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ORGANISMIC SUPERCATEGORIES: II. ON MULTISTABLE SYSTEMS

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The representation of biological systems in terms of organismic supercategories, introduced in previous papers (*Bull. Math. Biophysics*, **30**, 625–636; **31**, 59–70) is further discussed. To state more clearly this representation some new definitions are introduced. Also, some necessary changes in axiomatics are made. The conclusion is reached that any organismic supercategory has at least one superpushout, and this expresses the fact that biological systems are multistable. This way a connection between some results of Rashevsky's theory of organismic sets and our results becomes obvious.

1. *Introduction*. In two recent papers (Băianu and Marinescu, 1968; Comoroşan and Băianu, 1969; herein afterwards referred to respectively as I and II), a new abstract representation of biological systems was introduced.

The purpose of the present paper is to make more rigorous and clear some of the points concerning the principle of choice (as introduced in I and II), and to derive some of its indirect consequences.

In I and II biological observables were not explicitly introduced. However, it seems that any biophysical model or representation has to introduce observables explicitly. The study of biological systems by means of "supercategories" can be approached from two distinct and complementary points of view. On the one hand, a biological system can be represented in terms of sequences of states, regarding the states as "primary" concepts. The hope is that the general *invariants* of these sequences of states will determine the general properties of biological systems. On the other hand, states will be defined in terms of observables on the basis of a logical analysis of biological knowledge. However, there is a connection between these approaches, and this connection will be discussed here.

A number of unclarities of our representation were pointed out by Professor Rashevsky, by Professor Rosen, and by Professor Arbib. To make matters clear I shall introduce some corrections and precise definitions. In order to do it some basic questions must be emphasized and some clarifications must be made. A crude and simple biological example will be quite sufficient. When we consider the relations among the neurons of the brain, it may be in a general sense said, that these relations are more complex than those among the liver cells. A better idea is given by separating groups of cells which have the same biological activity, and then consider the relations among these (Rashevsky 1967c, 1969). One should like to look for a few essential relations, and then to express all other relations as compositions of the essential ones. This is a very simple way which leads to an *algebraic structure*, and which may give a simplified insight into the biological activity of organisms. However, other properties which refer to the whole organism may be of interest, such as connectedness (Rosen, 1965). A formalization of these properties leads to a *topological structure*. The algebraic structure which should be assigned to the brain in order to account for its biological activity will be different from that one which should be assigned to the liver. Even more, there are some relations between the brain and the liver that should be taken into account. Thus, in order to represent the whole organism, one has to consider aggregates of distinct types of structures and connections among structures.

Aggregates of this kind were called "supercategories". "Supercategories" may appear in many other cases when a mathematical study of complex systems is involved. These superstructures provide a better understanding of differences in structural complexities of systems. Also, any optimality principle which would consider the organism as a structure in space-time (Rashevsky, 1966, p. 293), could not avoid such superstructures. Thus, in growth and differentiation processes the degree of complexity of the organisms increases with time, although one should refer to the organism as being the *same* system. In order to get a very crude idea of the fact, consider the following intuitive image (Fig. 1). From a *geometric* point of view one would recognize these three figures as squares; in this case, the shape is a geometric invariant—all the figures are alike. However, these have not the same number of components, and thus, from a *topological* point of view, they are different. In order to represent both aspects, invariance and change, one should have to adopt both points of view.

A few words must be added about mathematical structures. Any unification of mathematical theories makes explicit or implicit use of the basic notion of a

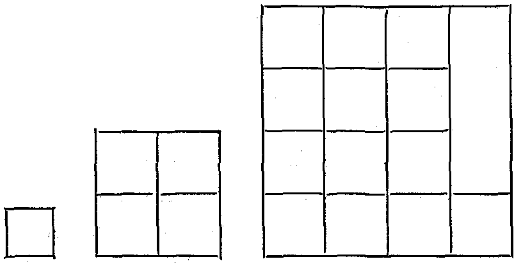
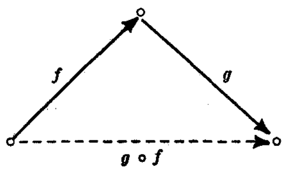


Figure 1. (Explanations are given in text)

structure. "The theory of structures over sets admits a more general and axiomatic form within the theory of categories and functors, and this theory of categories seems to be the most characteristic trend in present day mathematics . . ." (Ehresmann, 1966, p. 5).

Generally speaking, a *category* is *a class together with a partially defined law of composition satisfying some axioms* (*loc. cit.*). An element of a category is called a *morphism* and this generalizes the old notion of a mapping which was considered by Dedekind, Eilenberg and MacLane as the basic tool of mathematics (Ehresmann, *loc. cit.*; Eilenberg and MacLane, 1945). However, the elementhood relation $e \in S$, with e denoting an element and S denoting a set, is not essential in modern mathematical constructions and may be avoided in a foundation of mathematics (Lawvere, 1966). Lawvere's foundation makes explicit use of the category of categories. *Supercategories* of diagrams were independently introduced as general representations of systems (axioms I–IV of I, p. 629). A concrete example will provide an intuitive basis for the understanding of the necessity of Lawvere's axiomatic foundation, and for its utility in defining *supercategories*. Let us discuss diagram 1 of I (p. 631). Essentially the same notations are used here and are explained in more detail.

Hormonal control is established as a result of certain connections among the components of an organism. Some centers from the diencephalon (A_1) produce by neurosecretion the corticotropin-releasing-factor which is then transported to adenohypophyse (A_2) where it stimulates the release of ACTH (adrenocorticotropic hormone). ACTH acts on corticosuperrenala (A_3), initiating the secretion of corticosteroids, which are then transported by the circulatory flow to the tissues. Through a feedback mechanism (v_{32}) the activity of adenohypophyse depends on the level of corticosteroids concentration in the circulatory flow. Also, it affects the level of neurosecretion (v_{31}). There is a morphism u in diagram 1 which connects the activity of A_3 with both feedback circuits. From this diagram and from equation (12) of I it may be inferred that a finer control of this activity could be performed by an upper cycle $A_1 \overset{u_{21}}{\leftrightarrows} A_2$, where u_{21} would represent a neural path which must have only *stimulatory* synapses with some centers from A_1. Let us discuss in more detail the underlying concepts of this diagram. From diagram 1 it may be seen that the main discussion concerns connectedness and *ordered connections* (arrows). If one goes further, then one has to consider some rules which define the composition of these connections. These rules may be formulated as conditions of coincidence for the vertices of the connections that are to be composed. Such rules are called laws of composition. The most simple of these laws is the categorical law of composition, which requires that the end of a morphism f must coincide with the source of another morphism g if the two morphisms are composable. This law of composition is illustrated by the triangular diagram



(The morphism denoted by $g \circ f$ is dotted to indicate that it comes last, as a *construction* based on g and f.)

