# A Novel Heap based Optimizer for Scheduling of Large-scale Combined Heat and Power Economic Dispatch 

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#### Abstract

Cogeneration systems economic dispatch (CSED) provides an optimal scheduling of heat /power generating units. The CSED aims to minimize the whole fuel cost (WFC) of the cogeneration units taking into consideration their technical and operational limits. Then, the current paper examines the first implementation of dominant bio-inspired metaheuristic called heap-based optimization algorithm (HBOA). The HBOA is powered by an adaptive penalty functions for getting the optimal operating points. The HBOA is inspired from the organization hierarchy, where the mechanism consists of the interaction among the subordinates and their immediate boss, the interaction among the colleagues, and the employee's selfcontribution. Based on the infeasible solutions' remoteness from the nearest feasible point, HBOA penalizes them with various degrees. Four case studies of the CSED are implemented and analyzed, which comprise of 4, 24, 84 and 96 generating units. The HBOA is proposed to solve CSED problem with consideration of transmission losses and the valve point impacts. An investigation with the recent optimization algorithms, which are supply demand optimization (SDO), jellyfish search optimization algorithm (JFSOA), and marine predators' optimization algorithm (MPOA), the improved MPOA (IMPOA) and manta ray foraging (MRF), is developed and elaborated. From the obtained results, it is clearly observed that the optimal solutions gained, in terms of WFC, reveal the feasibility, capability, and efficiency of HBOA compared with other optimizers especially for large-scale systems. case


INDEX TERMS Cogeneration systems economic dispatch, fuel cost minimization, heap based optimization algorithm, distribution reconfiguration, valve point impacts, transmission losses.

## Nomenclature:

| $a_{i}, b_{i}, \& c_{i}$ | The $i^{\text {th }}$ power plant cost coefficients |  |  |
| :--- | :--- | :--- | :--- |
| $a_{j}, b_{j}, \& c_{j}$ | The ${ }^{\text {th }}$ heat plant cost coefficients | $H_{d}$ | Heat demand in the system |
| $a_{k}, b_{k}, c_{k}, d_{k}, e_{k}$ | The $k^{h h}$ unit cost coefficients | $N_{\mathrm{p}}, N_{\mathrm{h}}$ and $N_{c}$ | Number of power-only plants, heat- |
| $\& \& f_{k}$ |  | $P$ | only plants and cogeneration units |
| $C$ | Total production costs | $P_{d}$ | Output of power generation units |
| $C_{i}\left(P_{i}^{p}\right)$ | Fuel cost of power unit $i$ | Electric power demand |  |
| $C_{j}\left(H_{j}^{h}\right)$ | Fuel cost of $j^{h h}$ heat plant | $P_{\text {Loss }}$ | Transmission losses |
| $C_{k}\left(P_{k}^{c}, H_{k}^{c}\right)$ | Operational cost of $k^{t h}$ cogeneration | $\lambda_{i} \& \rho_{i}$ | Valve-point cost coefficients |
|  | unit | $H$ | Output of heat generation units |

## I. INTRODUCTION

## A. MOTIVATION AND INCITEMENT

Conversion from fossil fuels to electricity in the conventional units are the main cause for low energy efficiency of these units that leads to significant wasted energy amount. However, cogeneration systems economic dispatch (CSED) can save up to $40 \%$ of the generation costs, and achieve $90 \%$ energy efficiency [1]. Additional advantage of cogeneration units for the environment is the associated decrease in contaminating gas emissions, which is generally assessed by $13-18 \%$ [1]. The importance of CSED is evident in achieving the minimum operating costs of cogeneration units with optimum scheduling of heat and power units as well with keeping of operational constraints, which are heat and power balance constraint, valve-point effect, and generation capacity limits which take into consideration combined heat and power (CHP) units' nonconvex feasible operating areas. With the growing size, CSED has become a distinctive non-convex, non-linear, and large-scale global optimization issue in the viewpoint of theories and engineering applications [1].

## B. LITERATURE REVIEW

A plethora of conventional and mathematical approaches have been developed to solve CSED optimization problem such as sequential quadratic programming (SQP) [2], lagrangian relaxation (LR) [3], benders decomposition (BD) [4] and LR with surrogate subgradient (LRSS) multiplier updates [5]. These optimization techniques may converge to a local optimum, which is highly dependent on the initial starting points. Furthermore, the inclusion of more nonconvexity, non-linear, and non-smooth cost functions increases the complexity of many of them [3], [6].

Nowadays, various efficient heuristic and meta-heuristic optimization algorithms have been developed to the CSED problem for their capability of dealing with such complex problem. The researchers have employed many optimization algorithms for achieving the best possible scheduling of heat and electricity producing units with least cost such as whale optimization algorithm (WOA) [7], harmony search (HS) [8], differential evolution (DE) algorithm [9], quantum optimization (QO) [10], and particle swarm optimization (PSO) [11]. A hybrid PSO and weighted vertices based (WVO), has been applied in [12] to obtain the optimal solution of CSED problem. In [13], particle swarm optimization with time-varying acceleration coefficients (TVAC-PSO) has been employed with adding a sinusoidal term to the polynomial cost function to represent the effect of the valve point. The objective function of pollutant gas emissions was combined with the operational cost to generate a multi-objective CSED issue to be addressed in [14]. In [15], PSO was used to simulate the functioning of a coal-fired CSED that was coupled to heat and power generating units. In [16], the security of the electricity
network was examined in a multi-objective formulation used for CSED management, which takes into account the cost of pollutant emissions. The CSED problem while retaining the dependability of micro-grids and operational restrictions was provided in [17].
Moreover, multi-player harmony search (MPHS) [18], oppositional teaching learning-based optimizer (OTLBO) [19], line-up competition optimizer [20], non-dominated sorting genetic algorithm (NSGA) [21], bee colony optimization (BCO) [22], salp swarm algorithm (SSA) [23], multi-verse optimizer (MVO) [24], equilibrium optimizer (EO) [25], and stochastic fractal search algorithm [26], [27]. have been presented to solve this problem with lesser computational effort.

In [28], a cuckoo search optimization, with emerged sorting process in a descending order based on the fitness value and new operator to update the individuals, has been applied for the CSED problems. In [29], an improved genetic algorithm with two types of crossover operators for the CSED issue. As well, hybrid non-dominated sorting genetic algorithm with multi-objective PSO [30], multi-verse optimization (MVO) [24], and an enhanced shuffle frog leaping optimizer [31] have been efficiently applied for the same purpose but their validations were restricted to just small-scale applications of 5 -units and 7-units systems. In [32], a novel Kho-Kho Optimization (KKO) for tackling the CSED challenge was described, although it requires a feasibility assessment because several obtained operational points did not meet their given limitations. Squirrel search algorithm (SSA) has been employed for solving complicated multi-region combined heat and power economic dispatch problem with consideration of thermal generators and solar and wind power uncertainty [33].

Efforts have not ceased to get new reliable and effective techniques and develop the existing techniques for optimal solution of such complex problems [34]. One of these new effective optimization techniques is the heap-based optimization algorithm (HBOA). HBOA is inspired from the organization hierarchy. This can be seen when a team working for achieving their goal arrange themselves in a hierarchy which is named corporate rank hierarchy (CRH) to organize the search agents based on their fitness in a hierarchy using the heap data structure.

## C. CONTRIBUTION AND PAPER ORGANIZATION

The paper presents a solution to the combined heat and power economic dispatch problem using a heap-based optimizer. The objective is to find the optimal schedule of generating units such that heat and power, both demands are met from cogeneration units, in an optimal manner. In this paper, HBOA is developed to solve the CSED issue while considering the valve point effects and other practical restrictions. This paper contributions are reviewed as:

- HBOA is designed with an adapted penalty formulas to find an optimal feasible operating coordinate for the

CSED complex problem. Based on the distance between the infeasible option and the next feasible option, HBOA penalizes them with various degrees, which give it the opportunity to easily reach optimal solutions even in complex problems.

- The CSED model is inspected considering valve point impacts and transmission losses.
- HBOA is effectively employed with high superiority to previous techniques on small-scale systems such as the 4units, and 24-unit systems with technical and operational constraints fulfillment.
- HBOA feasibility, scalability and validity are verified and assessed for large-scale systems such as the 84-unit and 96-unit systems.
- For all systems and studied cases, HBOA improves the solution quality and capability of finding feasible optimal operating points of all units (heat only units, power only units and cogeneration units).
The remaining of the paper is organized as follows: The CSED problem is illustrated in Section II. Additionally, in Section III, HBOA is described for obtaining the optimal CSED solution. In Section IV, the simulation results and discussion are introduced. Finally, Section V concludes this work.


## II. PROBLEM FORMULATION

The main objective of the CSED problem aims to minimize the whole fuel costs (WFC) supplying the cogeneration, heat only and power only units that satisfy the power and heat demands. This can be represented as follows [1]:

$$
\operatorname{Min} \sum_{i=1}^{N_{p}} C_{i} P_{i}^{p}+\sum_{j=1}^{N_{h}} C_{j} H_{j}^{h}+\sum_{k=1}^{N_{c}} C_{k}\left(P_{k}^{C}, H_{K}^{C}\right)
$$

The terms of generation costs given in Eq. (1) can be written as follows [7]:

$$
\begin{align*}
& C_{i}\left(P_{i}^{p}\right)=a_{i}\left(P_{i}^{p}\right)^{2}+b_{i} P_{i}^{p}+c_{i}+\left|\lambda_{i} \sin \left(\rho_{i}\left(P_{i}^{p_{\min }}-P_{i}^{p}\right)\right)\right|  \tag{2}\\
& \quad C_{j}\left(H_{j}^{h}\right)=a_{j}\left(H_{j}^{h}\right)^{2}+b_{j} P_{j}^{p}+c_{j}(\$ / \mathrm{h})  \tag{3}\\
& C_{k}\left(P_{k}^{c}, H_{k}^{c}\right)=a_{k}\left(P_{k}^{c}\right)^{2}+b_{k} P_{k}^{p}+c_{k}+d_{k}\left(H_{k}^{c}\right)^{2}+ \\
& e_{k} H_{k}^{c}+f_{k} H_{k}^{c} P_{k}^{c} \tag{4}
\end{align*}
$$

The cost function of power-only plant is described in Eq. (2) which comprises a quadratic and sinusoidal terms, where the sinusoidal term manifests the valve-point impacts. The valve point impacts make the CSED as non-differentiable and non-convex problem. The cost of heat only is represented in Eq. (3). Additionally, for Eq. (4) represents the cogeneration units cost function, where the $H^{c}$ and $P^{c}$ are the heat output and power output, respectively.

