A Dichotomy for Homomorphism-Closed Queries on Probabilistic Graphs

Antoine Amarilli[1] and İsmail İlkan Ceylan[2]

September 16, 2020

[1]Télécom Paris

[2]University of Oxford

Uncertain data management

In this talk, we manage data represented as a labeled graph

Uncertain data management

In this talk, we manage data represented as a labeled graph

WorksAt	
Antoine	Télécom Paris
Antoine	Paris Sud
Benny	Paris Sud
Benny	Technion
İsmail	U. Oxford

Uncertain data management

In this talk, we manage data represented as a labeled graph

WorksAt	
Antoine	Télécom Paris
Antoine	Paris Sud
Benny	Paris Sud
Benny	Technion
İsmail	U. Oxford

MemberOf	
Télécom Paris	ParisTech
Télécom Paris	IP Paris
Paris Sud	IP Paris
Paris Sud	Paris-Saclay
Technion	CESAER

Uncertain data management

In this talk, we manage data represented as a labeled graph

WorksAt	
Antoine	Télécom Paris
Antoine	Paris Sud
Benny	Paris Sud
Benny	Technion
İsmail	U. Oxford

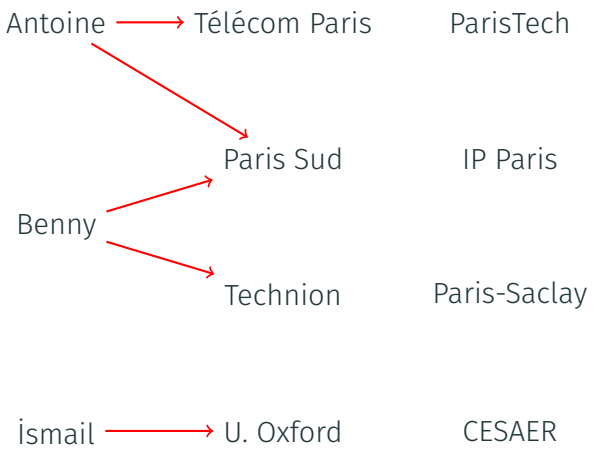
MemberOf	
Télécom Paris	ParisTech
Télécom Paris	IP Paris
Paris Sud	IP Paris
Paris Sud	Paris-Saclay
Technion	CESAER

Uncertain data management

In this talk, we manage **data** represented as a **labeled graph**

WorksAt	
Antoine	Télécom Paris
Antoine	Paris Sud
Benny	Paris Sud
Benny	Technion
İsmail	U. Oxford

MemberOf	
Télécom Paris	ParisTech
Télécom Paris	IP Paris
Paris Sud	IP Paris
Paris Sud	Paris-Saclay
Technion	CESAER

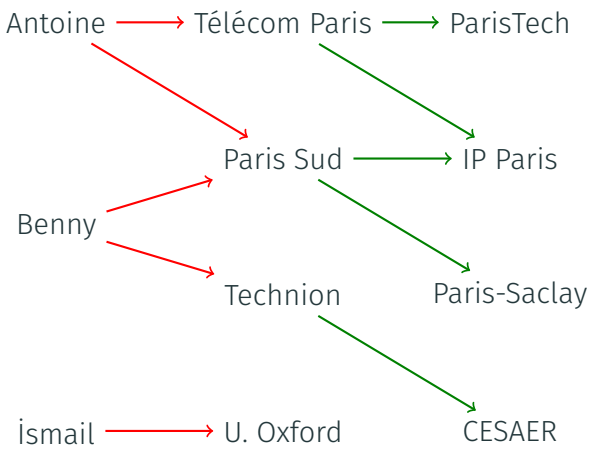


Uncertain data management

In this talk, we manage **data** represented as a **labeled graph**

WorksAt	
Antoine	Télécom Paris
Antoine	Paris Sud
Benny	Paris Sud
Benny	Technion
İsmail	U. Oxford

MemberOf	
Télécom Paris	ParisTech
Télécom Paris	IP Paris
Paris Sud	IP Paris
Paris Sud	Paris-Saclay
Technion	CESAER

