easily. In order to finish the plan of yearly working time, the vulnerable parts are ordered in a quantity of $Q$ parts/cycle every fixed-period, $T_c$. The total cost includes a fixed replenishment order cost $C_f$, a holding cost $C_h$, a purchasing cost $C_{pc}$ and the penalty cost $C_{pt}$. Thus, within a procurement cycle $TC$ can be calculated and expressed as

$$TC = C_f + C_h + C_{pc} + C_{pt} \tag{9}$$

Let the total working time of a single machine in a year be $T_W$. Then, as defined in equation (8), the expected continuous working time without parts changing time is $E(T)_{A+B}$. Let the number of vulnerable parts to be changed every time be $n$ because there are totally $n$ same type of parts in a machine. So, the total demand of vulnerable parts $D_p$ is

$$D_p = \frac{nT_w}{E(T_M)_{A+B}} \tag{10}$$

The ordering cycle time, $T_c$ (year/cycle) is a decision variable, and $c_f$ is the purchasing cost each cycle, so the fixed purchasing cost in a year, $C_f$, can be calculated by

$$C_f = \frac{c_f}{T_c}. \tag{11}$$

The annual holding cost, $C_h$ is composed of unit holding cost $c_h$ multiplied by average inventory that is expressed as $\bar{I} = Q/2 = D_p T_c / 2$. So, replacing $D_p$ with equation (10), the annual total holding cost $C_h$ is given by

$$C_h = \frac{nT_w c_h T_c}{2E(T_M)_{A+B}}. \tag{12}$$

Using equation (10) and the unit purchase price $c_u$, the annual parts purchasing cost, $C_{pc}$ can be computed

$$C_{pc} = \frac{nT_w c_u}{E(T_M)_{A+B}}. \tag{13}$$

Yearly penalty cost is reflection of the failure of a machine within one year. The cumulative probability of the machine failure $G(T_M^{\max})$ is for the stopping time, $T_M^{\max}$, and using $G(T_M^{\max})$ =1- $[1-F(T_M^{\max})]^n$ from equation (2), the annual total penalty cost, $C_{pt} = c_p D_p G(T_M^{\max})$, combining with equation (10), yields

$$C_{pt} = \frac{nT_w c_p\{1-[1-F(T_M^{\max})]^n\}}{E(T_M)_{A+B}} \tag{14}$$

(14)

Therefore, the total cost, $TC$, on combining equations (9), (11), (12), (13) and (14), can be written as

$$TC(T_M^{\max}, T_c) = \frac{c_f}{T_c} + \frac{nT_w c_h T_c / 2 + nT_w c_u + nT_w c_p\{1-[1-F(T_M^{\max})]^n\}}{E(T_M)_{A+B}} \tag{15}$$

According to the above formula, the total cost can be expressed as the sum of two functions. One is a function of the $T_c$, (purchasing cycle), and the second is a function which involves holding, purchasing, penalty costs and the maximum maintenance intervals expectation of the machine, $E(T_M)_{A+B}$. The above two functions could make it more notable about the solution structure and functional behavior of the cost function.

## III. OPTIMIZATION OF THE COST ESTIMATION FUNCTION

According to equation (15), obtaining the solution to this problem is difficult because of the involvement of the multi-facet probabilistic functions in evaluating the functional value, so finding a closed-form solution to this problem is apparently impossible. Thus, a search procedure is employed to identify the stationary point(s) within reasonable solution boundaries. To find the stationary point(s) of the function, a differentiation method is now employed to ease the computational scheme in this study.

### A. PROBLEM DEFINITION

In order to locate the stationary points for the objective function expressed in equation (15), partial differentiation on both $T_c$ and $T_M^{\max}$ are required to identify the solution. Thus, by differentiating the expression in (15) with respect to $T_c$, $\partial TC(T_M^{\max}, T_c)/\partial T_c = 0$ and its subsequent simplification yields

$$T_c = \sqrt{\frac{2c_f\left\{\int_0^{T_M^{max}} t\left\{n\left[1-F(T_M^{\max})\right]^{n-1} f(t)\right\}dt + T_M^{max}\left[1-F(T_M^{max})\right]^n\right\}}{n\, c_h\, T_w}} \tag{16}$$

Similarly, $\partial TC(T_M^{\max}, T_c)/\partial T_M^{\max} = 0$ yields a function (See Appendix B for detailed deduction process) given as follows:

$$nc_p f(T_M^{max})\left\{\int_0^{T_M^{max}} t\left\{n\left[1-F(T_M^{\max})\right]^{n-1} f(t)\right\}dt + T_M^{max}\left[1-F(T_M^{max})\right]^n\right\} - \left\{\frac{c_h T_c}{2} + c_u + c_p\{1-[1-F(T_M^{max})]^n\}\right\}\left[1 - F(T_M^{max})\right] = 0 \tag{17}$$