The CSED problem could be optimized with subject to the following constraints for feasible solutions:

$$
\begin{equation*}
\sum_{i=1}^{N_{\mathrm{p}}} P_{i}^{\mathrm{p}}+\sum_{j=1}^{N_{\mathrm{c}}} P_{j}^{\mathrm{c}}=P_{d} \tag{5}
\end{equation*}
$$

$$
\begin{gather*}
\sum_{j=1}^{N_{c}} H_{j}^{c}+\sum_{k=1}^{N_{h}} H_{k}^{h}=H_{d},  \tag{6}\\
P_{i}^{p_{\min }} \leq P_{i}^{p} \leq P_{i}^{p_{\max }} \quad i=1, \ldots, N_{p},  \tag{7}\\
H_{j}^{h_{\min }} \leq H_{j}^{h} \leq H_{j}^{h_{\max }} \quad j=1, \ldots, N_{h},  \tag{8}\\
P_{k}^{c_{\text {min }}}\left(H_{k}^{c}\right) \leq P_{k}^{c} \leq P_{k}^{c_{\max }}\left(H_{k}^{c}\right) \quad k=1, \ldots, N_{c},  \tag{9}\\
H_{k}^{c_{\min }}\left(P_{k}^{c}\right) \leq H_{k}^{c} \leq H_{k}^{c_{\text {max }}}\left(P_{k}^{c}\right) \quad k=1, \ldots, N_{c}, \tag{10}
\end{gather*}
$$

Equation (5) illustrates the balance of power generation and demand. Equation (6) manifests the heat generation and demand balance. Moreover, power-only plants capacity limits are demonstrated in Eq. (7), whereas Eq. (8) shows the heat-only units generation limits. Additionally, the cogeneration units' capacity limits are described in Eqs. (9) and (10).

The transmission losses are added to the power balance constraint, which introduces extra non-linearities into the model. It can be evaluated as signified in Eq. (11) [35]. Therefore, the equality balance constraint of Eq. (5) could be changed as characterized in Eq. (12).

$$
\begin{align*}
& P_{\text {Loss }}=\sum_{i=1}^{N_{\mathrm{p}}} \sum_{m=1}^{N_{\mathrm{p}}} B_{i n} P_{i}^{\mathrm{p}} P_{m}^{\mathrm{p}}+\sum_{i=1}^{N_{\mathrm{p}}} \sum_{j=1}^{N_{\mathrm{c}}} B_{i j} P_{i}^{\mathrm{p}} P_{j}^{\mathrm{c}}+\sum_{j=1}^{N_{\mathrm{c}}} \sum_{n=1}^{N_{\mathrm{c}}} B_{i n} P_{j}^{\mathrm{c}} P_{n}^{\mathrm{c}}  \tag{11}\\
& \quad \sum_{i=1}^{N_{\mathrm{p}}} P_{i}^{\mathrm{p}}+\sum_{j=1}^{N_{\mathrm{c}}} P_{j}^{\mathrm{c}}=P_{d}+P_{\text {Loss }} \tag{12}
\end{align*}
$$

## III. HEAP BASED OPTIMIZATION ALGORITHM FOR CSED problem

The heap-based algorithm is inspired from organizations hierarchy. This can be seen when a team working arrange themselves in a hierarchy for achieving their goal, which is named corporate rank hierarchy (CRH). In this regard, the concept of CRH is to organize the search agents based on their fitness in a hierarchy using the heap data structure to map this concept. Three elements are the main pillars of HBOA. The first element is the collaboration among the assistants and their immediate boss. While the second element is the interaction among the colleagues. The third element is the self-contribution of the employees. Four steps are developed for mapping the heap concept as follows:

## A. MODELING THE CORPORATE RANK HIERARCHY

The CRH model is developed with the heap data structure which is similar to tree-shaped data structure. Therefore, the full CRH manifests the population while the search agent represents a heap node. The search agent's fitness is the master of the heap node, and the population index of the search agent is the value of the heap node.

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## B. FIRST PILLAR: MODELING OF THE INTERACTION WITH IMMEDIATE BOSS

In the centralized organizational structure, the policies and rules are set from the upper levels, whereas subordinates must execute the instruction from their direct supervisors. It can be described through updating the agent position of each search using the following equation:

$$
\begin{equation*}
x_{i}^{k}(t+1)=B^{k}+\gamma(2 r-1)\left|B^{k}-x_{i}^{k}(t)\right| \tag{13}
\end{equation*}
$$

where; $t$ indicates the current iteration, $k$ signifies the $k^{\text {th }}$ vector component of, and $\|$ refers to the absolute value. The term ( $2 r-1$ ) represents the $k^{\text {th }}$ component of vector $\vec{\lambda}$, which is produced randomly as illustrated in Eq. (14):

$$
\begin{equation*}
\lambda^{k}=2 r-1 \tag{14}
\end{equation*}
$$

where; $r$ exemplifies a random number from the range $[0,1]$ which is generated according to uniform distribution. However, $\gamma$ is calculated according to Eq. (15).

$$
\begin{equation*}
\gamma=\left|2-\frac{\left(t \bmod \frac{T}{C}\right)}{\frac{T}{4 C}}\right| \tag{15}
\end{equation*}
$$

where; $T$ exemplifies the total iterations' number, and $C$ is a user-defined parameter which controls the variation in the values of $\gamma(2 r-1)$. However, the parameter $C$ will complete in $T$ iterations and can be represented as follows:

$$
\begin{equation*}
C=\lfloor T / 25\rfloor \tag{16}
\end{equation*}
$$

## C. THE SECOND PILLAR: MATHEMATICAL MODELING OF THE INTERACTION BETWEEN COLLEAGUES

The colleagues with the same level are considered as the nodes and each agent $\overrightarrow{x_{i}}$ updates its position with respect to its randomly designated colleague $\overrightarrow{S_{r}}$ :

$$
x_{i}^{k}(t+1)= \begin{cases}S_{r}^{k}+\gamma \lambda^{k}\left|S_{r}^{k}-x_{i}^{k}(t)\right|, & f\left(\overrightarrow{S_{r}}\right)<f\left(\overrightarrow{x_{i}}(t)\right)  \tag{17}\\ x_{i}^{k}+\gamma \lambda^{k}\left|S_{r}^{k}-x_{i}^{k}(t)\right|, & f\left(\overrightarrow{S_{r}}\right) \geq f\left(\overrightarrow{x_{i}}(t)\right)\end{cases}
$$

where; $f$ describes the fitness of the search agent.

## D. THE THIRD PILLAR: MODELING OF THE SELFCONTRIBUTION OF AN EMPLOYEE

The self-contribution of an employee is mapped in this phase as manifested in the following equation:

$$
\begin{equation*}
x_{i}^{k}(t+1)=x_{i}^{k}(t) \tag{18}
\end{equation*}
$$

## E. MERGING THE THREE PILLARS

This subsection shows the merging procedure of the position updating equations into one equation. The probabilities of selection use a roulette wheel to balance both exploration and
exploitation through splitting the proportions into $p_{1} ; p_{2}$, and $p_{3}$. The selection of the proportion $p_{I}$ enables a search agent to update its position using Eq. (18), where the bound of $p_{I}$ can be calculated as follows:

$$
\begin{equation*}
p_{1}=1-\frac{t}{T} \tag{19}
\end{equation*}
$$

The selection of the proportion $p_{2}$ enables a search agent to update its position using Eq. (13), where the bound of $p_{2}$ can be calculated as follows:

$$
\begin{equation*}
p_{2}=p_{1}+\frac{1-p_{1}}{2} \tag{20}
\end{equation*}
$$

The selection of the proportion $p_{3}$ enables a search agent to update its position using Eq. (17), where the bound of $p_{3}$ can be calculated as follows:

$$
\begin{equation*}
p_{3}=p_{2}+\frac{1-p_{1}}{2}=1 \tag{21}
\end{equation*}
$$

Consequently, Eq. (22) presents a general position updating mechanism of HBOA as follows:

$$
x_{i}^{k}(t+1)=\left\{\begin{array}{l}
x_{i}^{k}(t), \quad p \leq p_{1}  \tag{22}\\
B^{k}+\gamma \lambda^{k}\left|B^{k}-x_{i}^{k}(t)\right|, p_{1}<p<p_{2} \\
S_{r}^{k}+\gamma \lambda^{k}\left|S_{r}^{k}-x_{i}^{k}(t)\right|, p_{2}<p \leq p_{3} \text { and } f\left(\overrightarrow{S_{r}}\right)<f\left(\overrightarrow{x_{i}}(t)\right) \\
x_{r}^{k}+\gamma \lambda^{k}\left|S_{r}^{k}-x_{i}^{k}(t)\right|, p_{2}<p \leq p_{3} \text { and } f\left(\overrightarrow{S_{r}}\right) \geq f\left(\overrightarrow{x_{i}}(t)\right)
\end{array}\right.
$$

where $p$ represents a produced randomly number $[0,1]$.
To handle the CSED problem, the HBOA is illustrated in Fig. 1. For mutual-dependent cogeneration units, the second form is shown in Fig. 2. Depending upon on penalty component inside the fitness under consideration, they are dealt utilizing quadratic penalized terms. As a result, the whole objective to be minimized ( F ) is formally defined as shown in (23):

$$
\begin{equation*}
\mathrm{F}=\mathrm{TF} C+\psi_{\mathrm{v}} \sum_{k=1}^{N_{C}} B I \cdot\left\{P_{k}^{C}\left(H_{k}^{C}\right)-P_{k}^{\text {CLimit }}\left(H_{k}^{C}\right)\right\} \tag{23}
\end{equation*}
$$

where; the term $\left(P_{k}^{c_{\text {Limit }}}\left(H_{k}^{c}\right)\right)$ reflects the power limit to the set heating output for the cogeneration $(k)$. Moreover, the symbol ( $B I$ ) manifests a binary coefficient which equals 1 for violation state and zero else, whilst $\psi_{\mathrm{v}}$ shows a penalized factor related to the cogeneration operating point violation ( $\psi_{\mathrm{v}}=50000$ ).


