The Complexity of Bounded Context Switching with Dynamic Thread Creation

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HIGHLIGHTS 2020 (originally ICALP 2020)

Dynamic Networks of Concurrent Pushdown Systems (DCPS)

Model Features:

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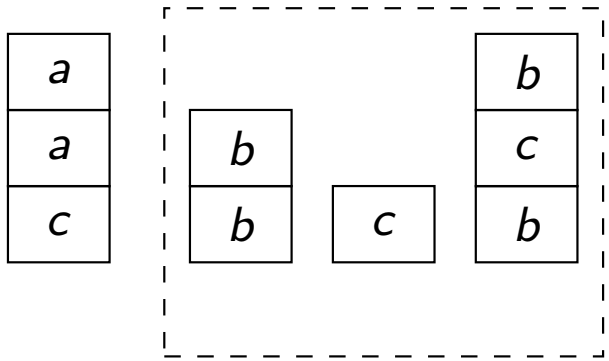

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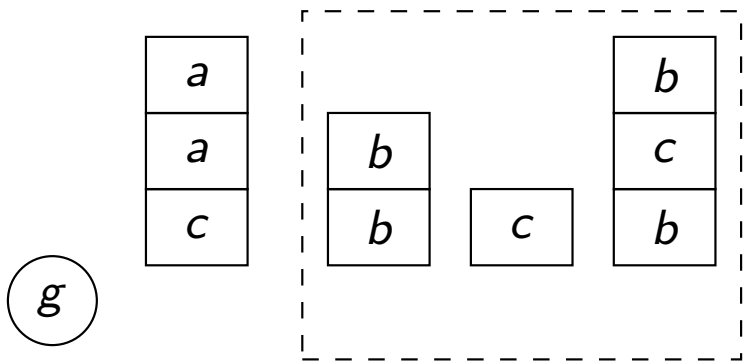

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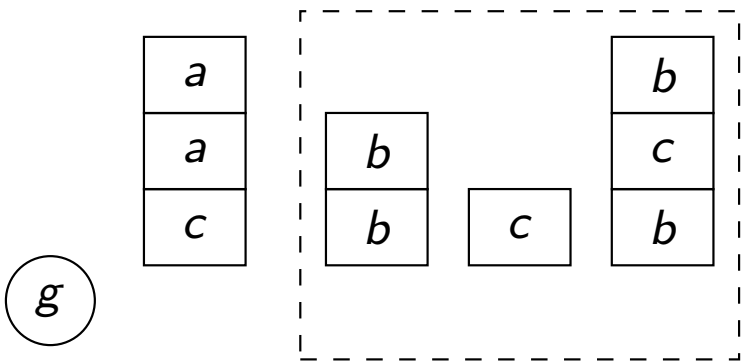

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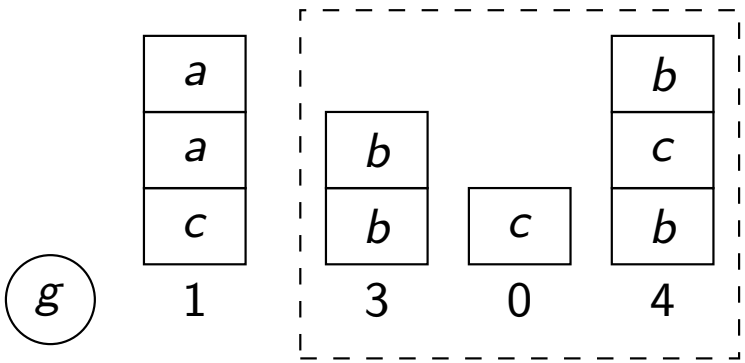
- Bound K on context switches per thread (avoids undecidability).



Safety Verification

K-bounded state reachability problem for DCPS (SRP[K])

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SRP[0] is EXPSPACE-complete.

- Shown by Ganty and Majumdar (2012).