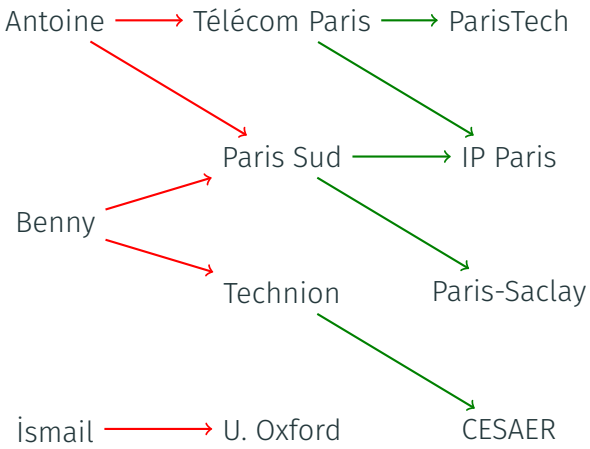


Uncertain data management

In this talk, we manage **data** represented as a **labeled graph**

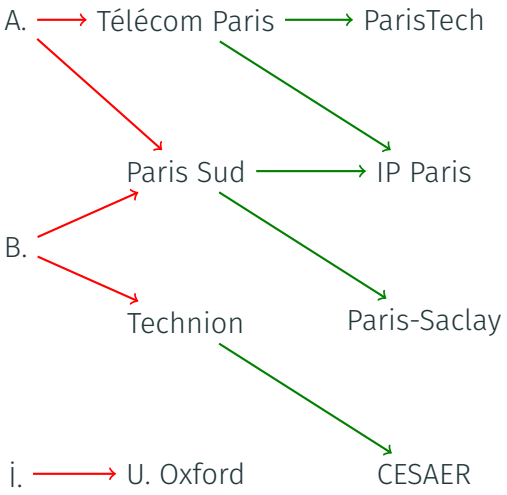
WorksAt	
Antoine	Télécom Paris
Antoine	Paris Sud
Benny	Paris Sud
Benny	Technion
İsmail	U. Oxford

MemberOf	
Télécom Paris	ParisTech
Télécom Paris	IP Paris
Paris Sud	IP Paris
Paris Sud	Paris-Saclay
Technion	CESAER



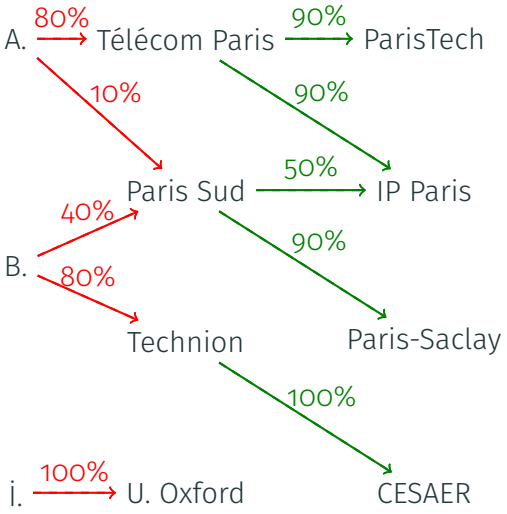
→ **Problem:** we are not **certain** about the true state of the data

Uncertain data model



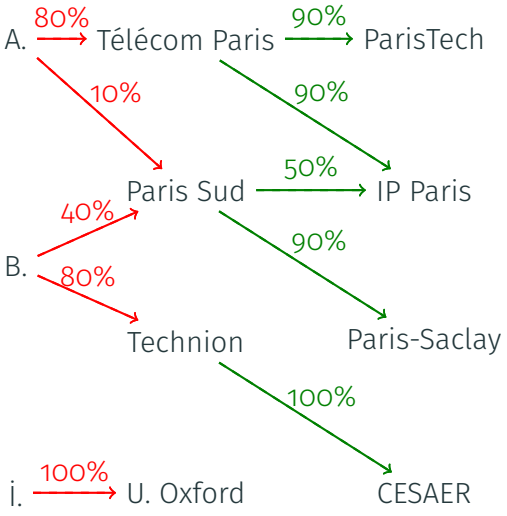
- Uncertain data model: **TID**, for **tuple-independent database**
- Each fact (edge) carries a **probability**

