Effect of the lateral resistance of fabric on the critical load of the sewing needle in sewing technology part A: Theoretical approach

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Abstract The needle of the industrial sewing machine is its most important part. During the sewing process, the needle is subjected to various actions and interactions between these actions. One of the factors that are most related to these interactions is the lateral elastic resistance – spring effect – of the sewn fabric on the needle during its penetration in the layers of fabric. In the present work, this effect has been investigated via a special formula, where the spring constant is incorporated. It was found that the equivalent length coefficient of the sewing needle (γ) ranged between 0.7, where the lower end of the needle still has not penetrated the fabric and 2, where the lower end has already penetrated the fabric. The elastic stability factor (η) was changed from 2.46 to 20.35, to create a critical axial compressive load $P_{cr} = 5$ for a 39 cN of the sewing needle. This means that the spring constant (S) changed from $S = 0$ to $S = \infty$, which enhanced the critical load by 8 folds. For $S = 0$, the angle has two options, the first is $\leq \frac{\pi}{2}[(\lambda l)^o]$ and the second $\geq \frac{\pi}{2}[(\lambda l)^\phi]$. The values of γ, η, and P_{cr} for $(\lambda l)^o$ were not reasonable, while values of these variables for $(\lambda l)^\phi$ were acceptable. The future vision of the present work is to design and construct a stand to experimentally check the theoretical work suggested in this paper and to find its experimental values.

1. Introduction

Feodosev [1] studied the stability of equilibrium of deformable systems, such as the Euler problem, the large displacement of a slender bar, the stability of a bar in the presence of plastic strains, the stability of rings and tubes under external pressure, the effect of end conditions on the critical force, etc.

Chen and Lui [2] studied the bifurcation approach applied in stability analysis and the energy used for the stability analysis of elastic columns, the load deflection curves of a perfectly straight column or for a column with small/large initial crookedness, with some eccentricity, with an imperfectly straight shape, or with bending at the onset of loading. The tangent or reduced modulus theory is not applicable any more for such shapes. In these cases, the ultimate load P_u and the numerical analysis are commonly used.

Pisarenko et al. [3], in their work, tabulated different cases of cantilevers and cantilever beams with various types of axial

compressive loading, for ideal and imperfect columns. Also, the end conditions of these beams were studied and tabulated. These tables were later considered a reference for researchers and other scientists interested in the area of structural stability in mechanical engineering.

ElGholmy and Elhawary [4] applied Pisarenko et al.'s [3] technique to calculate the critical load of the needles used in the garment and apparel sewing technology. It was found that the value of the critical load for the industrial sewing machines' needle was 62 cN. Thus, the working resistance force during the sewing process must be less than P_{cr}, to avoid the loss of needle stability. This is controlled by the safety factor of the elastic stability ($m = P_{cr}/P_w$) (where where P_{cr} – is the elastic buckling load or critical elastic buckling load, P_w is the penetration force of the needle during the sewing process, and η is the elastic stability factor). For denim fabrics, P_w is 120 cN (fabric mass per sq.m. = 120). The value of η changed to about 0.0284, which can be neglected.

ElGholmy and Elhawary [5] studied the eccentricity of the needle of the industrial sewing machine via the secant formula. They found that the secant formula for the sewing needle of the industrial sewing machine has the following shape:

$$\sigma_{max} = \frac{P}{A} \left[1 + \frac{e \cdot c}{k^2} \left(sec \frac{\pi}{2} \sqrt{\frac{P_a}{P_{cr}}} \right) \right]$$

The symbols in this equation are explained in the text.

They also found that the sewing needle's secant safety factor (S.F.) was:

$$S.F. = \left[1 + \frac{e \cdot c}{k^2} \left(sec \frac{\pi}{2} \sqrt{\frac{P_a}{P_{cr}}} \right) \right]$$

In their research, Debabrara and Adhijit [6] stated that the case of needles with pin-shaped ends – column – is the most common, where this column is known as the fundamental column. Buckling is supposed to happen within a proportional limit or an elastic limit (from a practical point of view, they are the same point). They mentioned that the column's imperfection can include the eccentricity of the load and the crookedness of its initial geometry.