SRP[K] is EXPSPACE-hard and in 2EXPSPACE for every $K \geq 1$.

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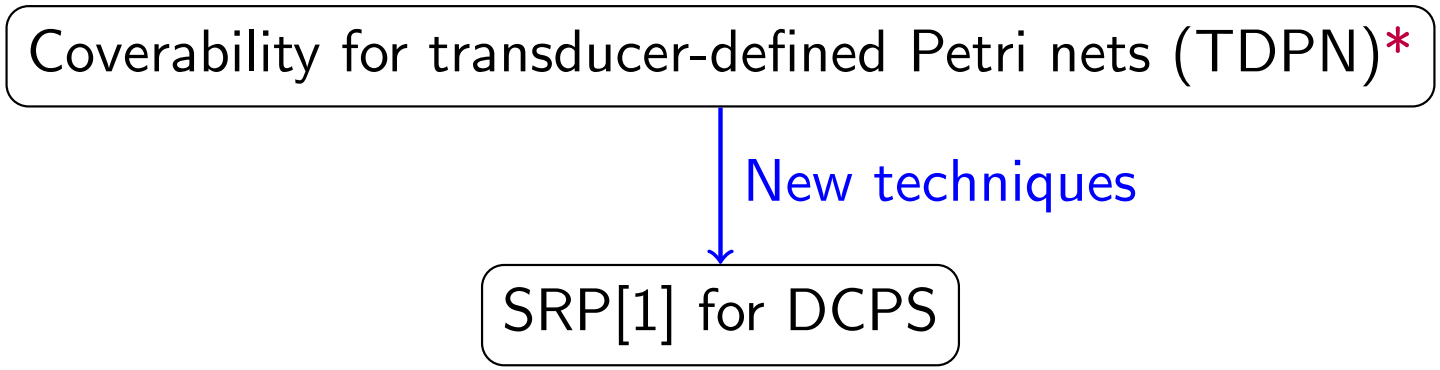
Our main result

SRP[K] is 2EXPSPACE-hard for every $K \geq 1$.