Uncertain data model



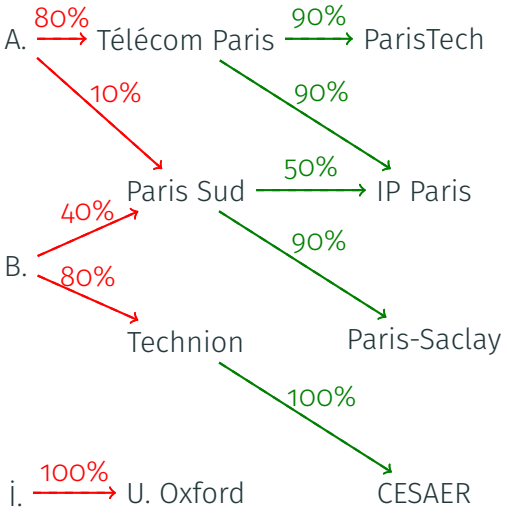
- Uncertain data model: **TID**, for **tuple-independent database**
- Each fact (edge) carries a **probability**

Uncertain data model



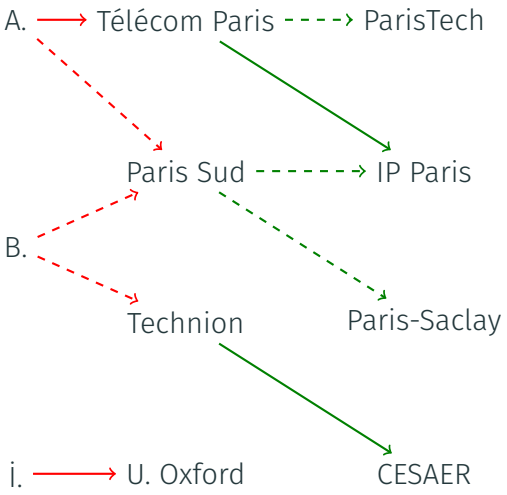
- Uncertain data model: **TID**, for **tuple-independent database**
- Each fact (edge) carries a **probability**
- Each fact exists with its given **probability**
- All facts are **independent**

Uncertain data model



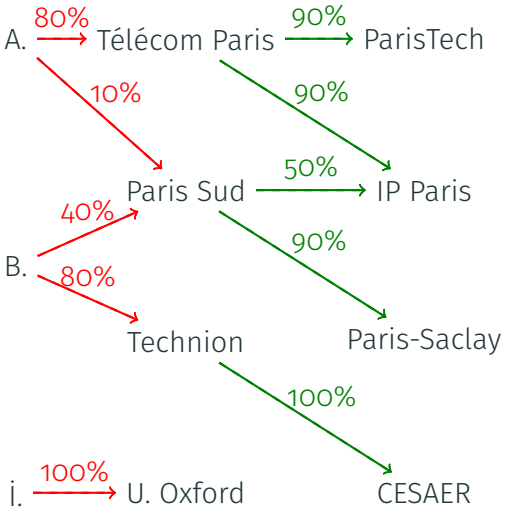
- Uncertain data model: **TID**, for **tuple-independent database**
- Each fact (edge) carries a **probability**
- Each fact exists with its given **probability**
- All facts are **independent**
- **Possible world *W*:** subset of facts

Uncertain data model



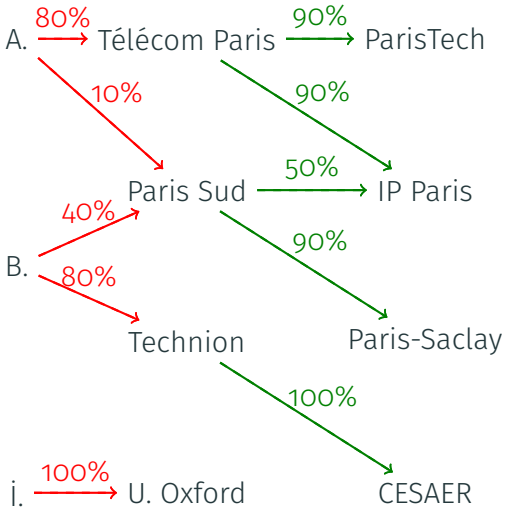
- Uncertain data model: **TID**, for **tuple-independent database**
- Each fact (edge) carries a **probability**
- Each fact exists with its given **probability**
- All facts are **independent**
- **Possible world *W*:** subset of facts

Uncertain data model



- Uncertain data model: **TID**, for **tuple-independent database**
- Each fact (edge) carries a **probability**
- Each fact exists with its given **probability**
- All facts are **independent**
- **Possible world *W*:** subset of facts
- **Probability** of *W*:

Uncertain data model



- Uncertain data model: **TID**, for **tuple-independent database**
- Each fact (edge) carries a **probability**
- Each fact exists with its given **probability**
- All facts are **independent**
- **Possible world *W*:** subset of facts
- **Probability** of *W*:

$$\Pr(W) = \left(\prod_{F \in W} \Pr(F)\right) \times \left(\prod_{F \notin W} (1 - \Pr(F))\right)$$

Homomorphism-closed queries

- **Query:** maps a graph (**without probabilities**) to YES/NO

Homomorphism-closed queries

- **Query:** maps a graph (**without probabilities**) to YES/NO
- **Conjunctive query** (CQ): can I find a match of a **pattern**? e.g., $x \longrightarrow y \longrightarrow z$

Homomorphism-closed queries

- **Query:** maps a graph (**without probabilities**) to YES/NO
- **Conjunctive query** (CQ): can I find a match of a **pattern**? e.g., $x \longrightarrow y \longrightarrow z$
- **Union of conjunctive queries** (UCQ): does **one of the CQs** match?

Homomorphism-closed queries

- **Query:** maps a graph (**without probabilities**) to YES/NO
- **Conjunctive query** (CQ): can I find a match of a **pattern**? e.g., $x \longrightarrow y \longrightarrow z$
- **Union of conjunctive queries** (UCQ): does **one of the CQs** match?

→ **Homomorphism-closed query** Q: if G satisfies Q and G has a homomorphism to G' then G' also satisfies Q

Homomorphism-closed queries

- **Query:** maps a graph (**without probabilities**) to YES/NO
- **Conjunctive query** (CQ): can I find a match of a **pattern**? e.g., $x \longrightarrow y \longrightarrow z$
- **Union of conjunctive queries** (UCQ): does **one of the CQs** match?

→ **Homomorphism-closed query** Q: if G satisfies Q and G has a homomorphism to G' then G' also satisfies Q

They generalize **CQs** and **UCQs**, but also **regular path queries** (RPQs), **Datalog**, etc.

Problem statement: Probabilistic query evaluation (PQE)

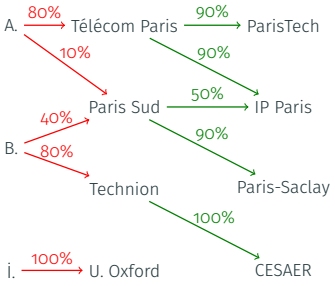
Here is the problem $\mathrm{PQE}(Q)$:

- We fix a query Q: $x \longrightarrow y \longrightarrow z$

Problem statement: Probabilistic query evaluation (PQE)

Here is the problem $\mathrm{PQE}(Q)$:

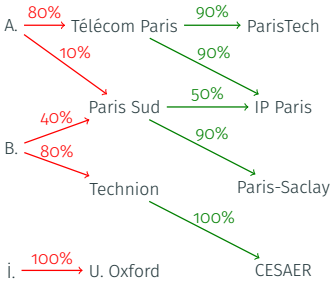
- We **fix** a query Q: $x \longrightarrow y \longrightarrow z$
- The **input** is a TID D:



Problem statement: Probabilistic query evaluation (PQE)

Here is the problem $\mathrm{PQE}(Q)$:

- We **fix** a query Q: $x \longrightarrow y \longrightarrow z$
- The **input** is a TID D:

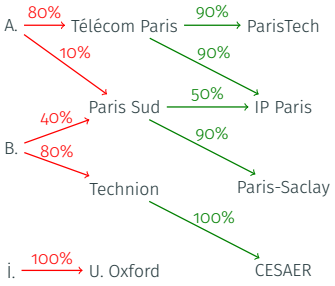


- The **output** is the **probability** that the query is true

Problem statement: Probabilistic query evaluation (PQE)

Here is the problem $\mathrm{PQE}(Q)$:

- We **fix** a query Q: $x \longrightarrow y \longrightarrow z$
- The **input** is a TID D:



- The **output** is the **probability** that the query is true

→ **Question**: What is the complexity of $\mathrm{PQE}(Q)$ depending on the query Q?