FIGURE 1. Flowchart of the developed HBOA.


FIGURE 2. Dependency between power and heat for cogeneration unit.

It is illustrated according to Eq. (23) and Fig. 2, the value of the penalized component increases as the infeasible points are moved away from the next regarding borders. As a result, the HBOA provides a greater capacity for searching for viable sites. Furthermore, a stopping criterion is implemented in which the optimum result is acquired when a specified number of iterations is attained. Based on the infeasible solutions' distance from the next border, HBOA penalizes them with various degrees.

## IV. SIMULATION RESULTS

In this section, the HBOA is applied on four test systems, which are 4 -units, 24 -unit, 84 -unit and 96 -unit test systems. The number of iterations ( T ) and individuals ( $\mathrm{n}_{\mathrm{pop}}$ ) are 300 and 50 , respectively, for the 4 -units' systems while they are 3000 and 100 , respectively, for the 24 -unit, 84 -unit and 96unit systems.

## A. THE 4-UNIT TEST SYSTEM

It involves single conventional power-only unit, two cogeneration units and one heat-only unit. The system demands of power and heat are 200 MW and 115 MWth , respectively [36]. The proposed HBOA optimizer is implemented and tested for optimal solution of CSED optimization problem and compared with other efficient mathematical approaches such as LR [3], SQP [2], LRSS [5] and BD [4] as depicted in Table 1. Additionally, recent techniques such as manta ray foraging (MRF) [37], jellyfish search optimization algorithm (JFSOA) [34], and supply demand optimization (SDO) are applied for fair comparison. As shown, from the obtained results, the effectiveness and robustness of the employed HBOA optimizer are demonstrated with a minimum WFC of 9257.0694 \$. Ultimately, from the economic perspective, the yearly savings with the application of the proposed HBOA as compared with the WFC obtained by other conventional methods, LR [3], SQP [2], LRSS [5] and BD [4], is about 268.056 \$/year. The convergence characteristics, shown in Fig. 3, clearly shows that HBOA is capable to find feasible operating points of all units and to improve the solution quality with respect to the recent techniques such as MRF[37], SDO and JFSOA.

## B. SIMULATION RESULTS OF THE 24-UNIT TEST SYSTEM

The load and heat demand of this test system are respectively 2350 MW and 1250 MWth. Additionally, it includes 5 heat units, 13 thermal units, and 6 CHP units as obtained from [38]. The proposed HBOA is implemented and applied on this test system as tabulated in Table 2. By simulating the results, it can be observed that the HBOA gives optimal solution with WFC of 57994.51 \$. Other reported techniques such as grey wolf optimization (GWO) [38], teaching learning-based optimization (TLBO) [19], oppositional TLBO (OTLBO) [19], group search optimization (GSO) [39], improved version of GSO (IGSO) [39], TVAC-PSO [13] and CPSO [13] are also applied, which give WFC of 57846.84 \$, 58006.999 \$, 57856.2676 \$, 58225.745 \$, 58049.01 \$, 58122.746 \$ and 59736.2635 \$, respectively. Also, recent techniques MRF, SDO and JFSOA are applied on this test system which give WFC of 58173.93 \$, 58208.0267 \$ and 58739.5241 \$, respectively. It is observed from the reported WFC ( $\mathrm{WFC}^{\mathrm{R}}$ ) given in this table that the WFC obtained from GWO, TLBO and OTLBO overwhelmed the proposed HBOA for achieving minimum costs. However, by verifying the operating points of these methods, great violation of the operating point of CHP unit- 6 is detected as shown in Figs.

4, 5 and 6 for GWO, TLBO and OTLBO techniques, respectively. As shown in these figures the operating point of CHP unit-6 is (31.47 MW and 18.39 MWth), (31.46 MW and 18.38 MWth), and (31.98 MW and 18.22 MWth) for GWO, TLBO and OTLBO, respectively. In addition, it is observed that small deviation between calculated WFC ( $\mathrm{WFC}^{\mathrm{C}}$ ) and the reported value. Accordingly, the comparison of the proposed method with the GWO, TLBO and OTLBO techniques, in this case, is not fair comparison. While, in comparison with other techniques given in Table 2 , the proposed method is considered the best.

Fig. 7 shows the convergence rates of the proposed technique and other recent optimization techniques. It is clear from this figure that HBOA is capable to find feasible operating points of all units and to improve the solution quality and finally reach the least WFC of $287933.8131 \$$. In addition, achieving all constraints with $100 \%$ accuracy.

## C. SIMULATION RESULTS OF 84-UNIT TEST SYSTEM

The load and heat demand of this the 84-unit system are 12700 MW and 5000 MWth, respectively. Additionally, it includes 20 heat units, 40 thermal units, and 24 CHP units as obtained from [7]. Table 3 gives the optimal unit scheduling using the proposed techniques as well as other relevant techniques. By simulating the result, it is observed that the obtained optimal solution achieved by HBOA is lower than the reported techniques which are WOA [7] and MPHS [18] as well as the recent techniques applied in this article which are MPOA, IMPOA, MRF, SDO and JFSOA.


FIGURE 3. Convergence rates of HBOA versus other recent techniques for the CSED of 4-units system.


FIGURE 4. GWO based operating point of CHP unit-6 of the 24-unit test system [38].


FIGURE 5. TLBO based operating point of CHP unit-6 of the 24 -unit test system [19].


FIGURE 6. OTLBO-based operating point of CHP unit-6 of the 24-unit test system [19].

TABLE 1. Optimal schedluing of CSED problem for the 4 -units system using HBOA and other techniques.

| Unit | LR [3] | SQP [2] | BD [4] | LRSS [5] | MRF | JFSOA | SDO | HBOA |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| $\operatorname{Pg} 1$ (MW) | 0 | 0 | 0 | 0 | 0 | 0 | 0 |  |
| $\operatorname{Pg} 2$ (MW) | 160 | 160 | 160 | 160 | 160 | 160 | 0 |  |
| Hg2 (MWth) | 40 | 40 | 40 | 40 | 40 | 40 | 160 | 40 |
| Pg3 (MW) | 40 | 40 | 40 | 40 | 39.9991 | 39.9991 | 39.9991 | 40 |
| Hg3 (MWth) | 75 | 75 | 75 | 75 | 75.0009 | 75.0009 | 75.0009 | 70.9991 |
| Hg4 (MWth) | 0 | 0 | 0 | 0 | 0 | 0.0000 | 0.0000 | 0.0009 |
| WFC (\$) | $\mathbf{9 2 5 7 . 1}$ | $\mathbf{9 2 5 7 . 1}$ | $\mathbf{9 2 5 7 . 1}$ | $\mathbf{9 2 5 7 . 1}$ | $\mathbf{9 2 5 7 . 0 6 9 4}$ | $\mathbf{9 2 5 7 . 0 6 9 4}$ | $\mathbf{9 2 5 7 . 0 6 9 8}$ | $\mathbf{9 2 5 7 . 0 6 9 4}$ |



FIGURE 7. Convergence characteristics of HBOA versus other recent optimizaion techniques for the CSED problem of the 24-unit system.

TABLE 2. Optimal solution of CSED problem of the 24-unit system using HBOA and other techniques.