Proof Outline

SRP[1] for DCPS

Proof Outline



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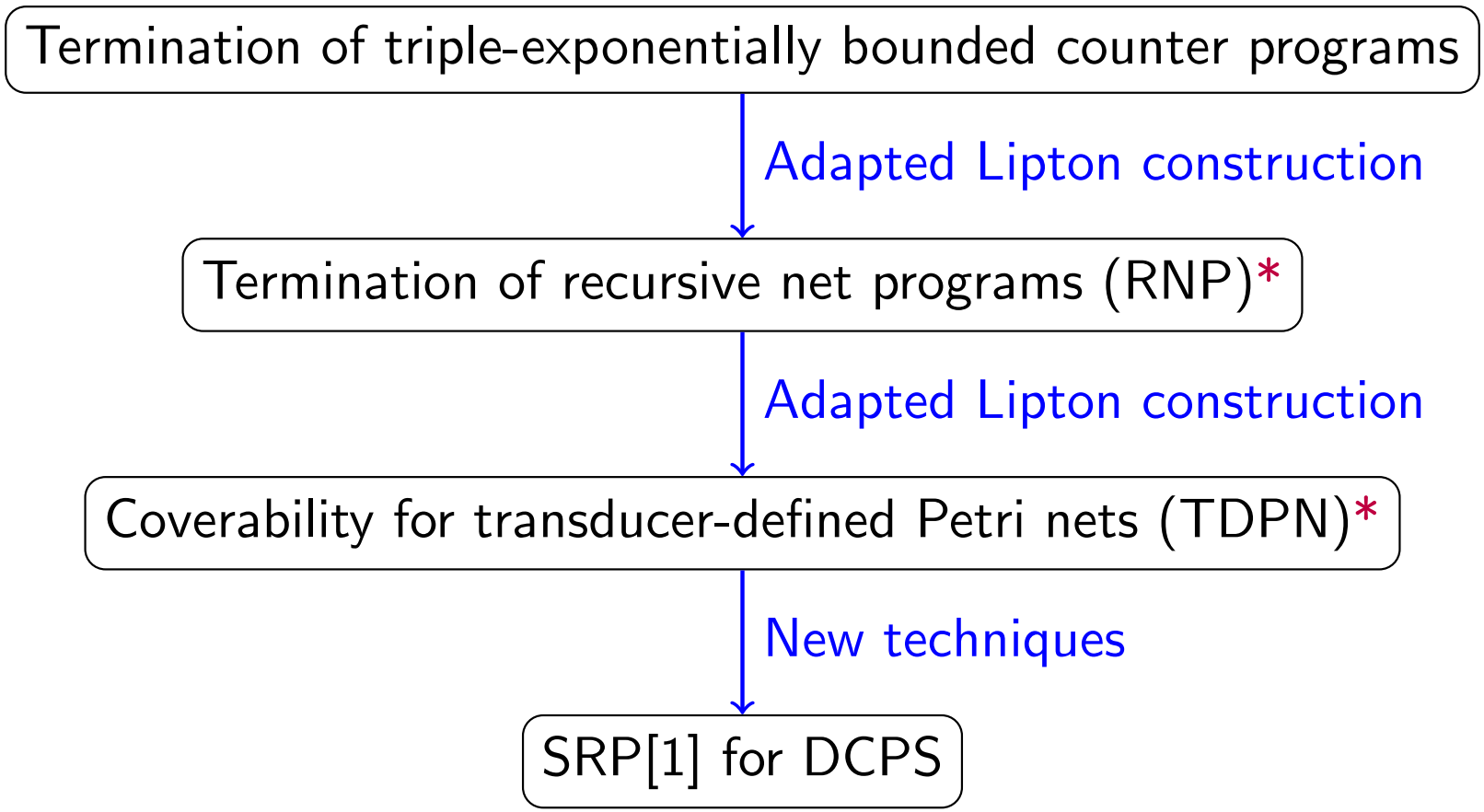