In order to transform the function in (17) to a single variable function, equations (16) and (17) are combined together to obtain an expression with only one decision variable, $T_M^{\max}$, given by

$$\left\{nc_p f(T_M^{max}) - \frac{c_f}{nT_w}\left[1-F(T_M^{max})\right]\right\} \times \left\{\int_0^{T_M^{max}} t\left\{n\left[1-F\left(T_M^{\max}\right)\right]^{n-1} f(t)\right\}dt + T_M^{max}\left[1-F(T_M^{max})\right]^n\right\} - \left\{c_u + c_p\{1-[1-F(T_M^{max})]^n\}\right\}\left[1-F(T_M^{max})\right] = 0 \tag{18}$$

Equation (18) cannot be written in a closed-form expression to obtain the decision variable, $T_M^{\max}$; but by replacing the order cycle, $T_c$, in function (15) with equation (16), a simplified total cost function can be derived so that $T_c$, one of the decision variables, is eliminated to decrease the computational burden, and thus, equation (15) is transformed into

$$TC(T_M^{max}) = c_f\sqrt{\frac{n\, c_h\, T_w}{2c_f\left\{\int_0^{T_M^{max}} t\left\{n\left[1-F(T_M^{max})\right]^{n-1} f(t)\right\}dt + T_M^{max}\left[1-F(T_M^{max})\right]^n\right\}}} + \frac{\sqrt{\frac{1}{2}c_f n T_w c_h\left\{\int_0^{T_M^{max}} t\left\{n\left[1-F(T_M^{max})\right]^{n-1} f(t)\right\}dt + T_M^{max}\left[1-F(T_M^{max})\right]^n\right\}}}{E(T_M)_{A+B}} + \frac{nT_w c_u + nT_w c_p\{1-[1-F(T_M^{max})]^n\}}{E(T_M)_{A+B}}. \tag{19}$$

So, an objective-oriented search (OOS) algorithm is proposed to find an approximate solution. With further calculation, decision variable $T_c$, the order size $Q$ (which depends on $T_c$), and minimum total cost $TC$, can be computed accordingly.

### B. OBJECTIVE-ORIENTED SEARCH (OOS) ALGORITHM FOR CEM MODEL

The minimum value of $TC(T_M^{\max})$ is determined in this search process by iteratively computing the result of right hand side of equation (19) in terms of $T_M^{\max}$. For a detailed description of the search method, an OOS algorithm is developed to complete this task. In this algorithm, when the iteration begins, the search step size of the algorithm is set to $\Delta$. In the algorithm iteration process, once the value of $TC(T_M^{\max})$ - $TC^*(T_M^{\max})$ is positive, a smaller step (halved) of $\Delta \leftarrow \Delta/2$ is given and the algorithm will search in opposite direction. By repeatedly calculating the value of $TC(T_M^{\max})$ with the changed value of $T_m$ which depends on the step size of $\Delta$, the minimal value of $TC(T_M^{\max})$ is approached. Let $\Re(T_M^{\max}) = TC(T_M^{\max}) - TC^*(T_M^{\max})$, if $\Re(T_M^{\max}) \le S$, the predetermined threshold, the algorithms can stop iteration and the final $T_M^{\max}$ is the optimal value that we want to find. The algorithm is given below, and the summary steps used for the OOS algorithm is illustrated through the flow chart shown in Figure 4.

***OOS Algorithm:***

*Step* 1: Initialize $c_f$, $c_p$, $c_h$, $c_u$, $T_w$, $\Delta$ (step size lifespan of *T*) and distribution type of lifespan $T$. Input initial value of $T$ and parametric values for distribution of $T$, $f(t)$. Initialize stop condition of iteration value $S \approx 0$ for $\Re^*(T_M^{\max})$. Set $\Re(T_M^{\max}) \leftarrow \infty$, $T_c \leftarrow 0$, $T \leftarrow 0$.

*Step 2*: Input initial value values $T = T + \Delta$.

*Step* 3: Calculate $F(x)$ and compute expected lifespan $E(T)_{A+B}$ with stopping time is $T_M^{\max} \leftarrow T$ using Equation (5), (7) and (8).

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*Step 4:* Compute $T_M^{\max} = T_M^{\max} + \Delta$ by equation (16).

*Step 5*: Compute $TC(T_M^{\max})$ by using equation (19).

If $\Re(T_M^{\max}) < 0$, set $TC^*(T_M^{\max}) = TC(T_M^{\max})$, $T_M^{\max *} = T$, $T_c^* = T_c$, set $T = T + \Delta$, go to *step* 3,

else if $\Re(T_M^{\max}) > 0$ and $\Re(T_M^{\max}) > S$, set $T = T - \Delta$, $\Delta \leftarrow \Delta / 2$, return *Step 3*,

else let $\Re^*(T_M^{\max}) = \Re(T_M^{\max})$, $T_M^{\max *} = T$, $T_c^* = T_c$.