| Unit | GWO[38] | $\begin{gathered} \hline \text { TLBO } \\ {[19]} \\ \hline \end{gathered}$ | $\begin{gathered} \hline \text { OTLBO } \\ {[19]} \\ \hline \end{gathered}$ | GSO [39] | $\begin{gathered} \hline \text { IGSO } \\ {[39]} \\ \hline \end{gathered}$ | $\begin{aligned} & \text { TVAC- } \\ & \text { PSO [13] } \end{aligned}$ | CPSO [13] | MRF | JFSOA | SDO | HBOA |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Pg1 (MW) | 538.844 | 628.324 | 538.5656 | 627.7455 | 628.152 | 538.5587 | 680 | 538.5601 | 538.0895 | 449.2756 | 538.55874 |
| Pg2 (MW) | 299.3423 | 227.3588 | 299.2123 | 76.2285 | 299.4778 | 224.4608 | 0 | 149.1049 | 224.8625 | 149.6789 | 300.2175 |
| Pg3 (MW) | 299.3423 | 225.9347 | 299.122 | 299.5794 | 154.5535 | 224.4608 | 0 | 224.2988 | 301.8762 | 202.5620 | 301.08255 |
| Pg4 (MW) | 109.9653 | 110.3721 | 109.992 | 159.4386 | 60.846 | 109.8666 | 180 | 109.8616 | 158.7306 | 109.8603 | 159.77792 |
| Pg5 (MW) | 109.9653 | 110.2461 | 109.9545 | 61.2378 | 103.8538 | 109.8666 | 180 | 159.4583 | 110.5483 | 109.9316 | 63.217359 |
| Pg6 (MW) | 109.9653 | 160.1761 | 110.4042 | 60 | 110.0552 | 109.8666 | 180 | 109.8041 | 112.2867 | 159.7364 | 60.688902 |
| Pg 7 (MW) | 109.9653 | 108.3552 | 109.8045 | 157.1503 | 159.0773 | 109.8666 | 180 | 109.8188 | 107.4217 | 160.0083 | 160.20652 |
| Pg8 (MW) | 109.9653 | 110.5379 | 109.6862 | 107.2654 | 109.8258 | 109.8666 | 180 | 159.5256 | 109.6984 | 159.7429 | 111.5383 |
| Pg9 (MW) | 109.9653 | 110.5672 | 109.8992 | 110.1816 | 159.992 | 109.8666 | 180 | 109.7499 | 109.9874 | 109.8345 | 161.25395 |
| Pg10 (MW) | 77.6223 | 75.7562 | 77.3992 | 113.9894 | 41.103 | 77.521 | 50.4304 | 76.9438 | 77.0731 | 77.3900 | 40 |
| Pg11 (MW) | 77.6223 | 41.8698 | 77.8364 | 79.7755 | 77.7055 | 77.521 | 50.5304 | 75.4655 | 41.9398 | 77.4070 | 40.000265 |
| Pg12 (MW) | 55 | 92.4789 | 55.2225 | 91.1668 | 94.9768 | 120 | 55 | 92.196 | 58.4508 | 92.3672 | 55.657936 |
| Pg13 (MW) | 55 | 57.514 | 55.0861 | 115.6511 | 55.7143 | 120 | 55 | 91.88 | 91.8472 | 92.3952 | 55.284532 |
| Pg14 (MW) | 83.465 | 82.5628 | 81.7524 | 84.3133 | 83.9536 | 88.3514 | 117.4854 | 85.705 | 81.9727 | 115.8210 | 87.944171 |
| Pg15 (MW) | 40 | 41.4891 | 41.7615 | 40 | 40 | 40.5611 | 45.9281 | 50.039 | 42.2700 | 40.9645 | 41.266255 |
| Pg16 (MW) | 82.7732 | 84.771 | 82.273 | 81.1796 | 85.7133 | 88.3514 | 117.4854 | 100.9829 | 83.4650 | 114.8737 | 84.034893 |
| Pg17 (MW) | 40 | 40.5874 | 40.5599 | 40 | 40 | 40.5611 | 45.9281 | 40.4323 | 41.8894 | 69.3012 | 43.143673 |
| Pg18 (MW) | 10 | 10.001 | 10.0002 | 10 | 10 | 10.0245 | 10.0013 | 10.0284 | 12.2384 | 10.1338 | 11.082469 |
| Pg19 (MW) | 31.4568 | 31.0978 | 31.4679 | 35.097 | 35 | 40.4288 | 42.1109 | 56.1452 | 45.3522 | 48.7160 | 35.044029 |
| Hg14 (MWth) | 106.0991 | 105.6717 | 105.2219 | 106.6588 | 106.4569 | 108.9256 | 125.2754 | 107.4333 | 104.5803 | 124.2764 | 108.69733 |
| Hg15 (MWth) | 75 | 76.2843 | 76.5205 | 74.998 | 74.998 | 75.4844 | 80.1174 | 83.6669 | 76.6280 | 75.7112 | 76.092716 |
| Hg16 (MWth) | 105.789 | 106.9125 | 105.5142 | 104.9002 | 107.4073 | 108.9256 | 125.2754 | 116.0123 | 104.7995 | 123.8075 | 106.47627 |
| Hg17 (MWth) | 75 | 75.5061 | 75.4833 | 74.998 | 74.998 | 75.484 | 80.1174 | 75.3723 | 75.5878 | 100.2936 | 77.714606 |
| Hg 18 (MWth) | 40 | 39.9986 | 39.9999 | 40 | 40 | 40.0104 | 40.0005 | 40.0026 | 40.9191 | 40.0036 | 40.464341 |
| Hg19 (MWth) | 18.3782 | 18.2205 | 18.3944 | 19.7385 | 20 | 22.4676 | 23.2322 | 29.6101 | 24.5560 | 26.2104 | 20.020468 |
| Hg20 (MWth) | 469.7337 | 468.2278 | 468.9043 | 469.3368 | 466.2575 | 458.702 | 415.9515 | 437.9153 | 463.4714 | 399.9754 | 460.53781 |
| Hg 21 (MWth) | 60 | 59.9867 | 59.9994 | 60 | 60 | 60 | 60 | 59.994 | 59.9228 | 59.9258 | 60 |
| Hg22 (MWth) | 60 | 59.9814 | 59.9999 | 60 | 60 | 60 | 60 | 59.9953 | 59.9511 | 59.9020 | 60 |
| Hg23 (MWth) | 120 | 119.6074 | 119.9854 | 119.6511 | 120 | 120 | 120 | 119.9982 | 119.6168 | 119.9138 | 119.99644 |
| Hg24 (MWth) | 120 | 119.603 | 119.9768 | 119.7176 | 119.8823 | 120 | 120 | 119.9996 | 119.9674 | 119.9803 | 120 |
| Sum(Pg) | 2350.26 | 2350 | 2350 | 2350 | 2350 | 2350.0002 | 2349.9 | 2350 | 2350.0000 | 2350.0000 | 2350.00 |
| Sum(Hg) | 1250 | 1250 | 1250 | 1250 | 1250 | 1250 | 1250 | 1250 | 1250.0000 | 1250.0000 | 1250.00 |
| WFC ${ }^{\text {R }}$ | 57846.84 | 58006.999 | 57856.2676 | 58225.745 | 58049.01 | 58122.746 | 59736.2635 | 58173.93 | 58208.0267 | 58739.5241 | 57994.51 |
| WFC ${ }^{\text {C }}$ | 57851.76* | 58007* | 57856.26* | 58225.74 | 58048.56 | 58122.7494 | 59733.8271 | 58173.93 | 58208.0267 | 58739.5241 | 57994.51 |
| Deviation | 4.92 | 0.0008 | 0.0076 | 0.005 | 0.45 | 0.0034 | 2.4364 | - | - | - | - |

The superscript " $R$ " refers to reported value and the superscript " $C$ " refers to calculated value

In addition to that, an assessment of the operating points introduced in Table 3, it is found that the results reported by WOA [7] and MPHS [18] include a great violation on the operating point of many units. It is clearly observed from this assessment that the operating points of CHP units $42,44,45$, $50-53$ and 58-63, which obtained by WOA [7], are outside their acceptable limits. Fig 8 shows sample of violated operating points of CHP units $58-60$. Also, the operating
points provided by MPHS [18] for CHP units 43-45, 47, 5052, 53, 55 and 59-62 are outside their acceptable limits. Fig. 9 provides sample of violated operating points of CHP units 58-60. The convergence characteristics shown in Fig. 10 manifest the superiority, stability and efficiency of the HBOA in finding feasible operating points of all the units and to improve the solution quality in this large system with all constraints achievement.

TABLE 3. Optimal scheduling results of CSED problem of 84-unit system using HBOA and other techniques.