Results on PQE

Existing dichotomy on the **unions of conjunctive queries** (UCQs):

Theorem [Dalvi and Suciu, 2012]

- *Some UCQs* Q *are* ***safe*** *and* $\mathrm{PQE}(Q)$ *is in* ***PTIME***
- *All others are* ***unsafe*** *and* $\mathrm{PQE}(Q)$ *is* ***#P-hard***

Results on PQE

Existing dichotomy on the **unions of conjunctive queries** (UCQs):

Theorem [Dalvi and Suciu, 2012]

- *Some UCQs Q are **safe** and $\mathrm{PQE}(Q)$ is in **PTIME***
- *All others are **unsafe** and $\mathrm{PQE}(Q)$ is **#P-hard***

We study PQE for **homomorphism-closed queries** and show:

Results on PQE

Existing dichotomy on the **unions of conjunctive queries** (UCQs):

Theorem [Dalvi and Suciu, 2012]

- *Some UCQs Q are **safe** and $\mathrm{PQE}(Q)$ is in **PTIME***
- *All others are **unsafe** and $\mathrm{PQE}(Q)$ is **#P-hard***

We study PQE for **homomorphism-closed queries** and show:

Theorem [Amarilli and Ceylan, 2020]

*For any **query Q closed under homomorphisms**:*

- *Either Q is equivalent to a **safe UCQ** and $\mathrm{PQE}(Q)$ is in **PTIME***

Results on PQE

Existing dichotomy on the **unions of conjunctive queries** (UCQs):

Theorem [Dalvi and Suciu, 2012]

- *Some UCQs Q are **safe** and $\mathrm{PQE}(Q)$ is in **PTIME***
- *All others are **unsafe** and $\mathrm{PQE}(Q)$ is **#P-hard***

We study PQE for **homomorphism-closed queries** and show:

Theorem [Amarilli and Ceylan, 2020]

*For any **query Q closed under homomorphisms**:*

- *Either Q is equivalent to a **safe UCQ** and $\mathrm{PQE}(Q)$ is in **PTIME***
- *In all other cases, $\mathrm{PQE}(Q)$ is **#P-hard***

Results on PQE

Existing dichotomy on the **unions of conjunctive queries** (UCQs):

Theorem [Dalvi and Suciu, 2012]

- *Some UCQs Q are **safe** and $\mathrm{PQE}(Q)$ is in **PTIME***
- *All others are **unsafe** and $\mathrm{PQE}(Q)$ is **#P-hard***

We study PQE for **homomorphism-closed queries** and show:

Theorem [Amarilli and Ceylan, 2020]

*For any **query Q closed under homomorphisms**:*

- *Either Q is equivalent to a **safe UCQ** and $\mathrm{PQE}(Q)$ is in **PTIME***
- *In all other cases, $\mathrm{PQE}(Q)$ is **#P-hard***

So **bad news**: all homomorphism-closed queries are **hard** except safe UCQs

What's next?

- The result only applies to graphs, not higher-arity databases
  - We conjecture that it holds for arbitrary arity

What's next?

- The result only applies to **graphs**, not higher-arity databases
  - We **conjecture** that it holds for arbitrary arity
- Adapting to **unweighted** PQE, where all probabilities are $1/2$?
  - We have a recent result on **non-hierarchical self-join-free CQs** [Amarilli and Kimelfeld, 2020]
  - Recent paper by Suciu and Kenig [Kenig and Suciu, 2020]

What's next?

- The result only applies to **graphs**, not higher-arity databases
  - We **conjecture** that it holds for arbitrary arity
- Adapting to **unweighted** PQE, where all probabilities are $1/2$?
  - We have a recent result on **non-hierarchical self-join-free CQs** [Amarilli and Kimelfeld, 2020]
  - Recent paper by Suciu and Kenig [Kenig and Suciu, 2020]

Thanks for your attention!

tcs4f.org

nofreeviewnoreview.org

References i

Amarilli, A. and Ceylan, I. I. (2020).
A dichotomy for homomorphism-closed queries on probabilistic graphs.
In *ICDT*.

Amarilli, A. and Kimelfeld, B. (2020).
Uniform reliability of self-join-free conjunctive queries.
arXiv preprint arXiv:1908.07093.

Dalvi, N. and Suciu, D. (2012).
The dichotomy of probabilistic inference for unions of conjunctive queries.
J. ACM, 59(6).

References ii

Kenig, B. and Suciu, D. (2020).
A dichotomy for the generalized model counting problem for unions of conjunctive queries.
arXiv preprint arXiv:2008.00896.