*Step 6*: Stop and current values of $T_M^{\max *}$ and $T_c^*$ are the best solution and $TC^*(T_c^*, T_M^{\max *}) = TC^*(T_M^{\max})$ is the minimum total cost.

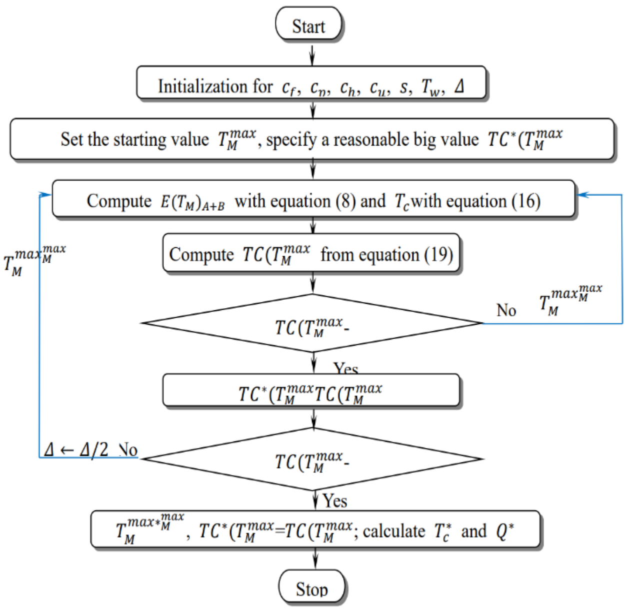


**FIGURE 4. Simplified flow chart of OOS algorithm**

## IV. CASE STUDY

In the industry, the failure of vulnerable parts is very common, such as adapting and stressed parts assembled in a machine, gaskets in equipment, tires of vehicles, etc. this study takes the valve gasket as an example to describe the case.

### A. CALCULATE LIFESPAN AND CYCLE TIME

In this context, according to employing OOS search algorithm, the rationality of the CEM model is verified with an actual case of vulnerable gaskets, which are part of a sophisticated inspection equipment. The main purpose of this equipment is electromagnetic valve detection. This equipment includes three compound gaskets that are necessary for the detection process. Since the equipment needs to be used frequently, vulnerable parts installed on the equipment are frequently worn out and replaced. In the following example, the lifespan of gaskets obeys normal distribution, $T \sim N(\mu_P, \sigma)$, where $\mu_P$ and $\sigma$ represent the expected value and standard deviation of the lifespan of vulnerable parts, respectively.

For the lifespan distribution of these vulnerable parts in this case, its mean and variance have been determined by experiment, $\mu_p$ = 150 (hours) and $\sigma$ = 5. The inspection equipment needs to work for 4380 hours in a year in this mass production system representing the stable demand during the working time, at least for one year. In the procurement scenario of this study, the fixed ordering of one order and unit variable cost of a part are given by actual case, $c_f$ = \$20/order and $c_u$ = \$15/unit. While this machine is working, unexpected failure for each of the 3 parts has potential damages to the testing work piece and results in a potential penalty cost of $c_p$ =\$1/unit, and consequently all the 3 parts need to be changed to avoid further damage. For every part in the stock, there is an annual holding cost of $c_h$=\$4/unit/year. Detailed parameters are shown in Table 1.

TABLE 1. CASE PARAMETERS SURVEY TABLE

| Parameters | value |
| --- | --- |
| The number of vulnerable parts in an equipment (n) | 3 |
| Lifespan distribution | N (150, 5) |
| Fixed cost per order ($c_f$) | \$20/order |
| Variable cost per unit ($c_u$) | \$15/unit |
| Penalty cost per unit ($c_p$) | \$1/unit |
| Holding cost per unit ($c_h$) | \$4/unit/year |

By running the OOS program with an initial step size $\Delta$ =10 hours and stopping criteria $S$ = \$0.01, the final optimization result is shown in Table 2, $T_M^{\max}$ =141.3125 hours, an order cycle $T_c$ = 0.327 years$\approx$119 days (about 4 months), an order size of $Q$ = 10.189$\approx$10 units and the minimized total cost $TC^*$ =\$1,535.24.

Table 2 shows that both the CEM model and the OOS algorithm proposed in this study are very effective, since it converges to the stationary points very quickly within few iterations. The convergence of the algorithm is proved to be robust in a real case of purchasing vulnerable parts.