Composition laws may also be formulated by means of some unary function symbols, and Lawvere's axiomatics proceeds this way. The advantage is that his axiomatic system may be extended in such a way as to define other composition laws, distinct from the categorical composition law. Now, making use of Lawvere's elementary theory of abstract categories (*loc. cit.*) I shall give precise definitions instead of definitions 6, 7 and 9 of I. For a better understanding, Lawvere's axiomatic system is reproduced almost in its original form.

Axioms of the elementary theory of abstract categories (ETAC-Lawvere, *loc. cit.*).

0. For any letters x, y, u, A, B, the following are *formulas* $\Delta_0(x) = A$, $\Delta_1(x) = B$, $\Gamma(x, y; u)$, $x = y$ (Δ_0 and Δ_1 are unary function symbols). These are to be read respectively, "A is the domain of x," "B is the codomain (range) of x," "u is the composition x followed by y" and "x equals y."

1. If Φ and Ψ are formulas, then "[Φ] and [Ψ]", "[Φ] or [Ψ]," "[Φ] $\Rightarrow$ [Ψ]," "not [Φ]" are also formulas.

2. If Φ is a formula and x is a letter, then "$\forall x$[Φ]," "$\exists x$[Φ]" are also formulas. These are to be read "for every x, Φ" and "there is an x such that Φ," respectively.

3. A string of symbols is a formula of ETAC iff it follows from 0, 1, 2, above.

By a *sentence* is meant any formula in which every occurrence of each letter x is within the scope of a quantifier $\forall x$ or $\exists x$. The theorems of ETAC are all those sentences which can be derived by logical inference from the following axioms.

4. $\Delta_i(\Delta_j x)) = \Delta_j(x) \quad i, j = 0, 1.$
5. a) $\Gamma(x, y; u)$ and $\Gamma(x, y; u') \Rightarrow u = u'$;
   b) $\exists u[\Gamma(x, y; u)] \Rightarrow \Delta_1(x) = \Delta_0(y)$;
   c) $\Gamma(x, y; u) \Rightarrow \Delta_0(u) = \Delta_0(x)$ and $\Delta_1(u) = \Delta_1(y)$.
6. Identity axiom:

$$\Gamma(\Delta_0(x), x; x) \quad \text{and} \quad \Gamma(x, \Delta_1(x); x).$$

7. Associativity axiom

$$\Gamma(x, y; u) \quad \text{and} \quad \Gamma(y, z; w) \quad \text{and} \quad \Gamma(x, w; f) \quad \text{and} \quad \Gamma(u, z; g) \Rightarrow f = g.$$

Now, with these axioms in mind, it may be seen that axioms C_1, C_2, C_3, C_4 in definition 1 of I comprise *abbreviated formulas* of ETAC. Thus $A \xrightarrow{f} B$ means $\Delta_0(f) = A$ and $\Delta_1(f) = B$; $g \circ f$ means $\Gamma(f, g; h)$;

$$\begin{tikzcd} A \arrow[r, "f"] \arrow[dr, "h"'] & B \arrow[d, "g"] \\ & C \end{tikzcd} \quad \textit{commutes}$$

means

$$\Delta_0(f) = \Delta_0(h) = A \quad \text{and} \quad \Delta_1(f) = \Delta_0(g) = B \tag{1}$$

and

$$\Delta_1(g) = \Delta_1(h) = C \quad \text{and} \quad \Gamma(f, g; h).$$

Commutative diagrams are regarded as abbreviated formulas, signifying the associated systems of equations as (1) above. These diagrams have the advantage of a *geometric*-intuitive image of the underlying equations. For example, the associativity axiom C_3 of def. 1 of I becomes clear on contemplat-

$$\begin{tikzcd} \circ \arrow[r, "x"] \arrow[dr, "u"'] \arrow[drr, bend left, "f"] \arrow[drr, bend right, "g"'] & \circ \arrow[d, "y"] \arrow[dr, "w"] & \\ & \circ \arrow[r, "z"] & \circ \end{tikzcd}$$

ing the following commutative diagram made up of four elementary triangles of the above sort.

This simple example given by Lawvere (*loc. cit.*) corresponds precisely to the less intuitive diagram of C_3 of I. However, axiom C_4 of def. 1 of I must be completed; a complete formulation is given by the identity axiom (6) or equivalently by the following commutative diagrams:

$$1_A \text{ such that } \begin{tikzcd} \circ & A \arrow[l, "f"'] \\ & A \arrow[u, "1_A"'] \arrow[ul, "f"] \end{tikzcd} \qquad \begin{tikzcd} \circ \arrow[r, "g"] \arrow[dr, "g"'] & A \arrow[d, "1_A"] \\ & A \end{tikzcd}$$

for any A, f, g, as in the above diagrams. (1_A is completely defined by these diagrams.)

Obj (A) is another abbreviated formula which means:

a) $A = \Delta_0(A) = \Delta_1(A)$,

b) $\exists x[A = \Delta_0(x)]$ or $\exists y[A = \Delta_1(y)]$,

c) $\forall x \forall u[\Gamma(x, A; u) \Rightarrow x = u]$ and $\forall y \forall v[\Gamma(A, y; v) \Rightarrow y = v]$.

These three formulas express a common property of A, that of being an *object*. Ob C of def. 1 of I contains only objects of this type. In these terms a *category* is understood intuitively as any structure which is an interpretation of ETAC. A *functor* is understood as a triple consisting of two categories and of a rule F which assigns to each morphism x of the first category, a unique morphism $F(x)$ of the second category in such a way that conditions F_1, F_2 of def. 2 of I are fulfilled. These conditions are interpretations of the corresponding conditions from ETAC (see p. 4, *loc. cit.*). In order to proceed further the category of all functors is considered; the world of all functors is in fact a metacategory, or a large category (supercategory, in the classical terminology). Lawvere has shown which axioms must be added to ETAC in order to get a starting point for further investigations into the structure of categories. According to Lawvere, a *set* is defined as a *discrete category*. (That is, every morphism in a set is an object (*loc. cit.*).) This definition suggests that sets are relatively poor structures. However, it must be noticed here that Rashevsky's theory of organismic sets deals with more complex structures over sets which are sometimes implicit in the logical formulations. A specific example will be given on pages 550–552. For this reason I shall prefer to introduce explicitly the basic structures. An advantage of this choice, will be the gain in operationality.

