Coordinates Series A, no. 9

Juan de la Cosa's Projection: A Fresh Analysis of the Earliest Preserved Map of the Americas

Persistent URL for citation: http://purl.oclc.org/coordinates/a9.htm

Luis A. Robles Macias

Date of Publication: May 24, 2010

Luis A. Robles Macías (lroblesm@uoc.edu) is employed as an engineer at Total, a major energy group. He is currently pursuing a Masters degree in Information and Knowledge Management at the Universitat Oberta de Catalunya.

Abstract: Previous cartographic studies of the 1500 map by Juan de La Cosa have found substantial and difficult-to-explain errors in latitude, especially for the Antilles and the Caribbean coast. In this study, a mathematical methodology is applied to identify the underlying cartographic projection of the Atlantic region of the map, and to evaluate its latitudinal and longitudinal accuracy. The results obtained show that La Cosa's latitudes are in fact reasonably accurate between the English Channel and the Congo River for the Old World, and also between Cuba and the Amazon River for the New World. Other important findings are that scale is mathematically consistent across the whole Atlantic basin, and that the line labeled *cancro* on the map does not represent the Tropic of Cancer, as usually assumed, but the ecliptic. The underlying projection found for La Cosa's map has a simple geometric interpretation and is relatively easy to compute, but has not been described in detail until now. It may have emerged involuntarily as a consequence of the mapmaking methods used by the map's author, but the historical context of the chart suggests that it was probably the result of a deliberate choice by the cartographer.

Keywords: maps; cartography; Juan de la Cosa; cartographic projections; obliquity; latitude; ecliptic; Tropic of Cancer; spherical trigonometry; gnomonic projection; unnamed projection; Christopher Columbus

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1. Introduction: The Map of Juan de la Cosa

The map or chart of Juan de la Cosa is a manuscript map painted on a 93 x 183 cm parchment and signed by its author in 1500 in El Puerto de Santa María (now in the province of Cádiz, Spain). It was rediscovered in Paris in 1832, and is currently preserved at the Museo Naval of Madrid.

This map is the earliest preserved undisputed representation of the Americas, and consequently it has received a great deal of scholarly attention since the nineteenth century, particularly in the Spanish language. [1] It shows the lands known to Iberian navigators by the end of the fifteenth century: Europe, the Mediterranean and Africa in the middle; an incomplete representation of Asia to the east; and, to the west, a large green mass of land and adjacent islands of what would later be called the "New World" or "America."

From a cartographic point of view, the map displays two non-overlapping circles of compass roses [2], with no apparent grid of latitude or longitude. However, three orthogonal straight lines were drawn by the cartographer: one red line labeled *circulo cancro* (also labeled just *cancro*) that runs east-west roughly along the central axis of the map; another red east-west line farther south named *circulo equinocial* (and also linia *equinocial* and *equinocial*); and a green north-west line in the middle of the Atlantic labeled *liña meridional*.

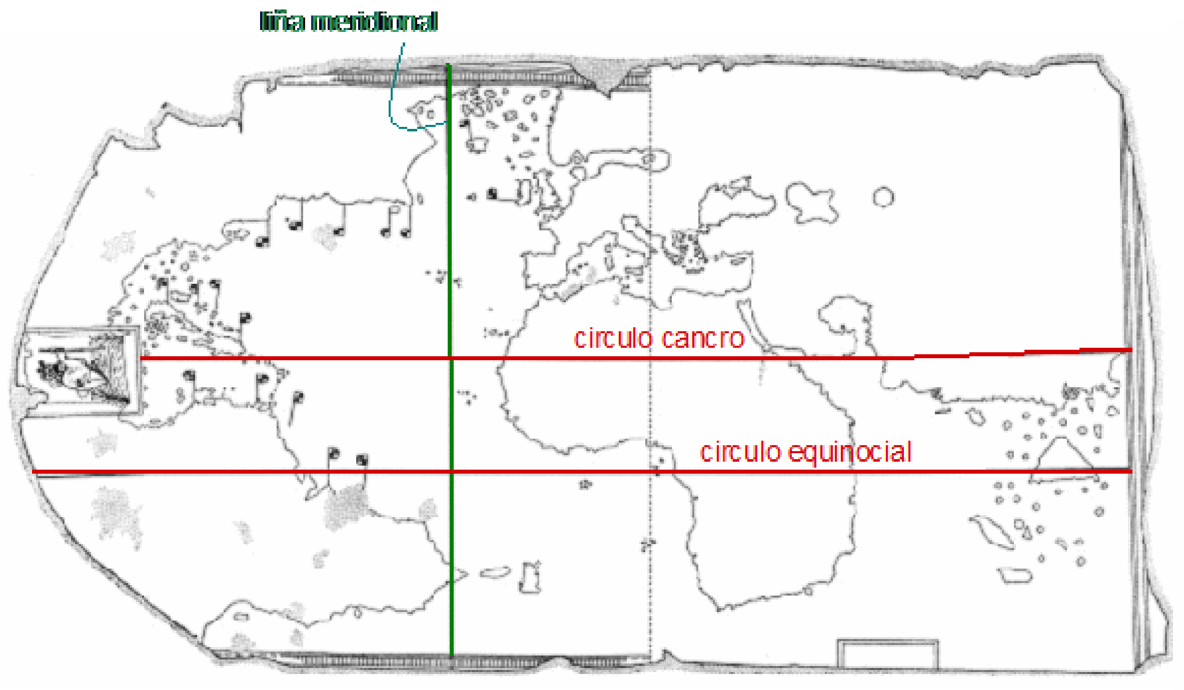

Fig. 1: Outline of the La Cosa map with its three main lines: cancro, equinocial and meridional.[3]

The chart appears to have been compiled from different sources of cartographic information:[4]

•The contours of Europe and the Mediterranean-Black Sea are similar to those on medieval portolan charts.

• Asia comes in great part from Ptolemy's *Geographia*, but other sources may also have been used. Furthermore, the map mentions the arrival of Portuguese ships in India.

• Regarding Africa, the toponyms are clearly different between the Atlantic basin, where they are mostly Portuguese and Castilian, and the Indian Ocean coast, which is labelled exclusively in local languages. This may be an indication of the use of two different cartographic sources for the territories west and east of the Cape of Good Hope.

• Central and South America's Atlantic coast and islands had already been explored when the map was drawn by Christopher Columbus's first three voyages and by at least four other expeditions authorized by the Crown of Castile. It is confirmed that Juan de la Cosa took part in three of them: in Columbus's first and second voyages to the Antilles, and in the expedition led by Alonso de Ojeda in 1499 to the northern coasts of continental South America. He may have also participated in Columbus's third voyage, according to some authors.[5] Furthermore, Portuguese ships touched the easternmost tip of South America in 1500 and named their discovery "island of Vera Cruz," a land that appears as an island (*isla descubierta por portugal*) on La Cosa's chart.

• Finally, the information on North America comes most probably from the English-sponsored expedition by John Cabot in 1497.

The visual aspect of the map is noticeably different between the Americas, which were drawn in green with sparse decoration, and the rest of the world. There are also signs that some parts of the map were not finished, especially the Asian mainland.[6] It is now widely recognized that the map was not the work of Juan de la Cosa alone, although for the sake of simplicity La Cosa will be mentioned in the rest of this article as though he had been the sole author of the map. According to Prof. Jesús Varela, the chart was probably ordered by Bishop Juan Rodríguez de Fonseca, the super-minister in charge of geographic discoveries at the court of the Catholic Monarchs. Several key political decisions reflected in the map (flags, toponyms, positions) were probably dictated directly by Fonseca. Varela has also pointed out that another Castilian cartographer, Andrés de Morales, worked for Fonseca at the time, and may have prepared small-scale maps of the American coastline that were later put together on a single parchment by La Cosa.[7]

Summing up, the 1500 chart should be seen as the fruit of the collective work of several cartographers who collected information from a variety of disparate sources and assembled it into a single world map. This process has been fittingly called "cartographic welding".[8]

2. The Problem of Latitude on the La Cosa Map

Cartographic studies of the La Cosa map have mainly focused on its western half— i.e. the territories of the Atlantic basin. The scale or scales of the chart have been the object of much scrutiny and controversy—with several studies concluding that La Cosa used two different scales, one for America and another for the Old World, while other authors disagree.[9] The most striking cartographic problem is, however, the latitude of the American lands, in particular the Antilles, which most scholars have found to be unexplainably erroneous. The present study attempts to provide a new cartographic interpretation of the chart that answers these open questions.

The issue of the erroneous latitudes has been neatly formulated by Ilaria Luzzana:

"El error de más de 12º por su latitud [de Cuba] y por la de Haití constituye un arcaísmo difícil de justificar en la fecha de esta carta, incluso teniendo en cuenta la diferencia de escala adoptada entre la parte oriental y la parte occidental del planisferio." (Author's translation: The error of more than 12º in the latitudes of Cuba and Haiti represents an archaism that is difficult to justify given the date of this chart, even taking into account the difference of scale between the eastern and western parts of the planisphere.)[10]

Where does the figure of 12º come from? Most analysts of the La Cosa map have identified the thick red line labelled *circulo cancro* with the Tropic of Cancer and the *circulo equinoccial* with the Equator. They have then superimposed on the map a "real image" of the world based on a grid of perpendicular parallels and meridians, i.e. some kind of cylindrical projection, in such a way that the two mentioned lines match their supposed equivalents on the La Cosa map; and they have then measured the longitude and latitude of each territory.[11] Such studies have found that the Antilles are quite right in longitude but appear to be far more to the north than they should, lying strangely to the north of the Tropic instead of south of it. Such a conclusion is bizarre because it is generally accepted that fifteenth-century navigators had less difficulty in determining latitude than longitude.

Ricardo Cerezo Martínez, 1992-1994

Between 1992 and 1994, Ricardo Cerezo Martínez published a series of three extensive articles on La Cosa's chart. Although Cerezo clearly stated that this map "lacks any relationship with the *plate carrée* type of chart, with meridians and parallels," he nevertheless implicitly assumed for his analysis a *plate carrée* projection based on identifying the *cancro* line with the Tropic of Cancer.[12] (The *plate carrée* projection is also known as the equirectangular or 'plain chart' projection.)

Like previous scholars, Cerezo observed that the Greater Antilles presented a large apparent error in latitude, and proposed an original explanation: as the cartographer oriented his map according to magnetic north, a particularly strong magnetic declination in the Caribbean would have introduced a clockwise rotation of the whole area of about 11 degrees. A graphic representation of this hypothesis is shown in Figure 2. This explanation presents, in my opinion, two weak points: 1) no conclusive measurement of the value of magnetic declination across the Caribbean around 1500 is available, and 2) even if we accepted Cerezo's hypothesis, a still-substantial mismatch of around 5 degrees in latitude would remain to be explained.

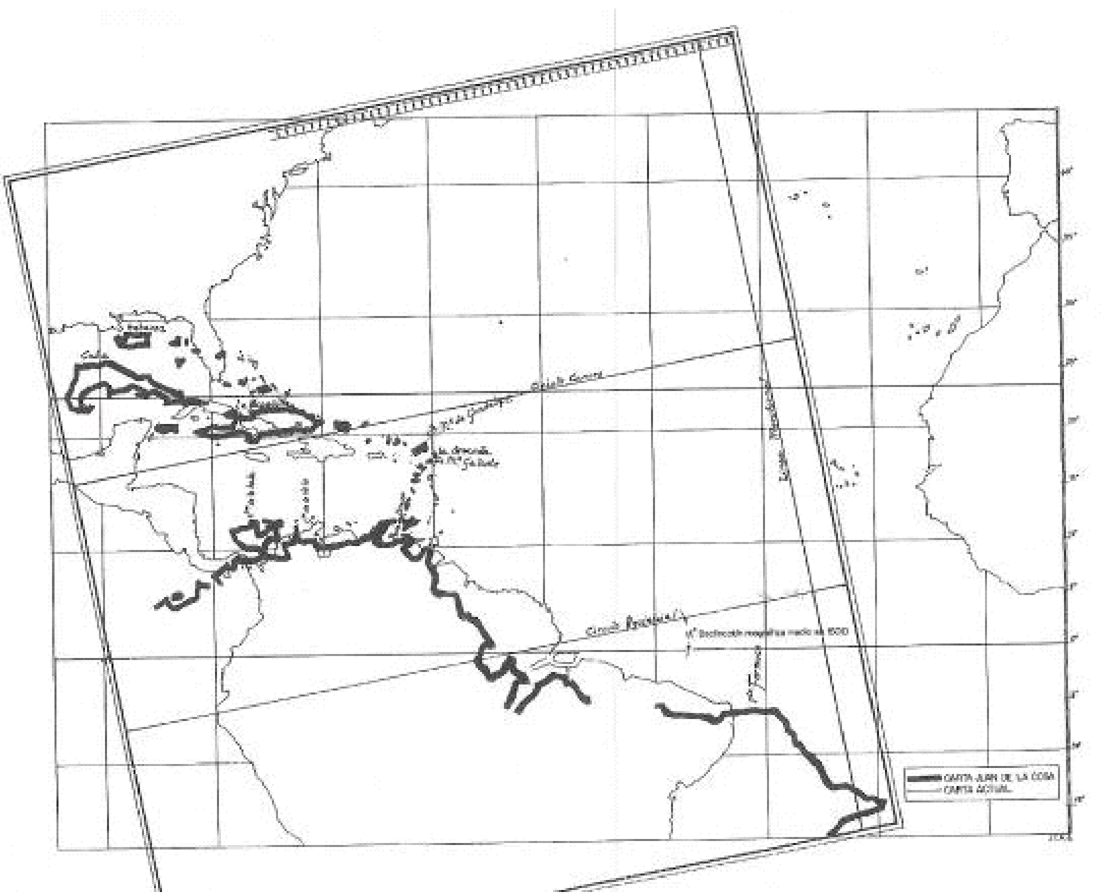

Fig. 2: Ricardo Cerezo's explanation of the apparent error in latitude of the Greater Antilles by means of a 11º rotation. The real coastline has been drawn using a Mercator projection.[13]

Fernando Silió Cervera, 1995

In the most comprehensive cartometric study of La Cosa's chart to date, Fernando Silió Cervera reached conclusions similar to Cerezo's regarding the alleged errors in latitude for the Antilles, and proposed the same explanation based on magnetic declination.[14] In addition, he observed that for the Old World latitudes were inexact from the Canaries to the north and "totally erroneous in Northern Europe."[15] This is rather surprising given that the latitudes of these regions were well known by 1500.

Probably the most original observation made by Silió was the apparent coexistence on La Cosa's chart of two different assumptions about the length of a degree (or, inversely, of the radius of the earth): a module of 17½ leagues per degree (computed from the distance between the *equinocial* and *cancro* lines), which fits well with the Western Africa coastline, and another module of 16 2/3 leagues per degree, which seems to apply to the north-south axis of the Euro-Mediterranean region.[16] This is one of the reasons why Silió believes that La Cosa's chart was composed by 'welding' different local maps into a large mappamundi.