### B. SENSITIVENESS OF SYSTEM AND PART PARAMETERS

In this research, the number of vulnerable parts of a machine, $n$, has an impact on the effective working time. As mentioned earlier, the maximum maintenance interval of an equipment, affected by the combined lifespan distributions of all parts, is determined by the vulnerable part which has the shortest lifespan. In addition, from the perspective of business operation, holding excess parts and the machine downtime penalty both have a significant impact on order size and machine downtime, and eventually

TABLE 2. SEARCH RESULT FOR CEM WITH OOS ALGORITHM

| Iteration or Step No. | Stopping time, $T_M^{\max}$ | Stopping criteria, $S$ | Step size $\Delta$ | Total cost, $TC$ |
|---|---|---|---|---|
| 1 | 21.0000 | 180,194.60 | 10.000000 | 198,549.97 |
| 2 | 31.0000 | 8,653.24 | 10.000000 | 18,355.36 |
| 3 | 41.0000 | 3,083.64 | 10.000000 | 9,702.12 |
| … | ,… | … | … | … |
| 16 | 146.0000 | 2282.48 | 5.000000 | 3,817.72 |
| 17 | 143.5000 | 170.82 | 2.500000 | 1,706.06 |
| 18 | 142.2500 | 22.79 | 1.250000 | 1,558.03 |
| 19 | 141.6250 | 3.31 | 0.625000 | 1,538.56 |
| 20 | 141.4688 | 0.41 | 0.156250 | 1,535.46 |
| 21 | 141.3906 | 0.12 | 0.078125 | 1,535.17 |
| 22 | 141.3516 | 0.04 | 0.039063 | 1,535.09 |
| 23 | 141.3320 | 0.02 | 0.019531 | 1,535.07 |
| 24 | 141.3125 | 0.01 | 0.019531 | 1,535.05 |

TABLE 3. COMPARISON OF IMPACT ON $T_m$ FOR DIFFERENT VALUES OF $C_p$ AND $C_h$

| Holding cost $C_h$ (\$/unit) | Penalty cost $C_p$ (\$/unit) | Stopping time $T_M^{\max *}$ (Hours) | Order cycle $T_c^*$ (Days) | Order size $Q^*$ | $TC^*$ (\$) |
|---|---|---|---|---|---|
| 4.00 | 0.50 | 141.66 | 119 | 10 | 1,529.19 |
| 4.00 | 1.00 | 141.31 | 119 | 10 | 1,535.24 |
| 4.00 | 2.00 | 141.00 | 119 | 10 | 1,544.97 |
| 3.00 | 1.00 | 141.33 | 138 | 12 | 1,518.68 |
| 5.00 | 1.00 | 141.33 | 107 | 9 | 1,549.49 |

have a consequential impact on the total cost, $TC$. Thus, an analysis of $C_h$, $C_p$ and the parts number, $n$ is conducted to study their effect on the total cost.

**(1) Sensitiveness of system and part parameters**

To figure out the impact of penalty cost and holding cost on total cost, this study tested a set of penalty cost, $C_p$ and holding cost, $C_h$ and compared the difference between the optimal value of $T_m$. By adopting the same example in the case study, testing parameters, $C_p$ is set to 0.5, 1.0, and 2.0 dollars per unit, and $C_h$ is set correspondingly to 3.00, 4.00, and 5.00 dollars per unit per year. The computational results with OOS algorithm are shown in Table 3.

Table 3 shows that as the penalty cost, $C_p$ increases, replacing parts early helps minimize Total cost. The reduction in order size is preferred when unit holding cost, $C_h$ increases which is intuitively expected. However, scientific decision with appropriate values of $C_p$ and $C_h$ can decrease the total cost to a relatively low level.

(2) **Influences of number of parts, $n$**

As mentioned earlier, the quantity of vulnerable parts, $n$ has an influence on the maximum maintenance interval of the machine. Equation (3) illustrates the probability density function of failure time for the $n$ parts.

For the parts with the same properties as in the case study problem, the probability density function, $g(t)$ is plotted in Figure 5 for $n$ = 1, 2, 3, 4 and 8. It can be clearly seen from the figure that when the number of vulnerable parts increases, the continuous working time of the machine decreases, although the function value changes little. Since the machine is affected by $n$ parts, the allowable maximum working time decreases accordingly. Under the assumption of a normal distribution, this is also an illustrative verification for Theorem 2.

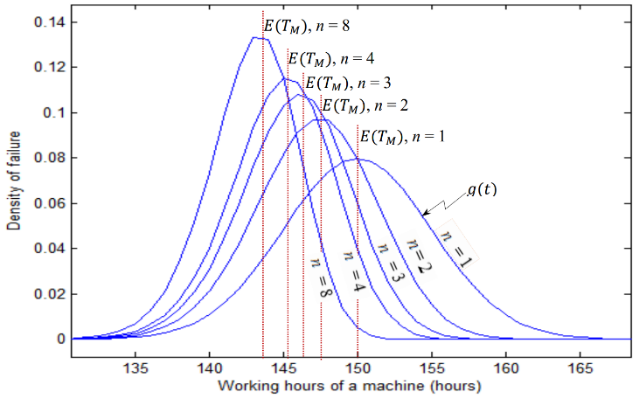


**FIGURE 5. Comparison of density function with different amount of vulnerable parts**

When $n$ = 1, $g(t)$ is a special case that the optimal replacement time for a machine is the same as that for the only one vulnerable part, and the working time for the machine follows the same distribution with vulnerable part.