Hussien et al. [7], in their study, recorded that the penetrating force of the sewing needle ranged from 10 to 37 cN for a 1 Nm, G70 needle, for piqué samples with a fabric mass per square meter = 200 g/m², while the force was in the range of 20 cN to 110 cN, for single jersey fabric with a fabric mass per square meter= 111 g/m², for a 1 Nm, G70 needle. The more pronounced value of the penetrating force was 180 cN, for 5H satin stripe samples at a mass per square meter equals to 156 g/m² and a needle size of 1 Nm, G65. In general, according to Debabrara et al. [6], the range of the penetrating force of the needle is from 10 to 180 cN. These values were measured experimentally.

Practically, the prediction of the irregularity of the stitch length, before production, was achieved by statistically computing the percentage of the coefficient of vernation (C.V. %) of the feeding force, using a computer-based measuring system that could be used as a testing equipment to differentiate between the ideal sewing parameters of different types of fabric. This was done to improve sewing quality [8]. Stylo et al. [9]assembled a sewing machine equipped with certain measuring instruments to measure the penetration force of

the sewing needle, to find the degree of damage during the sewing process.

From the previous discussion, it is clear that the damage that occurs to the needle during the sewing process can cause severe problems. Thus, predicting the needle's critical load, before work, can prevent these problems. This introduction generally explains the different factors concerning the elastic buckling load (P_{cr}) of the machine's elements, especially the sewing needle. The present work studied a unique factor, during the sewing operation, which plays a role in the determination of the elastic buckling load.

2. Mathematical approach

Fig. 1 shows the sewing needle of an industrial type sewing machine, subjected to an axial compressive force, during the sewing process.

It can be seen that the lower end of the needle is moved or displaced over distance f_B, under elastic resistance, i.e., the spring effect of the fabric being sewn. This spring effect is represented by the spring of a coil that exerts a reaction of R_B value, described as follows:

$$R_B = S \cdot f_B$$

where S is the spring's stiffness or constant. The differential equation of the sewing needle's free part is described as follows:

$$EI \frac{d^2y}{dx^2} = M$$

$$= P[f_B - y_x] - R_B(\ell - x)$$

$$\therefore EI \frac{d^2y}{dx^2} =$$

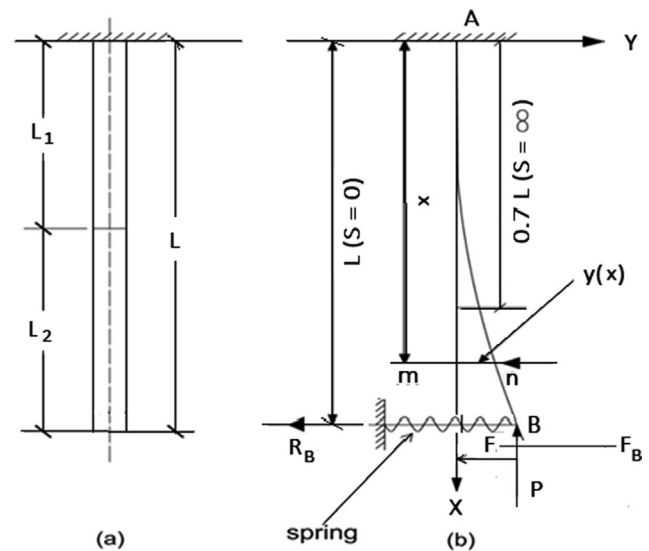$$= P_{cr}[f_B - y] - S \cdot f_B[\ell - x] \qquad (1)$$

Fig. 1   The buckling curve of the free section of a sewing needle, subjected to lateral elastic fabric resistance (spring effect).

where $I^{'}$ is the minimum sewing needle inertia cross-section, f_B is the elastic displacement of the sewing needle's lower end, y_x is the general displacement of the buckling curve of the sewing needle at distance x from the upper end of the needle, X is the axis of the needle, S is the spring constant or stiffness of the fabric's elastic reaction or resistance, and p_{cr} is the axial critical compressive force on the free end of the needle.