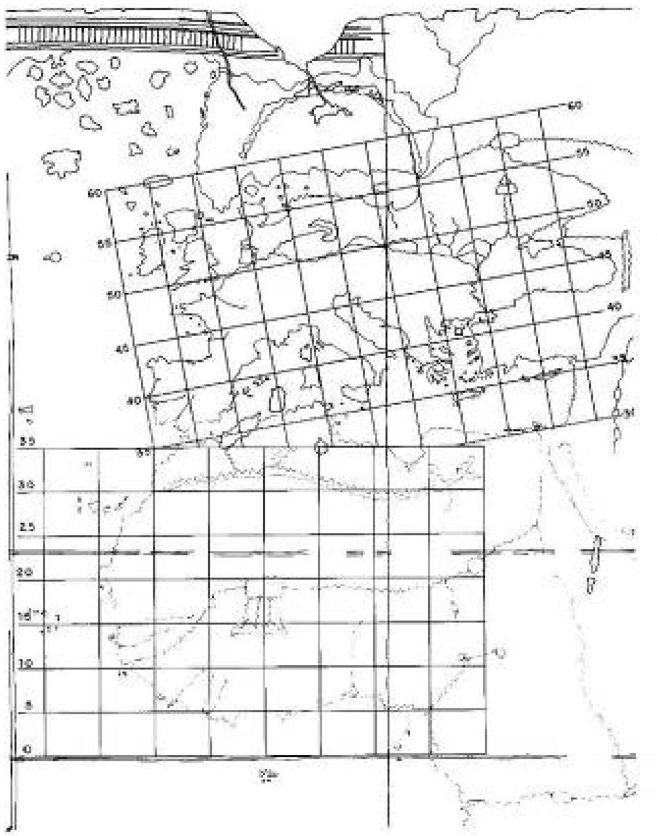

Fig. 3: Double grid of meridians and parallels proposed by Fernando Silió for Africa and the Euro-Mediterranean region.[17]

Ángel Paladini Cuadrado, 1994

At the same time that Silió was writing his book, Ángel Paladini Cuadrado published an insightful analysis of the latitude of American territories on La Cosa map.[18] He chose to ignore the *cancro* line, and calculated the length of a degree by measuring the distance between the *equinocial* and well-identified points of the continental Caribbean coastline. He then built a traditional equirectangular grid only for that region, and found out that the latitudes of the Antilles on the La Cosa map were far more accurate than previously thought, with errors of less than 2 degrees. Paladini also discovered that the error in latitude appeared to increase systematically with longitude, in what he called an almost linear correlation, and proposed that this systematic error might be due to the effect of magnetic declination, not taken into account by the cartographer.

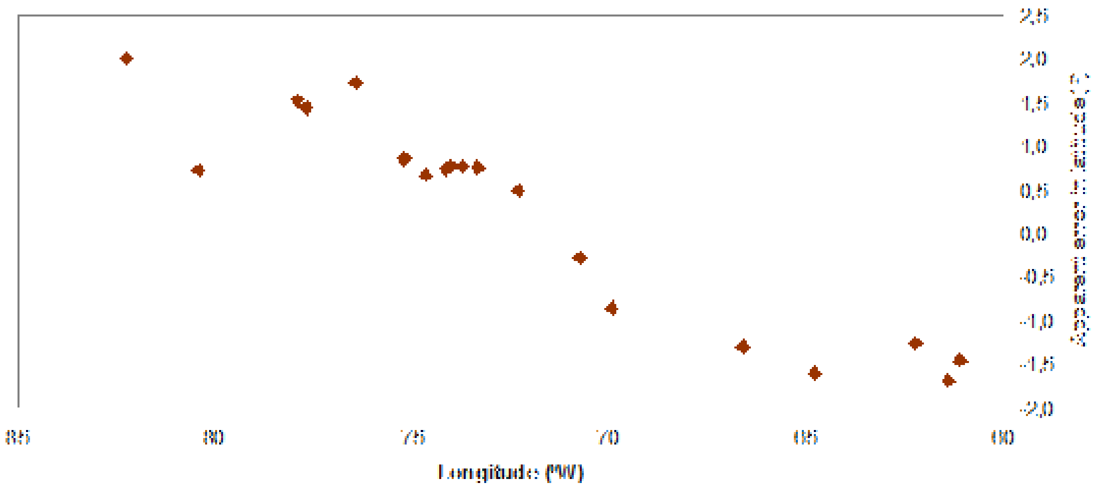

Fig. 4: Apparent error in latitude of several Caribbean locations as a function of their real longitude, according to Ángel Paladini Cuadrado.

Summing up, those scholars who identify La Cosa's *cancro* line with the Tropic of Cancer inevitably conclude that the map presents huge errors in latitude. On the other hand, Ángel Paladini chose to ignore the *cancro* line and came up with the opposite conclusion—i.e. that latitudes are quite accurate in the Caribbean region drawn by La Cosa. This shows how decisive assumptions about the nature of that line are for the analysis of the map, and raises the question: is it possible that the *cancro* line does not represent the Tropic but something else?

3. Identifying the Underlying Cartographic Projection

In order to answer the question of whether the latitudes of La Cosa's Atlantic territories are grossly erroneous or, on the contrary, remarkably accurate, I have carried out an analysis of this part of the map to determine its underlying cartographic projection.

Some readers may be surprised by this proposal, as it has often been written that La Cosa's chart was drawn "with no projection". I would like at this point to quote from Waldo Tobler's classical article on the projections of ancient maps : "it is correct to say that [an ancient map] is not based on a map projection only in the sense that the cartographer involved was not consciously employing a map projection. But, as one learns from any elementary work on map making, every map requires a map projection. The ancient maps therefore are implicitly referred to some map projection."[19]

—Methodology

This study follows the approach for identifying the underlying cartographic projection of old maps proposed by Tobler in his previously mentioned article of 1966. The method consists of the following steps:

1. Postulate one possible projection. The choice of the candidate projection(s) can be based on historical information or on geographical data contained in the ancient map itself.

2. Draw a 'real' image of the world in the postulated projection, i.e. using the mathematical equations that describe it.

3. Superimpose the real image over the ancient map, which will probably involve some scaling and shifting of the image.

4. Compute the error— i.e. some numerical measure of the discrepancy between the real coordinates of a set of known place-names and the coordinates inferred for those place-names on the ancient map.

5. Iterate until the error found is sufficiently small.

When successfully applied, this approach yields in the end a set of explicit mathematical equations that represent reasonably well the ancient map.

In later years, Tobler's method was generalized so that the underlying projection could be described not with explicit equations but with a table of numerical values corresponding to the statistical best fit between the map's place-names and their associated real coordinates.[20] A recent application of this generalized method is the study performed by Joaquim Alves Gaspar to identify the underlying "empirical projection" of medieval portolan charts.[21]

Numerical methods where a statistical best fit is attempted require identifying the real locations of a large set of place-names evenly distributed over the map. Otherwise, the results might not be statistically representative.[22] These methods are therefore best suited for maps of well-known regions with plenty of clearly-identified toponyms— Mediterranean portolans being good examples of such maps. An advantage of Tobler's earlier approach is that, while it still requires the identification on the ancient map of some place-names to use them as control points to measure the accuracy of the match, the number of such control points can be smaller than in numerical methods. Besides, in cases where toponyms are particularly scarce, the accuracy can also be qualitatively estimated from the comparison of coastal profiles between maps.

In the Juan de la Cosa chart, the Mediterranean and Black Sea regions are described with 635 toponyms and would therefore be well suited for best-fit methods. On the other hand, the whole South American coastline contains 66 and North America only 19 place-names.[23] Furthermore, while it is easy to assign real coordinates to the Mediterranean toponyms, in the Americas the identification of many place-names is uncertain and often a matter of controversy. This is why the present study uses the Tobler's earlier approach rather than statistical methods to ascertain the underlying cartographic projection of the Atlantic basin of the La Cosa chart.

In the following pages, several possible mathematical projections will be considered to explain the shape of the Atlantic territories on the La Cosa chart. Candidate projections will be chosen based on observations about the nature of the *cancro* and *meridional* lines as explained below. The discrepancy in latitude and longitude of a set of place-names will then be evaluated for two of these projections.

The precision of this study is limited by several factors. To begin with, many toponyms in the La Cosa chart are not precisely associated with a single point of the map. Secondly, no high-resolution digital image of the map is currently available, so researchers have to work with partial photographic reproductions that may introduce some distortions. Finally, the deformation of the supporting parchment could also be a source of error, although previous studies have shown that they are minimal and limited to the western edge of the map.[24] For all these reasons, discrepancies of less than one degree in longitude or latitude will be considered satisfactorily small in the comparison.

—A New Interpretation of La Cosa's "Cancro" Line

Before tackling the issue of the *cancro* line, let us first review briefly what the other two main lines on the map are believed to mean. Everybody agrees that the thick red east-west line labeled *equinocial* represents the equator, and there is no reason to doubt it. Indeed, La Cosa placed his equator very close to the real position, unlike Columbus (who believed it passed over the Portuguese fortress of Sao Jorge da Mina, actually at 5 degrees north), and unlike later maps like those of Waldseemüller (1507), Apiano (1520) or Honter (1542).[25] Regarding the thick green north-south line labeled *liña meridional*, it has been debated whether it represents the meridian mentioned by Pope Alexander VI in his bull *Inter Caetera* (1493), the reference line of the Treaty of Tordesillas of 1494,[26] the place where Columbus found magnetic declination to be zero,[27] or the latter two things at the same time.[28] Notwithstanding the exact interpretation, all authors agree that it represents a meridian. Given that the line has been drawn to the west of Cape Verde islands and crosses the archipelago of the Azores between the islands Terceira and S. Miguel, it can be estimated that the *liña meridional* corresponds to a meridian approximately 26 degrees west of Greenwich.

At this point we should also review the concept of the *ecliptic*. It is usually defined as the great circle formed by the intersection of the earth's surface with its orbital plane, which is inclined by around 23.4º with respect to the planet's axis of rotation. The ecliptic is tangent both to the Tropic of Cancer and to the Tropic of Capricorn. It should also be recalled that, due to the earth's rotation, the ecliptic changes place continuously, moving around the planet approximately once every day. The point of tangency between the ecliptic and the Tropic of Cancer will thus find itself at any given meridian once per day; on the summer solstice it will pass over each meridian exactly when it is solar noon at that meridian.

On Figure 5 the green curve represents meridian 26ºW, and the red one the ecliptic at the point in time when it is tangent to the Tropic of Cancer precisely at meridian 26ºW. The inset shows a fragment of La Cosa's map showing the *cancro* line in the Caribbean region.

It can be seen that, while the real Tropic of Cancer lies to the north of the Greater Antilles, the ecliptic passes directly above the island of Puerto Rico and remains south of Hispaniola, Jamaica and Cuba. On the La Cosa map the *cancro* line follows a path similar to the real ecliptic, crossing Puerto Rico and the Caribbean Sea and staying south of Hispaniola and Jamaica.

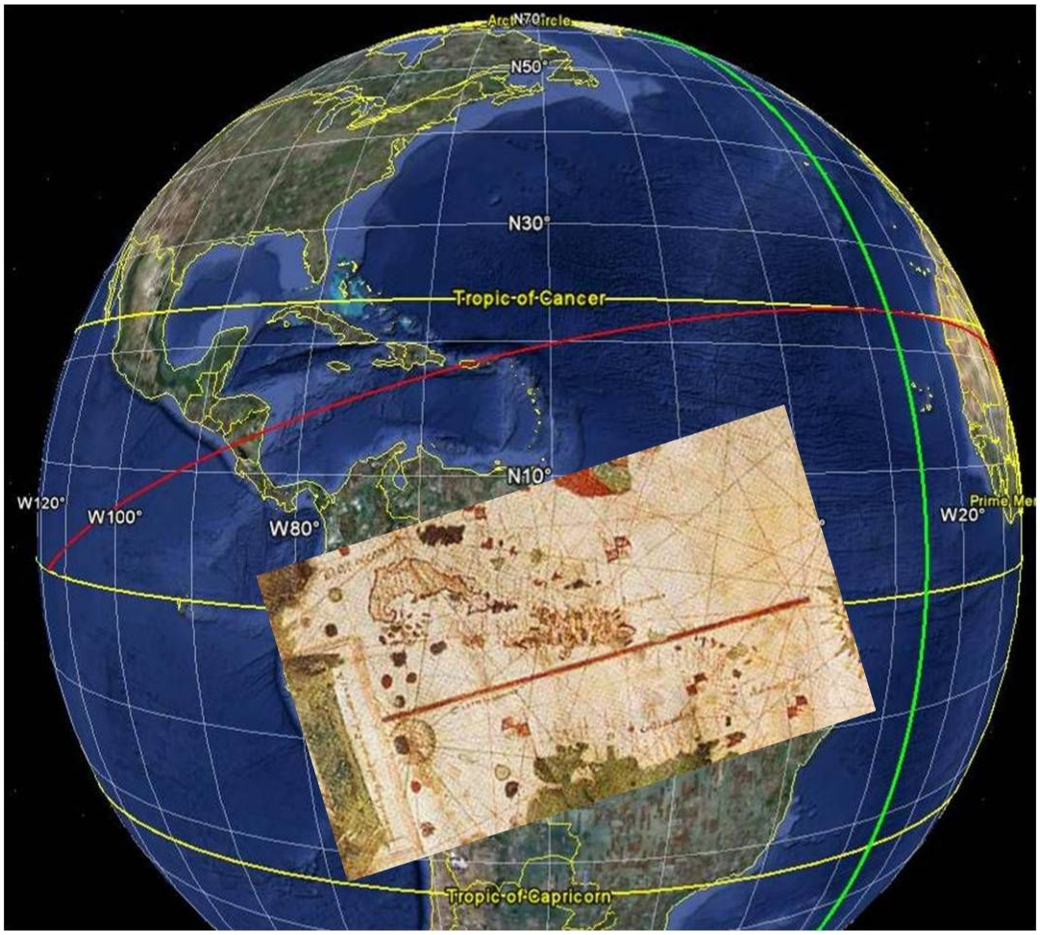

Fig. 5: Background: Perspective view of the Earth with meridian 26ºW in green, the Tropic of Cancer in yellow and the ecliptic in red. Inset: La Cosa's Caribbean region with cancro line in red.

Figure 6 shows the area from La Cosa's meridian line to the point in the Indian Ocean where the ecliptic intersects the equator—again with the meridian 26ºW in green and the ecliptic in red. It can be seen that, while the real Tropic of Cancer crosses North Africa, the Red Sea and the Arabian Peninsula, La Cosa's *circulocancro* lies substantially more to the south and does not intersect the Red Sea. This again corresponds better with the ecliptic line, which passes south of Bab el Mandeb, the strait linking the Red Sea and the Indian Ocean. Further east, however, the *cancro* line seems to match correctly the real Tropic of Cancer, as both cross the Indian subcontinent at similar locations.