Let us reconsider diagram 1 of I. In the terminology of Eilenberg and MacLane (1945), the composition law (p. 632 of I) is defined as a mapping $D(A_3) \times D(A_3) \xrightarrow{\Delta} R(A_3)$ with $D(A_3)$ the set of morphisms which have the same range A_3, and $R(A_3)$ the set of morphisms which have the same range A_3. Correspondingly, there may be introduced two new unary function symbols Δ_0 and Δ_1 which express this composition law by the equations:

$$\Delta_0(v_{23}^+) = \Delta_0(v_{13}^+) = \Delta_1(u_v),$$

or equivalently v_{13}^+, $v_{23}^+ \in D(A_3)$ and $u_v \in R(A_3)$. (See p. 632 of I.) The symbol $\square$ must be replaced by the categorical composition law inasmuch as we have $\Delta_1(u_v) = \Delta_1(i_3)$ and $\Delta_0(u_v) = \Delta_0(u_{23})$. Also the composition law ∇ may be defined as a mapping $C(A_2, A_3) \times C(A_2, A_3) \to C(A_2, A_3)$ where $C(A_2, A_3)$ stands for the set of morphisms between A_2 and A_3 [it is sometimes denoted by $H(A_2, A_3)$ or by hom (A_2, A_3)]. The symbol ∇ may be also defined by equations of the type

$$\Delta_0(u_{23}^0) = \Delta_0(u_{23}') = \Delta_0(u_{23})$$

and

$$\Delta_1(u_{23}^0) = \Delta_1(u_{23}') = \Delta_1(u_{23}).$$

This simple example suggests that a *supercategory* must comprise categories, terms which do not contain the elementhood relation and some composition laws to operate on its morphisms. The basic idea is to start with metadiagrams of the type

$$\begin{tikzcd} \circ \arrow[r] \arrow[d] & \circ \arrow[r, dashed] \arrow[d] & \circ \arrow[d] & \circ \arrow[l] \arrow[d] \\ \circ \arrow[r] & \circ \arrow[r, dashed] & \circ & \circ \arrow[l] \end{tikzcd}$$

where the two outer diagrams belong to distinct categories and the dotted morphisms simply connect these diagrams but do not belong to any of these. A specific example is provided by the morphisms which assign to a given topological space its corresponding homological groups.

Now, if we consider the property of being self-reproducing as an essential property of biological organisms or of pairs of biological organisms, then we have to introduce this property axiomatically. However, in an organism there are some highly specialized cells which do not reproduce themselves after the specialization is completed. Consequently, we must allow the construction of new objects which are not self-reproducing, from the self-reproducing ones.

Thus, triples which consist of categories, entities which are self-reproducing and terms which do not contain the elementhood relation, will be introduced first. Then, *supercategories* are constructed from such triples and morphisms among these. Definitions follow.

(D1). a) A "class" (when the word is used in quotation marks) is a triple $K = (C, \Pi, N)$, where

C is an arbitrary category;
Π is a category of π-entities;
N is a non-atomic expression.
Explanations follow.

b) π-entities were introduced by Löfgren as "complete self-reproducing entities" subject to the negation of the axiom of restriction:

$$\exists S: \quad (S \neq \emptyset) \quad \& \quad \forall u: [(u \in S) \Rightarrow \exists v: (v \in u) \quad \& \quad (v \in S)$$

which is known to be independent from ordinary logical-mathematical-biological reasoning. The above axiom says that there exists a nonempty set S, such that for any element u of S, there is an element v of u which is also an element of S. An *atomically self-reproducing entity* is a unit class relation π such that:

$$\begin{aligned} &\pi\pi \langle \pi \rangle \quad (\pi \text{ stands in the relation } \pi \text{ to } \pi), \\ &\pi\pi \langle \pi, \pi \rangle, \ \pi\pi \langle \pi, \pi, \pi \rangle, \text{ etc.} \end{aligned} \tag{2}$$

(Löfgren, 1968). A *symbiotically self-reproducing pair* of distinct entities π_1 and π_2 is defined by the equalities

$$\pi_1(\pi_2) = \langle \alpha, \beta, \ldots, \pi_2 \rangle; \qquad \pi_2(\pi_1) = \langle \alpha', \beta', \ldots, \pi_1 \rangle. \tag{3}$$

That is, π_1 is the behavior function of an automaton that reproduces the entity π_2. From the input π_2 the automaton π_1 thus produces a sequence of constructs $\alpha, \beta, \ldots$, that ends up in the final output π_2 (Löfgren, 1968). I shall introduce first a few changes in this formalism. Let $\pi_1, \pi_2, \ldots, \pi_n$ be objects in a category Π. The identity morphisms $1_{\pi_1}, 1_{\pi_2}, \ldots, 1_{\pi_n}$ will be interpreted as *descriptions* of $\pi_1, \pi_2, \ldots, \pi_n$, respectively. A morphism $\pi_1 \xrightarrow{f} \pi_2$ will represent the fact that the automaton π_1 produces finally the output π_2. A morphism in an epsilon-graph, from an object X to an object Y means $X \in Y$. The composition of morphisms has natural interpretations according to the axiom of restriction.

c) A *connected* category C^+ is a category in which any two objects are connected by a chain of morphisms, no matter how the morphisms of the chain are oriented. A connected category in which no morphism is interpreted as an elementhood relation, will be called a *non-atomic expression.*

Now, suppose that there are some "classes" $K_0 = (C_0, \Pi_0, N_0)$ such that $(C_0, \Pi_0, N_0) = C_0$, for some categories C_0. This equality will be taken as an axiom and will be referred to as K_1. Def. 6 of I is now replaced by D1 above (see a). Making use of ETAC and D1, an elementary theory of abstract *supercategories* (ETAS) will be introduced. As a consequence def. 7 of I will have a precise formulation. Let us consider first a biological example.

If F represents a cell which after some transitions leads to a symbiotically self-reproducing pair $\langle \pi_1, \pi_2 \rangle$ and if after z steps it is transformed in a pair which is not self-reproducing, the situation may be represented by a transition of a "class" which comprises the self-reproducing pair into another "class" which comprises a pair which is not self-reproducing. Equivalently, the process may be represented by the following diagram

$$\begin{tikzcd}
 & & T \\
F \arrow[r, dashed] & \pi_1 \arrow[d] & \langle \pi_2, \langle \cdots, \pi_2 \rangle\rangle \arrow[l] \arrow[u, dashed] \arrow[d] \\
 & \langle \pi_1, \langle \cdots, \pi_1 \rangle\rangle \arrow[r] \arrow[d] \arrow[ur, phantom, "\circlearrowright z"] & \pi_2 \\
 & Z &
\end{tikzcd}$$

where (T, Z) is the pair which is not self-reproducing. For example, (T, Z) would correspond to higher metazoan brain cells which do not reproduce themselves after the specialization is completed (Rashevsky, personal communication). The symbiotically self-reproducing pair may correspond to the DNA $\sim$ DNA-polymerase system in a cell. Let us denote the above introduced "classes" by K_+ and K^+. Thus, $K_+ = (F, (\pi_1, \pi_2), N_0)$, and $K^+ = ((T, Z), \Pi_0, N_0)$, with N_0 and Π_0 being void categories, that is, which have no morphisms. This representation introduces a new morphism $K_+ \dashrightarrow K^+$ (the transition from the "class" K_+ to the "class" K^+), and consequently a new type of structure. The expression $K_+ \dashrightarrow K^+$ will be called a *supercategory*. Now follows the general definition of a *supercategory*, which will be given in terms of the elementary theory of supercategories.

