reachability in fixed dimension VASS

vector addition systems
with states

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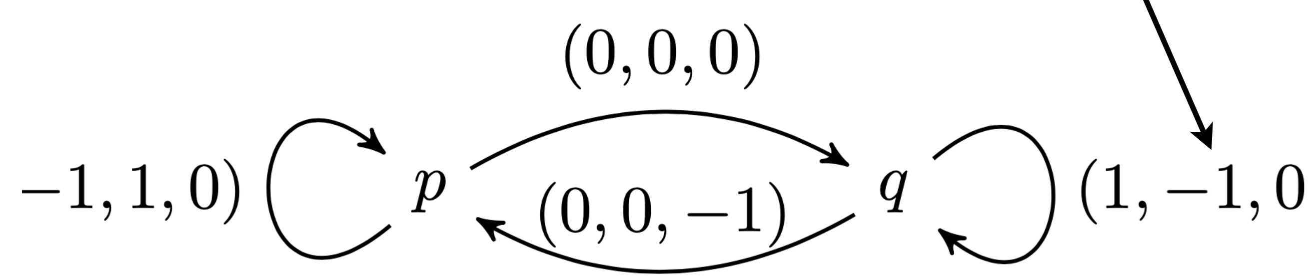
numbers given
in binary or in unary

a VASS of dimension 3:



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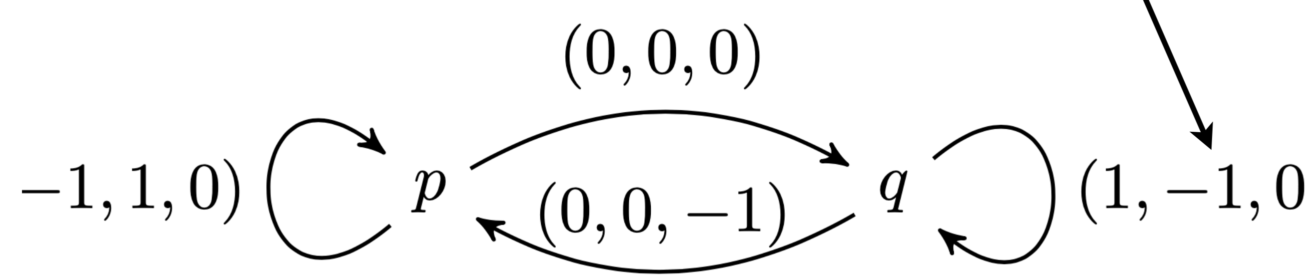

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the length of **the shortest run**
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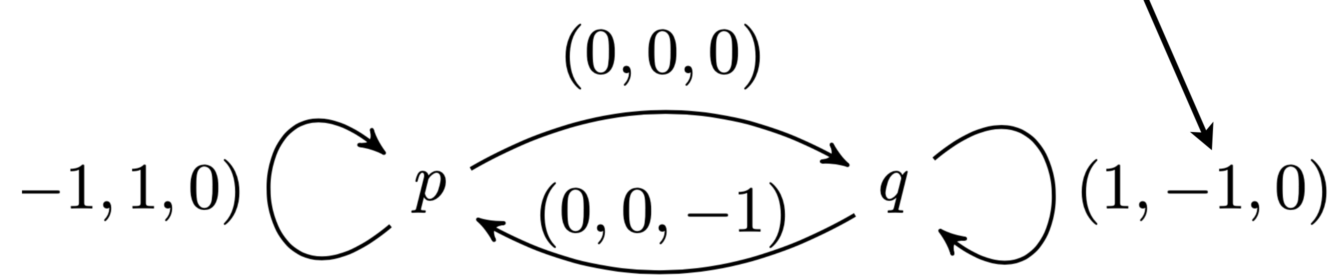