Eq. (1) could be rearranged and rewritten as follows:

$$\frac{d^2y}{dx^2} = \lambda^2(f_B - y) - \frac{Sf_B}{EI^{'}}(\ell - x) \qquad (2)$$

where

$$\lambda^2 = \frac{p_{cr}}{EI^{'}}$$

Eq. (2) can be rewritten as follows:

$$\frac{d^2y}{dx^2} + \lambda^2 y = \lambda^2 . f_B\left(1 - \frac{S.\ell}{p_{cr}}\right) + \lambda^2 \frac{Sf_B}{p_{cr}}.x \qquad (3)$$

The solution for Eq. (3) is:

$$y = C.\sin \lambda_x + D\cos \lambda_x + f_B\left(1 - \frac{S}{p_{cr}}\ell\right) + \frac{S}{p_{cr}}f_B.x \qquad (4)$$

The integration constants C and D and the critical load (P_{cr}) can be calculated from the sewing needle's boundary conditions, i.e.,

$$\left. \begin{array}{l} y(0) = y_A = 0 \\ = \frac{dy}{dx}(0) = 0 \end{array} \right\} \qquad (5)$$

$$\therefore D = -f_B\left(1 - \frac{S}{p_{cr}}\ell\right)$$

Then, when $x = \ell$

$$y(\ell) = y_B = f_B$$

From Eq. (4), $\frac{dy}{dx}$ can be calculated,

$$\frac{dy}{dx} = \lambda C \cos \lambda x - \lambda D \sin \lambda x + \frac{S}{p_{cr}}f_B$$

When $x = 0$,

$$\therefore \lambda C + \frac{S}{p_{cr}}f_B = 0$$

$$\therefore C = -\frac{S}{\lambda p_{cr}}f_B$$

By substituting the values of C and D in Eq. (4), then:

$$y(x) = -\frac{S}{\lambda p_{cr}}f_B \sin \lambda x - f_B\left(1 - \frac{S}{p_{cr}}\ell\right)\cos \lambda x$$
$$+ f_B\left(1 - \frac{S}{p_{cr}}\ell\right) + \frac{Sf_B}{p_{cr}}x \qquad (6)$$

By substituting $x = \ell, y(\ell) = f_B$ in Eq. (6)

$$\therefore y(\ell) = -\frac{S}{\lambda p_{cr}}f_B \sin \lambda l - f_B\left(1 - \frac{S}{p_{cr}}\ell\right)\cos \lambda l + F_B\left(1 - \frac{S}{p_{cr}}\right)$$
$$+ \frac{S}{p_{cr}}f_B\ell = f_B$$

$$\therefore \frac{S}{p_{cr}}\sin \lambda l - \left(1 - \frac{S}{p_{cr}}\ell\right)\cos \lambda l = 0$$

Then,

$$\tan \lambda l = \lambda l\left(1 - \frac{p_{cr}}{S\ell}\right) \qquad (7)$$

From Eq. (7), two cases will be studied. In the first case, $S = 0$, i.e., no spring effect from the fabric. Then the second case will be studied.

$$\tan \lambda l = \infty$$

$$\therefore \lambda l = \frac{\pi}{2}$$

In this case, the sewing needle has a free lower end. So, the critical load (P_{cr}) is:

$$P_{cr} = \frac{\pi^2}{(\gamma\ell)^2}EI^{'}$$

$$= \frac{\pi^2}{(2)^2}\frac{EI^{'}}{\ell^2}$$

$$\frac{\pi^2}{4}\frac{EI^{'}}{\ell^2}$$

i.e., $\gamma = 2$

$$P_{cr} = \eta\frac{EI^{'}}{\ell^2}, \eta = \frac{\pi^2}{4} = \frac{\pi^2}{\gamma^2} = 2.47$$