In order to reveal the influence of the quantity of the vulnerable parts, different quantity of parts (2, 3, 4, 8) are input into the CEM model by computing with OOS

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algorithm. Table 4 shows the comparison of the impact of $TC^*$ with different values of $n$ as the quantity of vulnerable parts increase, the maximum allowable stopping time, $T_M^{\max *}$ decreases. The change with $T_M^{\max *}$ also reflects the decrease of a jointly overall expected working time with the vulnerable parts.

### C. COMPUTATIONAL RESULTS WITH DIFFERENT DISTRIBUTIONS

To test the adaptability of the CEM model proposed in this study, two other distributions, uniform and Gamma are used to simulate the model, and their performances are compared. The summary results for three different distributions for the CEM model are given in Table 5. The mean values of $\mu$ for uniform, normal and Gamma distributions are all assumed to be 150 hours and the total amount of the vulnerable parts is $n$ = 3. Conclusion that can be drew from Table 5 that different lifespan distributions influence the $T_M^{max}$ (the maximum stopping time), $T_c$ (order cycle) and the $TC$ (total cost), there is almost no influence to the order size.

In general, the quantity of vulnerable parts has significant influence on the allowable working time in a machine. Since the machine works with a maximum time of the minimum lifespan of $n$ parts, so the parts have a serial impacts on the effective working time. On the other hand, the quantity of vulnerable parts in a machine is predetermined. Under this condition, the total cost is determined mainly by unit penalty cost, $c_p$ and holding cost, $c_h$ .

TABLE 4. COMPARISON OF IMPACT ON $TC^*$ WITH DIFFERENT QUANTITY $n$

| Quantity of parts $n$ (units) | Stopping time $T_M^{\max *}$ (Hours) | Order cycle $T_c^*$ (Days) | Order size $Q^*$ | $TC^*$ ($) |
|---|---|---|---|---|
| 2 | 143.50 | 147 | 12 | 1,034.99 |
| 3 | 141.31 | 119 | 10 | 1,535.24 |
| 4 | 141.00 | 103 | 9 | 2,036.26 |
| 8 | 141.00 | 72 | 6 | 4,115.77 |

TABLE 5. OPTIMAL SOLUTIONS FOR DIFFERENT DISTRIBUTIONS (WITH $N$ = 3)

| Distributions of vulnerable parts | OOS search algorithm Stopping time $T_M^{\max *}$ (Hours) | Order cycle $T_c^*$ (Days) | Order size $Q^*$ | $TC^*$ ($) |
|---|---|---|---|---|
| Uniform (145, 155) | 145.00 | 121 | 10 | 1,479.73 |
| Normal (150, $5^2$) † | 141.31 | 119 | 10 | 1,535.24 |
| Gamma (500, 0.3) ‡ | 133.50 | 115 | 11 | 1,629.35 |

† $N(\mu,\sigma^2)$ ; ‡ Gamma ($\alpha,\beta$) with mean $\mu_G = \alpha\beta$ .

## V. CONCLUSIONS

Procurement policy is tightly related to the actual requirement of a production system or supply chain of an enterprise. Accordingly, research on the affected procurement policy is more significant when the influence cannot be neglected. This paper studies the replacement strategy of vulnerable parts, which is necessary to the normal operation of a machine. By combining the lifespan distribution of each single vulnerable part, the lifespan distribution of the machine is studied. Under a preemption of machine working time, the expectation of the transformed distribution of the machine is studied. Then, by considering penalty cost, holding cost, fixed purchasing cost, and part unit cost, a novel CEM model is proposed for vulnerable parts. To solve the model, an OOS algorithm was proposed in this study. According to a real case with normally distributed vulnerable parts, the CEM model and OOS algorithm are proved to be effective. Finally, according to an actual case, we conduct sensitivity analysis on each parameter involved in the model and try to explore the deeper understanding and relationship between optimal maximum allowable working time of a machine, cycle time and total cost. In this case, the original purchase cost was $1,706 and the optimized purchase cost is $1535.05. With the CEM, the total cost is reduced by 10%. The results show that the use of the CEM model and OOS algorithm could greatly reduce procurement total cost of vulnerable parts.

The joint maintenance planning and replacement part management problems for multiple vulnerable parts is an important branch of intelligent manufacturing decisions. It is essential to the operation of the enterprise. The CEM model described above applies to most of vulnerable parts under jointly distributed lifespans in manufacturing system, which could raise efficiency of supply chain and reduce the operation cost of enterprise.

In fact, many machines or systems lifespan estimates are based on experience. For future research, on the one hand, according to the idea of the above model, more accurate lifespan assessment can be implemented for some machines or systems composed of some parts. Then more convincing cost assessment model could be obtained. On the other hand, this model may extent to the case of a product or equipment with different types of vulnerable parts and, therefore, with different probability laws. In addition, some machines consist of vulnerable parts that have parallel or compound impacts on the performance of the whole machine. For such a situation, it might be necessary to do further study on the effects of such system.