| Unit | WOA [7] | MPHS [18] | MPOA | IMPOA | MRF | JFSOA | SDO | HBOA |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Pg1 (MW) | 110.8794 | 113.9557 | 111.4996 | 111.8429 | 110.8514 | 111.1626385 | 113.5593533 | 113.9999976 |
| Pg2 (MW) | 112.2931 | 113.2521 | 113.9898 | 113.4366 | 112.3562 | 112.5520392 | 109.0508668 | 113.115563 |
| Pg3 (MW) | 98.1159 | 98.8762 | 106.8479 | 97.7637 | 100.7567 | 107.7765112 | 100.2258404 | 103.8372469 |
| Pg4 (MW) | 129.8682 | 179.7497 | 180.728 | 182.4321 | 140.3075 | 179.7375644 | 162.4975687 | 184.8069538 |
| Pg5 (MW) | 88.6586 | 95.803 | 96.9592 | 93.9162 | 91.0444 | 87.78776144 | 91.53858887 | 89.50520793 |
| Pg6 (MW) | 139.9998 | 140 | 106.125 | 139.9973 | 139.9861 | 139.9948775 | 106.3733811 | 106.6482728 |
| Pg7 (MW) | 196.1145 | 268.7403 | 300 | 300 | 260.4044 | 266.7806527 | 259.7062335 | 256.2554544 |
| Pg8 (MW) | 295.0226 | 285.4014 | 289.0542 | 299.6063 | 284.5362 | 290.8186132 | 285.8462781 | 297.0513115 |
| Pg9 (MW) | 284.6146 | 286.4166 | 288.3886 | 288.4402 | 284.6324 | 284.6025908 | 284.6018609 | 299.9953945 |
| Pg10 (MW) | 279.6016 | 205.0934 | 220.5232 | 136.1618 | 204.8627 | 207.4135385 | 207.9820774 | 130 |
| Pg11 (MW) | 318.4002 | 168.8124 | 243.6735 | 243.7775 | 168.3955 | 243.5827337 | 243.2637849 | 169.3090338 |
| Pg12 (MW) | 318.4004 | 168.8851 | 169.2619 | 319.3076 | 243.5983 | 318.3996454 | 316.302264 | 306.094109 |
| Pg13 (MW) | 394.2803 | 394.224 | 484.0939 | 304.5266 | 394.3207 | 304.5241963 | 393.3344712 | 394.5008241 |
| Pg14 (MW) | 484.2888 | 484.035 | 394.2923 | 394.2702 | 484.0415 | 304.5184491 | 393.7099759 | 393.7355999 |
| Pg15 (MW) | 484.0382 | 394.2865 | 394.2815 | 394.2756 | 394.287 | 394.322214 | 394.2903111 | 305.5366609 |
| Pg16 (MW) | 304.6382 | 394.2512 | 394.4309 | 394.3378 | 393.7834 | 304.521412 | 394.0107241 | 394.4500635 |
| Pg17 (MW) | 489.6014 | 489.3601 | 489.4321 | 489.547 | 489.7002 | 489.2867798 | 400.1520446 | 500 |
| Pg18 (MW) | 489.2782 | 489.3589 | 490.2527 | 489.3544 | 399.5418 | 399.4800955 | 490.2136591 | 490.8920464 |
| Pg19 (MW) | 511.9256 | 511.9201 | 331.7721 | 421.5186 | 511.7246 | 511.432115 | 511.2793736 | 514.6259351 |
| Pg20 (MW) | 511.3778 | 511.34 | 511.3554 | 511.3658 | 421.521 | 511.3041636 | 511.0039475 | 525.3542635 |
| Pg21 (MW) | 433.5245 | 525.5076 | 523.4301 | 523.3126 | 525.9069 | 523.3475504 | 521.1386228 | 550 |
| Pg22 (MW) | 433.5316 | 523.5785 | 433.7107 | 549.9264 | 523.3683 | 523.2824448 | 523.1449807 | 548.5299466 |
| Pg23 (MW) | 523.2806 | 523.4343 | 523.6605 | 523.4014 | 523.2229 | 523.2914355 | 523.4973647 | 550 |
| Pg24 (MW) | 523.2888 | 523.7584 | 523.5275 | 525.0759 | 537.5339 | 523.3169144 | 532.1422657 | 521.6137338 |
| Pg25 (MW) | 523.2889 | 523.7573 | 523.4639 | 529.0869 | 523.2575 | 523.306813 | 525.425869 | 522.5635051 |
| Pg26 (MW) | 524.059 | 523.8777 | 523.6618 | 524.6181 | 523.363 | 523.2792844 | 524.750045 | 549.3197361 |
| Pg27 (MW) | 10 | 10.0039 | 52.0712 | 10.3208 | 10.045 | 10.00833641 | 14.82149808 | 14.54018441 |
| Pg28 (MW) | 10 | 10.0903 | 32.943 | 11.976 | 10.0584 | 10.00401295 | 17.55317247 | 10.09827847 |
| Pg29 (MW) | 11.2485 | 10.0012 | 42.5597 | 12.0379 | 10.091 | 10.02394824 | 10.15981108 | 10.90987678 |
| Pg30 (MW) | 91.0437 | 96.9912 | 96.6457 | 91.331 | 95.7828 | 96.95242919 | 88.64142595 | 96.99993948 |
| Pg31 (MW) | 189.9732 | 189.9995 | 190 | 190 | 189.9726 | 181.241749 | 167.7277666 | 180.3914692 |
| Pg32 (MW) | 189.9997 | 189.9932 | 190 | 189.994 | 189.9393 | 189.9953502 | 110.3709413 | 189.8297956 |
| Pg33 (MW) | 163.6655 | 189.9954 | 190 | 189.9604 | 189.9865 | 159.7499057 | 189.7311867 | 181.720502 |
| Pg34 (MW) | 165.1037 | 169.3255 | 174.713 | 184.4522 | 170.7813 | 199.9449699 | 154.7321767 | 199.9999763 |
| Pg35 (MW) | 166.7651 | 199.9967 | 199.9973 | 173.6353 | 199.897 | 199.6705009 | 168.5102131 | 182.9159828 |
| Pg36 (MW) | 165.8941 | 189.6863 | 167.6098 | 165.3069 | 166.564 | 199.991963 | 167.5306785 | 200 |
| Pg37 (MW) | 89.7967 | 110 | 89.3805 | 66.2837 | 90.1237 | 89.86684403 | 95.80067815 | 109.9993619 |
| Pg38 (MW) | 109.9979 | 109.9998 | 60.2502 | 109.9931 | 109.5728 | 109.9917663 | 94.59496126 | 110 |
| Pg39 (MW) | 109.9994 | 109.9919 | 97.059 | 109.9853 | 57.7381 | 94.16960534 | 100.4361961 | 89.83998575 |
| Pg40 (MW) | 516.5065 | 511.4807 | 421.8636 | 511.3409 | 511.31 | 511.3076161 | 511.4129671 | 550 |
| Pg41 (MW) | 112.3421 | 97.1804 | 157.4488 | 155.524 | 146.3775 | 132.3982552 | 88.08453564 | 126.911668 |
| Pg42 (MW) | 50.4459 | 43.397 | 162.3604 | 139.4474 | 112.5172 | 144.4049507 | 176.3682171 | 126.6515375 |
| Pg43 (MW) | 131.6591 | 89.6268 | 119.564 | 147.3521 | 166.6346 | 88.99398926 | 133.3407891 | 115.3838013 |
| Pg44 (MW) | 57.3384 | 45.1847 | 103.2307 | 112.9782 | 101.4222 | 105.2551322 | 82.43229679 | 133.2788296 |
| Pg45 (MW) | 10.0991 | 21.2282 | 41.2788 | 65.9401 | 78.014 | 93.63589835 | 84.39909467 | 42.80196981 |
| Pg46 (MW) | 44.2424 | 48.3113 | 91.2561 | 68.5323 | 102.0881 | 49.07352085 | 80.59064342 | 43.67911921 |
| Pg47 (MW) | 103.2198 | 128.4079 | 87.1336 | 40.2656 | 72.7479 | 76.78719074 | 40.12150597 | 77.28023842 |
| Pg48 (MW) | 40.2287 | 59.2361 | 56.8732 | 74.6718 | 81.453 | 55.66222673 | 40.39987198 | 74.81841571 |
| Pg49 (MW) | 127.0797 | 96.9275 | 101.7154 | 161.1207 | 114.0001 | 167.415806 | 128.5747866 | 99.51927201 |
| Pg50 (MW) | 65.5205 | 56.1951 | 115.7489 | 92.7839 | 115.9933 | 170.7899652 | 153.1704573 | 116.0935991 |
| Pg51 (MW) | 20.6157 | 19.4934 | 111.1813 | 131.9444 | 105.0388 | 167.4168782 | 134.9154112 | 109.3199759 |
| Pg52 (MW) | 56.4012 | 58.3874 | 146.7759 | 129.1619 | 110.138 | 125.329591 | 169.0986537 | 106.0198439 |
| Pg53 (MW) | 168.9969 | 137.1533 | 54.607 | 49.6165 | 58.5187 | 60.42436073 | 57.48988444 | 60.72297981 |
| Pg54 (MW) | 40.4449 | 54.4494 | 69.7127 | 46.5981 | 88.4724 | 79.5458983 | 73.5303264 | 52.58094411 |
| Pg55 (MW) | 101.1777 | 135.4933 | 73.5156 | 48.279 | 90.3362 | 56.36733616 | 47.84016687 | 43.69127173 |
| Pg56 (MW) | 55.7835 | 73.2057 | 63.6804 | 68.7759 | 98.8981 | 66.27287672 | 41.59481573 | 56.18677799 |
| Pg57 (MW) | 10.056 | 22.5826 | 29.6084 | 17.0945 | 11.9271 | 10.526294 | 39.568433 | 12.984927 |
| Pg58 (MW) | 64.4182 | 48.4507 | 32.8538 | 20.8543 | 10.2473 | 13.966558 | 30.601921 | 29.500822 |
| Pg59 (MW) | 152.2648 | 114.4764 | 24.4815 | 20.437 | 10.7945 | 10.242311 | 15.455255 | 10.224582 |
| Pg60 (MW) | 56.1164 | 61.8714 | 34.4688 | 23.6739 | 10.8036 | 18.416857 | 28.854667 | 13.318664 |

