The Strahler number of a Parity Game

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Highlights - 2020

Parity Games

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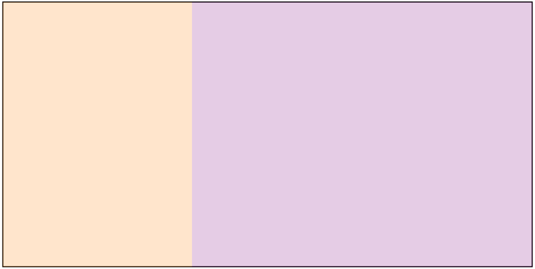
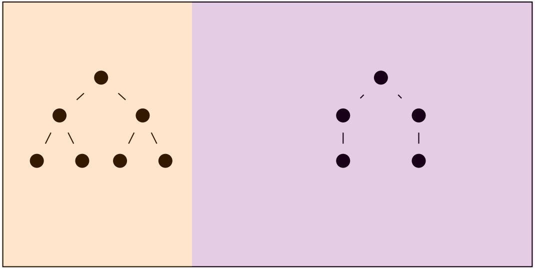
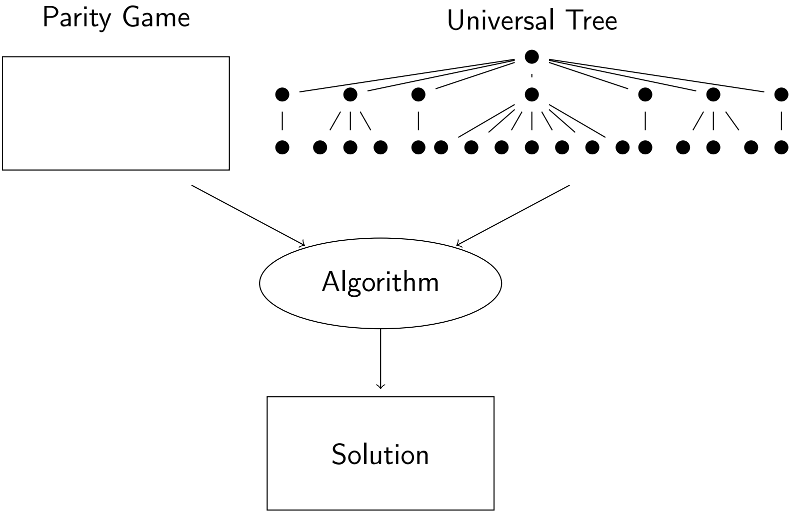


Parity Games



Quasipolynomial Algorithms for Parity Games

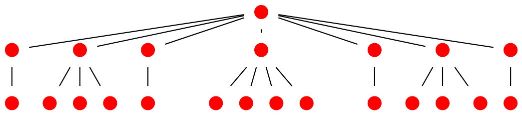


Quasipolynomial Algorithms for Parity Games

An *ordered* tree is (n, h)-Universal if any *ordered* tree embeds into it as long as it has

- height at most h
- at most n leaves.

Example of a $(4, 2)$-Universal tree



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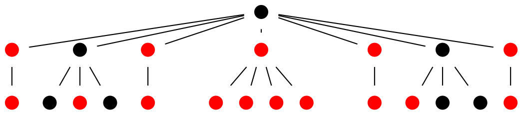

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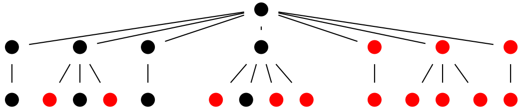

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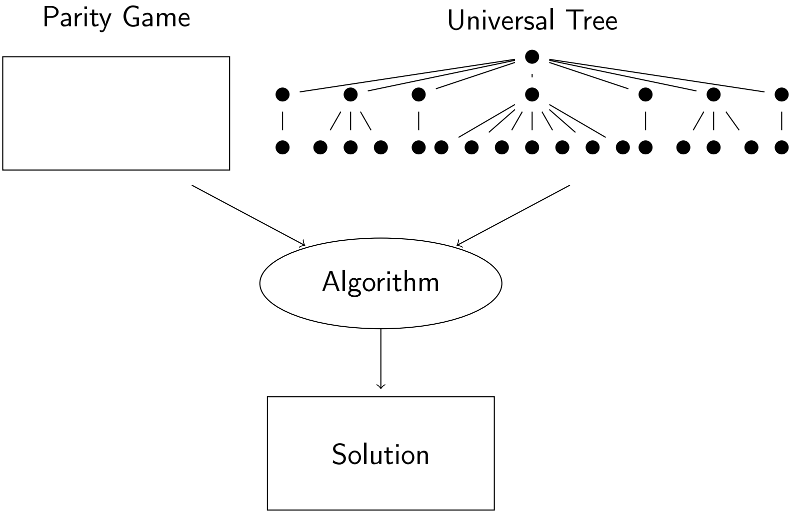