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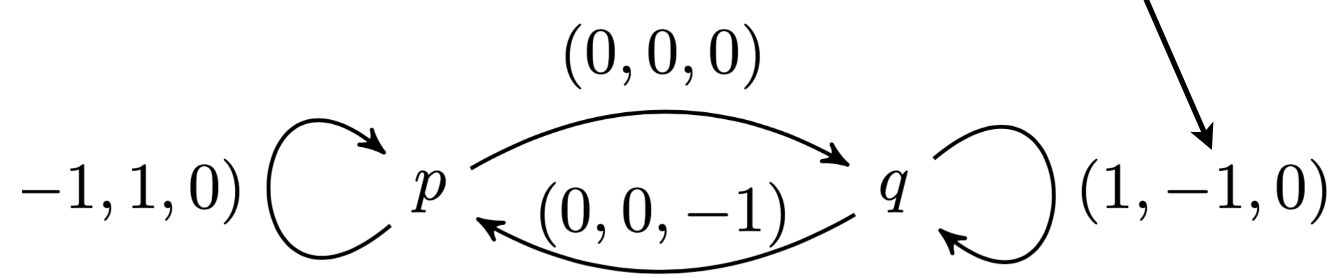
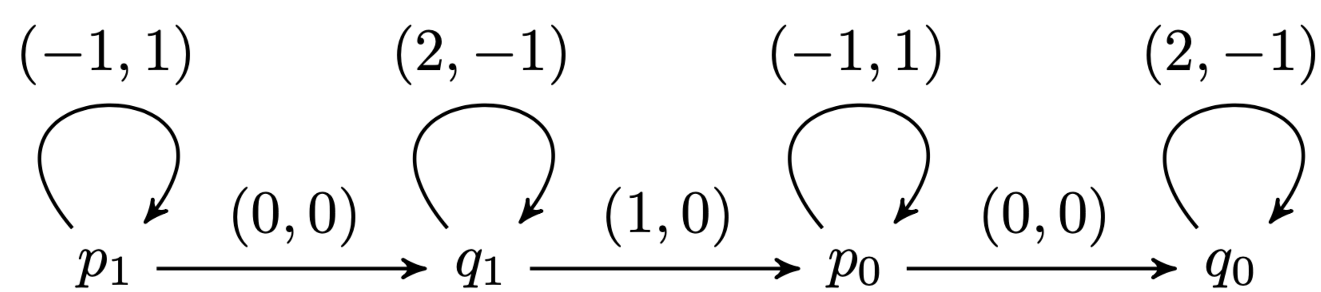


a **flat** (no nested loops) VASS of dimension 2:



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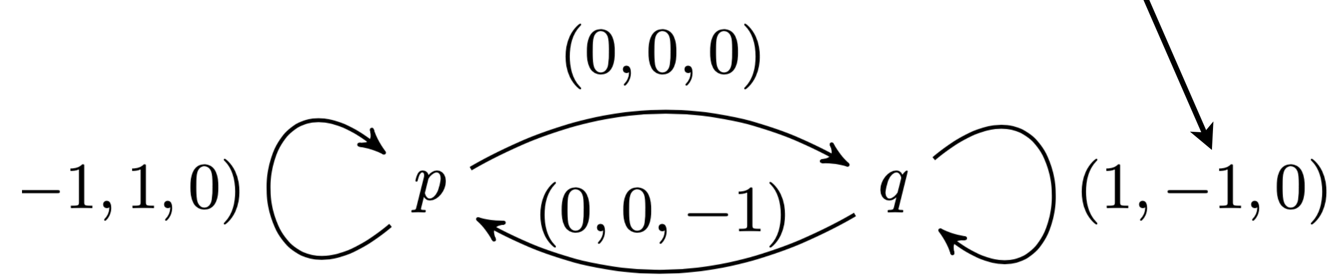