## APPENDIX A

### CUMULATIVE PROBABILITY FUNCTION OF A MACHINE WITH $n$ VULNERABLE PARTS

Since $T_M = \min(T_i, i=1,2,\ldots n)$, $T_M^{\max}$ is the stopping time for the machine, and the cumulative probability, $G(T_M^{\max})$, can be given as

$$G(T_M^{\max}) = P\{\min(T_i, i=1,2,\ldots n) < T_M^{\max}\} \qquad \text{(A.1)}$$

(A.1)

$$= 1- P\{\min(T_i, i=1,2,\ldots n) > T_M^{\max}\}. \qquad \text{(A.2)}$$

(A.2)

In equation (A.2), for every parts in a machine, $P\{\min(T_i, i=1,2,\ldots n) > T_M^{\max}\}$ = $P\{T_1 > T_M^{\max}\}$ $P\{T_2 > T_M^{\max}\} \ldots P\{T_n > T_M^{\max}\}$. All the vulnerable parts follow the same identical distribution, so the probability of $P\{\min(T_i, i=1,2,\ldots n) > T_M^{\max}\}$ is the same value for $i=1,2,\ldots,n$. As a result, $P\{\min(T_i, i=1,2,\ldots n) > T_M^{\max}\}$ = $[P\{\min(T\,) > T_M^{\max}\}]^n$, and equation (A.2) can be rewritten as

$$G(T_M^{\max}) = 1- [P\{\min(T\,) > T_M^{\max}\}]^n \qquad \text{(A.3)}$$

On further deduction, equation (A.3) yields

$$G(T_M^{\max}) = 1-[1-P\{\min(T\,) < T_M^{\max}\}]^n \qquad \text{(A.4)}$$

(A.4)

By simplification, the final expression is given by

$$G(T_M^{\max}) = 1-[1-F(T_M^{\max})]^n. \qquad \text{(A.5)}$$

(A.5)

where $F(T_M^{\max}) = P\{\min(T\,) < T_M^{\max}\} = P\{T_M < T_M^{\max}\}$, that is, $T_M$ ~ f($t_M$).

## APPENDIX B

### PARTIAL DIFFERENTIATION ON $T_M^{\max}$ WITH OBJECTIVE FUNCTION

The total cost objective function is described in equation (15), by partial differentiation method, $\partial TC(T_M^{\max}, T_c)/\partial T_M^{\max}$ = 0, it gives

$$\frac{\partial}{\partial T_M^{max}}\left[\frac{\frac{1}{2}T_c n T_w c_h + nT_w c_u + nT_w c_p\{1-[1-F(T_M^{max})]^n\}}{\left\{\int_0^{T_M^{max}} t\left\{n\left[1-F\left(T_M^{\max}\right)\right]^{n-1} f(t)\right\}dt + T_M^{max}[1-F(T_M^{max})]^n\right\}}\right] = 0. \qquad \text{(B.1)}$$

Since $\dfrac{\partial}{\partial T_M^{\max}}\left(nT_w c_h T_c\ /2 + nT_w c_u + nT_w c_p\{1-[1-F(T_M^{\max})]^n\}\right)$

$$= n^2 T_w c_p [1-F(T_M^{\max})]^{n-1} f(T_M^{\max}),$$

and

$$\frac{\partial}{\partial T_M^{max}}\left\{\int_0^{T_M^{max}} t\{\mathrm{n}[1-F(x)]^{n-1} f(t)\}dt + T_M^{max}[1-F(T_M^{max})]^n\right\}$$

$$= [1-F(T_M^{\max})]^n,$$

thus, equation (B.1) can be simplified as

$$\frac{n^2 T_w c_p [1-F(T_M^{max})]^{n-1} f(T_M^{max})\left\{\int_0^{T_M^{max}} t\{\mathrm{n}[1-F(x)]^{n-1} f(t)\}dt + T_M^{max}[1-F(T_M^{max})]^n\right\}}{\left\{\int_0^{T_M^{max}} t\{\mathrm{n}[1-F(x)]^{n-1} f(t)\}dt + T_M^{max}[1-F(T_M^{max})]^n\right\}}$$

$$-\frac{\left[\frac{1}{2}T_c n T_w c_h + nT_w c_u + nT_w c_p\{1-[1-F(T_M^{max})]^n\}\right][1-F(T_M^{max})]^n}{\left\{\int_0^{T_M^{max}} t\{n[1-F(T_M^{max})]^{n-1} f(t)\}dt + T_M^{max}[1-F(T_M^{max})]^n\right\}} = 0, \quad (B.2)$$

which yields

$$n^2 T_w c_p [1-F(T_M^{max})]^{n-1} f(T_M^{max}) \left\{\int_0^{T_M^{max}} t\{n[1-F(T_M^{max})]^{n-1} f(t)\}dt + T_M^{max}[1-F(T_M^{max})]^n\right\} - \left[\frac{1}{2}T_c n T_w c_h + nT_w c_u + nT_w c_p\{1-[1-F(T_M^{max})]^n\}\right][1-F(T_M^{max})]^n = 0. \quad (B.3)$$