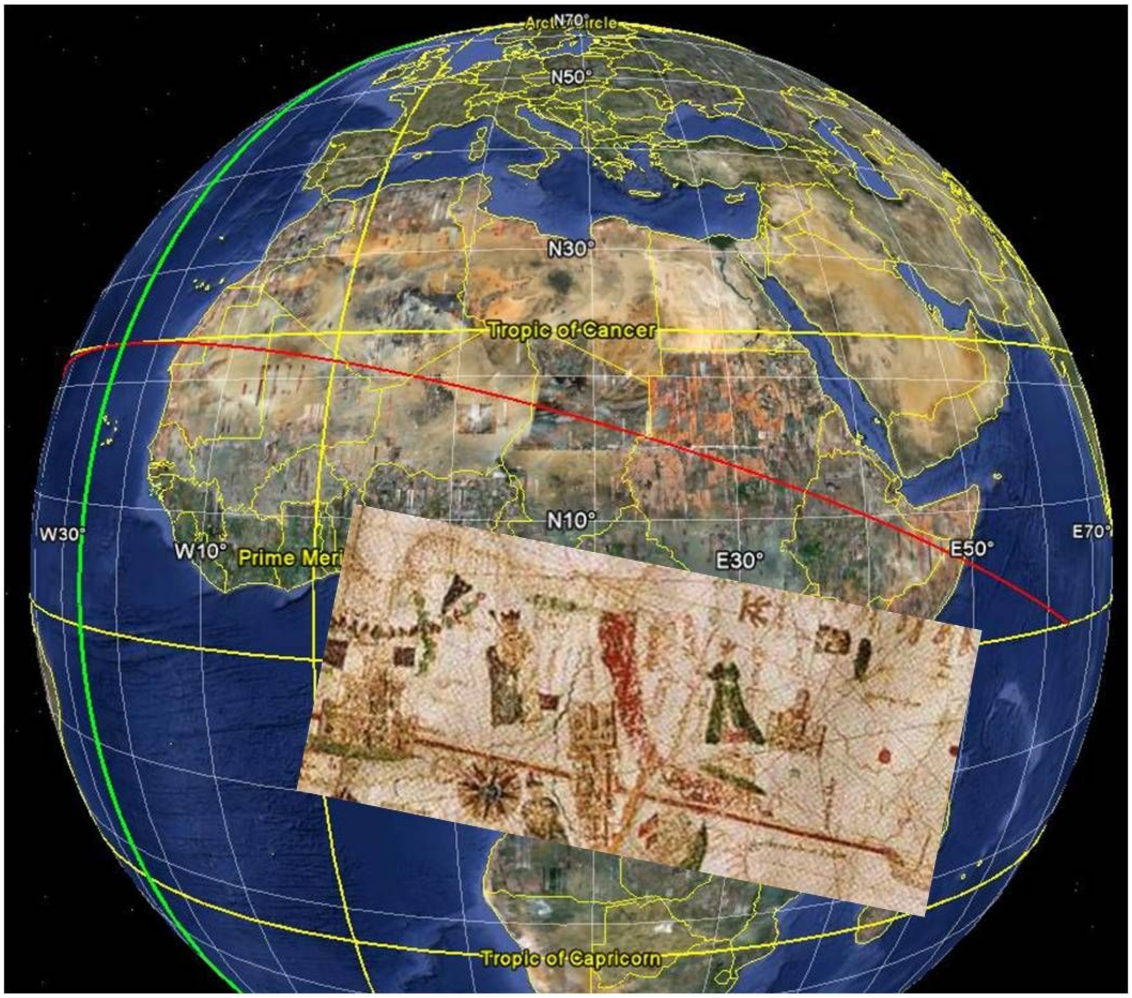

Fig. 6: Background: Perspective view of the Earth with meridian 26ºW in green and the ecliptic in red. Inset: La Cosa's Northeast Africa and Arabia with cancro line in red.

These observations lead me to postulate the following hypotheses:

1. La Cosa's *liña meridional* represents the meridian 26ºW Greenwich.

2. On the western half of the map (the portion covering the Americas, Europe and Africa) La Cosa's *cancro* line does not represent the Tropic of Cancer but the ecliptic, drawn at the point in time when it is tangent to the Tropic of Cancer at the meridian labeled *liña meridional*.

3. And, of course, La Cosa's *equinocial* represents the Equator.

—Mathematical Derivation of Candidate Projections

A system of coordinates has been defined on the La Cosa chart as shown on figure 7. Axis X represents the east-west direction and axis Y the north-south. The intersection between the *meridional* and the *equinocial* lines is taken as the origin of coordinates.

Candidate cartographic projections will be those that transform the three great circles identified with La Cosa's main lines (meridian 26ºW, equator, and ecliptic) into a set of three orthogonal straight lines. The word *projection* is used here in a broad sense, meaning any set of equations X = X (LAT, LON) and Y = Y (LAT, LON), where LAT stands for latitude and LON for longitude.

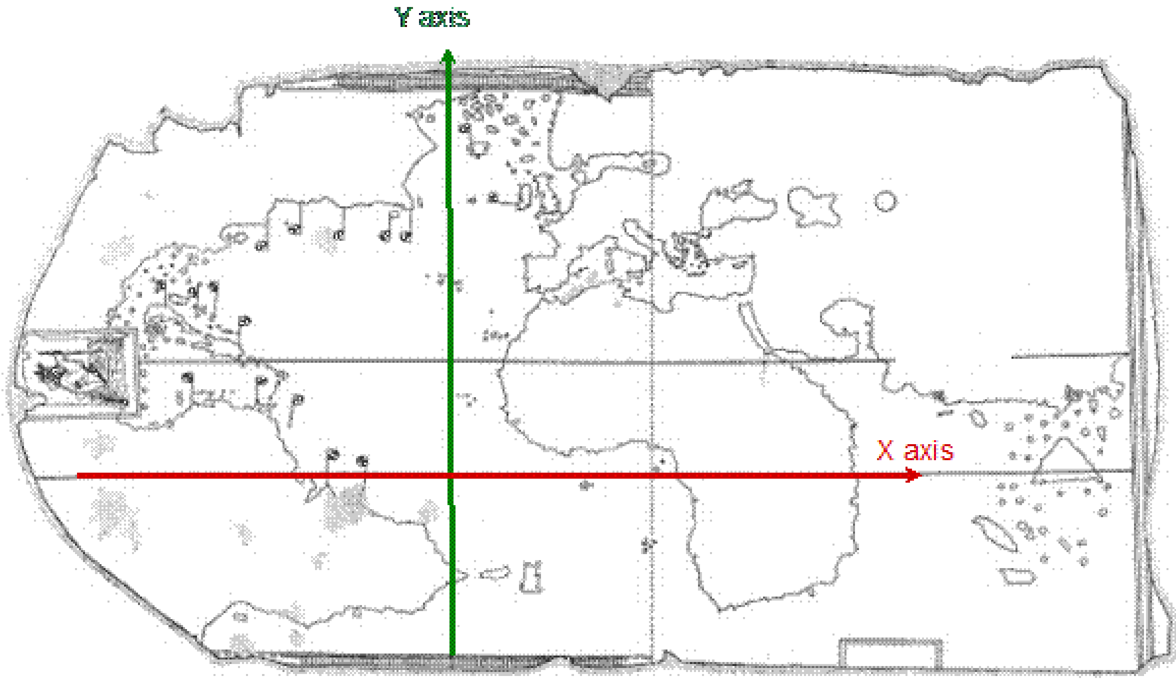

Fig. 7: Axes of reference X and Y defined along the equinocial *and* meridional *lines, respectively.*

The fact that the *equinocial* line represents the equator is expressed mathematically as $Y_{LAT=0} = 0$. Similarly, assuming that *liña meridional* is equivalent to meridian 26ºW (which has been called LON_0) results in the constraint $X_{LON=LON0} = 0$.

Regarding the ecliptic, if we make the assumption that the earth is a perfect sphere, it can be described by the following equation:

$$\tan LAT = E\cos(LON - LON_0)$$

where parameter E is a constant equal to tan (23.4º) and LON_0 is the longitude of the meridian at which the ecliptic is tangent to the Tropic of Cancer, equal to 26ºW in this case.

The projection we are looking for has to transform the ecliptic into a straight line of equation Y = constant. This will be verified if the Y component of the candidate projection is an exclusive function of the ratio $\tan LAT / \cos(LON - LON_0)$.

Table 1 summarizes the mathematical conditions deduced from the hypotheses on the three main lines of La Cosa map.

Liña meridional = meridian 26ºW	$X_{LON=LON_0} = 0$
Cancro = ecliptic	$Y = Y\left(\dfrac{\tan LAT}{\cos(LON - LON_0)}\right)$
Equinocial = equator	$Y_{LAT=0} = 0$

Table 1: Mathematical conditions imposed by the three hypotheses on the nature of the main lines of Juan de la Cosa's map

In theory there exist infinite mathematical projections that satisfy the three conditions listed in Table 1, but in fact few of them have ever been described in the literature. The best-known one is the gnomonic projection, which will be discussed in next section. It will be shown, however, that a map of the Atlantic plotted in the gnomonic projection does not fit well with the coastlines drawn by La Cosa. This will lead to testing a second projection that is far less well known, but provides a substantially better match.

4. Gnomonic Projection

The gnomonic projection (GP), also known as gnomic or central, is a perspective projection obtained by plotting the sphere from its center onto a plane tangent to its surface. Its main property is that all great circles of the sphere are represented as straight lines on the plane. While it was probably known at least in theory since antiquity, no map in gnomonic projection dated before 1600 has been preserved. [29]

The mathematical equations of the gnomonic projection in the equatorial case are:

$$X = \tan(LON - LON_0)$$

$$Y = \frac{\tan LAT}{\cos(LON - LON_0)}$$

where LON stands for longitude, LAT for latitude and X and Y represent the horizontal and vertical coordinates, respectively, of the flat projected map. Longitude is measured with respect to a certain meridian of reference, LON_0. Both LAT and ($LON – LON_0$) are restricted to the range (-90º, +90º). These equations verify the conditions of Table 1.

A real image of the Atlantic basin has been plotted in the gnomonic projection, using the point LON_0 = 0º, LON_0 = 26ºW as the centre of projection. The image was built with Microsoft Excel and, in order not to overload the graphs, only selected coastal features and islands were plotted. The result has been superimposed over La Cosa map. In addition, the discrepancies in latitude and longitude of a set of control points have been computed numerically and reported in Annex 1.

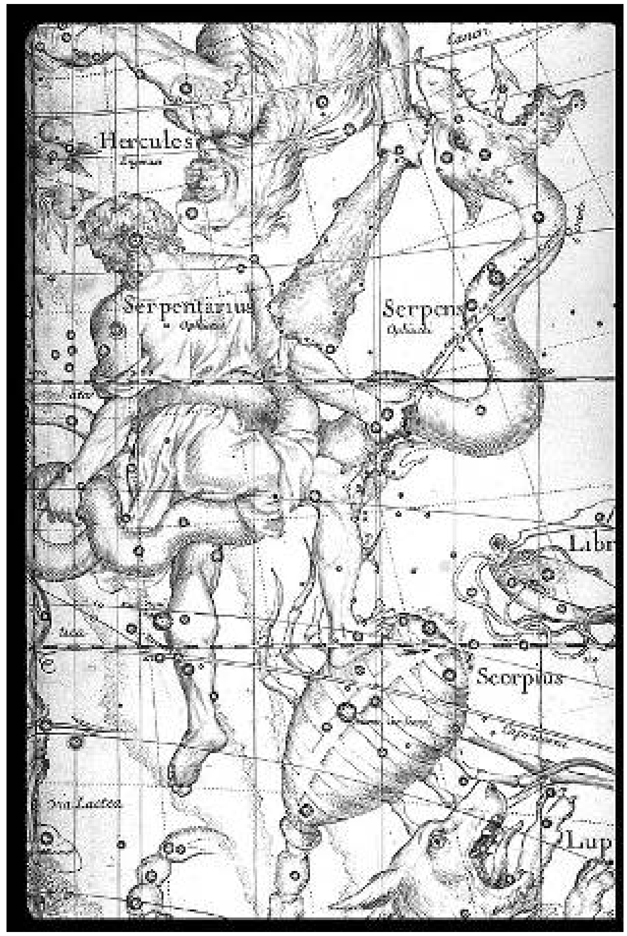

Fig. 8: Star map of 1674 in gnomonic projection. The ecliptic and the celestial equator are parallel horizontal lines in this particular case of the projection.[30]

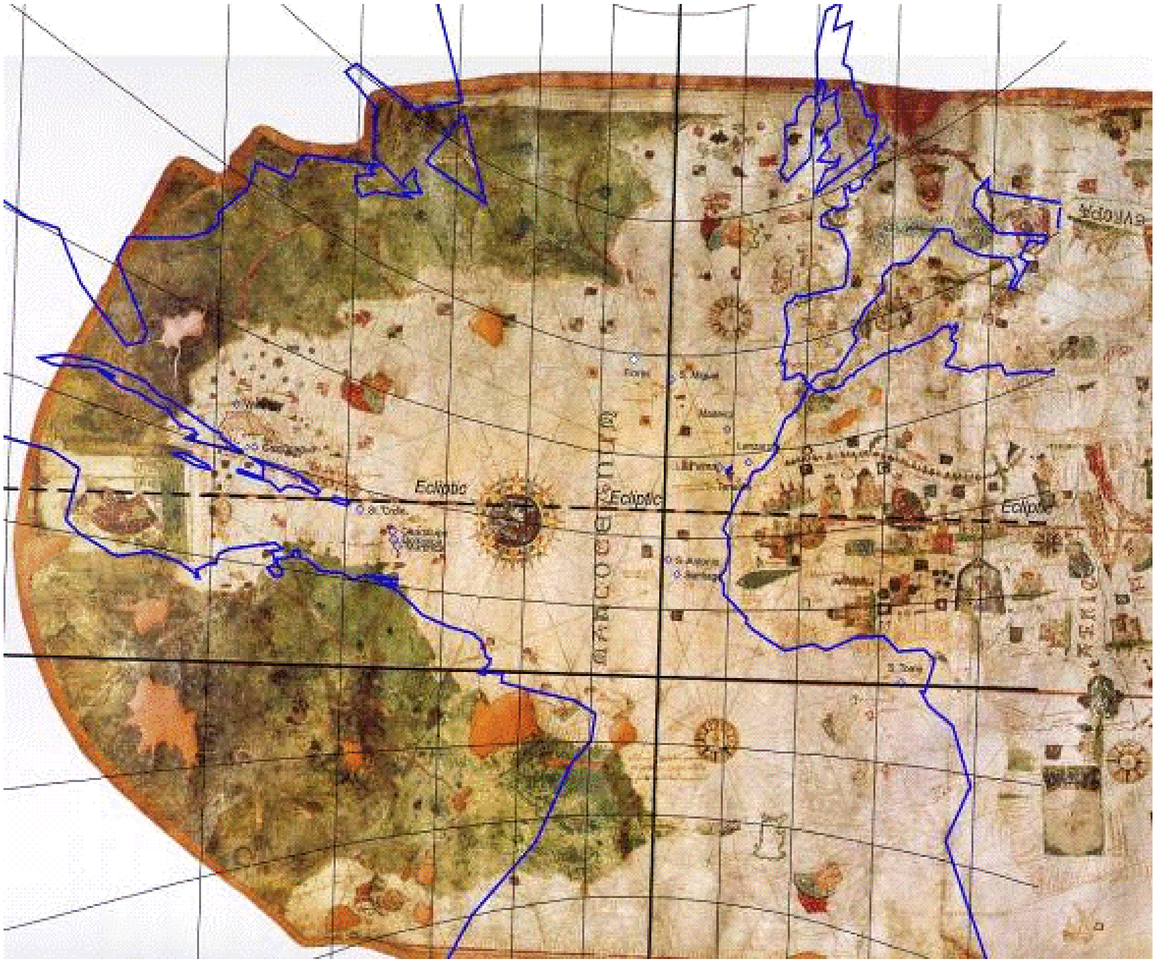

Fig. 9: Superimposition over the map of Juan de la Cosa of a real image of the Atlantic basin in gnomonic projection with reference meridian 26ºW of Greenwich.[31]

Figure 9 and the numeric results show that some regions like the western coast of Africa present a good fit with the map of Juan de la Cosa, and also that the Antilles are located close to their latitudes if a gnomonic projection is assumed. On the other hand, the Iberian Peninsula, whose latitude was well known to Spanish cartographers by 1500, presents a substantial error of up to 6 degrees in latitude. The mismatch is even greater for Britain and Ireland (ca. 9 degrees) or Iceland (ca. 14 degrees).

The huge latitude errors observed in Europe are the main reason to conclude that the Atlantic basin of La Cosa map was not drawn using a gnomonic projection. Another projection that satisfies the conditions of Table 1 should therefore be postulated.