Axioms of the elementary theory of "supercategories" (ETAS).

S_1) For any "classes" K_+, K^+ and any letters k_+, k^+, z, the following are *formulas*:

$$\bar{\Delta}_0(k_+) = K_+, \qquad \bar{\Delta}_1(k^+) = K^+; \qquad \bar{\Gamma}(k_+, k^+; z);$$
$$\bar{\Gamma}(w; x, y, \ldots), (x, y, \ldots \text{ in } K_+ \text{ or in } K^+), \quad k_+ = k^+.$$

These formulas are to be read as in axiom 0 of ETAC, replacing the letters A and B by the "classes" K_+ and K^+. $\bar{\Gamma}(w; x, y, \ldots)$ is to be read: "for a given letter w there is an assignment $w \to (x, y, \ldots)$, with $x, y, \ldots$ being in K_+ or in K^+."

S_2). a) If $\Delta_0(z) = K_+$, there is a letter x in K_+ such that

$$\Delta_0(k_+) = \Delta_1(x) = A, \text{ with } A \text{ being an object in } K_+;$$

b) If $\Delta_1(z) = K^+$, there is a letter y in K^+ such that

$$\Delta_1(k^+) = \Delta_0(y) = B, \text{ with } B \text{ being an object in } K^+.$$

S_3) If $\bar{\Phi}$ and $\bar{\Psi}$ are formulas of ETAS, and k is a letter, then: "$[\bar{\Phi}]$ and $[\bar{\Psi}]$," "$[\bar{\Phi}]$ or $[\bar{\Psi}]$," "$[\bar{\Phi}] \Rightarrow [\bar{\Psi}]$," "not $[\bar{\Phi}]$," "$\forall k[\bar{\Phi}]$," "$\exists k[\bar{\Phi}]$" are also formulas.

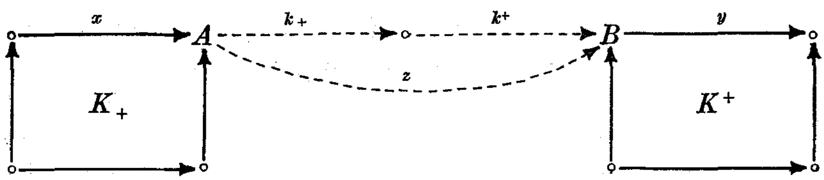
S_4) A string of symbols is a formula of ETAS iff this follows from S_1, S_2, S_3. The theorems of ETAS are all those sentences which can be derived by logical inference from these axioms and from the following axiom:

S_5) a) $\bar{\Gamma}(k_+, k^+, z)$ and $\bar{\Gamma}(k_+, k^+, z') \Rightarrow z = z'$;

b) $\exists z[\bar{\Gamma}(k_+, k^+; z)] \Leftrightarrow \Delta_1(k_+) = \Delta_0(k^+)$;

c) $\bar{\Gamma}(k_+, k^+; z) \Rightarrow \Delta_0(z) = \Delta_0(k_+)$ and $\Delta_1(z) = \Delta_1(k^+)$.

Axioms S_1, S_2, S_5 become clear on contemplating the following metadiagram (*supercategorical* diagram):



(D2). Any structure which is an interpretation of ETAS will be called a *supercategory*.

In order to obtain a simplified insight into the structure and dynamics of a "complex" system such as a biological or a social system, one has to start with a rather simple structure and then to construct from this simple structure more complicated structures. The next section introduces a formal mean to deal with such problems on the basis of the above definitions.

2. *Generators*. Besides the basic idea of a structure (Bourbaki, 1958), the concept of a *generation* of higher type structures and larger classes is very im-

portant. This concept may be used to obtain a simplified insight into the dynamics of systems. The generation of structures of higher types often implies the presence of some adjoint functors (Lawvere 1967, 1969). Now, it is usual in algebra to define theories by *generators and relations* (Eilenberg and Wright, 1967). A rather simple example is provided by a group. Some groups may be generated by *multiplication* from a set of their elements. The elements of this set are called *generators*. Such considerations may lead to more general concepts concerning generating procedures. One of these is introduced here.

(D3). A "class" K together with some rules of transformation (which may be as well functors), will be called a *generating class*.

If we take $K = K_0 = C_0$, and if C_0 is a discrete category then a *T-algebra* A (Eilenberg and Wright, 1967, p. 3) may be defined as a particular generating class (A, R) with $A = C_0$ and R a rule which assigns to each $\phi\colon [n] \to [p]$ in the theory T and to each p-tuple $(x_1, \ldots, x_p)$ of elements of A, an n-tuple $(x'_1, \ldots, x'_n) = (x_1, \ldots, x_p)\phi$ of elements of A, which are also subject to some axioms. (Here by $[n]$ was denoted the set $\{1, \ldots, n\}$.) Another example is provided by the representation of analogous systems in terms of observables (Rosen, 1968a). The one-dimensional harmonic oscillator was shown to be represented analogously in terms of observables of the system comprising a free gravitating particle. The equations of motion of the two systems are, respectively:

$$\begin{cases} k\dot{P} = Q \\ \dot{Q} = P \end{cases} \quad \text{and} \quad \begin{cases} \dot{p} = h; \\ \dot{q} = p. \end{cases} \tag{4}$$

Let there be two small categories, the morphisms of which are, respectively, $S_1 \xrightarrow{Q} R$, $S_1 \xrightarrow{P} R$, and $S_2 \xrightarrow{q} R$, $S_2 \xrightarrow{p} R$, with S_1, S_2 the corresponding state spaces of the two systems, and R the set of real numbers. The quantities Q, P and q, p, h are observables of the two systems $(S_1, \{f_t\})$ and $(S_2, \{g_t\})$. Two diagrams may be constructed with these morphisms as objects:

$$\begin{tikzcd} Q \arrow[d, "Y"'] \arrow[dr, "+\frac{1}{k}Y^2"] & \\ P \arrow[r, "+\frac{1}{k}Y"'] & Q \end{tikzcd} \qquad \begin{tikzcd} q \arrow[d, "Y"'] \arrow[dr, "Y^2"] & \\ p \arrow[r, "Y"'] & h \end{tikzcd}$$