in dimension 2,
every VASS is **flattable**

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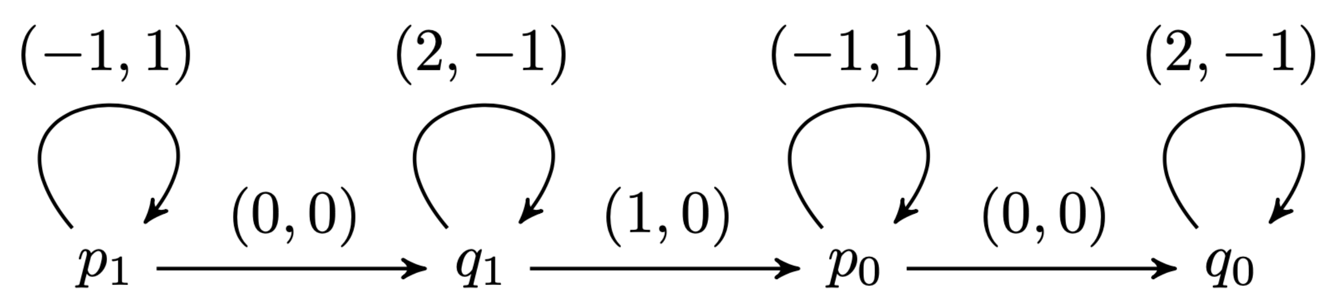

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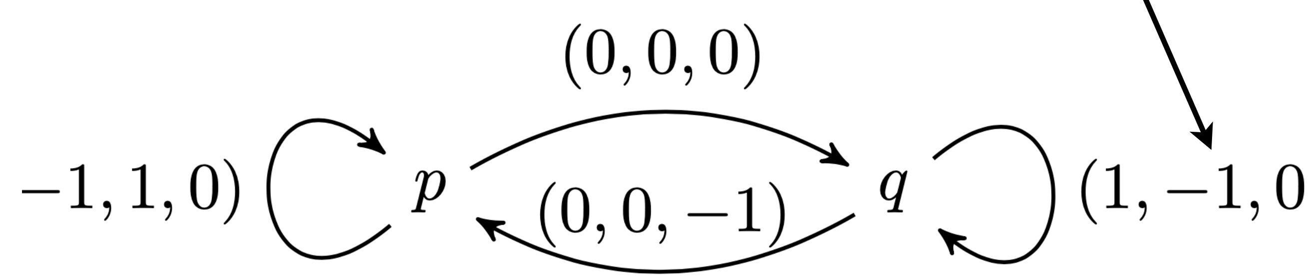

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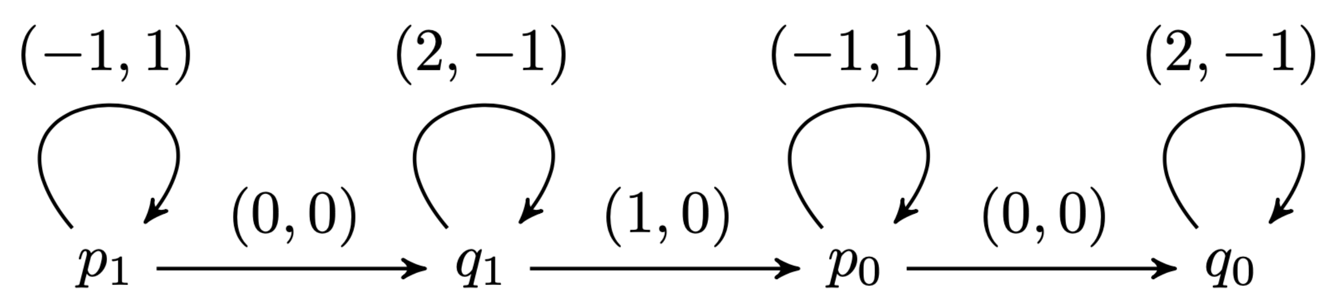

our results: improved lower bounds

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the shortest run:

	unary flat	unary	binary
dim 1	**poly**$^{\vee}$	**poly**$^{\vee}$	**exp**$^{\vee}$
dim 2	**poly**$^{\vee}$	**poly**$^{\vee}$	**exp**$^{\vee}$
dim 3	**exp**$^{\vee}$		
dim 4	**exp**$^{\vee}$		
dim 5	**exp**$^{\vee}$		
…	**exp**$^{\vee}$		
dim 13	**exp**$^{\vee}$	**exp^**	**2-exp^**
dim 14	**exp**$^{\vee}$	**2-exp^**	**3-exp^**
dim 15	**exp**$^{\vee}$	**3-exp^**	**4-exp^**
…	**…**	**…**	**…**

} due to flattability (dim 1, dim 2)

} underlies TOWER lower bound (dim 13 onwards)