5 . Unnamed Projection

Equations

Assuming that the earth is a sphere, let us define a mathematical cartographic projection by the following equations:

$$X = LON - LON_0$$

$$Y = \arctan\left(\frac{\tan LAT}{\cos(LON - LON_0)}\right)$$

where both LAT and ($LON - LON_0$) are restricted to the range (-90º, +90º). It can be easily verified that these equations satisfy the conditions listed in Table 1.

This projection has, as we will see, a simple geometric interpretation. Despite this, it has received very little attention from cartographers, and has not been given any specific name. In the rest of this article it will be called the Unnamed Projection (UP).

Geometric Interpretation

On a spherical Earth, let us draw the equator and a reference meridian. Through any given point P on the surface of the sphere, one and only one great circle can be drawn that is perpendicular to the reference meridian. This great circle cuts the equator at two points, diametrically opposed. If this operation is repeated through any other point P' the corresponding great circle will cut the Equator at exactly the same two points, which we will henceforth call "west pole" and "east pole." We will call "**east-west meridians**" the great circles that converge on the west and east poles. So as to avoid confusion, we will call "**north-south meridians**" the great circles that are usually called meridians.

Let us define **obliquity (OBL)** of point P as the angle between the plane that contains the east-west meridian of P and the plane of the equator; this is equivalent to the latitude of the point where the east-west meridian of P intersects the reference north-south meridian. It is evident that the obliquity of any point on the Equator is 0º.

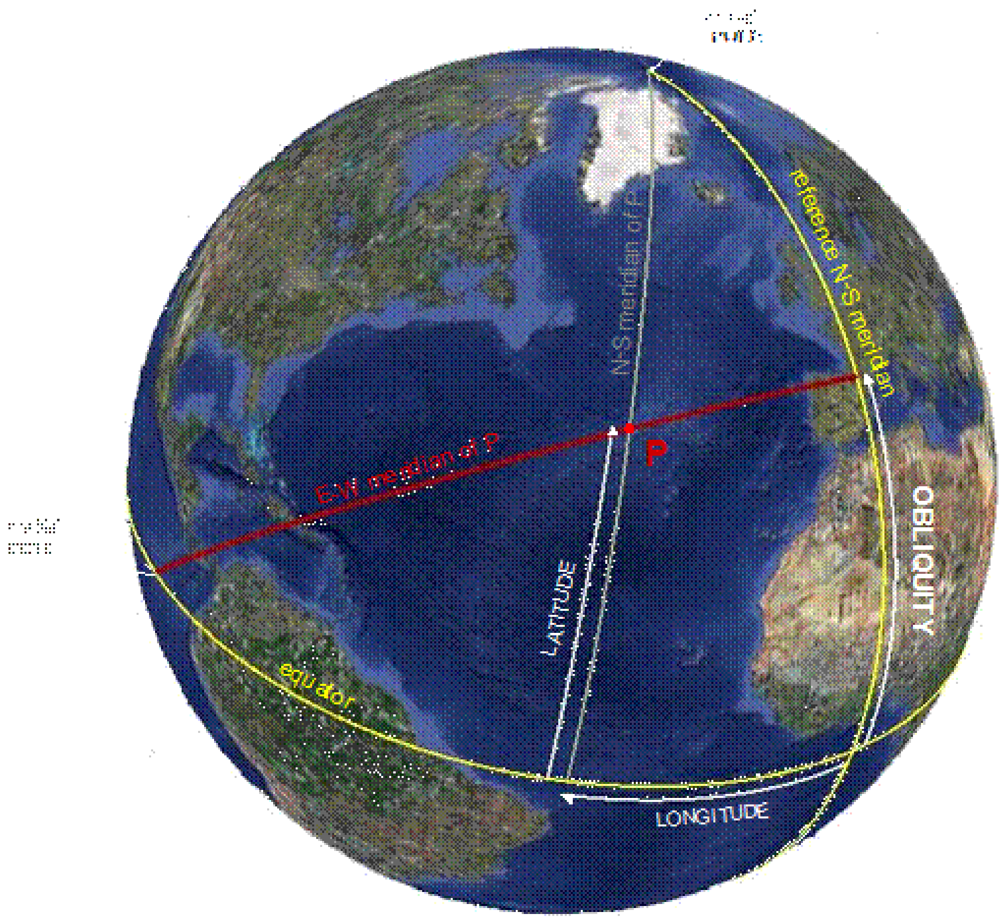

Fig. 10: Visualization of the concepts of east-west meridian and obliquity.

I initially chose to call this angle "obliquity" just because it was consistent with the commonly-used phrase "the obliquity of the ecliptic". To my surprise, I later found out that the seventeenth-century author Samuel Sturmy already used the word obliquity for a very similar concept in his discussion of great-circle sailing.[32]

The mathematical formulae for calculating obliquity from latitude (LAT) and longitude (LON) can be obtained by applying spherical trigonometry:

$$\tan OBL = \frac{\tan LAT}{\cos(LON - LON_0)}$$

where subscript “0” indicates as usual the reference north-south meridian.

Using this formula for obliquity, we can rewrite the equations that define the Unnamed Projection, obtaining:

$$X = LON - LON_0$$

$$Y = OBL$$

The UP can thus be visualized as the result of stretching north-south meridians and east-west meridians into a grid of vertical and horizontal straight lines equally spaced between them. This projection is therefore similar to the *plate carrée* or equirectangular projection except that horizontal lines represent constant obliquity instead of constant latitude.

—Cartographic Properties

Figures 11 and 12 show a sketch of the Atlantic basin drawn in the Unnamed Projection using two different reference meridians: 26ºW and 50ºW (0º = Greenwich).

It can be seen that, while north-south meridians are straight vertical lines, parallels of constant latitude appear as curves, all of them perpendicular to the reference meridian. As in the gnomonic projection, changing the reference meridian affects substantially the shape of the lands represented on the map, due to the fact that the vertical coordinate Y depends explicitly on the value of LON_0. The horizontal dotted line represents the ecliptic, which naturally appears as a horizontal straight line in this projection.

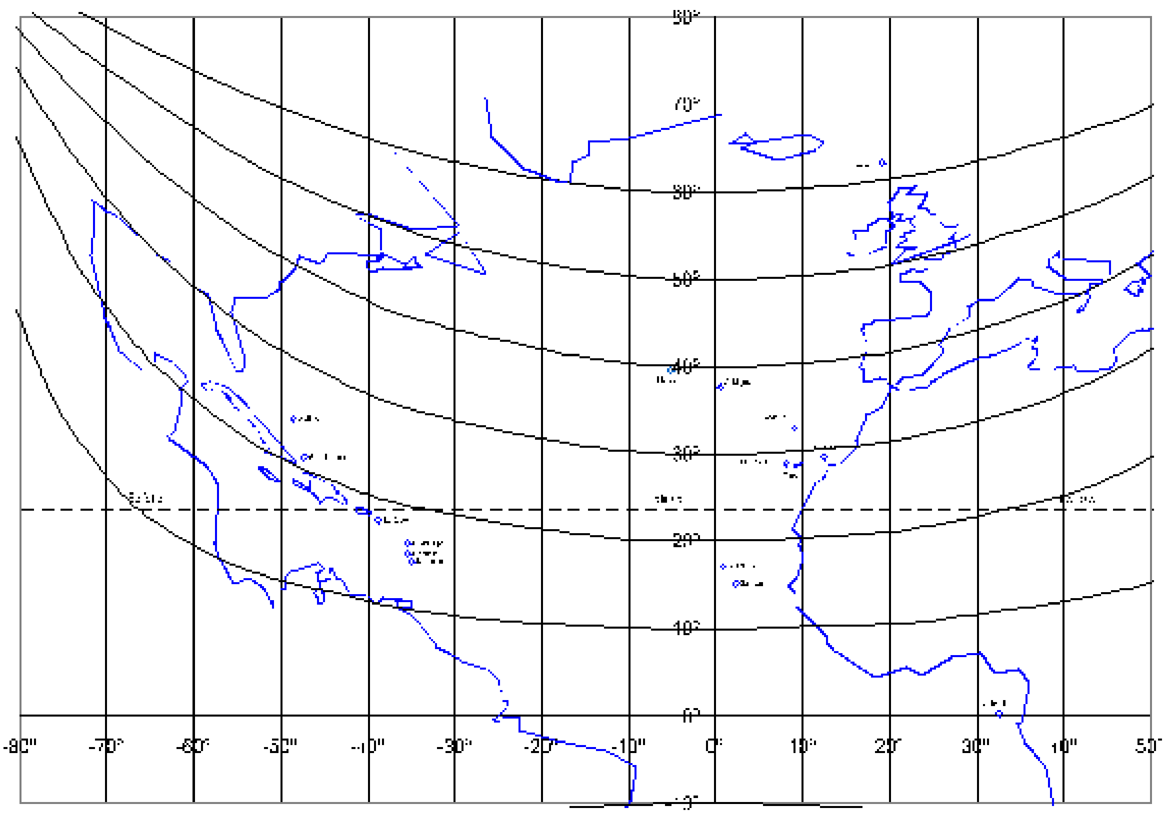

Fig. 11: Unnamed Projection with reference meridian = 26ºW.

Fig. 12: Unnamed Projection with reference meridian = 50ºW.

A mathematical analysis shows that the UP is neither equivalent nor conformal, except for a single point: the origin of coordinates. Demonstrations can be found in annex 2. Nevertheless, the UP presents some interesting properties (listed in Table 2) compared with those of other projections that were used or theoretically known in the early 16th century.

One of the advantages of the UP over cylindrical projections is that the pattern of deformation is equal along the east-west and north-south directions; in mathematical jargon, it presents a rotational symmetry of order 4. This property may be useful when drawing a world map because it does not give priority to north-south over east-west or vice versa.

In the UP, both vertical and horizontal lines are great circles. This is better than the *plate carrée* projection, where meridians are the only orthodromic lines drawn straight, but of course is inferior to the gnomonic projection, where all great circles are straight lines.

In theory the UP can represent a whole hemisphere, but in practice distortions become enormous for the regions near the North, South, West and East poles. Therefore, its use should be limited to an area reasonably close to the center of projection, say not more than 70 or 80º away.

Finally, the two main axes of coordinates of the UP are drawn in real magnitude. This is an advantage over azimuthal projections, in particular the gnomonic, which distorts scale badly along those axes.

	Unnamed	**Plate carrée**	**Gnomonic**	**Stereographic**
Conformal?	no	no	no	YES
Equivalent?	no	no	no	no
Equal deformation N-S and E-W?	YES	no	YES	YES
Straight lines = great circles?	Vertical and horizontal only	Vertical only	YES	Through origin only
Axes of coordinates in real magnitude?	YES	YES	no	no
Maximum scope?	Less than one hemisphere	Whole world	Less than one hemisphere	Usually one hemisphere

Table 2: Comparison of properties of the UP with other cartographic projections.

—History

The first author (and the only one up to now) to describe the concept behind the UP was the Spanish aeronautical engineer José Antonio Hurtado García in a paper published in 2006 and entitled "La 'longitud del occidente' y la 'latitud del equinoccial': un sistema de coordenadas geográficas, ortogonal, inédito."[33] Hurtado defends the controversial theory that whenever Christopher Columbus wrote 'distancia a la equinoccial' he did not mean 'latitude', as believed by all historians, but the angle that has been called here 'obliquity.' This would explain the bizarre values of 'distancia a la equinoccial' reported by Columbus for Cuba and Hispaniola, which have puzzled scholars up to now.[34]

Hurtado also hinted that Juan de la Cosa made use of the 'distancia a la equinoccial' in his map taking as reference for longitude the meridian of Hierro, in the Canaries. However, Hurtado acknowledged that the relative positions of the lands shown in the 1500 map did not fit his theory and blamed it on a mistake or a manipulation by Columbus.

In the following chapter it will be shown that Hurtado's intuition may in fact be right: the Atlantic basin of the map of Juan de la Cosa looks as if it had been built using an equirectangular grid of longitude and obliquity (i.e. an Unnamed Projection), albeit taking as reference not the meridian of Hierro, but another eight degrees to the west.

6. Comparison with the La Cosa map

A real image of the Atlantic basin has been plotted in Unnamed Projection, using the point LAT_0 = 0º, LON_0 = 26ºW as the center of projection, and the result has been superimposed over La Cosa map. In addition, the discrepancies in latitude and longitude of a set of control points have been computed numerically and can be found in Annex 1. The results are discussed below, first region by region, and then in a global overview.

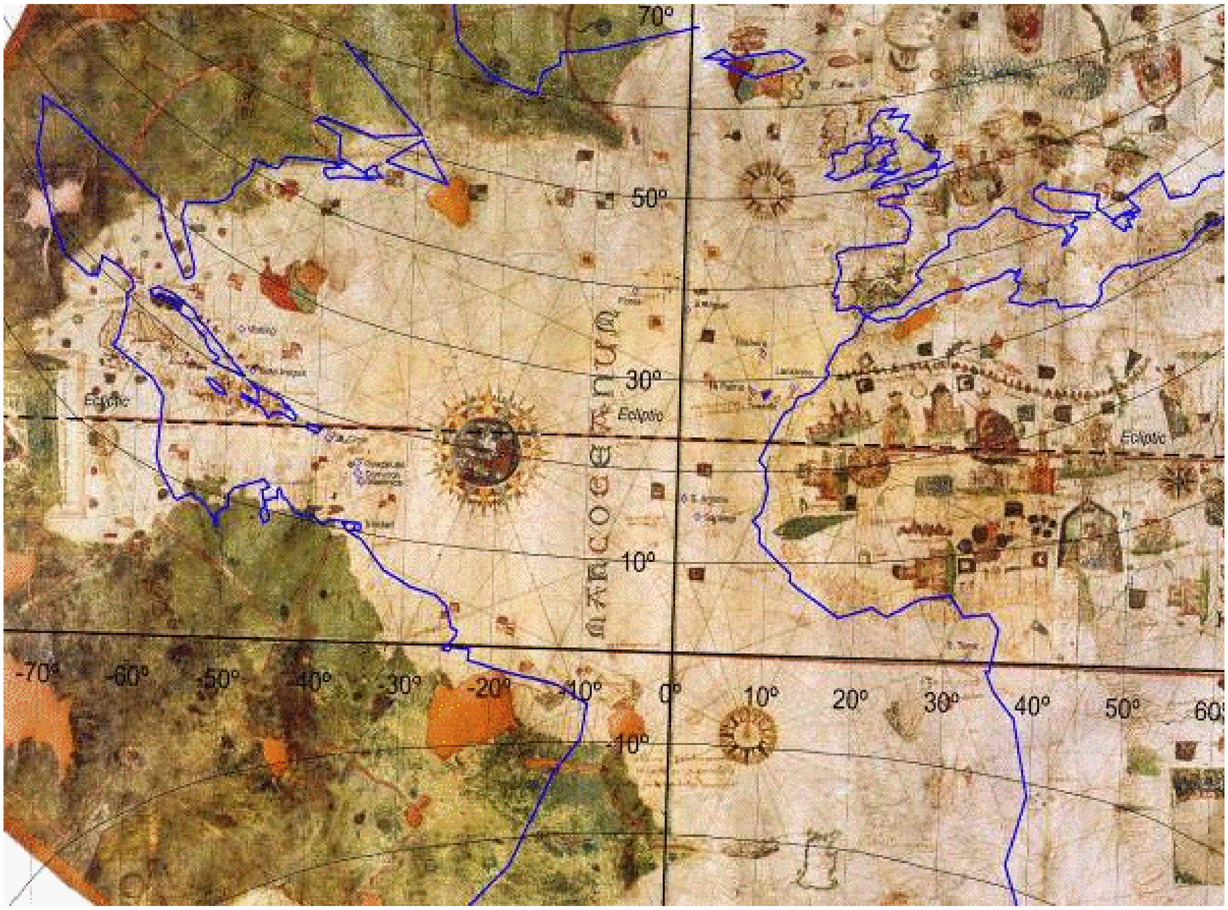

Fig. 13: Superimposition over the map of Juan de la Cosa of a real image of the Atlantic basin in Unnamed Projection with reference meridian 26ºW of Greenwich.