TABLE 3. Coninued

| Unit | WOA | MPHS | MPOA | IMPOA | MRF | JFSOA | SDO | HBOA |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Pg61 (MW) | 123.1718 | 134.5865 | 87.7235 | 39.1262 | 46.0268 | 46.39822 | 49.290726 | 37.731189 |
| Pg62 (MW) | 46.9576 | 58.702 | 43.6068 | 52.6339 | 50.9742 | 77.241845 | 74.333519 | 55.419047 |
| Pg63 (MW) | 11.9343 | 14.2665 | 71.0358 | 39.5056 | 46.4064 | 53.473728 | 83.330785 | 38.524705 |
| Pg64 (MW) | 57.1203 | 45.9687 | 46.6292 | 35.7657 | 81.0028 | 57.218278 | 35.547811 | 58.369607 |
| Hg 41 (MWth) | 122.389 | 113.7627 | 147.6562 | 146.6201 | 141.47 | 133.6418 | 108.7418 | 130.36047 |
| Hg 42 (MWth) | 84.017 | 77.8321 | 150.4384 | 137.5875 | 122.4835 | 140.38155 | 157.70193 | 130.22022 |
| Hg 43 (MWth) | 133.2295 | 109.4936 | 126.4326 | 142.0364 | 152.8414 | 109.2841 | 131.03854 | 123.54322 |
| Hg44 (MWth) | 89.9664 | 79.4565 | 117.2753 | 122.7413 | 116.2061 | 118.40557 | 98.006315 | 134.13526 |
| Hg 45 (MWth) | 40.0414 | 44.7352 | 76.1032 | 97.3837 | 107.8045 | 121.30091 | 113.23396 | 77.305255 |
| Hg46 (MWth) | 24.2001 | 26.025 | 119.2437 | 99.63 | 128.5812 | 82.83207 | 110.00064 | 78.021767 |
| Hg 47 (MWth) | 115.7242 | 131.1739 | 115.6884 | 75.23 | 103.2457 | 106.75568 | 75.067676 | 107.1827 |
| Hg48 (MWth) | 75.1974 | 91.5396 | 89.5394 | 104.929 | 110.7559 | 88.52056 | 75.295738 | 105.05633 |
| Hg49 (MWth) | 130.6574 | 113.4407 | 116.4089 | 149.7545 | 123.2778 | 153.28427 | 125.33879 | 115.19298 |
| Hg50 (MWth) | 97.0303 | 88.9429 | 124.3008 | 111.4069 | 124.4197 | 155.14832 | 145.2591 | 124.37381 |
| Hg51 (MWth) | 44.5495 | 44.0508 | 121.7187 | 133.3892 | 118.2344 | 153.29586 | 135.01731 | 120.69336 |
| Hg52 (MWth) | 29.7277 | 30.6049 | 141.5828 | 131.7908 | 121.1505 | 129.39075 | 154.20294 | 118.7926 |
| Hg53 (MWth) | 154.183 | 136.0749 | 87.5945 | 83.2834 | 90.9128 | 92.630772 | 88.899597 | 92.889872 |
| Hg54 (MWth) | 75.3759 | 87.4415 | 100.6428 | 80.6963 | 116.7821 | 109.13766 | 103.85151 | 85.79143 |
| Hg55 (MWth) | 116.1224 | 135.2714 | 103.9151 | 82.1301 | 118.4447 | 89.1249 | 81.352437 | 78.186666 |
| Hg56 (MWth) | 88.6239 | 103.6541 | 95.4345 | 99.841 | 125.8343 | 97.677172 | 76.35822 | 88.97404 |
| Hg57 (MWth) | 39.7995 | 45.3681 | 48.3974 | 43.0398 | 40.8103 | 40.225735 | 52.660052 | 41.279683 |
| Hg 58 (MWth) | 33.3716 | 25.9449 | 49.7936 | 44.6247 | 40.0514 | 41.699032 | 48.821315 | 48.357485 |
| Hg 59 (MWth) | 144.7767 | 123.3418 | 46.1677 | 44.4733 | 40.3305 | 40.103128 | 41.545489 | 40.096243 |
| Hg60 (MWth) | 88.9117 | 93.8406 | 50.4183 | 45.8603 | 40.3306 | 43.604568 | 48.016149 | 41.229832 |
| Hg61 (MWth) | 128.4364 | 134.6699 | 43.9544 | 21.857 | 25.0017 | 25.175411 | 26.454331 | 21.184123 |
| Hg62 (MWth) | 81.0055 | 91.1235 | 23.9094 | 27.9878 | 27.233 | 39.198929 | 37.819152 | 27.168868 |
| Hg63 (MWth) | 40.8289 | 41.6933 | 36.3734 | 22.0418 | 24.4381 | 28.384611 | 41.713801 | 21.602582 |
| Hg64 (MWth) | 30.0546 | 24.7914 | 21.4939 | 17.154 | 40.905 | 30.097467 | 13.095769 | 25.847086 |
| Hg65 (MWth) | 383.1144 | 389.4986 | 282.7322 | 327.5015 | 340.3042 | 347.86975 | 334.4616 | 397.96444 |
| Hg66 (MWth) | 59.9997 | 59.9997 | 369.2619 | 390.4617 | 338.8826 | 349.52939 | 394.53059 | 394.08612 |
| Hg67 (MWth) | 60 | 59.998 | 386.0055 | 392.0802 | 340.7755 | 349.73186 | 358.54799 | 400.20396 |
| Hg68 (MWth) | 119.9855 | 119.9399 | 367.6143 | 384.5644 | 339.2679 | 344.06369 | 394.30008 | 401.44516 |
| Hg69 (MWth) | 119.999 | 119.9995 | 60 | 59.9984 | 59.9823 | 59.994889 | 59.69648 | 60 |
| Hg 70 (MWth) | 402.653 | 388.5823 | 59.9999 | 59.9836 | 59.8608 | 59.977035 | 59.639208 | 59.363243 |
| Hg 71 (MWth) | 59.8978 | 59.9997 | 59.9974 | 60 | 59.9941 | 59.992207 | 59.908384 | 59.861172 |
| Hg72 (MWth) | 59.9998 | 59.989 | 59.9976 | 59.9957 | 59.9949 | 59.814627 | 59.623394 | 60 |
| Hg73 (MWth) | 119.9996 | 119.8977 | 59.9994 | 59.9999 | 59.9795 | 59.983075 | 59.847266 | 58.883967 |
| Hg74 (MWth) | 119.9999 | 119.9265 | 60 | 60 | 59.9874 | 59.999159 | 59.859142 | 59.541711 |
| Hg75 (MWth) | 385.534 | 385.8941 | 59.9985 | 60 | 59.9795 | 59.900677 | 59.848149 | 59.812732 |
| Hg76 (MWth) | 58.9464 | 59.9999 | 59.9998 | 59.9969 | 59.8799 | 59.980839 | 59.705253 | 60 |
| Hg 77 (MWth) | 59.9995 | 59.9967 | 119.9998 | 119.9984 | 119.9313 | 119.99507 | 119.87857 | 120 |
| Hg78 (MWth) | 119.7982 | 119.997 | 119.9925 | 120 | 119.9494 | 119.99397 | 119.36566 | 120 |
| Hg79 (MWth) | 119.9988 | 119.8126 | 119.9821 | 119.9984 | 119.8432 | 119.99531 | 114.40442 | 119.95288 |
| Hg 80 (MWth) | 381.8642 | 402.4022 | 119.9979 | 119.9414 | 119.9641 | 119.93083 | 119.90201 | 120 |
| Hg 81 (MWth) | 59.9999 | 59.9995 | 119.9999 | 120 | 119.9284 | 119.95831 | 119.48437 | 119.99997 |
| Hg 82 (MWth) | 59.9991 | 59.9995 | 119.9829 | 120 | 119.9682 | 119.99584 | 119.97149 | 119.4424 |
| Hg83 (MWth) | 119.998 | 120 | 119.9556 | 120 | 119.998 | 119.99688 | 118.70615 | 119.99989 |
| Hg84 (MWth) | 119.9982 | 119.8582 | 119.9993 | 119.9905 | 119.9835 | 119.99574 | 118.82723 | 111.95649 |
| Sum (Pg) | 12700 | 12700.01 | 12700 | 12700 | 12700 | 12700 | 12700 | 12700 |
| Sum (Hg) | 5000.005 | 5000.064 | 5000 | 5000 | 5000 | 5000 | 5000 | 5000 |
| WFC (\$) | 290123.97* | 288157.43* | 294717.7 | 289903.8 | 291225.6 | 290323.82 | 292788.5 | 289822.39 |



FIGURE 8. Sample of violated operating point of CHP units 57-60 by WOA.


FIGURE 9. Sample of violated operating points of CHP units 57-60 by MPHS.


FIGURE 10. Convergence characteristics of HBOA versus other recent optimizaion techniques for the CSED problem of the 84-unit system.

## D. SIMULATION RESULTS OF LARGE-SCALE TEST SYSTEM

The 96 -unit system represents a large-scale test system, which can be used to assess the scalability, stability and efficiency of the proposed technique. The load and heat demand of this test system are 12700 MW and 5000 MWth, respectively. Additionally, it includes 20 heat units, 52 thermal units, and 24 CHP units as obtained from [7]. Table 4 gives the optimal unit scheduling using the proposed techniques as well as other relevant techniques such as WOA [7], WVO_PSO [12], MRF [40], MPOA, IMPOA, SDO and

JFSOA. By simulating the result, it can be observed that the obtained optimal WFC (235102.65 \$) achieved by the proposed HBOA is lower than the other reported techniques. The calculated WFC of other techniques WOA, WVO_PSO, MRF, MPOA, IMPOA, SDO and JFSOA are, respectively, 236702.97 \$, 235789.2 \$, 235541.4 \$, 236283.1 \$, 235260.3 \$, 236185.18 \$ and 235277.05 \$.

Similar to previous test systems, the operational points for the findings presented in Table 4 by WOA [7] and WVO_PSO [12] are reviewed. This evaluation demonstrates that the operating point supplied by WOA [7] is possible
with precise WFC since the difference between its stated and calculated values is negligible. In contrast, however WVO_PSO [12] provides suitable operating point for all units, a significant difference is remarked between the stated WFC value of 235789.2 \$ and the computed 238005.79 \$.

The convergence characteristics, shown in Fig. 11, ensures that the proposed HBOA is capable to find feasible operating points accurately for all units and to improve the solution quality for such large-scale system.

TABLE 4. Optimal scheduling of the CSED problem for 96-unit system by HBOA and other techniques.