Diagram D_1 Diagram D_2

such that equations (4) are obtained from the diagrams taking $Y = d/dt$. Now, if R is organized as a discrete category, then covariant functors F, G

from D_1 and D_2 to R may be considered. Thus, natural transformations (or functorial morphisms (p. 628, def. 3 of I) $F_i \xrightarrow{\eta_{ij}} F_j$, $G_k \xrightarrow{\varepsilon_{kl}} G_1$ will define in fact the 1-parameter families of natural transformations $\{f_t\}$, $\{g_t\}$ with f_t: $S_1 \to S_1$ and $g_t: S_2 \to S_2$. The couples (D_1, η_{ij}), (D_2, ε_{kl}) are particular cases of generating classes. It may be noted that there is a monomorphism Ob $D_1 \xrightarrow{\rho}$ Ob D_2 and an isomorphism Fl $D_1 \xrightarrow{s}$ Fl D_2. Even more, the type of the operators involved ($Y = d/dt$) is left invariant (is *preserved* by a functor $D_1 \xrightarrow{(\rho,s)} D_2$. These may be equivalent conditions to those of Lemma 2 and Lemma 4 (Rosen, 1968a) which ensure the existence of an analogy between systems S_1 and S_2. Thus, the generating class $(\rho(\text{Ob } D_1), s^{-1})$, with $\rho(\text{Ob } D_1) \subset \text{Ob } D_2$, and conforming to the above mentioned conditions, allows the generation of D_1. This shows that the generating diagram of the first system is, up to an isomorphism, among the diagrams generated by the generating diagram of the second. (Note that $\bar{h}$ is a constant mapping $S_2 \xrightarrow{h} R$ such that $\bar{h}(s) = h$ for any $s \in S_2$.) The above example suggests a more general definition of an observable: as a *morphism*. Sometimes, it will be more convenient to consider this morphism in the category of categories, that is, to define an observable as a functor. These observables connected in a diagram, lead to a generating class of the system under consideration. Also, this representation provides a natural formalization of linked observables (Rosen, personal communication). The next example shows that this definition may be applied in a specific form to the study of organismic sets, revealing their basic structure. Let S_0 be an organismic set of N elements e_i, $i = 1, 2, \ldots, N$. Then according to axiom 1a) of (Rashevsky, 1967a) each $e_i \in S_0$ is characterized by m_i potential activities $a_{i_1}, \ldots, a_{i_{m_i}}$. Let $S_i^{(a)}$ be the set of all activities: $S_i^{(a)} = \{a_{i_1}, \ldots, a_{i_{m_i}}\}$ of e_i. According to 1b of Rashevsky (1967) the sets $S_i^{(a)}$ may result in a number r_i of products (in the economic sense), $p_{i_1}, \ldots, p_{i_{r_i}}$. Let $S_i^{(p)} = \{p_{i_1}, \ldots, p_{i_{r_i}}\}$ be the set of all products p_{i_k}. These sets are subject to a number of conditions (axioms 1c–1f, and postulates 1–6, *loc. cit.*). I shall construct with these sets two other sets

$$S^{(a)} = \{S_i^{(a)} \mid i = 1, 2, \ldots, N\}$$

and

$$S^{(p)} = \{S_i^{(p)} \mid i = 1, 2, \ldots, N\}.$$

Then, if we consider the one-to-one mapping $S_0 \xrightarrow{\theta} S^{(a)} \times S^{(p)}$ it may be noticed that it assigns precisely to each e_i, the set $S_i^{(p)}$. However, according to 1c (Rashevsky, 1967) at any given moment only a subset of $S_i^{(a)}$ and a subset of

$S_i^{(p)}$ are actually assigned to the corresponding e_i. Thus, we are forced to consider the products

$$S_i^{(a)} \times S_i^{(p)}, (S_i^{(a)} \times S_i^{(p)} \times S_i^{(a)}),$$
$$(S_i^{(a)} \times S_i^{(a)} \times S_i^{(p)} \times S_i^{(p)}), \ldots, \underbrace{(S_i^{(a)} \times S_i^{(a)} \times \ldots)}_{m-\text{times}} \times \underbrace{(S_i^{(p)} \times S_i^{(p)} \ldots)}_{r-\text{times}},$$

where $m = \max m_i (i = 1, \ldots N)$ and $r = \max r_i; (i = 1, \ldots, N)$. Denote $S_i^{(a)} \times S_i^{(p)}$ by S_{ap}^{i1}; denote $(S_i^{(a)} \times S_i^{(a)} \times S_i^{(p)} \times S_i^{(p)})$ by $S_{ap}^{(i)22}$, and so on. S_{ap}^{0} will be denoted by $\mathbf{1}$ (that is when each $S_i^{(a)} = \emptyset$, and each $S_i^{(p)} = \emptyset$). With these sets an *algebraic theory* $\underline{S}$ may be constructed. Its objects are $1, S_{ap}, \ldots, S_{ap}^{mr}$, and its morphisms are $S_{ap} \leftarrow 1, \ldots, S_{ap}^{mr} \rightarrow S_{ap}$, acting separately on each member of the products which is written between parenthesis. Now, if S_0 is considered as a discrete category, there are some functors $S_0 \xrightarrow{F} \underline{S}$ which define each e_i, at each moment, in terms of activities and products according to 1a–1f (*loc. cit.*). This, in turn, implies that functors $S_0 \xrightarrow{F} \underline{S}$ may be seen as coarse observables of S_0. The category $S_0^{\underline{S}}$ of covariant functors F^{-1} (which commute with products), from the algebraic theory $\underline{S}$ to the discrete category S_0, is an algebraic category (Lawvere, 1963), as it may be seen by comparation with the following definition.

(D4). The category of covariant functors (which commute with products) from the algebraic theory A to the category of sets, the morphisms of which are the natural transformations between these functors, is called the *associated algebraic category* of A.