—Caribbean

Puerto Rico 's latitude and longitude appear to be exact (i.e. error <1º). The Lesser Antilles are very accurately located as well, with errors of less than 1.5 degrees. The Bahamas have exact latitudes and small errors in longitude (between one and three degrees).

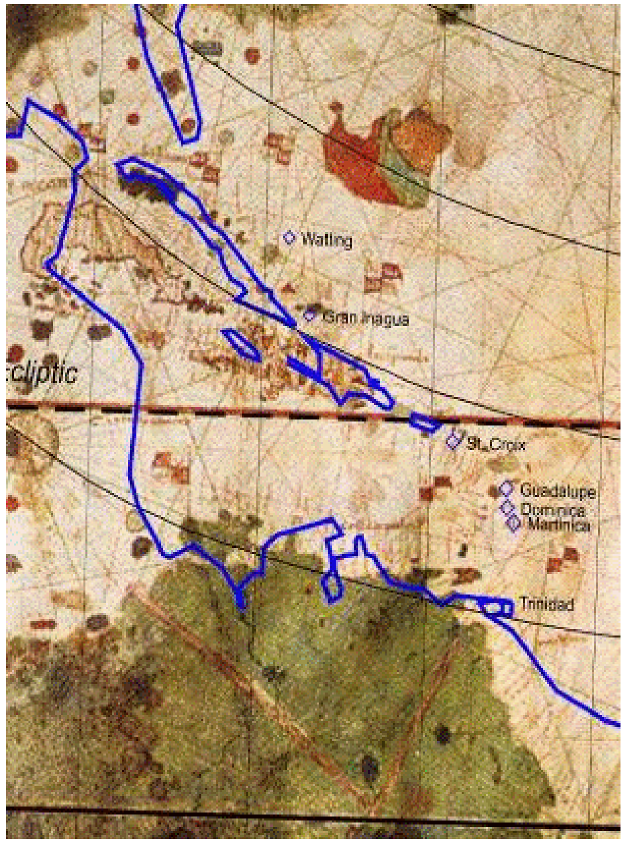

Fig. 14: Detail of the Caribbean region.

La Cosa's Hispaniola is correct in latitude but its longitude presents an interesting situation: the eastern side (cape Engaño, Puerto Plata) is accurately located, but the western end shows a substantial deviation in longitude of around 5 degrees. It looks as if the shape of the island had been stretched by around 75% along its east-west axis.

La Cosa's Jamaica has the right size, but it has been plotted around 5 degrees to the west of its real position. Similarly, the southeast of Cuba is essentially correct in shape and in latitude, but lays around 5 degrees too far to the west. These errors in longitude are just the consequence of having drawn Hispaniola around 5 degrees longer than it should be on the east-west direction. The accuracy of La Cosa's drawing of Cuba decreases as we move north until it becomes rather erroneous at the northwestern tip of the island, which La Cosa most probably never explored himself.

The continental coast of the Caribbean west of Trinidad has been drawn on the right latitude too (the maximum error is 1.2º) but its longitude has been shifted about 3 degrees westwards. Besides, the length of the coastline appears to have been overestimated by around 25% (13.1 degrees on La Cosa's chart between *boca del drago* and *C de la bela* instead of 10.4 degrees in reality).

Regarding the continental coastline of Central America, there is absolutely no correspondence between La Cosa's map and reality. This confirms the general opinion that this part of the map is totally conjectural.

—South America

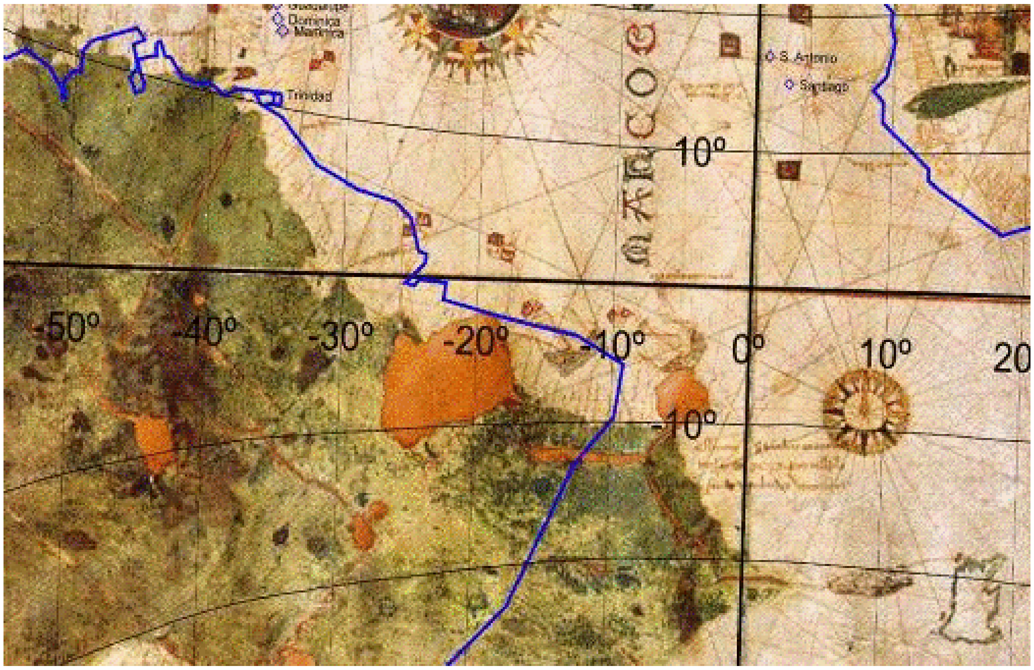

Fig. 15: Detail of South America.

To the east of Trinidad, the continental coastline of the Guianas shows significant deviations in longitude but its general shape is correct.

The mouths of the Amazon are correctly depicted as lying right under the equator. From there to the east, however, a systematic error of around 6º in latitude seems to exist. Errors in longitude are difficult to assess in this part of the map because of the lack of coastal features or toponyms that can be unequivocally identified. For example, Ricardo Cerezo identified the *rº do se falla una cruz* ("river where a cross is found") with the Acarau, in which case its longitude would be around two degrees wrong; however, it could also correspond to the Jaguaribe river and then its longitude error would be close to zero.

To the east of *p:fermoso*, La Cosa's continental coastline becomes devoid of toponyms. This has led scholars like Fernando Silió to believe that this point marked the end of the coast really explored by Castilian navigators, while from there to the east the coastline added by La Cosa was merely conjectural. The comparison with the real image of South America in the UP tends to corroborate Silió's hypothesis.

—West Africa and Atlantic Islands

The Canary islands and Madeira appear near their exact positions (errors < 1º). The Azores and Cape Verde islands are well placed, with errors in latitude and longitude typically smaller than 2 degrees.

The African coastline is quite accurate between the Strait of Gibraltar and Cape Verde (errors < 2º). To the south of that important coastal feature, longitude becomes increasingly inaccurate because the gulf of Guinea was drawn by La Cosa substantially longer than it is in reality: the difference of longitude between Cape Verde and Cape Lopez appears to be around nine degrees too long, i.e. 33%. Latitude however remains remarkably accurate all along this stretch of coast and also in neighboring islands like Sao Tomé.

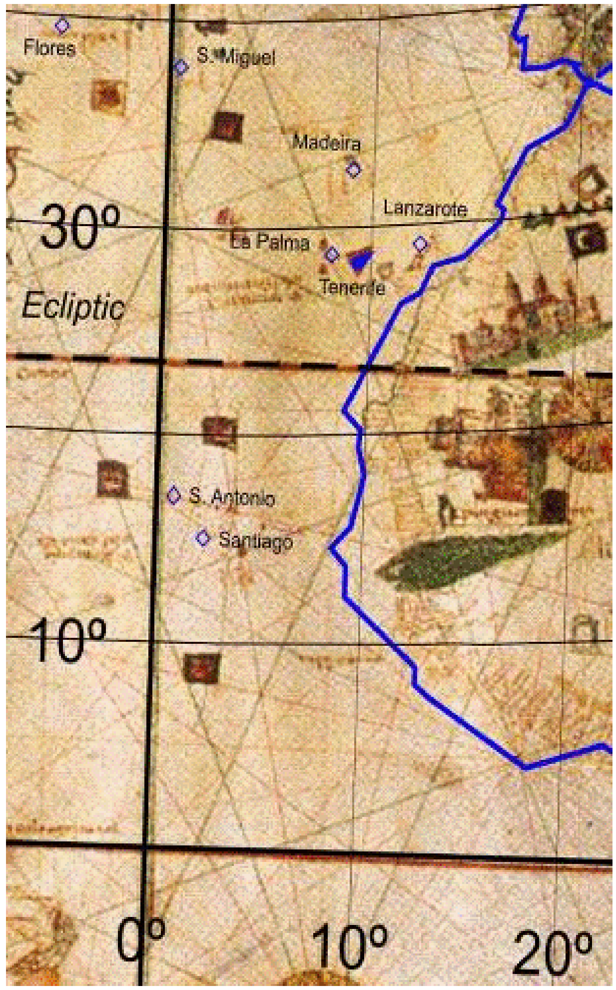

Fig. 16: Detail of West Africa and Atlantic islands.

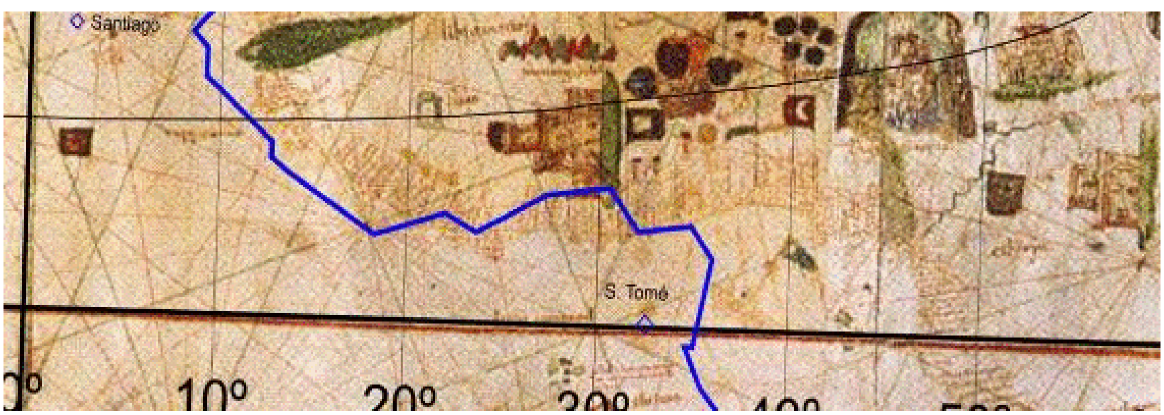

Fig. 17: Detail of the Gulf of Guinea.

—Equatorial and Southern Africa

The African coast between the Equator and the mouth of Congo River is correct in latitude and presents an homogeneous error of ca. 11 degrees in longitude.

South of the Congo, however, latitude becomes badly distorted. This region of the world was therefore probably not drawn by La Cosa with the Unnamed Projection, or else his sources of information were very erroneous.

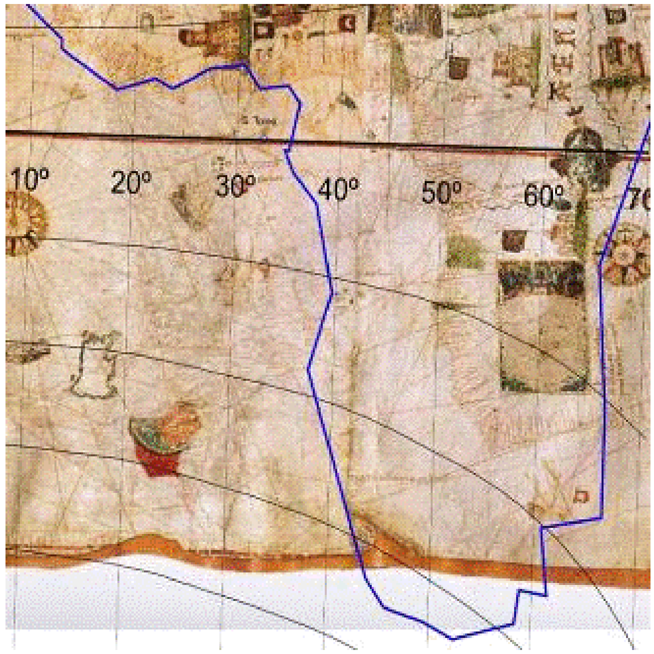

Fig. 18: Detail of Equatorial and Southern Africa.

—Europe and the Mediterranean

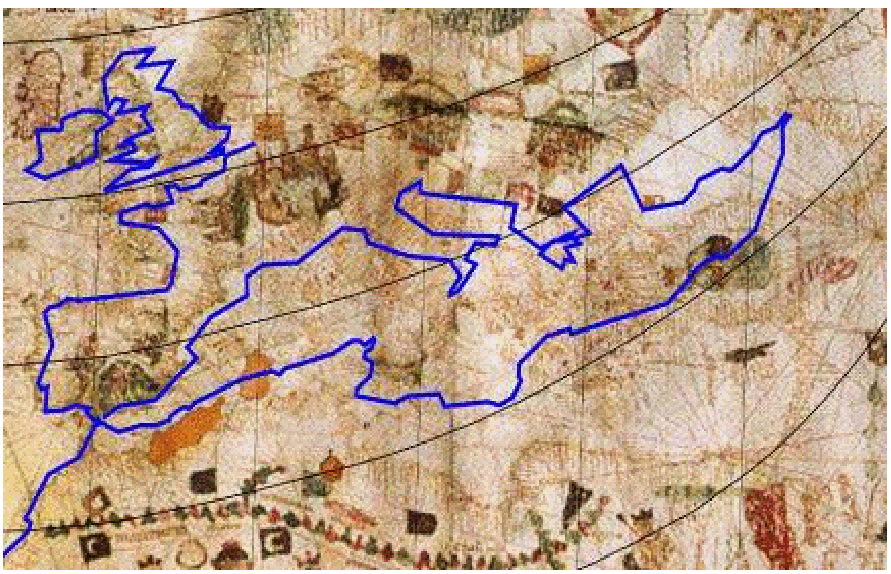

Fig. 19: Europe and the Mediterranean.

There is a clear mismatch between La Cosa's map and the image obtained with the Unnamed Projection for the Mediterranean basin. The axis of La Cosa's Mediterranean Sea forms an angle of around 10º with the vertical axis of the map, a value that is consistent with contemporary portolan charts.[35] On the other hand, the European Atlantic coast fits quite well with the UP, latitude being correct (error < 1º) all the way between Gibraltar and the English Channel. Britain and Ireland present larger errors in latitude and longitude (between 1 and 4 degrees) but their scale and shape are relatively accurate.

The clear contrast between the Mediterranean and Atlantic regions of the La Cosa chart is in agreement with Fernando Silió's hypothesis that the Mediterranean region was drawn by the cartographer from a separate map, which he later 'welded' to the map of the Atlantic basin.