| Unit | WOA | WVO_PSO | MRF | MPOA | IMPOA | SDO | JFSOA | HBOA |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Pg1 (MW) | 361.6989 | 179.5201 | 538.5341 | 628.3176 | 448.7995 | 448.7294 | 628.1374 | 537.254715 |
| Pg2 (MW) | 225.5924 | 149.7683 | 224.4012 | 224.5265 | 299.2027 | 224.5646 | 224.3853 | 341.5738074 |
| Pg3 (MW) | 224.3946 | 229.3011 | 299.1921 | 149.5909 | 224.4507 | 149.8704 | 224.5230 | 151.1408777 |
| Pg4 (MW) | 110.8030 | 109.8671 | 109.9002 | 60.0035 | 63.4146 | 109.7272 | 109.8640 | 109.0977628 |
| Pg5 (MW) | 110.0061 | 109.8682 | 109.9178 | 159.7684 | 109.7907 | 160.7416 | 109.8715 | 64.10866612 |
| Pg6 (MW) | 159.6156 | 116.8034 | 109.8556 | 66.8003 | 159.8008 | 159.1329 | 159.7649 | 110.0480942 |
| Pg7 (MW) | 109.8526 | 159.7348 | 109.8524 | 111.3558 | 111.3255 | 114.5665 | 159.7332 | 94.33638081 |
| Pg8 (MW) | 110.3586 | 159.7467 | 109.8904 | 109.8511 | 161.0556 | 109.7220 | 110.0519 | 60.00000879 |
| Pg9 (MW) | 160.0915 | 109.8668 | 159.7286 | 109.8603 | 160.0759 | 109.4680 | 110.3982 | 108.2875293 |
| Pg10 (MW) | 114.8006 | 114.8000 | 77.3946 | 77.3244 | 116.7355 | 87.2949 | 110.6273 | 115.4310336 |
| Pg11 (MW) | 114.9807 | 77.4926 | 77.4030 | 40.2655 | 75.2831 | 79.0857 | 77.4864 | 49.01342421 |
| Pg12 (MW) | 92.3721 | 92.4009 | 55.0881 | 118.7892 | 91.1026 | 92.7083 | 92.3981 | 92.02655803 |
| Pg13 (MW) | 92.4525 | 92.4049 | 92.3770 | 92.3969 | 92.4673 | 92.5555 | 92.4291 | 55.18840658 |
| Pg14 (MW) | 359.0854 | 452.2910 | 449.1396 | 628.3182 | 359.0546 | 359.0618 | 358.9390 | 360.9754061 |
| Pg15 (MW) | 225.1433 | 227.3241 | 224.4004 | 224.3978 | 75.3023 | 220.8337 | 224.4711 | 299.3860657 |
| Pg16 (MW) | 229.5487 | 0.0000 | 149.5993 | 224.4013 | 224.4307 | 299.2436 | 227.8984 | 359.9221971 |
| Pg17 (MW) | 164.3339 | 159.7347 | 109.8854 | 159.5379 | 61.8602 | 109.9312 | 109.9684 | 159.7120077 |
| Pg18 (MW) | 160.2151 | 159.7471 | 109.8918 | 159.7402 | 61.3299 | 159.7770 | 109.9616 | 109.3602599 |
| Pg19 (MW) | 159.9402 | 109.8670 | 109.8124 | 159.8021 | 60.5627 | 109.5956 | 109.8707 | 110.4137284 |
| Pg20 (MW) | 109.9636 | 111.7779 | 109.8538 | 159.8253 | 159.7782 | 159.5821 | 110.4373 | 101.8946306 |
| Pg21 (MW) | 146.0127 | 109.8666 | 155.7131 | 109.7960 | 60.0354 | 159.6577 | 109.9683 | 109.8648707 |
| Pg22 (MW) | 120.0047 | 109.8668 | 109.9640 | 113.2404 | 109.9942 | 162.2520 | 109.8613 | 179.4592083 |
| Pg23 (MW) | 77.9996 | 40.1131 | 77.4163 | 59.6964 | 71.2542 | 78.1863 | 114.7916 | 40.13034648 |
| Pg24 (MW) | 77.4223 | 77.4045 | 77.4663 | 113.7356 | 114.7235 | 80.1193 | 77.4448 | 77.22962911 |
| Pg25 (MW) | 92.4849 | 92.4197 | 92.4246 | 89.9064 | 56.5095 | 89.5740 | 92.4135 | 66.61283247 |
| Pg26 (MW) | 93.9975 | 119.9998 | 55.0084 | 92.7438 | 62.1826 | 92.9623 | 92.4438 | 91.02277492 |
| Pg27 (MW) | 359.1802 | 359.0494 | 359.0726 | 359.0086 | 628.3175 | 359.4461 | 359.4659 | 359.4498313 |
| Pg28 (MW) | 299.9979 | 360.0000 | 299.1270 | 150.2002 | 299.3138 | 150.0325 | 149.5652 | 299.4259674 |
| Pg29 (MW) | 224.7272 | 299.2987 | 224.4124 | 149.6189 | 299.2129 | 299.2876 | 224.7536 | 289.8301184 |
| Pg30 (MW) | 159.8239 | 159.7332 | 159.7033 | 159.9275 | 159.9439 | 109.8718 | 109.9432 | 161.92495 |
| Pg31 (MW) | 160.1449 | 109.8760 | 159.7413 | 159.6800 | 165.6381 | 110.2902 | 109.7440 | 107.7669533 |
| Pg32 (MW) | 124.3426 | 113.2579 | 109.8501 | 109.9803 | 159.7031 | 109.8811 | 109.8997 | 159.4640725 |
| Pg33 (MW) | 159.7681 | 159.7332 | 160.5856 | 109.9906 | 149.1821 | 159.9922 | 112.3437 | 162.5783378 |
| Pg34 (MW) | 109.9525 | 60.0000 | 109.8423 | 110.0943 | 159.9684 | 111.7247 | 118.7835 | 159.9001838 |
| Pg35 (MW) | 110.0723 | 109.8666 | 109.8916 | 109.4116 | 109.9135 | 109.8613 | 109.9580 | 60.03756814 |
| Pg36 (MW) | 78.0018 | 77.5396 | 77.7225 | 46.7352 | 40.3663 | 78.5714 | 114.8045 | 113.7051781 |
| Pg37 (MW) | 77.4827 | 77.4056 | 77.4192 | 53.6586 | 44.5586 | 77.7615 | 77.4993 | 114.3819608 |
| Pg38 (MW) | 66.6749 | 92.8907 | 92.4623 | 75.0300 | 55.1223 | 92.3722 | 92.6306 | 94.29271341 |
| Pg39 (MW) | 92.4487 | 92.4059 | 55.0310 | 106.6037 | 57.4248 | 92.6213 | 92.3919 | 92.65458803 |
| Pg40 (MW) | 180.3641 | 359.0391 | 448.7942 | 90.1856 | 538.5549 | 449.2151 | 448.8086 | 448.3942915 |
| Pg41 (MW) | 224.5898 | 299.6630 | 149.6504 | 299.1623 | 224.4471 | 224.4453 | 150.3797 | 297.8196029 |
| Pg42 (MW) | 224.4866 | 227.5653 | 224.4160 | 299.2613 | 312.9394 | 231.2957 | 299.2035 | 146.2329552 |
| Pg43 (MW) | 109.6226 | 110.1121 | 109.8406 | 109.8768 | 60.8300 | 109.5049 | 111.2975 | 110.6346294 |
| Pg44 (MW) | 109.8456 | 109.8666 | 109.7623 | 171.5053 | 72.3844 | 109.5823 | 110.3086 | 161.7767837 |
| Pg45 (MW) | 65.8336 | 159.7363 | 126.5520 | 159.7403 | 109.9107 | 111.7111 | 159.5163 | 61.66518066 |
| Pg46 (MW) | 159.6880 | 109.8794 | 109.9766 | 116.2378 | 60.2285 | 60.5083 | 109.8988 | 108.9737886 |
| Pg47 (MW) | 116.6052 | 159.7332 | 109.8817 | 114.3100 | 113.1684 | 109.8581 | 109.9034 | 110.4947542 |
| Pg48 (MW) | 159.2737 | 159.7339 | 109.8483 | 109.8951 | 159.8220 | 159.0074 | 110.2078 | 109.9050818 |
| Pg49 (MW) | 78.0268 | 76.2477 | 77.4184 | 95.4460 | 111.7253 | 78.2198 | 77.3975 | 42.02833221 |
| Pg50 (MW) | 77.5176 | 77.4035 | 77.4013 | 44.6056 | 45.3943 | 77.3337 | 77.3324 | 72.25178996 |
| Pg51 (MW) | 92.8301 | 92.4021 | 92.4220 | 119.9983 | 92.5237 | 106.8704 | 92.7631 | 93.0608812 |
| Pg52 (MW) | 85.2843 | 92.4705 | 92.6009 | 113.3472 | 63.6064 | 86.4035 | 93.1231 | 92.57317097 |
| Pg53 (MW) | 111.0331 | 82.5222 | 97.9354 | 117.3331 | 104.0096 | 118.8776 | 104.9832 | 104.4403419 |
| Pg54 (MW) | 68.5580 | 68.4792 | 58.3116 | 68.6504 | 67.0915 | 53.4688 | 45.1316 | 47.60359863 |
| Pg55 (MW) | 95.6342 | 119.0174 | 114.8980 | 102.2794 | 83.8374 | 107.2267 | 100.2247 | 88.44497134 |
| Pg56 (MW) | 60.8717 | 88.1603 | 46.6792 | 58.7652 | 50.0228 | 65.5615 | 51.7195 | 50.14389995 |
| Pg57 (MW) | 23.7299 | 10.3817 | 26.4381 | 14.1789 | 24.4103 | 31.9267 | 19.0094 | 12.03402877 |
| Pg58 (MW) | 41.3490 | 45.4655 | 36.6545 | 40.9762 | 36.0253 | 61.9135 | 56.3032 | 45.6270289 |
| Pg59 (MW) | 139.9448 | 118.4227 | 96.2230 | 113.1150 | 115.7964 | 90.7926 | 125.4849 | 91.41295489 |
| Pg60 (MW) | 46.8012 | 71.0473 | 63.5999 | 53.7449 | 64.8510 | 43.2561 | 51.9022 | 51.86266145 |