In the second case, $S = \infty$, i.e., the lower end of the sewing needle is completely fixed, then:

$$\tan \lambda_\ell = \lambda_\ell$$

$$\therefore \lambda_\ell = \frac{\pi}{0.7}$$

$$\therefore P_{cr} = \frac{\pi^2}{\gamma\ell^2}EI^{'}$$

$$= \left(\frac{\pi}{0.7}\right)^2 EI^{'}$$

$$= \eta\frac{EI^{'}}{\ell^2}$$

where

$$\eta = \frac{\pi^2}{\gamma^2} = \frac{\pi^2}{(0.7)^2}$$

Then, the equivalent length coefficient (γ) ranges from 0.7 to 2. The two previous cases are mentioned as an explanation of the state of the sewing needle. From a practical point of view, it is recommended to find the spring constant (S) of the elastic resistance of the fabric being sewn, via intensive experimental work. Fig. 2, while taking into consideration data from Table 1, represents a graphical solution of Eq. (7), when the lateral elastic resistance of the sewn fabric (spring constant) $S = \infty$, i.e., when the equivalent length of the needle is 0.7ℓ. This means that the coefficient of equivalent length (γ) is equal to 0.7.
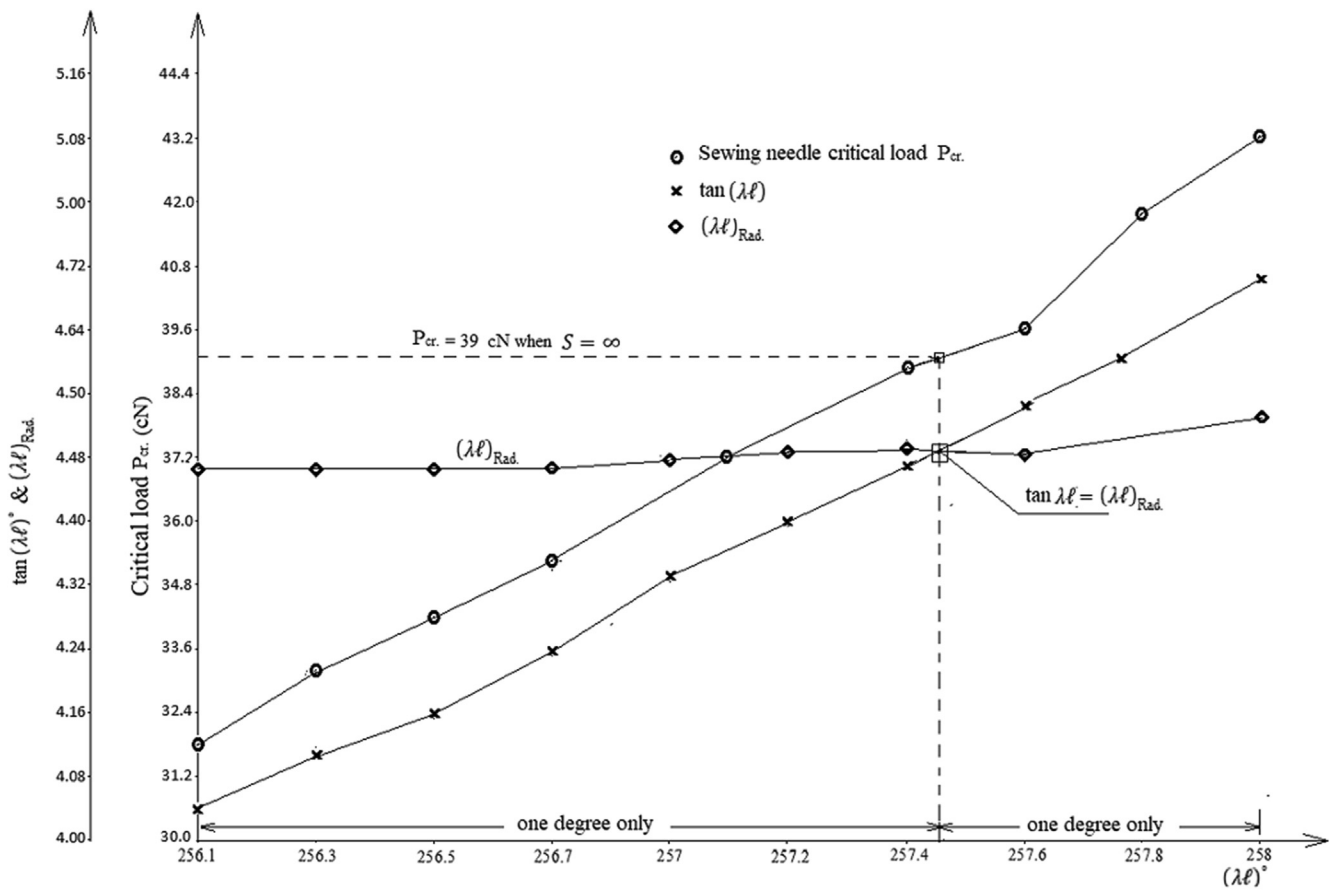
Fig. 2 Graphical solution of Eq. (7) for S = ∞. [S – Elastic lateral resistance or spring constant – for the sewn fabric]