$^{\vee}$upper bounds

^lower bounds

our results: improved lower bounds

the shortest run:

	unary flat	unary	binary
dim 1	**poly**$^{\vee}$	**poly**$^{\vee}$	**exp**$^{\vee}$
dim 2	**poly**$^{\vee}$	**poly**$^{\vee}$	**exp**$^{\vee}$
dim 3	**exp**$^{\vee\wedge}$	**exp^**	
dim 4	**exp**$^{\vee}$		**2-exp^**
dim 5	**exp**$^{\vee}$		
…	**exp**$^{\vee}$		
dim 13	**exp**$^{\vee}$	**exp^**	**2-exp^**
dim 14	**exp**$^{\vee}$	**2-exp^**	**3-exp^**
dim 15	**exp**$^{\vee}$	**3-exp^**	**4-exp^**
…	**…**	**…**	**…**

} due to flattability (dim 1–2)

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^lower bounds

our results: improved lower bounds

the shortest run:

	unary flat	unary	binary
dim 1	poly$^{\vee}$	poly$^{\vee}$	exp$^{\vee}$
dim 2	poly$^{\vee}$	poly$^{\vee}$	exp$^{\vee}$
dim 3	exp$^{\vee\wedge}$	exp^	
dim 4	exp$^{\vee}$		2-exp^
dim 5	exp$^{\vee}$		
…	exp$^{\vee}$		
dim 13	exp$^{\vee}$	exp^	2-exp^
dim 14	exp$^{\vee}$	2-exp^	3-exp^
dim 15	exp$^{\vee}$	3-exp^	4-exp^
…	…	…	…

} due to flattability (dim 1–2)

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$^{\vee}$upper bounds

^lower bounds

complexity of reachability:

	unary flat	unary	binary
dim 1	NL*	NL*	NP*
dim 2	NL*	NL*	PSPACE*
dim 3			
dim 4			
dim 5	NP*		
…	NP*		
dim 13	NP*	PSPACE^	EXPSPACE^
dim 14	NP*	EXPSPACE^	2-EXPSPACE^
dim 15	NP*	2-EXPSPACE^	3-EXPSPACE^
…	…	…	…

*complete

^hard

our results: improved lower bounds

the shortest run:

	unary flat	unary	binary
dim 1	**poly**$^\vee$	**poly**$^\vee$	**exp**$^\vee$
dim 2	**poly**$^\vee$	**poly**$^\vee$	**exp**$^\vee$
dim 3	**exp**$^{\vee\wedge}$	**exp^**	**?**
dim 4	**exp**$^\vee$		**2-exp^**
dim 5	**exp**$^\vee$		
…	**exp**$^\vee$		
dim 13	**exp**$^\vee$	**exp^**	**2-exp^**
dim 14	**exp**$^\vee$	**2-exp^**	**3-exp^**
dim 15	**exp**$^\vee$	**3-exp^**	**4-exp^**
…	**…**	**…**	**…**

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	unary flat	unary	binary
dim 1	**NL***	**NL***	**NP***
dim 2	**NL***	**NL***	**PSPACE***
dim 3	**?**	**?**	**?**
dim 4	**?**	**?**	**?**
dim 5	**NP***	**?**	**?**
…	**NP***	**?**	**?**
dim 13	**NP***	**PSPACE^**	**EXPSPACE^**
dim 14	**NP***	**EXPSPACE^**	**2-EXPSPACE^**
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…	**…**	**…**	**…**

*complete
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our results: improved lower bounds

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	unary flat	unary	binary
dim 1	**poly**$^{\vee}$	**poly**$^{\vee}$	**exp**$^{\vee}$
dim 2	**poly**$^{\vee}$	**poly**$^{\vee}$	**exp**$^{\vee}$
dim 3	**exp**$^{\vee}$**^**	**exp^**	**?**
dim 4	**exp**$^{\vee}$		**2-exp^**
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dim 1	**NL***	**NL***	**NP***
dim 2	**NL***	**NL***	**PSPACE***
dim 3	**?**	**?**	**?**
dim 4	**?**	**?**	**?**
dim 5	**NP***	**?**	**?**
…	**NP***	**?**	**?**
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…	**…**	**…**	**…**

*complete

^hard

thank you!