—Northern Regions

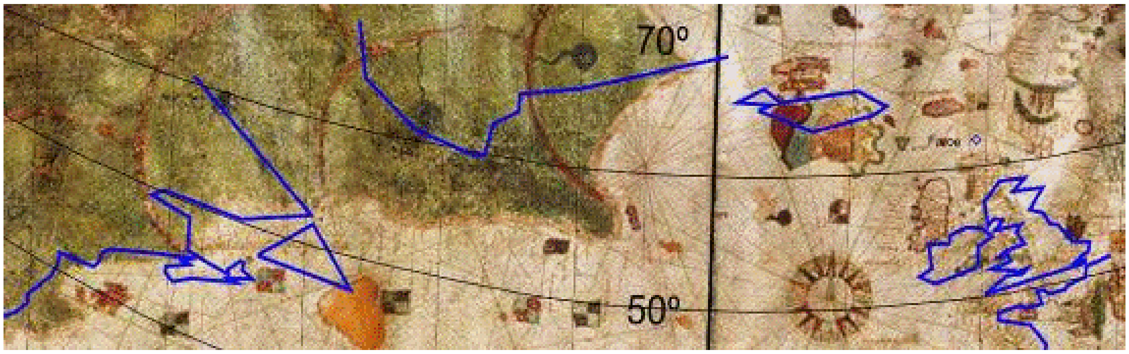

Fig. 20: Detail of the regions of the North Atlantic.

Iceland (*frislanda* on the chart) is close to its real position, only about 2 degrees too far north. However, its east-west extension is far shorter than it should be.

On the North American side of La Cosa's map, the stretch of coast "discovered by the English" and adorned with flags could correspond to anything. The grossly inaccurate position of this coastline tends to confirm the theory that La Cosa drew it based on some indirect account of Cabot's voyage, but with insufficient cartographic information.[36] The easternmost toponym, *y^a berde*, would lie at 55ºN latitude if the Unnamed Projection were valid for this part of this map. With the same assumption, the distance between the two extreme English flags would be around 2,300 km.

—Overall View

Taking into account all the above observations, the first key finding is that, when viewed through the prism of the UP, the positions of La Cosa's Antilles and Caribbean coast are quite accurate in both latitude and longitude, as are those of the Canaries, West Africa and the Iberian Peninsula. This is a significant finding, which contradicts the previous studies, which conclude that the Caribbean islands in the La Cosa map were grossly erroneous in latitude. Actually, latitude is accurate (error < 2º) all the way between the English Channel and Congo river for the Old World, and from southeast Cuba to the Amazon river for the New World.

The relative distances and sizes between the territories on both sides of the Atlantic are correctly represented, which implies a coherent scale throughout the whole Atlantic basin. It is true that several regions present distortions of scale along the east-west axis, notably Hispaniola, which appears to have been drawn by La Cosa around 75% longer than it is in reality. Other examples are the Caribbean coastline (25%), the gulf of Guinea (33%) and Iceland (-65%). These distortions are local, however, and the fact that they all occur on the east-west direction hints that they had more to do with the difficulty of measuring longitude in the fifteenth century than with a desire to use two different scales for the New and the Old World.

It is also interesting to note that the Atlantic basin on the La Cosa map fits well with an image of the world drawn using geographic north as reference. It has often been said that the whole of La Cosa's chart was drawn taking magnetic north as reference.[37] In my opinion it would be more prudent to consider separately the different pieces that compose La Cosa's puzzle. On one hand, La Cosa's Euro-Mediterranean region shows the typical rotation of medieval portolan charts. On the other hand, La Cosa's African and American coastlines seem to be correctly aligned with geographic north when viewed through the prism of the UP.

7. Discussion

The previous section has shown that there is a good fit between La Cosa's 1500 map and the image of the Atlantic basin drawn in the Unnamed Projection. While the agreement is particularly good in latitude, some longitudes are quite accurate too, most notably the overall difference of longitude between Africa and the Caribbean.

This good fit could be:

1. a deliberate use of the UP by the cartographer,
2. just a coincidence, or
3. an intermediate case, in which the mapmaking method used resulted in the involuntary emergence of a UP-like shape.

—Just a Coincidence?

Sheer luck has sometimes been invoked in the literature to explain the apparent concordance between an old map and a cartographic projection. For example, in the 20th century several authors found a good match between the profiles of portolan charts of the Mediterranean and a modern map of that region drawn in a transverse Mercator projection. At least one of those authors, Ángel Paladini, explained the fact as a coincidence due to magnetic declination. He postulated that in the Late Middle Ages magnetic declination varied smoothly along the Mediterranean area, changing from around 4 degrees at the meridian of Greenwich to 17 degrees at the meridian 35ºE. This would have deformed the implicit meridians of the original portolan chart exactly to the point where the map became conformal.[38]

When the La Cosa map is compared with the UP image it turns out that the match is best for those territories of which La Cosa had first-hand knowledge: the Antilles, Caribbean coast, Spain, West Africa, and Atlantic archipelagos. Inversely, the Brazilian shores, which were known to La Cosa only through second-hand reports from two expeditions, do not fit well with the UP. In Africa, the correspondence with the UP degrades as one moves away from Europe and the seas where La Cosa is known to have sailed. If luck were the main factor behind the apparent match, one would expect the concordance between La Cosa’s chart and the UP to be random, sometimes better and sometimes worse, irrespective of the amount of actual data available to the cartographer. As this is not the case, randomness does not suffice to explain away the possible use of the UP by La Cosa.

—Involuntary Emergence

A more solid case could be built on the hypothesis that the authors of the La Cosa map did not have the UP in mind, but applied mapmaking methods that resulted in the natural emergence of a shape that happens to look like a UP. In a recent paper, Gaspar has proposed such an interpretation to explain the shape of Africa on the Cantino map of 1502.[39]

Gaspar postulates that the author of the Cantino map (and some other cartographers of the time) applied “…the so-called ‘planimetric method,’” which involves the use of a constant scale, and “plotting directly on the plane the latitudes, magnetic directions and distances observed at sea, as if the earth was flat.” [40] Both distances and angles were plotted in real magnitude, which of course is erroneous, since it ignores the curvature of the earth. As a result, quoting Gaspar: “when the planimetric method is used to represent large areas of the Earth surface, the inconsistencies that result from ignoring its curvature can be enormous and the resulting representations become strongly dependent on the tracks used to plot the places on the chart.”[41]

Gaspar attempted to reconstruct the Cantino map using this planimetric method, following certain procedures, which he thinks were used by contemporary map makers. Most notably, he supposed that the author(s) had a table of estimates of distances, rhumbs and latitudes, which he treated for his reconstruction as error free. Another key assumption was that magnetic declination was ignored everywhere outside of the Mediterranean. Gaspar found out that under these conditions the planimetric method explains features of the Cantino map that up to now had been considered errors, namely the length of the gulf of Guinea and the breadth of the Isthmus of Suez.

However, Gaspar did not include in his analysis the Caribbean region of the Cantino map, which has great similarity with that drawn by La Cosa, because Cuba and Hispaniola appear clearly to the north of the “Tropicus Cancri” line. The planimetric method locates points of equal latitude along straight horizontal equally-spaced lines. Applying it to the Caribbean would have resulted in a clear mismatch between the Cantino chart and the real image of that region. Therefore, this method does not explain the location of Caribbean territories on the Cantino chart and *a fortiori* on the La Cosa chart.

It is important not to forget that La Cosa's chart was a world map, made to be exhibited at the Court, and was neither intended nor suitable for navigation. Gaspar's planimetric method is consistent with navigation manuals of later decades in the sixteenth century, and may have been successfully used for smaller-scale marine charts, but when cartographers composed world maps they faced different issues and did not necessarily apply the same methods as when drawing marine charts. Nonetheless, use of the planimetric method could be an explanation for the shape of the African coastline drawn by La Cosa, as he is widely assumed to have copied it from some Portuguese source.

The following sections of this article will discuss whether a deliberate use by La Cosa of the UP could be consistent with the historical context of Castile in 1500.

—Spirit of Innovation in Cartographic Projections

The fifteenth and sixteenth centuries were a period of effervescence in the invention or rediscovery of new cartographic methods and projections. In order to integrate the newly discovered lands with known geography, mapmakers tested new mathematical and empirical solutions. A far-from-exhaustive list would include the trapezoidal projection by Donnus Nicolaus Germanus (1466), the globular projection of the Deutsche Ptolomäus (1490), the cordiform projection invented by Johannes Stabius (possibly before 1502, published in 1514), the stereographic projection by Walther Ludd (1507), the modified equidistant projection by Johannes Ruysch (1508), and the octant projection developed by Leonardo da Vinci (1514).[42]

Most of the projections and techniques experimented with at this time were discarded as they were superseded by better or simpler alternatives. The Unnamed Projection could have been one such innovation that was tested and abandoned shortly thereafter.

Spain was not an exception in this period of diversity and innovation in cartography. Two decrees issued by the Crown of Castile, first in 1508 and again in 1512, criticized the fact that there existed at the time many charts of the Indies made by diverse techniques and by diverse mapmakers with substantially different results, and ordered a single, official master chart ("Padrón General" or "Padrón Real") of the Indies to be drawn and updated regularly. The Padrón General would thenceforth be publicly exhibited in the Casa de la Contratación in Seville, and all pilots would have to carry with them and follow a copy of that master chart. Furthermore, pilots would be prevented from using any other map.[43] The Unnamed Projection may have been one of the projections left aside in the movement towards cartographic standardization initiated by these decrees.

—Determination of Geographical Coordinates

A map can be built in two ways: either from raw numeric data or from graphical information, i.e. previous partial maps. From what has been said up to now, it is safe to assume that La Cosa's chart is of the second type: it was compiled from a number of partial charts, like a puzzle made of pieces of very different origins. The question is, then, how the cartographer drew each of those separate pieces, and in particular what numeric data were originally used for the fragment that contains the coasts of the Atlantic.

If La Cosa applied the Unnamed Projection in a deliberate way, then logically he must have computed first a table with the longitude and obliquity (calculated from latitude and longitude) of each geographical feature, and must have then plotted these coordinates on the easy-to-draw square grid of the UP. How longitude and obliquity/latitude data may have been obtained is discussed below.

Basic positional data

The navigators of the fifteenth century had developed a number of techniques to collect data on the position of their ships. They could calculate roughly the distance sailed every day and night by estimating the speed of a vessel. Several methods have been described for this, like the "log and line," but probably the most practiced one was just the pilot's feel based on his experience.[44] In addition, the magnetic needle, already several centuries old by 1500, provided the heading of the ship, and therefore gave an indication of the rhumb followed along the journey.

Furthermore navigators took advantage of any other information provided by the natural environment that surrounded them: wind direction and force, water color, presence and type of algae, the occasional bird. The firmament brought some information too: the position of the Pole Star indicated the true geographic north, and allowed them to check the deviation of the needle; and the relative duration of days and nights could give the order of magnitude of latitude. The two latter methods were reportedly used by Columbus in his first voyage to the Indies and must have been known to Juan de la Cosa, for he was the owner and master of the ship which Columbus was sailing.

Latitude

Since the mid fifteenth century, the Portuguese had been learning the methods used to determine latitude by astronomic observations. Navigator Diogo Gomes claimed to have used instruments to measure the height of the Pole Star in Guinea in 1460-62,[45] and in the last decades of the century there appeared the first known manual of astronomical navigation, which contains instructions for measuring latitude both from the Pole Star and through the observation of the sun at noon.[46]

There is, however, controversy among historians regarding the extent to which such astronomical methods were actually put into practice, and also regarding the accuracy which they provided. Felipe Fernández-Armesto, for instance, is quite skeptical that astronomic instruments played any important role in fifteenth-century navigation, while at the same time pointing out that "seasoned navigators had skills we have now lost and could make impressive judgment of relative latitude by observing the sun or the Pole Star with the naked eye."[47] On the contrary, Joaquim Alves Gaspar affirms that "the introduction of astronomical navigation by the Portuguese, in the middle of the fifteenth century, proved to be an adequate and durable solution for the problem [of determining latitude at sea]."[48] W.G.L. Randles provides yet another point of view: astronomical methods may well have been used by the Portuguese, but there is no proof that by 1500 the instruments could provide more accurate readings of latitude than what could be calculated by estimating distances in the traditional way.[49] In an interesting nuance, Douglas T. Peck makes a distinction between navigation, for which he sees no use of astronomic methods at those times, and map making, which could have taken advantage of occasional measurements of latitude performed on land by expert cosmographers so as to better place the new discoveries on general maps of the world.[50]

In any case, all authors agree that the Atlantic coastlines on Portuguese maps became much more accurate in latitude in the period elapsed between the first known charts of 1480s and the Cantino map of 1502. Whatever the methods used —distance estimates, astronomical instruments or naked-eye celestial observations—the knowledge of the methods and the numerical results may have diffused to or been developed simultaneously in the neighboring kingdom of Castile, which would explain the high accuracy of latitudes in La Cosa's chart.

Longitude

In his review of the different methods used by Iberian navigators to estimate longitude in the sixteenth century, Randles concluded that at sea the only reasonable technique was to estimate distance and rhumb, and then calculate the change in longitude mathematically.[51] On land, knowledgeable astronomers could measure longitude by observing the moon's position or its eclipses, but all known attempts done in the early exploration of the Americas reportedly gave large errors. Only in the second half of the century did astronomic measurements become reasonably accurate, thanks to the careful observation of eclipses from fixed locations, such as Mexico City.

It may be useful to remember that the famously accurate measurement of longitude performed by al-Biruni (973-1048) between Baghdad and Ghazna was not based on moon's eclipses, but on the combination of distance estimates, astronomic determination of latitude and the use of spherical trigonometry to take into account the real shape of the Earth.[52] The combined use of astronomic measurements of latitude with distance estimates to better assess the position of a ship on a chart is explained in many manuals of the sixteenth century but, unlike al-Biruni, they assume a flat Earth and therefore use much simpler and less accurate mathematics.[53] A further limitation of this technique is that latitude data cannot improve longitude estimates between points that lie on the same parallel.

For the La Cosa chart, it is most probable that longitudes were estimated from distances and rhumbs, not from lunar observations. However, the fact that on this chart the longitude errors are higher along east-west coastlines (gulf of Guinea, Hispaniola, the Caribbean), hints that astronomic measurements of latitude may have been used to correct longitude estimates in directions other than east-west.

—Trigonometry

One doubt that could be cast on the explicit use of the Unnamed Projection by La Cosa is whether he could have had sufficient knowledge of spherical trigonometry to derive the formulae and to compute the values of the different mathematical functions involved. In particular, tables of the tangent function would have been required to compute numerically the vertical coordinate of the UP, but such tables were not printed in Latin until the work by Rheticus (1514-1574). Nonetheless, as we will see, spherical trigonometry was actively practiced in the Iberian Peninsula in the Middle Ages, and may have reached the authors of the La Cosa chart via three different scientific traditions.

