Positionality and strategy improvement for continuous payoffs

A. Kozachinskiy

University of Warwick, Coventry, UK

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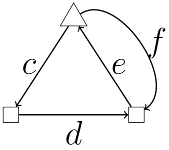
Infinite games on finite graphs

Fix a function $\phi\colon C^\omega \to \mathbb{R}$ (called a *payoff*). A ϕ-game consist of:

- 

$c, d, e, f \in C$
- Min and Max are shifting a pebble (Min in $\triangle$-nodes and Max in $\square$-nodes) along the edges. Infinitely many shifts.
- Zero sum: Min pays Max a fine of size $\phi(c_1 c_2 c_3 \ldots)$, where $c_1, c_2, c_3, \ldots$ are colors along trajectory of the pebble.

Definition

A payoff ϕ is <u>positional</u> if in all ϕ-games players can play optimally via a positional strategy.

Continuous payoffs

Definition

A payoff $\phi\colon C^\omega \to \mathbb{R}$ is continuous if for any $\alpha \in C^\omega$ and for any infinite sequence $\beta_1, \beta_2, \beta_3, \ldots \in C^\omega$ the following holds. Assume that for any $i \in \mathbb{N}$ we have that α and β_i coincide in the first i elements. Then

$$\phi(\alpha) = \lim_{i \to \infty} \phi(\beta_i)$$

Can be defined by the cylinder topology, which is compact.

Examples: (multi)discounted payoff is continuous, Parity and Mean Payoff are not.

Characterizing positional payoffs

A payoff $\phi\colon C^\omega \to \mathbb{R}$ is called **prefix-monotone** if there are no $x, y \in C^*$ and $\alpha, \beta \in C^\omega$ such that

$$\phi(x\alpha) > \phi(x\beta), \qquad \phi(y\alpha) < \phi(y\beta).$$

Theorem

Let $\phi\colon C^\omega \to \mathbb{R}$ be a continuous payoff. Then ϕ is positional if and only if ϕ is prefix-monotone.

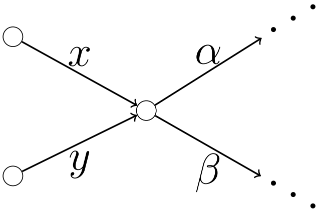


Figure: The "only if" part.

What else can be said

Generalizing some results for (multi)discounted payoffs.

- strategy improvement (all continuous positional payoffs)
- LP-type problems and subexponential randomized algorithms (all continuous positional payoffs).
- Strong bounds on strategy improvement (for generalized or non-linear discounted payoff).

What about stochastic games?

- Continuous + stochastically positional $\implies$ (multi)discounted.

Thank you!