Dividing both sides of eq. (B.3) by $nT_w[1-F(T_M^{\max})]^{n-1}$ it yields

$$nc_p f(T_M^{max}) \left[\int_0^{T_M^{max}} t\{n[1-F(T_M^{max})]^{n-1} f(t)\}dt + T_M^{max}[1-F(T_M^{max})]^n\right] - \left(\frac{T_c c_h}{2} + c_u + c_p[1-F(T_M^{max})]^n\right)[1-F(T_M^{max})] = 0 \quad (B.4)$$

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## REFERENCES

[1] Lina, C.C., and Wang, T.H., (2011), “Build-to-order supply chain network design under supply and demand uncertainties,” Transportation Research Part B-methodological, 45(8): 1162–1176SI.

[2] Volling T., Matzke A., Grunewald M. and Spengler T.S., (2013), “Planning of capacities and orders in build-to-order automobile production: A review,” *European Journal of Operational Research*, 224(2): 240–260.

[3] Yao, X., and Askin, R., (2019), “Review of supply chain configuration and design decision-making for new product,” *International Journal of Production Research,* 57(7): 2226-2246.

[4] Moonseong, H., Myles, S. F., and David, B. A., (1998), “Power and sample for survibal analysis under the Weibull distribution when the whole lifespan is of interest,” *Mechanisms of Aging and Development*, 102 (1998): 45-53.

[5] Chalkley, A. M., Billett, E., and Harrsion, D., Simpson, G., (2003), “Development of a method for calculating the environmentally optimum lifespan of electrical household products,” *Journal of Engineering Manufacture*, 217, DOI: 10.1243/095440503771909908.

[6] Lamond, B. F., and Sodhi, M. S., (2006), “Minimizing the expected processing time on a flexible machine with random tool lives,” *IIE Transactions,* 38(1): 1-11.

[7] Khan, M. A., Mittal, S. West, S. and Wuest, T., (2018), “Review on upgradability - A product lifespan extension strategy in the context of product service systems,” *Journal of Cleaner Production,* 20(2018): 1154-1168.

[8] Setiawan, A., Wangsaputra, R., Martawirya, Y. Y., Halim A. H., (2019), “An Object-Oriented Modeling Approach for Production Scheduling on CNC-Machines in Flexible Manufacturing System to Maximize Cutting Tool Utilization,” *Journal of Advanced Manufacturing Systems,* 18(2): 293-310.

[9] De, S. N., Aghezzaf, E., and Desmet, B., (2019), “Optimizing installation (R, Q) policies in distribution networks with stochastic lead times: a comparative analysis of guaranteed- and stochastic service models,” *International Journal of Production Research,* 57(13): 4148-4165.

[10] Uzunoglu, K. U. and Yalcin, B., (2020), “Continuous review (s, Q) inventory system with random lifespan and two demand classes,” *Opsearch,* 57(1): 104-118.

[11] Masahiro, O., Shinsuke, M., Tomohiro, T., Ichiro, D. and Seiji, H., (2010), “Lifespan of commoditied part II,” *Journal of Industrial Ecology,* DOI: 10.1111/j. 1530-9290.2010.00251.x.

[12] Munoz, R. and Llanos, J., (2012), “Estimation of the lifespan of agricultural tractor using a diffusion model at the aggregate level,” *Cienciae Investigación Agrarian*, 39(3): 557-562.

[13] Firoozi, Z. and Ariafar, S., (2017), “A supply chain network design model for random-lifespan products,” *Journal of Industrial and Production Engineering,* 34(2): 113-123.

[14] Li, Y., Liu, K. l., Foley, A. M., Zulke, A., Berecibar, M., Nanini-Maury, E., Van, M. J. and Hoster, H. E., (2019), “Data-driven health estimation and lifespan prediction of lithium-ion batteries: A review,” *Renewable and Sustainable Energy Reviews,* 113, DOI: 10.1016/j.rser.2019.109254.

[15] Gu, M., and Chen Y., (2019), “Two improvements of similarity-based residual life prediction methods,” *Journal of Intelligent Manufacturing,* 30(1): 303-315.

[16] Soltani, M., Kulkarni, R., Scheinost, T., G T. and Zimmermann, A., (2019), “A novel approach for reliability investigation of leds on molded interconnect devices based on fe-analysis coupled to Injection Molding Simulation,” *IEEE Access,* 7, DOI: 10.1109/ACCESS.2019.2913786.