Today, plane trigonometry is part of elementary or secondary education in most countries, whereas spherical trigonometry is taught only at the university level in specialized courses. Historically, however, spherical trigonometry developed first, and all classic works about trigonometry, e.g. by Hipparcus and Ptolemy, were concerned with spherical and not plane trigonometry.[54] This ancient knowledge was transmitted and enlarged by medieval Arab/Muslim scientists, who defined the trigonometric functions as we know them today and calculated tables of the tangent function (e.g. Abu'l Wafa, 10th century).[55] Among other applications, these mathematicians used spherical trigonometry to successfully solve the issue of *qibla* (i.e. how to orient mosques towards Mecca), a problem at least as complicated as the Unnamed Projection.[56]

The first book in the West entirely devoted to spherical trigonometry was written in Al-Andalus in the eleventh century by Ibn Muad, also known as Al Jayyani. This book contained seven key theorems and also a table of tangents, calculated as the ratio of sine over cosine.[57] In the following century, spherical trigonometry was popularized in a simpler form by Jabir Ibn Aflah, from Seville, whose work was translated into Latin soon thereafter. Later on—as Muslim Spain fragmented, was conquered by North African invaders, and eventually lost most of its territory to Christians—its science never reached again the peaks of achievement of the eleventh century. Nevertheless, the Kingdom of Granada preserved a good mastery of mathematics and in particular of spherical trigonometry at least well into the fourteenth century, as exemplified by the work of Ibn Raqqam (d.1315).[58] Little is known, however, about the fifteenth century, a period of general decadence in that kingdom, which culminated in the conquest by Castile in 1492.[59] Even so, Muslim scholarship still survived for a few years until in 1499-1502 cardinal Cisneros ordered around 5,000 manuscripts in Arabic to be burnt, and started the persecution that would eventually fully eradicate Muslim culture from Spain.[60]

While most Muslim scientists emigrated from the territories conquered by the Iberian Christian kingdoms from the twelfth century onwards, Jewish ones usually stayed in place and kept practicing mathematics and astronomy. Their use of Hebrew and Arabic as main languages of work and their connections with other Jewish centers in the Western Mediterranean led to the formation of a relatively closed scientific school.[61] The best-known Jewish astronomer of the fifteenth century, Abraham Zacuto, lived and worked in Castile but was forced into exile in 1492 along with all other Jews who refused the royal order to convert to Christianity.

There was yet a third path by which the advanced mathematics of Al-Andalus was transmitted down to the times of Juan de la Cosa. In the thirteenth century king Alfonso X of Castile organized the translation and completion of scientific works from Arabic into Castilian Romance, the ancestor of modern Spanish. One of the resulting books, the *Tratado del cuadrante sennero*, deals in depth with the theory of spherical trigonometry.[62] On a more practical level, the *Libros del Saber de Astronomía* give the solution of numerous trigonometric problems and provide tables for the functions tangent and arc tangent, among others.[63] A mathematical tradition in the Castilian language was thus established; it would flourish in the mid fifteenth century when the University of Salamanca founded a faculty of Astrology in which "spherical astronomy" was taught.[64]

It is improbable that Juan de La Cosa, a seafarer not a scholar, counted himself among the few Europeans who had learned trigonometry by 1500. But La Cosa did not work alone. The map that bears his name was in fact the result of cooperation among several cartographers. It is not at all impossible that at least one of them was sufficiently acquainted with spherical trigonometry through any of the three scientific traditions—Islamic, Jewish, and Christian—that co-existed in the Iberian Peninsula until the end of the fifteenth century, to be able to perform the calculations required by the Unnamed Projection.

—The Ecliptic on Maps

While the ecliptic line has always been very important for astronomers and appears prominently on any star chart, one might wonder at why the ecliptic should appear at all on a map of the Earth. The truth is, however, that several cartographic works of the fifteenth and sixteenth centuries include this curve, for instance Martin Behaim's globe,[65] and the Ottoman world map of Ali Macar Re'is.[66] Moreover, in the seventeenth century the ecliptic became a frequent feature of world maps.

Cartographers must have had some reason to draw this line given that in 1734 the English translation of Varenius' *Compleat System of General Geography* affirmed that the ecliptic "tho' 'tis not proper to the Earth; yet 'tis drawn on the terrestrial Globe, and is seen on Maps, because of it's great use."[67] It should be remarked that the ecliptic on terrestrial globes can be useful as it helps solve some astronomical problems; however, Varenius also mentions 'Maps', i.e. flat maps.

Summing up, it is difficult to explain for what practical purpose La Cosa would have wanted to highlight the ecliptic on his map, but it is also true that other cartographers of the Renaissance drew that line too, for some unknown reason.

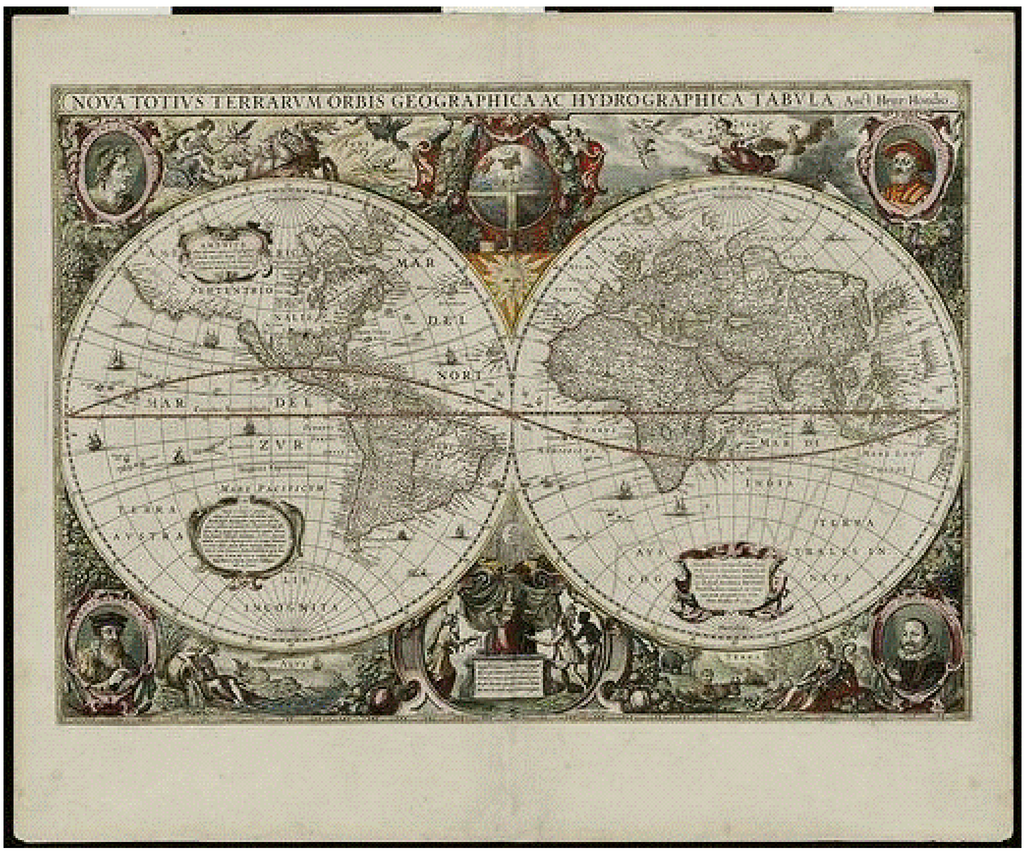

Fig. 21: World map of 1630 that shows the ecliptic in a prominent way.[68]

8. Conclusions and Open Questions

There is a good overall correspondence between La Cosa's 1500 map and a real image of the Atlantic basin drawn in the Unnamed Projection. The fact that the match is best for those territories of which La Cosa had first-hand knowledge (Antilles, Caribbean coast, West Africa) indicates that this is not the product of random chance. The UP may have emerged involuntarily as the natural consequence of the particular mapmaking method used by La Cosa, but if that is the case such method remains to be described. Rather, I believe that the historical context of the map—the effervescence of new cartographic projections around 1500, and the presence of several mathematical traditions familiar with spherical trigonometry in the Iberian Peninsula—suggests that the UP was deliberately conceived or selected by one of the cartographers who took part in the production of the La Cosa chart.

Interpreting La Cosa's map through the grid of analysis provided by the UP leads to the following conclusions:

1. La Cosa placed all of the following territories on or close to their real positions: the Caribbean region, the West African coast, the archipelagos of the Middle Atlantic, and the Iberian Peninsula. Latitude is reasonably accurate all the way from the English Channel to the Congo River for the Old World, and from Cuba to the Amazon for the New World. This solves the paradox raised by several previous studies of the map which, assuming cylindrical projections, had found substantial errors of latitude that were difficult to justify for a map dated 1500, and which were larger than the apparent errors of longitude on the same map.

2. Some longitudinal measurements are remarkably accurate, particularly the overall difference between the Lesser Antilles and the African coast. On the other hand, the longitudes of several stretches of coastline aligned on an east-west direction show large errors. This hints that astronomical measurements of latitude may have been used to improve the accuracy of longitude estimates.

3. The problem of the scale of the chart is more complicated than the hypothesis of two different scales for the New and the Old World put forward by some authors. The use of the UP as proposed in this study implies that scale is equal along the *equinocial* and *meridional* lines, and that it varies in a mathematically consistent way across the whole Atlantic basin.

4. La Cosa's Atlantic basin fits well with a projection that is oriented towards geographic North. It is not necessary to invoke magnetic declination to explain its shape. This is in contrast with the Mediterranean region of the same chart, which shows the typical rotation of contemporary portolan charts.

5. The good fit of the map with a real image of the world in UP confirms the initial hypothesis that the thick red horizontal line labeled *circulo cancro* represents the ecliptic and not the Tropic of Cancer, as usually assumed.

6. The inaccuracy of the South American coastline east of the Amazon on the La Cosa map reflects the fact that this region was not charted by La Cosa first hand, but drawn based on reports from other Castilian expeditions. Similarly, the largely erroneous depiction of the North American coastline shows that La Cosa was not able to integrate correctly his probably fragmentary data on Cabot's expedition with the rest of the map. La Cosa's Central America is confirmed to be purely conjectural.

7. The African continent south of the Congo River and the Mediterranean clearly do not fit with the UP used for the Atlantic basin. The reason must be that La Cosa (or another of his fellow mapmakers) drew or copied maps of these regions based on some other cartographic method and later "welded" them with the map of the Atlantic.

The results obtained in this study open new questions and possible lines of research:

- Do other maps of the fifteenth or sixteenth centuries fit well with the UP?
- Have other cartographers used obliquity as a coordinate instead of latitude?
- Has any instrument ever been designed to measure obliquity or to compute it indirectly?
- What purpose does it serve to draw the ecliptic on a flat terrestrial map?

More than 500 years after its creation and 178 after its rediscovery, the map of Juan de la Cosa may still hold surprises in store for historians of cartography.

Acknowledgements

I would like to give special thanks to Fernando Silió Cervera for his kind help and encouragement with this research. I would also like to thank Carmen Manso Porto and Enrique Rojo Fernández for their advice. Of course none of these persons necessarily supports the points of view expressed in this article and any errors, omissions or misinterpretations are exclusively mine.

Annex 1: Control points

Antilles

				Unnamed projection		Gnomonic projection		Plate carrée projection	
Actual toponym	**Real LAT**	**Real LON**	**La Cosa's toponym**	**Error LAT**	**Error LON**	**Error LAT**	**Error LON**	**Error LAT**	**Error LON**
Fort de France (Martinique)	14,60	61,08	ascension	0,6	1,2	1,4	-1,2	4,1	1,2
Roseau (Dominique)	15,30	61,38	dominica	0,6	0,4	1,4	-1,9	4,1	0,4
Montserrat	16,75	62,23	S mª de monsera	1,3	0,0	2,0	-2,4	5,3	0,0
St. Croix	17,75	64,78	S cruz	0,1	1,0	1,0	-2,4	5,0	1,0
Ponce (Puerto Rico)	18,02	66,60		0,4	0,7	1,3	-3,2	5,9	0,7
Santo Domingo (Hispaniola)	18,50	69,95		-0,3	2,1	1,1	-3,5	6,9	2,1
Puerto Plata (Hispaniola)	19,80	70,75	plata	0,4	0,6	1,6	-4,8	7,8	0,6
Port-au-Prince (Hispaniola)	18,55	72,33		-0,3	3,4	1,6	-3,7	8,4	3,4
Montecristi (Hispaniola)	19,88	71,65	Cst	0,3	2,0	1,7	-4,3	8,7	2,0
C. Engaño (Hispaniola)	18,60	68,32	C s rafael	0,4	0,6	1,5	-3,9	6,6	0,6
C. Dame-Marie (Hispaniola)	18,62	74,42	C de S: miguel	-1,6	5,2	1,1	-3,7	8,7	5,2
Gran Inagua	21,00	73,33	baaruco	0,3	2,1	1,9	-4,9	9,9	2,1
Cayo Samana	23, 05	73,75	samana	0,0	3,1	1,5	-4,5	10,9	3,1
Acklins Island	22,50	74,00	haiti	0,6	1,4	1,9	-5,6	10,8	1,4
Watling Island	24,00	74,67	guanahani	-0,1	2,3	1,3	-5,4	11,1	2,3
Port Antonio (Jamaica)	18,17	76,45	falso	-1,2	5,2	1,9	-4,6	10,2	5,2
Montego Bay (Jamaica)	18,45	77,93	c de Ileon	-2,0	4,9	1,4	-5,5	9,9	4,9
Cabo Maisi (Cuba)	20,27	74,17	punta de cuba	-0,8	3,5	1,3	-4,5	9,4	3,5
Guantánamo (Cuba)	20,15	75,23		-1,2	3,9	1,3	-4,7	9,6	3,9
Cabo Cruz (Cuba)	19,85	77,72	C de cruz	-1,8	4,1	1,4	-5,8	10,3	4,1
Gulf of Ana María (Cuba)	21,60	78,70		-2,3	4,2	1,0	-6,2	11,1	4,2
Peninsula of Zapata (Cuba)	22,40	82,15		-4,8	3,8	-0,5	-8,2	10,0	3,8

South-American continental coast west of Trinidad

				Unnamed projection		Gnomonic projection		Plate carrée projection	
Actual toponym	Real LAT	**Real LON**	**La Cosa's toponym**	**Error LAT**	**Error LON**	**Error LAT**	**Error LON**	**Error LAT**	**Error LON**
Cabo de la Vela	12,20	72,20	C de la bela	-0,7	5,3	1,4	-2,6	5,9	5,3
Guajira peninsula, East	12,10	71,10	C de espera	-0,4	3,9	1,4	-2,9	5,4	3,9
Maracaibo strait	10,70	71,60		-0,1	3,9	1,6	-3,1	5,3	3,9
Aruba	12,50	70,00	y de brasil	-0,3	3,1	1,3	-2,9	5,2	3,1
Morrocoy	10,90	68,20	c de la mata	-0,2	3,3	1,1	-2,1	4,1	3,3
Margarita	11,00	63,80	margalida	1,1	2,2	2,1	-1,3	4,6	2,2
Boca del Dragón	10,70	61,80	boca del drago	1,2	2,6	2,1	-0,5	4,3	2,6

South-American continental coast east of Trinidad

				Unnamed projection		Gnomonic projection		Plate carrée projection	
Actual toponym	**Real LAT**	**Real LON**	**La Cosa's toponym**	**Error LAT**	**Error LON**	**Error LAT**	**Error LON**	**Error LAT**	**Error LON**
Essequibo river	6,90	58,40	rº de holgança	1,3	3,8	2,0	1,4	3,3	3,8
Amazonas W mouth	1,40	49,90		0,3	7,8	0,5	6,5	0,6	7,8
Amazonas E mouth	-0,50	48,20		-1,5	5,1	-1,6	4,6	-1,7	5,1
Pindaré river	-2,40	44,30	rº de arboledas	-5,6	-2,5	-6,0	-2,0	-5,9	-2,5
Parnaiba river	-2,80	41,80	rº negro	-6,0	-2,7	-6,5	-2,1	-6,3	-2,7
Acarau river	-2,80	40,10	rº do se falla una cruz	-6,6	-2,1	-7,0	-1,6	-6,8	-2,1
Jaguaribe river	-4,40	37,80	rº de bazia bariles	-5,3	-3,2	-5,8	-2,8	-5,4	-3,2