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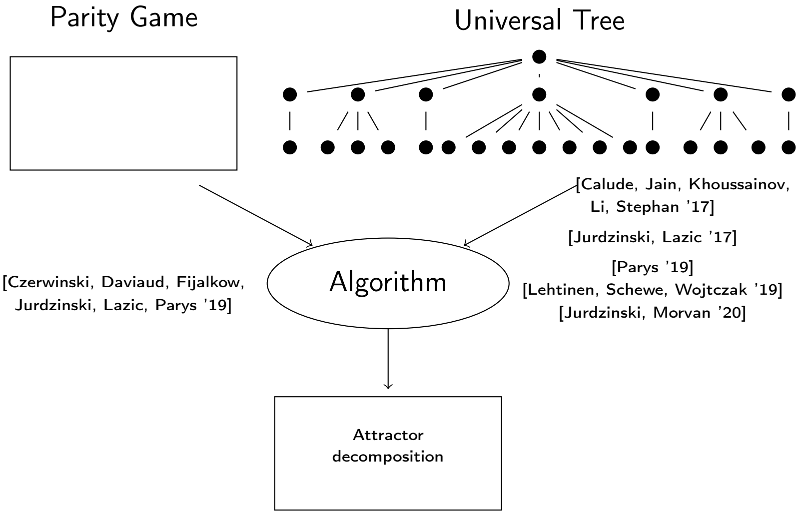
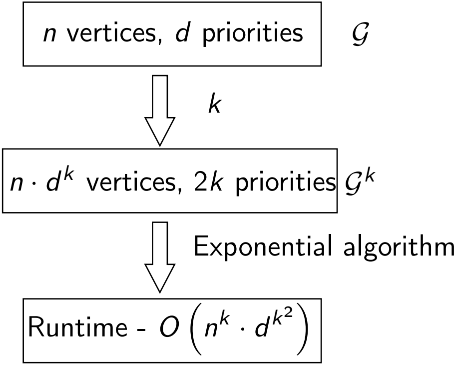


Quasipolynomial Algorithms for Parity Games

Register Games: [Lehtinen '18], [Lehtinen, Boker '19]



Corollary [Lehtinen '18]

There is an $O(n^{\log n} \cdot d^{\log^2 n})$ algorithm to solve parity games.

A result

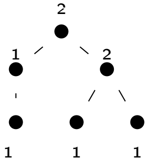
Theorem:

Register Number = Strahler Number

Strahler Number

Definition

The Strahler number of a rooted tree is the largest height of a perfect binary tree that is its minor.



$$str(u) = \begin{cases} \max\{str(v) \mid v \text{ is a child of } u\} & \text{if maximum is unique,} \\ \max\{str(v) \mid v \text{ is a child of } u\} + 1 & \text{otherwise} \end{cases}$$

Strahler Universal trees

Definition

An ordered tree $\mathcal{T}$ is k-Strahler (n, h)-Universal if any ordered tree with

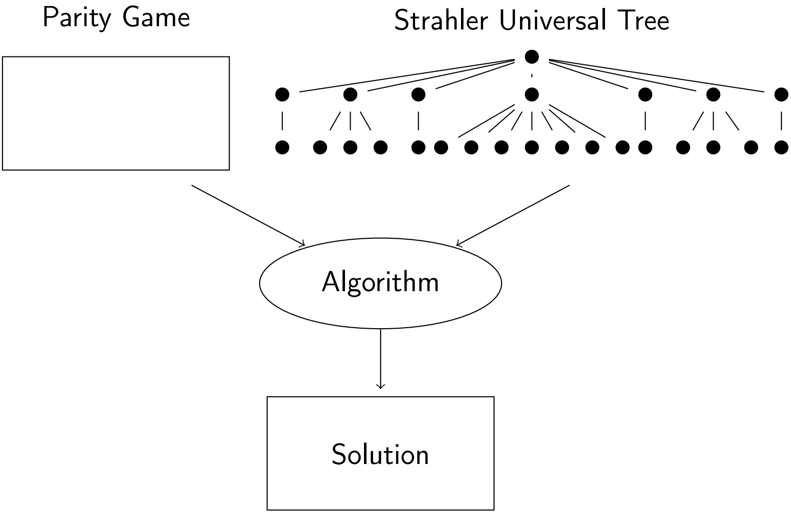
- at most n vertices,
- height at most h
- Strahler number no more than k

can be embedded in it.

Size

There are Strahler universal trees of size $O(poly(n) \cdot h^k)$

Algorithms for Parity Games



A polynomial time algorithm

Theorem

Given k, the Strahler number of a Parity game, we can find the winning sets for Audrey and Steven in time $poly(n) \cdot \left(\frac{d}{k}\right)^k$ and quasi-linear space.

Corollary

Solving parity games is polynomial if $k \cdot \lg\left(\frac{d}{k}\right) = O(\log n)$.
(Previously known for $k = O(1)$ and $d = O(\lg n)$)