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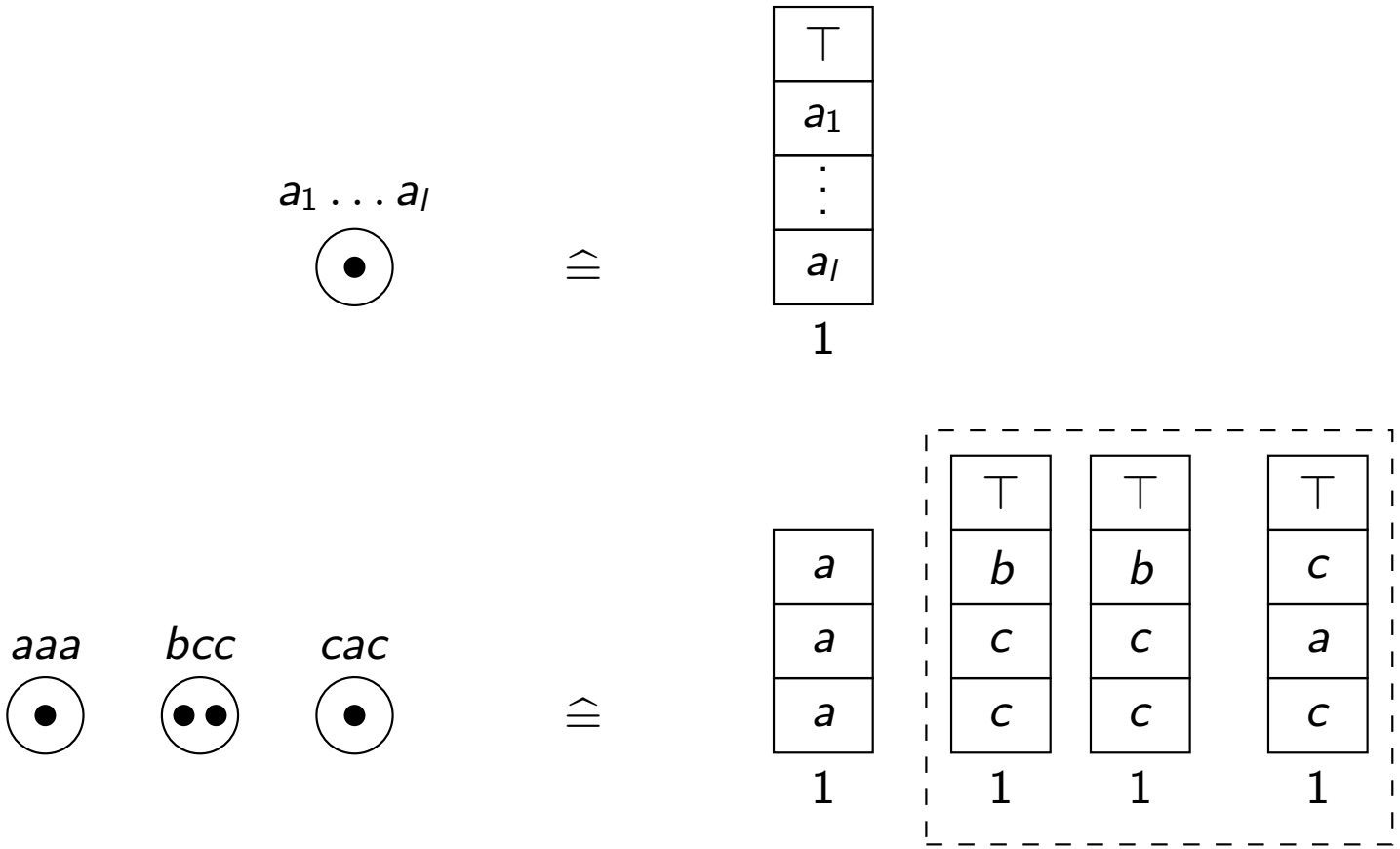
Thank you for your attention!

Any questions?

Appendix

Locking Inactive Threads



Lifting to 2EXPSPACE

We used $2^{2^d} = 2^{2^{d-1} \cdot 2} = \left(2^{2^{d-1}}\right)^2 = 2^{2^{d-1}} \cdot 2^{2^{d-1}}$.

- This means from one level to the next the bound gets squared.

$$(\cdots(2^{\overbrace{2}^{n\text{-times}}})^2 \cdots)^2 = 2^{2^n}$$

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Details of Known Results

Ganty and Majumdar (2012) consider threads running to completion.

- We can ensure that threads empty their stack in our model.
- This allows us to use their EXPSPACE-completeness result for $K = 0$.

Atig, Bouajjani, and Qadeer (2009) consider a slightly different DCPS:

- Each thread spawns with its parents cs-number plus 1.
- We can simulate our model in theirs using 2 more contex switches.
- Reduces our SRP[K] to their SRP[$K + 2$].
- This allows us to use their 2EXPSPACE-membership result.

Succinct Representation via Transducers

Use binary addresses $w = u.v$ for places:

- u: Role, i.e. which line, counter, or auxiliary place it is.
- v: Binary representation of recursion depth d.

Let the size of the RNP be h, the number of lines of code.

- Each counter appears in at least one line.
- Each line only needs at most one auxiliary place.
- Thus, the number of possibilities for u is linear in h.

Make the transducers distinguish each possible triple (pair) of prefixes u:

- Considering triples adds an exponent of 3, still poly in h.

Succinct Representation via Transducers

The recursion depth d changes by at most 1 at a time.

- Transducers have to check for equality or off-by-one on postfixes v.
- These checks require space linear in the number of bits.
- Since the maximum for d is 2^n, v has $\log(2^n) = n$ bits.

The triple (pair) of prefixes u tells us how the depths are related.

- Connect the paths for u with the appropriate checks at the end.

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