A

An algebraic category will be denoted by $\bar{A}$ or by *Ens*, with *Ens* being the category of all sets and mappings between sets. In order to pass from the coarse observables F to fine observables, like those introduced by Rosen (1968a), we have to consider *intensities* of the activities a_i, that is, mappings $S_i^{(a)} \xrightarrow{\varphi_i} R$, $S_i^{(p)} \xrightarrow{\psi_i} R$, with R being the set of real numbers. Thus, the connection between coarse observables and fine observables is given by the commutative diagram

$$\begin{tikzcd} & S \arrow[dr, "\Psi"] & \\ S_0 \arrow[ur, "F"] \arrow[rr, "X"'] & & \bar{R} \end{tikzcd}$$

with $\bar{R}$ being the algebraic theory whose objects are $1, R, R^2 = R \times R, \ldots, R^n = R \times R \times \cdots \times R$ (n times). In the above diagram functors $X = \Psi \circ F$

play the rôle of some generalized observables, and are *natural* in the sense of Eilenberg and MacLane (1945), that is, F stands as a functor for *any* category S_0 as defined above and for *any* categories S, $\bar{R}$, of the type indicated above; even more, this diagram joins objects which did not seem to be related previously. These observables remove partially an asymmetry of the triplet, physics, biology, sociology (Rashevsky, 1967, p. 151). The following theorem gives a characterization of the categories S_0^S which are associated with organismic sets.

Theorem. Any category S_0^S has generators, kernels, cokernels, products, coproducts, pullbacks, limits and colimits.

Proof. The proof is immediate by taking into account the fact that the category S_0^S conforms to definition (D4) and thus is an algebraic category. Any algebraic category has the above mentioned properties (Georgescu and Popescu, 1968: Corollary 2, Theorem 1, Corollary 1 and Theorem 3, respectively), and consequently, S_0^S has the aforesaid properties. Q.E.D.

This characterization gives the general algebraic properties of the coarse observables of an organismic set. Their particular interpretations may reveal interesting properties of organismic sets. In a subsequent paper, Rashevsky (1968a) introduced an organismic set S_0 as the union of three disjoint subsets S_{01}, S_{02} and S_{03}. The functioning of the *core* S_{01} is essential for the functioning of S_0. The discrete category S_{01} together with three functors G_1, G_2, G_3 may "generate" a complete organismic set S_0. Thus, if we take $G_1(S_{01}) = S_{02}$, $G_2(S_{02}) = S_{03}$ and G_3 as the classical union "$\cup$" of sets, then by sequential application of G_1, G_2 and G_3 a complete S_0 is obtained. Consequently, $[S_{01}; G_1, G_2, G_3]$ may be considered a generating class of S_0.

(D5). The generating classes of an organismic set will be called *organismic classes*.

Now, consider the case of an organismic set which is generated by an aggregate $(\bar{G}_1, \bar{G}_2, \ldots, \bar{G}_k)$ of organismic classes. If the members of this aggregate are in fact distinct categories together with functors, and if there exist some connections among these, then a *supercategory* may be constructed with these organismic classes. This *supercategory* will be called a *generating organismic supercategory* and will be denoted by S_σ. For example, the structural genes of a cell could be represented by sequential machines or by categories associated with semigroups of states of the sequential machines. The regulatory genes could be adequately represented by topological spaces (control spaces) or by subcategories of the category "Top" of topological spaces. The totality of structural and regulatory genes would be then represented by a generating organismic supercategory of semigroups and topological spaces.

Generating classes could have wide applications in developmental biology. Two examples will be given to illustrate this idea. The development of an organism from the ovum may be viewed as the generation of a *supercategory* from its generating classes. In doing this the term Π of the "class" will play a major role. The process of generation will involve the enlargement of the corresponding "state space" of the organism. A rather simple but important question is the following. Could we realize a mathematical construction such that this construction enlarges the state space on the one hand, and continuously maintains *stability* on the other hand?

It must be noted first that the enlargement of the state space may take place in different ways, some of which are equivalent, that is, lead to the same final result. Thus, if A is a fertilized ovum, while P is a stage of the organism developed from it, then diagrams of the type

$$\begin{tikzcd} P & A_2 \arrow[l] \\ A_1 \arrow[u] & A \arrow[l] \arrow[u] \end{tikzcd}$$

will correspond to a polymorphism depending on the route which the development of the ovum takes after a stage P (as in the case when from the same ova laid by a queen bee are obtained the other queens, drones and asexual workers; this example was suggested by Professor Rashevsky). In order to realize it A must be a fixed object, while P must have some additional properties. This example corresponds exactly to the categorical concept of a *pushout*.

(D6). Given two morphisms $A \xrightarrow{\alpha_1} A_1$ and $A \xrightarrow{\alpha_2} A_2$ with a common domain, a commutative diagram

$$\begin{tikzcd} P & A_2 \arrow[l, "\beta_2"'] \\ A_1 \arrow[u, "\beta_1"] & A \arrow[l, "\alpha_1"] \arrow[u, "\alpha_2"'] \end{tikzcd}$$

is called a *pushout* $\bar{P}$ for α_1 and α_2, if for every pair of morphisms $A_1 \xrightarrow{\beta_1'} P'$ and $A_2 \xrightarrow{\beta_2'} P'$ such that $\beta_1' \circ \alpha_1 = \beta_2' \circ \alpha_2$, there exists a unique morphism $P \xrightarrow{\gamma} P'$ such that $\beta_1' = \gamma \circ \alpha_2$ and $\beta_2' = \gamma \circ \beta_2$. If $\bar{P}'$ is also a pushout for α_1 and α_2, there is a morphism $P' \xrightarrow{\gamma'} P$ such that $\beta_1 = \gamma' \circ \beta_0'$ and $\beta_2' = \gamma' \circ \beta_2'$.

Now if we return to the previous example, then it may be seen that P corresponds to the stage when the routes separate in such a manner that P', P'', ... correspond to the final organisms developed from the same ova. If also $\bar{P}'$ is a pushout then the stage P' is isomorphic to the stage P, and this corresponds to the fact that in this case the ways which lead to P and P' are equivalent. In the above diagrams, morphisms represent transitions from one stage to the other.