Table 1 Calculations of the sewing needle when the lateral elastic resistance (S) = ∞ for the sewn fabric.

$(\lambda\ell)^\phi$	$\tan(\lambda\ell)^\phi$	$(\lambda\ell)_{Rad.}$	$\gamma = \frac{\pi}{\tan(\lambda\ell)}$	$\eta = \frac{\pi^2}{\gamma^2}$	$P_{cr.}$ (cN)
256.1	4.0408	4.4675	0.7771	16.327	31.8481
256.2	4.0713	4.4693	0.7713	16.8299	32.8183
256.3	4.1022	4.4715	0.7654	17.0879	33.3096
256.4	4.1335	4.4728	0.7596	17.088	33.3214
256.5	4.1653	4.4745	0.7538	17.3519	33.8362
256.6	4.1976	4.4762	0.7481	17.6173	34.3537
256.7	4.2303	4.78	0.7423	17.8937	34.8927
256.8	4.2635	4.4797	0.7365	18.1766	35.4444
257	4.3315	4.4832	0.7249	18.763	36.5879
257.1	4.3662	4.485	0.7142	19.06	37.167
257.2	4.4015	4.4867	0.7134	19.3728	37.777
257.3	4.4373	4.4885	0.7076	19.6917	38.3988
257.4	4.4737	4.4902	0.7019	20.0139	39.0273
257.5	**4.5107**	**4.4919**	**0.6961**	**20.3477**	**39.025**
257.6	4.5483	4.4937	0.6904	20.6851	39.678
257.7	4.5864	4.4954	0.6846	21.0371	41.0223
257.8	4.6252	4.4972	0.6789	21.3924	41.7152
257.9	4.6646	4.4989	0.6732	21.7585	42.4291
258	4.7046	4.5007	0.6674	22.1354	43.1651

Note: $(\lambda\ell)^\circ$; $(\lambda\ell)^{\{\circ}$ – marks angles in degrees and $(\lambda\ell)_{Rad.}$ – marks angles in radian.

where

$P_{cr} = \eta \frac{EI'}{\ell^2}$, E−206 GPa, $I' = I_4 = 1 - 9175 \times 10^{-16}$ m^4,

$\ell = 0.045$ m, $EI' = 3 - 9501 \times 10^{-5}$, $\frac{EI'}{\ell^2} = 0.0195$ N.

Table 2 Lateral resistance of fabric (S) = 0.