TABLE 4. Continued

| Unit | WOA | WVO_PSO | MRF | MPOA | IMPOA | SDO | JFSOA | HBOA |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| Pg61 (MWth) | 116.4269 | 101.8515 | 151.5302 | 86.5572 | 117.3994 | 81.8343 | 91.1831 | 110.2162351 |
| Pg62 (MWth) | 79.0704 | 51.5329 | 77.3787 | 65.4999 | 45.0354 | 59.3903 | 54.1472 | 47.62594622 |
| Pg63 (MWth) | 33.3173 | 10.0378 | 18.0933 | 24.1485 | 16.7891 | 25.694449 | 34.655934 | 18.272772 |
| Pg64 (MWth) | 59.7067 | 53.7073 | 53.0368 | 43.3566 | 55.8517 | 56.936262 | 75.627888 | 45.155884 |
| Pg65 (MWth) | 100.4416 | 121.9513 | 143.9952 | 112.1218 | 87.8497 | 87.130209 | 148.19343 | 88.829361 |
| Pg66 (MWth) | 74.1967 | 51.078 | 42.2496 | 71.6779 | 40.3985 | 42.186016 | 75.326734 | 43.74986 |
| Pg67 (MWth) | 120.2089 | 113.1537 | 109.9469 | 100.193 | 116.9941 | 133.38585 | 100.58341 | 95.340679 |
| Pg68 (MWth) | 61.1074 | 62.0878 | 79.6617 | 44.4974 | 56.0185 | 41.93974 | 42.252459 | 67.297804 |
| Pg69 (MWth) | 11.0113 | 11.5777 | 11.1333 | 22.2128 | 24.1531 | 14.52258 | 17.202374 | 11.638244 |
| Pg70 (MWth) | 69.0899 | 60.2592 | 43.7086 | 59.5941 | 38.9905 | 75.86932 | 47.915618 | 35.028678 |
| Pg71 (MWth) | 122.8034 | 175.0835 | 109.1474 | 99.2229 | 99.5488 | 120.0605 | 99.938915 | 87.384128 |
| Pg72 (MWth) | 49.235 | 72.1306 | 48.6566 | 41.7852 | 42.1765 | 54.261261 | 67.954604 | 53.523651 |
| Pg73 (MWth) | 115.1764 | 162.3515 | 86.7681 | 119.2587 | 112.1269 | 81.155384 | 88.257959 | 106.28473 |
| Pg74 (MWth) | 43.7841 | 103.5256 | 68.0141 | 81.821 | 47.7477 | 72.80805 | 53.319524 | 65.916093 |
| Pg75 (MWth) | 10.2339 | 15.3356 | 11.9507 | 17.8853 | 32.1539 | 37.150914 | 11.427801 | 12.449246 |
| Pg76 (MWth) | 36.5443 | 53.933 | 66.4527 | 53.6216 | 35.9992 | 48.038435 | 47.191073 | 35.002312 |
| Hg53 (MWth) | 121.6489 | 105.6543 | 114.2979 | 125.1893 | 117.711 | 125.89573 | 118.25717 | 117.8187 |
| Hg54 (MWth) | 99.6187 | 99.5846 | 90.7973 | 99.7322 | 98.3872 | 86.623498 | 79.425882 | 81.187872 |
| Hg55 (MWth) | 112.9517 | 126.1351 | 123.8053 | 116.7319 | 106.3889 | 119.40622 | 115.58344 | 108.9226 |
| Hg56 (MWth) | 92.9964 | 116.5743 | 80.764 | 91.1999 | 83.6523 | 97.065314 | 85.11551 | 83.024668 |
| Hg57 (MWth) | 45.8595 | 40.1635 | 47.0447 | 41.7913 | 46.1404 | 49.359884 | 43.852427 | 40.490709 |
| Hg58 (MWth) | 22.7261 | 24.7569 | 20.75 | 22.7167 | 20.4638 | 32.210403 | 29.679309 | 24.658925 |
| Hg59 (MWth) | 137.7969 | 125.8014 | 113.3423 | 122.8215 | 124.3203 | 110.03896 | 129.76254 | 109.94761 |
| Hg60 (MWth) | 80.8243 | 101.8014 | 95.3629 | 86.8657 | 96.4468 | 77.583713 | 85.274518 | 83.140535 |
| Hg61 (MWth) | 124.6482 | 116.5017 | 144.3816 | 107.9191 | 125.2277 | 105.26715 | 110.51193 | 120.77066 |
| Hg62 (MWth) | 108.7221 | 84.9557 | 107.2666 | 97.0135 | 79.3429 | 91.719635 | 87.148267 | 81.226938 |
| Hg63 (MWth) | 49.9249 | 40.0162 | 43.4612 | 46.0603 | 42.9032 | 46.723011 | 50.561873 | 39.67663 |
| Hg64 (MWth) | 31.1612 | 28.5033 | 28.1852 | 23.7898 | 29.4584 | 29.958547 | 38.46521 | 23.984424 |
| Hg65 (MWth) | 115.7042 | 127.7816 | 140.1516 | 122.2641 | 108.623 | 108.20305 | 142.50587 | 108.20515 |
| Hg66 (MWth) | 104.5144 | 84.563 | 76.942 | 102.3467 | 75.3435 | 76.881232 | 105.49604 | 77.96514 |
| Hg67 (MWth) | 126.7515 | 122.8444 | 121.0448 | 115.5599 | 124.9926 | 134.10702 | 115.78643 | 112.78318 |
| Hg68 (MWth) | 93.2126 | 94.0672 | 109.2356 | 78.881 | 88.8282 | 76.66424 | 76.943631 | 98.469051 |
| Hg69 (MWth) | 40.4142 | 40.6761 | 40.4834 | 45.2319 | 46.059 | 41.911104 | 43.086924 | 40.664318 |
| Hg70 (MWth) | 35.4892 | 31.4813 | 23.9564 | 31.165 | 21.8116 | 38.131156 | 25.870776 | 18.721909 |
| Hg71 (MWth) | 128.2524 | 157.5991 | 120.5929 | 115.0262 | 115.2096 | 126.69978 | 115.42154 | 106.59216 |
| Hg72 (MWth) | 82.9501 | 102.7366 | 82.4639 | 76.5419 | 76.8665 | 87.306805 | 99.131229 | 86.491633 |
| Hg73 (MWth) | 123.9395 | 150.454 | 108.0163 | 126.2709 | 122.2632 | 104.24372 | 108.86068 | 118.09871 |
| Hg74 (MWth) | 78.2418 | 129.8383 | 99.1825 | 111.1027 | 81.6887 | 103.31798 | 86.491059 | 97.014286 |
| Hg75 (MWth) | 39.1403 | 42.2867 | 40.3153 | 43.3791 | 49.4927 | 51.36668 | 40.610943 | 40.89038 |
| Hg76 (MWth) | 20.494 | 28.6059 | 34.2964 | 28.4563 | 20.4515 | 25.864199 | 25.540248 | 18.353784 |
| Hg77 (MWth) | 407.0662 | 348.9612 | 391.0097 | 375.3134 | 377.6032 | 403.45436 | 400.14747 | 385.57459 |
| Hg78 (MWth) | 59.7975 | 59.995 | 59.9818 | 60 | 59.9931 | 59.826923 | 59.999812 | 59.99772 |
| Hg79 (MWth) | 59.9911 | 60 | 59.9956 | 60 | 59.9998 | 59.981337 | 59.922567 | 60 |
| Hg80 (MWth) | 119.2458 | 120 | 119.997 | 120 | 119.9932 | 119.96359 | 119.99872 | 118.82035 |
| Hg81 (MWth) | 119.9549 | 119.9998 | 119.9965 | 119.9983 | 119.9901 | 119.43458 | 119.97171 | 119.99892 |
| Hg82 (MWth) | 390.1828 | 353.8826 | 389.1268 | 396.2707 | 428.6265 | 420.97925 | 402.77116 | 438.22049 |
| Hg83 (MWth) | 59.9729 | 59.9998 | 59.9812 | 60 | 59.9993 | 59.990183 | 59.993989 | 60 |
| Hg84 (MWth) | 59.9805 | 60 | 59.9953 | 59.9995 | 60 | 58.276324 | 59.997954 | 59.858041 |
| Hg85 (MWth) | 119.9912 | 120 | 119.9933 | 120 | 119.9995 | 117.79733 | 119.99343 | 119.95149 |
| Hg86 (MWth) | 119.6621 | 120 | 119.6262 | 120 | 119.9998 | 119.9557 | 119.99908 | 118.46015 |
| Hg87 (MWth) | 368.0236 | 410.0497 | 388.0796 | 400.6778 | 439.4129 | 383.92681 | 399.42417 | 450.50341 |
| Hg88 (MWth) | 59.9905 | 60 | 59.9083 | 59.9985 | 59.9992 | 59.689402 | 59.977283 | 59.784083 |
| Hg89 (MWth) | 59.9849 | 60 | 59.992 | 59.9999 | 60 | 59.968445 | 59.991776 | 59.830588 |
| Hg90 (MWth) | 119.9669 | 120 | 119.9921 | 120 | 120 | 119.9354 | 119.9983 | 119.99947 |
| Hg91 (MWth) | 119.905 | 119.8377 | 119.9924 | 120 | 119.9998 | 119.92609 | 119.99419 | 118.32769 |
| Hg92 (MWth) | 378.9529 | 323.882 | 386.2081 | 409.6858 | 412.3361 | 411.99124 | 398.46975 | 451.81766 |
| Hg93 (MWth) | 59.5063 | 60 | 59.9914 | 60 | 59.9984 | 59.908463 | 59.998636 | 60 |
| Hg94 (MWth) | 59.9907 | 59.9996 | 59.9989 | 59.9996 | 59.9998 | 59.996388 | 59.985181 | 59.916361 |
| Hg95 (MWth) | 119.9904 | 120 | 119.998 | 119.9997 | 119.9853 | 118.45831 | 119.98799 | 119.99988 |
| Hg96 (MWth) | 119.8607 | 120 | 119.9958 | 119.9999 | 119.991 | 119.99085 | 119.99335 | 119.83914 |
| Sum(Pg) | 9400.033 | 9399.99 | 9400 | 9400 | 9400 | 9400 | 9400 | 9400 |
| Sum(Hg) | 5000 | 4999.99 | 5000 | 5000 | 5000 | 5000 | 5000 | 5000 |
| WFC ${ }^{\text {R }}$ | 236,699.15 | 238005.79 \# | 235541.4 | 236283.1 | 235260.3 | 236185.18 | 235277.05 | 235102.65 |
| WFC ${ }^{\text {c }}$ | 236702.97 | 235789.2 | 235541.4 | 236283.1 | 235260.3 | 236185.18 | 235277.05 | 235102.65 |



FIGURE 11. Convergence characteristics of HBOA versus other recent optimizaion techniques for the CSED problem of the 96-unit system.

## V. CONCLUSION

This paper has been successfully implemented the HBOA for solving the CSED problem. This problem has an economic benefits and reduction of negative environmental effects in case of its optimal solution achievement. HBOA is designed using adaptable penalty formulas to find optimal and feasible operational conditions of heat or power only units and cogeneration combined heat and power units. Based on the infeasible solutions' distance from the next feasible border, it penalizes them with various degrees. Diverse pillars are studied in the CSED issue with inclusion of transmission losses and valve-point effects. HBOA is employed on 4, 24, 84 and 96 -unit systems with diverse power and thermal demands. HBOA efficacy for 4-unit and 24-unit test systems is proven. Also, HBOA is applied on the large-scale test systems, 84 and 96 -unit test systems, where the results ensure the scalability, efficiency and stability of the proposed techniques as compared with other techniques. In addition, the HBOA success in achieving the optimal solution without any violation of the operating point of any scheduled unit.

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