The next example shows that pushouts may be used as well to represent *stable* systems. However, a precise statement has to make explicit the meaning of *stability*, and this concept implies the idea of recovering of some states for certain intervals of time. A precise definition of stability will be given in the next section, after this example. A virus in an unfavourable environment may survive a very long time. This suggests that all transitions of states lead finally to a stable field $\{P', P'', \ldots\}$ as in the following diagram

$$\begin{tikzcd}
 & A_1 \arrow[dr] & & P' \\
A \arrow[ur] \arrow[rr] \arrow[dr] & & P \arrow[ll, dashed] \arrow[ur] \arrow[r] \arrow[dr] & P'' \\
 & A_2 \arrow[ur] & & \vdots
\end{tikzcd}$$

If this diagram is considered as a graph, then it may be seen that the graph has two cycles. Such graphs were called "kinematic" (Ashby, 1965), and give only a topological insight. Now, if a virus with a monostrand nucleic acid enters a cell, then some dynamical constraints are eliminated (the protein coat of the virus), thus generating a new stable field $\{P'_+, P''_+, \ldots\}$. Then, both $\bar{P}$ and $\bar{P}_+$ are pushouts, and transitions may take place between them: $P \rightleftarrows P_+$. Indeed, after the infection the virus is able to recover its stable field $\{P', P'', \ldots\}$. Apparently, the two systems (the virus and the cell), do not have to be considered as two systems in interaction, but as a single larger system. However, as far as the generating organismic supercategories of the two systems do *not* change as a result of their interaction, the virus and the cell may be considered as distinct systems although their first order structure (state space) is changed. Thus, the associated generating organismic supercategories are in fact second order structures which are *invariant* with respect to systems interactions. Only when these generating organismic supercategories are changed I shall speak of the change of the system as a whole. The same procedure could work in the case of evolutionary systems. This time, a third order structure, the generating organismic supercategory $\bar{\bar{S}}$ of a large class (in the biological sence) of organisms has to be considered. Then $\bar{\bar{S}}$ will be invariant for all organisms in the considered class, generating them one by one. Thus the idea of the fundamental biogenetic law or (exclusive) the principle of biological epimorphism (relational invariance), (Rashevsky 1967c, 1968c) is naturally obtained. Also, from the definition of a generating class, it may be seen that the number of relations among the elements of an organismic set increases with its development (*generation*). Realizations of these situations are left to the reader as an exercise. The above discussed fact was suggested by Rashevsky as a new biological principle (Rashevsky, 1968b). In a general model of biological systems generating diagrams of generating organismic supercategories would be considered and these would have to be connected in metadiagrams; morphisms in these metadiagrams would represent the connection between structures of distinct orders:

$$\begin{tikzcd} S_\sigma^n \arrow[rr] & & \bar{\bar{S}} \\ & S_\sigma^0 \arrow[ul, dotted] \arrow[ur] & \end{tikzcd}$$

In the above metadiagram by S_σ^0 denotes the generating organismic supercategory of zeroth order which generates an organismic set of zeroth order, and $S_\sigma^{(n)}$ denotes a generating organismic supercategory of nth order.

3. Multistable Systems. The previous discussion suggests a more general definition of dynamic systems in categorical terms. On the basis of such a definition, stability may be also defined in terms of observables rather than in terms of "equilibria of forces." Thus, a biological system may remain "stable" although its first order structure or *configuration* is changed. What remains invariant in this case is the associated supercategory S_σ. A specific example is provided by an organism which develops from an ovum. New cells may appear from the old ones, new relations arise among the cells and thus, new *configurations* appear. However, the whole organism is considered stable, regardless of the stage of its development. A change of configuration should be represented by changes in the associated type of algebraic and topological structures. Similarly, it may be said that the *configurations* are generated by an S_σ. Basically, a dynamic system has to be represented in terms of states and transitions among states. In doing this, observables must be also present to account for quantities which can be or are determined by experiments.

(D7) A *dynamic* system D is a commutative diagram of the type

$$\begin{tikzcd}
\bar{S} \arrow[r] & \tilde{R}^n \\
X \arrow[u] \arrow[r] \arrow[ur, dashed, "L"] & \bar{T} \arrow[u]
\end{tikzcd}$$

with: X—the *state space* of D (that is, a supercategory the objects of which are *states* of D and whose morphisms are transitions among these); $\tilde{R}^n$—the category the objects of which are elements from R, $R \times R, \ldots, R \times R \times \cdots \times R = R^n$, and whose morphisms are operators on real numbers; $\bar{S}$—a *supercategory* of generating classes and morphisms among these, and $\bar{T}$ a supercategory such that the structure of $\bar{S}$ depends primarily on the structure of T by setting a one-to-one correspondence $(\eta_{ij}: F_i \to F_j, L(s_i)) \to \bar{t}$ ($\bar{t}$ is an object in $\bar{T}$, s_i is an object in X and $F_{i,j}: \bar{S} \to R$). The object $\bar{T}$ will be called *temporal* for intuitive reasons. Consider λ and μ as fixed objects in R, and let $\mu > 0$. Let us consider a functor $G: X \to R$ such that:

a) $G(s) = V(\bar{t})$ for any $V: \bar{T} \to R$, s in X, and t in T;

b) $G(s') \geqslant \lambda$ and $G(s') - G(s'') < \lambda$ for some states $s', s'', \ldots$ in X.

c) $G(s^+) \leq \mu$ and $\mu \ll \lambda$ for some states s^+ in X. Then it will be noticed that an apparatus which measures time intervals with a precision $\delta \geqslant \lambda$ will observe only states $s', s'', \ldots$.

(D8) A category $\bar{X}$ which comprises only states $s', s'', \ldots$ of X will be subject to the foregoing conditions a), b) and will be called *stable* in X.

After intervals of time which are longer than the transitions which necessarily lead from a stable state s' to other stable states $s'', s''', \ldots$ according to condition b), $\bar{X}$ is stable in the algebraic sense, because s' and $s'', s''', \ldots$ are together in $\bar{X}$. If we take $\bar{S} = S_\sigma$ in (D7), then the corresponding X will be called *organismic*. It must be emphasized that a generating organismic supercategory S_σ is distinct from the corresponding organismic supercategory X_σ, although they are closely related. In the following some properties of X_σ will be presented.

(D9) If there are some distinct categories $X_d, X_e, \ldots, X_z$ which are stable in X, and if they form a supercategory, then X is called *multistable* of order z.

Returning to the example given on page 555, it should be noted that a virus may be called *ultrastable* because its state changes only when some observables take values which exceed some given limits, showing a succession of transient fields concluded by a terminal field which is always stable (Ashby, 1952). Then, a multistable system consists of "many ultrastable systems joined main variable to main variable" (Ashby, 1952), and this is adequately represented by a multistable supercategory together with its corresponding S_σ. As a consequence of this construction a multistable system gains "plasticity." The example suggests also that in the case of multistable systems there are present some distinct pushouts in X which are joined by morphisms, thus forming a *superpushout* in X. A geometric image of this situation is given below

$$\begin{tikzcd}
A_2 \arrow[r] & P \arrow[r, dashed] & B \arrow[r] \arrow[d] & B_1 \arrow[d] \\
A \arrow[u] \arrow[r] & A_1 \arrow[u] & B_2 \arrow[r] & P^+ \arrow[r, dashed] & \cdots
\end{tikzcd}$$

In constructing a mathematical model of a differentiating cell, it is useful to introduce "metastable" states s^* (Rosen, 1968), that is, states for which $G(s^*) \geqslant \varepsilon$, $\mu < \varepsilon < \lambda$. If A is a metastable field, B a stable state and $F(X)$, $F(Y)$ unstable fields, then a colimit may be constructed with these.