$(\lambda\ell)^\circ$	$\tan(\lambda\ell)^\circ$	$(\lambda\ell)_{\text{Rad.}}$	$\gamma = \frac{\pi}{\tan(\lambda\ell)^{/\circ}}$	$\eta = \frac{\pi^2}{\gamma^2}$	$P_{\text{cr.}}$ (cN)	N.B.
89	57.29	1.5526	0.0548	3283.2	64	Impractical values
89.1	63.6567	1.5543	0.04933	4051	79	
89.2	71.6151	1.556	0.04385	5127.67	100	
89.3	81.847	1.5578	0.03836	6700.4	130.6582	
89.4	95.4895	1.5595	0.03288	9120.02	177.84	
89.6	143.2	1.563	0.0219	20557.54	400.8719	
89.8	286.4777	1.5665	0.01096	82069.47	1600.35	
89.9	572.9572	1.5683	0.00548	328,320	6402.3	
90	**∞**	**1.57**	**2**	**2.46**	**4.81**	Not acceptable values
91	−57.29	1.5874	1.9781	2.5198	4.9136	
92	−28.6363	1.6049	1.9565	2.5757	5.0226	
93	−19.0811	1.6223	1.9355	2.6319	5.1322	
94	−14.3	1.6398	1.9149	2.6889	5.2433	
95	−11.4301	1.6572	1.8948	2.7462	5.3551	

Note: $(\lambda\ell)^\circ; (\lambda\ell)^{\{o}$ – marks angles in degrees and $(\lambda\ell)_{\text{Rad.}}$ – marks angles in radian.

From Fig. 2, it can be seen that the value of the critical load in this case is 39 cN. This happens when angle $(\lambda\ell)^{\{o}$ is equal to 257.5°. Seemingly, this type of needle is more suitable for light mass fabrics.

Taking the data in Table 2, representing Eq. (7), into consideration, it can be seen that there are two options for the angle; one where it is $\leq \frac{\pi}{2}$, i.e., $(\lambda\ell)^\circ$ and the second where it is $\geq \frac{\pi}{2}$, i.e., $(\lambda\ell)^{\{o} = $ from $\frac{\pi}{2}$ to 0.53π. For $(\lambda\ell)^\circ = (\lambda\ell)^{\{o} = \frac{\pi}{2}$, γ is equal to 2 (the coefficient of equivalent length) and η (the elastic stability factor) = 2.46. The case of $(\lambda\ell)^\circ$ cannot be applied in practice, as P_{cr} jumps exponentially. On the other hand, in the case of $(\lambda\ell)^{\{o}$, P_{cr} fluctuates between 5 and 6 cN, which is not acceptable practically for the sewing needle. It seems that Eq. (7) gives no satisfaction for S = 0.

3. Conclusion and future work

From the previous results and discussions, the following conclusions can be drawn out:

(1) The equation of the buckling curve of the free part of the sewing needle, in the presence of a lateral elastic resistance exerted by the fabric on the needle, can be expressed as follows:

1. $y(x) = -\frac{S}{\lambda P_{cr}} f_B \sin \lambda x - f_B \left(1 - \frac{S}{p_{cr}}\ell\right) \cos \lambda_x +$

   $f_B \left(1 - \frac{S}{p_{cr}}\ell\right) + \frac{S f_B}{p_{cr}} x$

2. The symbols of this equation were mentioned in the text. Eq. (7) can be used to draw the buckling curve.

(2) The equivalent length coefficient (γ), depending on the fabric's lateral elastic resistance coefficient (S), ranges from 2 (for S = 0) to 0.699 (for S = ∞). Consequently, the sewing needle's critical load (P_{cr}) ranges from 2.41 $\frac{EI'}{\ell^2}$ to 20.20 $\frac{EI'}{\ell^2}$. This means that the elastic stability coefficient (η) ranges from 2.4076 to 20.199.

(3) By reversing the procedure, the sewn fabric's lateral elastic resistance (S) can be predicted.
(4) The lateral elastic resistance exerted by the sewn fabric on the sewing needle leads to $\gamma = 0.7 - 2$ and $\eta = 2.46 - 20.35$, while p_{cr} falls in the range of 5–39.0 cN. This means that the increase in S from 0 to ∞ increases the critical load by 7.8 times.
(5) Future work should involve building a stand to check Eq. (7) and its results in a practical setting.
(6) Eq. (7) is not valid for spring stiffness S = 0.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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