[17] Huang, H., Tong, X., Cai, Y., and Tian, H., (2020), “Gap between discarding and recycling: Estimate lifespan of electronic products by survey in formal recycling plants in China,” *Resources Conservation and Recycling,* 156, DOI: 10.1016/j.resconrec.2020.104700.

[18] Wu, S., Yang, J., Peng, R., and Zhai, Q., (2021), “Optimal design of facility allocation and maintenance strategy for a cellular network,” *Reliability Engineering & System Safety*, 205(2021), DOI: 10.1016/j.ress.2020.107253.

[19] Weustink, I. F., Brinke, E. T., Streppel, A. H., Kals, H. J. J., (2000), “A generic framework for cost estimation and cost control in product design,” *Journal of Materials Processing Technology*, 103 (2000): 141-148.

[20] Safaei, N. and Zuashkiani, A., (2012), “Manufacturing system design by considering multiple machine replacements under discounted costs,” *IIE Transactions*, 44(12): 1100-1114.

[21] Hur, M., Keskin, B. B. and Schmidt, C. P., (2018), “End-of-life inventory control of aircraft spare parts under performance based logistics,” *International Journal of Production Economics,* 204: 186-203, DOI: 10.1016/j.ijpe.2018.07.028.

[22] Ketzenberg, M., Gaukler, Ga. and Salin, V., (2018), “Expiration dates and order quantities for perishables,” *European Journal of Operational Research,* 266(2):569-584.

[23] Venanzi, I., Castellani, R., Ierimonti, L. and Ubertini, F., (2019), "An automated procedure for assessing local reliability index and life-cycle cost of alternative girder bridge design solutions," *Advances in Civil Engineering,* 2019, DOI: 10.1155/2019/5152031.
[24] Kundu, A., Guchhait, P., Panigrahi, G. and Maiti, M., (2019), "An EOQ model for deteriorating item with promotional effort and credit linked demand," *European Journal of Industrial Engineering,* 13(3): 368-399.
[25] Jansen, B. W., van, S. A., Gruis, V., and van, B. G., (2020), "A circular economy life cycle costing model (CE-LCC) for building components," *Resources Conservation and Recycling,* 161, DOI: 10.1016/j.resconrec.2020.104857.
[26] Xia, T., Sun, B., Chen, Z., Pan, E., Wang, H., and Xi, L., (2021), "Opportunistic maintenance policy integrating leasing profit and capacity balancing for serial-parallel leased systems," *Reliability Engineering & System Safety,* 205(2021), DOI: 10.1016/j.ress.2020.107233.
[27] Blackhurst, J., Wu, T. and O'Grady, P., (2005), "PCDM: A decision support modeling methodology for supply chain, product and process design decisions," *Journal of Operations Management,* 23(3-4): 325-343.
[28] Cavalieri, S. Garetti, M. Macchi, M. and Pinto, R. (2008), "A decision-making framework for managing maintenance spare parts," *Production Planning & Control,* 19(4): 379-396.
[29] Wang, G. J. and Zhang, Y. L., (2016), "Optimal replacement policy for a two-dissimilar-component cold standby system with different repair actions," International Journal of Systems Science, 47(5), 1021-1031.
[30] Khan, M., Ahmad, A. and Hussain, M., (2019), "Integrated decision models for a vendor-buyer supply chain with inspection errors and purchase and repair options", *International Journal of Advanced Manufacturing Technology,* 104(9-12): 3221-3228.
[31] Iqbal, M. W. and Sarkar, B., (2019), "Recycling of lifespan dependent deteriorated products through different supply chains," *RAIRO-Operations Research,* 53(1):129-156.
[32] Bacchetti, A., Bertazzi, L., and Zanardini, M., (2020), "Optimizing the distribution planning process in supply chains with distribution strategy choice," *Journal of the Operational Research Society,* DOI: 10.1080/01605682.2020.1727785.
[33] Shen, J., Hu, J., and Ma, Y., (2020), "Two preventive replacement strategies for systems with protective auxiliary parts subject to degradation and economic dependence," *Reliability Engineering & System Safety,* 204(2020), DOI: 10.1016/j.ress.2020.107144.
[34] Ahmadi, R., (2019), "A New Approach to Maintenance Optimisation of Repairable Parallel Systems Subject to Hidden Failures," *Journal of the Operational Research Society,* 71(9): 1448-1465.
[35] Seif Z., Mardaneh E., Loxton R., and Lockwood A., (2020), "Minimizing equipment shutdowns in oil and gas campaign maintenance," *Journal of the Operational Research Society,* DOI: 10.1080/01605682.2020.1745699.
[36] Shi, Z., and Liu, S., (2020), "Optimal inventory control and design refresh selection in managing part obsolescence," *European Journal of Operational Research,* 287(1): 133-144.
[37] Chien, C., Chen, C., (2020), "Data-Driven Framework for Tool Health Monitoring and Maintenance Strategy for Smart Manufacturing," *IEEE Transactions on Semiconductor Manufacturing,* 33(4): 644-652.

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