(D10) Let D, C be two categories and $F\colon D \to C$ a covariant functor. A *colimit* of F is a pair $(A, \{u_X\})$ with A being an object of C and $u_X\colon A \to F(X)$ morphisms which are defined for each object X of D, such that for any morphisms $\alpha\colon X \to Y$ in D, any object B in C and morphisms $v_X\colon B \to F(X)$ (defined for each object X in D), there exists a *unique* morphism $v\colon B \to A$ which makes commutative the following diagram

$$\begin{tikzcd}
A \arrow[r, "u_x"] \arrow[dr, "u_y" near start] & F(X) \arrow[d, "F(\alpha)"] \\
B \arrow[u, dashed, "v"] \arrow[ur, "v_x"' near start] \arrow[r, "v_y"'] & F(Y)
\end{tikzcd}$$

In other words we have the following equations:

$$u_Y = F(\alpha) \circ u_X, \qquad F(\alpha) \circ v_X = v_Y, \qquad u_X \circ v = v_X. \tag{5}$$

(If the category D is *small*, i.e. Ob D is a set, it is also called a *scheme* and a functor $F\colon D \to C$ is called a *diagram* in C of scheme D.)

Now, if a differentiating cell is considered as a multistable system, supercolimits must be introduced to represent distinct fields of stable, unstable and metastable states. According to this, the general contouring of the state space X_σ of a differentiating cell may be visualized as below

$$
\begin{tikzcd}
A \arrow[dr] & B \arrow[l] \arrow[d] \arrow[dl] & \cdots\circ & G(Z) \arrow[l, dashed] & C \arrow[l] \arrow[d] \arrow[dl] \\
F(X) \arrow[u] \arrow[r] & F(Y) \arrow[rr, dashed] & & G(U) \arrow[u] & E \arrow[l] \arrow[ul]
\end{tikzcd}
$$

$$F : \underline{D} \longrightarrow \underline{C} \qquad\qquad G : \underline{E} \longrightarrow \underline{F}$$

A supercolimit may be defined as a metadiagram constructed from aggregates of colimits and the connecting morphisms among these (as morphism $F(Y) \to G(U)$). It is not difficult to see the connection between these definitions and the principle of choice. Previously, in II, we suggested that the mathematical form of the principle of choice would be

$$[\{M_k(d_k)\}_{k \in K}] = \cdots = \lim_{d_j \to d_k} [\{M_1(d_j)\}_{1 \in L}] \tag{6}$$

and

$$\frac{\partial o}{\partial s} = (m, p_m) \tag{6'}$$

where m is a certain numerical matrix, p_m is the corresponding matrix of probabilities and o, s are some essential observables of a dynamic system. Then some matrices M are assigned to each diagram and (6) is a condition of extremum. Qualitatively, the principle of choice asserts that each superdiagram in X has a superlimit or a supercolimit. From these considerations the following proposition is easily derived.

Proposition 1. Any organismic supercategory X_σ has at least one superpushout.

Proof. The principle of choice implies that any superdiagram in X has a supercolimit. Then, from the construction of a supercolimit, there results that each diagram in X has a colimit. Categories which have a finite number of colimits for all functors over a scheme Σ are called finitely Σ-cocomplete. If these categories are Σ-cocomplete for all diagram schemes Σ then these categories are called *finitely cocomplete.* I shall consider only the finite case. Generally, a category which is finitely cocomplete has pushouts. Now, it results from the construction of a superpushout that X_σ has at least one superpushout (as far as a superpushout is made up of interconnected pushouts). Q.E.D.

When interpreted, this assertion says that any biological system comprises at least one ultrastable subsystem. Systems with "suspended" life as protozoa

(Rosen, 1958) are reduced to the ultrastable subsystem. However, an X_σ is generally multistable. The generation of "undifferentiated" cells may be represented by the degeneration of a supercolimit into a superpushout. The superpushout of an ultrastable system would keep indefinitely and its associated stable field $\{P', P'', \ldots\}$ will be frozen such that the ultrastable system is "immortal" or almost "immortal." By contrast, a "differentiated" X_σ has more "plasticity," and is "mortal" because there is an increased probability for the system to stop between two consecutive stable fields or "steady states," when the system is in an unstable state. This unstable state would be reached when the system is placed in extremely difficult conditions. This is also one of the results of Rashevsky's theory of organismic sets (1968a, Theorem 3). Another connection between this representation and the above quoted theory is established by the following statement.

Proposition 2. If $T\colon A \to B$ is a full functor, then without any conditions on the supercategories A and B, T reflects superlimits and supercolimits.

Proof. This is only an extension of a theorem of Freyd (quoted from Mitchell, 1965, p. 56). The reasoning is similar to that one followed in the above quotation. The fact that T has to be full, shows that any transformation which preserves the stability properties of a biological system must also have this property. This may appear as a condition which must be added to the principle of relational invariance.

Proposition 3. A supercategory X is $\bar{\Sigma}$-cocomplete if and only if the functor $I\colon X \to [\bar{\Sigma}, X]$ has a coadjoint $L\colon [\bar{\Sigma}, \times] \to \bar{X}$. (Here $[\bar{\Sigma}, X]$ denotes the class of all superdiagrams in X over the superscheme $\bar{\Sigma}$ and this class can be made into a supercategory.)

Proof. Extending the concept of a scheme and the concept of an adjoint functor for the supercategorical case this proposition is a mere generalization of Mitchell's Proposition 12.1 and Corollary 12.1 (Mitchell, 1965, p. 67). An adjoint functor may be intuitively understood as a "close" comparation of two categories which preserves some important properties of the source category. This condition is necessary and sufficient for a biological system in order to reach *effectively* its steady-states.

4. Discussion. It is not difficult to see that the principle of choice as investigated here has *functional* aspects. One could ask if only these aspects are taken into account by this principle. Rosen (1959) suggested that in a convenient context the principle of adequate design could have structural or *morphological* implications. We have emphasized in II (Section D) such a structural implication of the principle of choice. However, when structural aspects are involved, there are some implicit or explicit assertions concerning the principle of G-relations, which make convenient the context. Here only limited assertions concerning this principle were made. As a consequence Rashevsky's theorems 1 and 2 (1968a) cannot be proved only on the basis of the principle of choice. Those theorems seem to be related to the principle of G-relations. Thus, a mathematical convenient expression of the principle of G-relations is necessary if one wants to study the connections between these principles and their possible consequences.

The full consequences of the principle of choice may be rich in concrete cases, and a detailed mathematical study of some particular systems would provide a mean to construct on this basis multistable systems.

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