Atlantic archipelagos

				Unnamed projection		Gnomonic projection		Plate carrée projection	
Actual toponym	**Real LAT**	**Real LON**	**La Cosa's toponym**	**Error LAT**	**Error LON**	**Error LAT**	**Error LON**	**Error LAT**	**Error LON**
Santa Maria	36,97	25,10	y s mª	1,3	-1,4	-1,7	-1,5	1,3	-1,4
Sao Miguel, East	37,81	25,14	y de s: miguel	1,6	-1,7	-1,7	-1,8	1,7	-1,7
Sao Miguel, West	37,84	25,85	y de s: miguel	1,6	-1,0	-1,7	-1,1	1,6	-1,0
Terceira	38,65	27,22	tercera	2,2	-0,4	-1,6	-0,4	2,2	-0,4
Faial	38,58	28,71	faial	1,1	-0,6	-2,3	-0,5	1,1	-0,6
Flores	39,45	31,21	illa…	1,7	-1,0	-2,2	-0,8	1,8	-1,0
Madeira, ponta do Pargo	32,82	17,26	punta del pargo	0,3	0,4	-1,4	0,0	0,6	0,4
Tenerife, center	28,25	16,61	tenerife	-0,1	0,3	-0,8	-0,2	0,2	0,3
La Palma	28,68	17,85	palma	0,3	0,6	-0,5	0,2	0,5	0,6
Lanzarote	29,00	13,63	lancar…	-0,3	-0,1	-1,2	-0,6	0,3	-0,1
Sao Tiago	15,10	23,65	s tiago	-0,7	-1,6	-0,2	-1,9	-0,7	-1,6
Santo Antao	17,07	25,15	s anton	-0,4	-1,0	0,0	-1,2	-0,4	-1,0
Sao Nicolau	16,61	24,24	s nicol…	-0,3	-1,7	0,2	-1,9	-0,3	-1,7
Fogo	14,92	24,38	fiogo	-0,9	-1,7	-0,3	-1,9	-0,8	-1,7
Boa Vista	16,10	22,81	buena bista	-0,5	-1,7	0,0	-2,0	-0,4	-1,7

Western Europe

				Unnamed projection		Gnomonic projection		Plate carrée projection	
Actual toponym	**Real LAT**	**Real LON**	**La Cosa's toponym**	**Error LAT**	**Error LON**	**Error LAT**	**Error LON**	**Error LAT**	**Error LON**
Gibraltar	36,13	5,34	gibaltar	-1,0	1,2	-3,5	0,9	0,6	1,2
Cape St Vincent	37,03	9,00	s: bicinte	-0,4	1,0	-3,2	0,5	0,7	1,0
Gran Berlinga	39,42	9,51	la berlinga	0,8	1,4	-3,2	0,9	1,7	1,4
Cape Finisterre	42,88	9,28	finistera	0,6	1,8	-4,5	1,3	1,6	1,8
Cape Peñas	43,66	5,85	peñas	-0,3	1,6	-5,5	1,2	1,2	1,6
Capbreton	43,64	1,44	c breton	0,0	2,1	-5,5	2,0	2,3	2,1
Gironde	45,60	1,12		-0,5	2,7	-6,6	2,5	1,7	2,7
Pointe du Raz	48,03	4,73		0,4	3,0	-6,8	2,6	1,9	3,0
Ile d'Ouessant	48,46	5,09		0,9	2,5	-6,8	2,1	2,4	2,5
Cherbourg	49,64	1,61	Xarabor	0,8	3,4	-7,6	3,2	2,7	3,4
Boulogne	50,74	-1,59	bolona	0,5	4,1	-8,4	4,1	2,9	4,1
Isle of Wight	50,66	1,29	bine	1,1	3,4	-7,9	3,2	3,1	3,4
London	51,50	0,12	Londre	1,5	3,8	-8,2	3,7	3,6	3,8
Land's End	50,07	5,70		1,5	2,6	-7,1	2,2	2,9	2,6
Isle of Man	54,21	4,54		1,5	3,3	-9,2	2,9	2,9	3,3
Donegal Bay	54,60	8,18		3,3	2,6	-8,3	2,1	4,2	2,6
Aran islands	53,11	9,70		3,2	1,6	-7,5	1,1	4,1	1,6
Dingle Bay	52,12	9,99		2,1	1,5	-7,6	1,0	3,0	1,5
Wexford	52,31	6,44		2,1	2,6	-7,9	2,1	3,3	2,6
Dundaik Bay	53,97	6,35		2,7	2,7	-8,4	2,2	3,9	2,7
Edinburgh	55,96	3,17		2,1	2,4	-10,0	2,1	3,7	2,4

Northern regions

				Unnamed projection		Gnomonic projection		Plate carrée projection	
Actual toponym	**Real LAT**	**Real LON**	**La Cosa's toponym**	**Error LAT**	**Error LON**	**Error LAT**	**Error LON**	**Error LAT**	**Error LON**
Iceland, East	65,08	13,54	frislanda	1,3	4,3	-14,5	3,8	1,6	4,3
Iceland, South	63,34	18,73	frislanda	1,9	1,2	-13,1	0,9	2,0	1,2
Iceland, West	64,84	24,03	frislanda	1,1	-2,6	-14,3	-2,8	1,2	-2,6
Iceland, North	66,09	18,43	frislanda	2,2	1,5	-14,6	1,2	2,3	1,5
Faroe islands	62,00	6,79	ysla de estilanda	2,0	2,9	-13,0	2,5	3,0	2,9
Oslo fjord	59,18	10,66		4,7	-11,5	-11,5	-11,0	7,2	-11,5
Island off North America??			y[a] berde	55,5	31,6				
North American coast, east			Cavo de ynglaterra	54,3	33,9				
North American coast, west			(last English flag)	46,0	63,6				

West Africa and Guinea

				Unnamed projection		Gnomonic projection		Plate carrée projection	
Actual toponym	**Real LAT**	**Real LON**	**La Cosa's toponym**	**Error LAT**	**Error LON**	**Error LAT**	**Error LON**	**Error LAT**	**Error LON**
Rabat	34,01	6,83		-1,2	1,2	-3,1	0,8	0,1	1,2
Safi	32,30	9,24		-0,7	0,8	-2,2	0,3	0,3	0,8
Agadir	30,43	9,61		-1,5	0,7	-2,5	0,2	-0,6	0,7
Cape Jubi	27,94	12,93		-1,4	1,0	-2,0	0,5	-0,9	1,0
Punta Sarga	23,60	15,99		-0,4	-1,1	-0,5	-1,6	0,0	-1,1
Cape Blanco	20,80	17,05		-0,6	-0,6	-0,4	-1,1	-0,3	-0,6
Cap d'Arguin	20,60	16,55	arguin	-0,7	-1,0	-0,5	-1,5	-0,4	-1,0
Cap Vert	14,75	17,52		-1,4	-1,9	-1,0	-2,4	-1,2	-1,9
Bissagos	11,30	16,00		-2,4	-3,2	-2,0	-3,8	-2,2	-3,2
Cape Palmas	4,35	7,55	c de palmas	-1,3	-5,7	-1,2	-5,6	-1,0	-5,7
Cape Three Points	4,74	2,06	tres puntas	-1,3	-5,6	-1,1	-4,7	-0,8	-5,6
Niger delta	4,50	-5,80		-1,1	-12,2	-0,6	-7,3	0,3	-12,2
Douala bay	3,91	-9,60		-0,8	-9,6	-0,3	-4,3	0,5	-9,6
Gabon river	0,38	-9,36	rº de gabon	0,1	-10,8	0,1	-5,1	0,3	-10,8
Cape Lopez	-0,64	-8,71	C de lope gonçal…	0,2	-10,7	0,2	-5,3	0,0	-10,7
Sao Tomé	0,23	-6,60		0,3	-9,5	0,4	-5,3	0,5	-9,5
Principe	1,60	-7,41		0,1	-9,9	0,3	-5,3	0,7	-9,9
Annobon	-1,44	-5,63		0,8	-10,7	0,7	-6,4	0,6	-10,7

Southern Africa

				Unnamed projection		Gnomonic projection		Plate carrée projection	
Actual toponym	**Real LAT**	**Real LON**	**La Cosa's toponym**	**Error LAT**	**Error LON**	**Error LAT**	**Error LON**	**Error LAT**	**Error LON**
Congo river	-6,06	-12,33	Rio arebatado y gran	2,2	-11,0	1,5	-4,0	0,2	-11,0
Cuanza river	-9,30	-13,20		4,3	-10,2	3,3	-3,2	1,6	-10,2
Cunene river	-17,25	-11,75		6,8	-10,4	5,2	-3,9	1,8	-10,4
Walvis bay	-22,94	-14,50	punta delgada	10,5	-9,1	8,6	-2,0	4,1	-9,1
Lüderitz bay ?	-26,60	-15,20	angra das boltas	11,1	-10,0	8,8	-2,2	2,7	-10,0
Cape of Good Hope	-34,37	-18,47	C de boa esperança	16,5	-10,0	13,6	-0,8	5,3	-10,0

Notes:

• The tables have been calculated assuming that the obliquity of the ecliptic is 23.4º and that the *liña meridional* represents meridian 26ºW.

• Signs of latitude and longitude: positive latitude = North; positive longitude = West.

• Signs of the errors: Latitude errors are positive if the value calculated from the chart is more to the North than the real value; longitude errors are positive if the value from the chart is more to the West than the real one.

Annex 2: Mathematical annex

Basic equations

The Cartesian coordinates of a point on the surface of the earth (assumed to be a perfect sphere of radius R) are related to the latitude (LAT) and longitude (LON) coordinates by the well known equations:

$$x = R \cos LAT \sin(LON - LON_0)$$

$$y = R \cos LAT \cos(LON - LON_0)$$

$$z = R \sin LAT$$

Using the definition of obliquity (OBL), we can transform these equations to express x, y and z as a function of obliquity and longitude:

$$\tan OBL = \frac{\tan LAT}{\cos(LON - LON_0)}$$

$$x = R \frac{\tan(LON - LON_0)}{\sqrt{1 + \tan^2 OBL + \tan^2(LON - LON_0)}}$$

$$y = R\frac{1}{\sqrt{1+\tan^2 OBL+\tan^2(LON-LON_0)}}$$

$$z = R\frac{\tan OBL}{\sqrt{1+\tan^2 OBL+\tan^2(LON-LON_0)}}$$

Non orthogonality of obliquity and longitude

For ease of writing I will rename the longitude and obliquity coordinates with letters *u* and *v* respectively:

$$u = LON - LON_0$$

$$v = OBL$$

It should be remembered that both u and v are restricted to the range (-90º, +90º).

Let us calculate the first Gaussian fundamental quantities of the coordinate system u,v:

$$e = \left(\frac{\partial x}{\partial u}\right)^2 + \left(\frac{\partial y}{\partial u}\right)^2 + \left(\frac{\partial z}{\partial u}\right)^2 = R^2\frac{1+\tan^2 v+\tan^2 u\tan^2 v+\tan^4 u}{\cos^4 u}$$

$$g = \left(\frac{\partial x}{\partial v}\right)^2 + \left(\frac{\partial y}{\partial v}\right)^2 + \left(\frac{\partial z}{\partial v}\right)^2 = R^2\frac{1+\tan^2 u+\tan^2 u\tan^2 v+\tan^4 v}{\cos^4 v}$$

$$f = \frac{\partial x}{\partial u}\frac{\partial x}{\partial v} + \frac{\partial y}{\partial u}\frac{\partial y}{\partial v} + \frac{\partial z}{\partial u}\frac{\partial z}{\partial v} = -R^2\frac{\tan u\tan v}{\cos^2 u\cos^2 v(1+\tan^2 u+\tan^2 v)}$$

For a coordinate system to be orthogonal, the value of f must be equal to 0 in every point. From the expression above it is clear that f is zero only on the points where u or v are zero, i.e. the axes of coordinates. It follows that this system of coordinates is not orthogonal.

Non conformality of the Unnamed projection

Using the variables u and v defined above, the Unnamed projection takes the form:

$$X = u$$

$$Y = v$$

Its Gaussian fundamental quantities are simple to calculate:

$$E = \left(\frac{\partial X}{\partial u}\right)^2 + \left(\frac{\partial Y}{\partial u}\right)^2 + \left(\frac{\partial Z}{\partial u}\right)^2 = 1$$

$$G = \left(\frac{\partial X}{\partial v}\right)^2 + \left(\frac{\partial Y}{\partial v}\right)^2 + \left(\frac{\partial Z}{\partial v}\right)^2 = 1$$

$$F = \frac{\partial X}{\partial u}\frac{\partial X}{\partial v} + \frac{\partial Y}{\partial u}\frac{\partial Y}{\partial v} + \frac{\partial Z}{\partial u}\frac{\partial Z}{\partial v} = 0$$

For a projection to be conformal (i.e. angles are preserved), its scale distortion coefficient *m* , defined as follows, has to be equal to one:

$$m^2 = \frac{E\left(\frac{du}{dv}\right)^2 + 2F\frac{du}{dv} + G}{e\left(\frac{du}{dv}\right)^2 + 2f\frac{du}{dv} + g}$$

Inserting the values E, F and G of the Unnamed projection we obtain:

$$m^2 = \frac{\left(\frac{du}{dv}\right)^2 + 1}{e\left(\frac{du}{dv}\right)^2 + 2f\frac{du}{dv} + g}$$

which is in general different from 1, therefore the projection is not conformal.

It is interesting to note however that the projection is conformal at the origin of coordinates (u = v = 0), given that e=1, f=0 and g=1 at that point.

Non equivalence of the Unnamed projection

For a projection to be equivalent (i.e. relative areas are preserved) it has to be verified that:

$$eg - f^2 = EG - F^2$$

In the case of the Unnamed Projection it is easy to calculate that the right-hand side of the equation is equal to 1. The left-hand side takes however a more complicated form:

$$eg - f^2 = \frac{\left(1 + \tan^2 v + \tan^2 u \tan^2 v + \tan^4 u\right)\left(1 + \tan^2 u + \tan^2 u \tan^2 v + \tan^4 v\right)}{\cos^4 u \cos^4 v} - \frac{\tan^2 u \tan^2 v}{\cos^4 u \cos^4 v (1 + \tan^2 u + \tan^2 v)^4}$$

As the latter expression is not equal to 1 in general, it can be concluded that the Unnamed projection is not equivalent. The origin of coordinates (u = v = 0) is an exception, as was the case for conformality too.

Note: All basic formulae for this annex have been taken from RICHARDUS, Peter and ADLER, Ron K. *Map Projections for Geodesists, Cartographers and Geographers*. North-Holland. 1 st edition 1972, 2 nd printing 1974. ISBN 0720450071 / ISBN 0444103627

Notes

1. It is not the purpose of this article to provide an exhaustive bibliography on the map of Juan de la Cosa, especially as the first works date back to the first half of the 19 th century with Alexander von Humboldt (*Examen critique de l'histoire de la geographie du nouveau continent*, 1837) and Ramón de la Sagra (*Parte correspondiente a la América de la carta general de Juan de la Cosa*, 1837). In 1995 Fernando Silió Cervera published a detailed description and discussion of the map together with the most exhaustive cartographic analysis to date: Fernando Silió Cervera, *La carta de Juan de la Cosa (1500): Análisis cartográfico* (Santander: Fundación Botín, 1995). For those who do not read Spanish, the articles cited below by Arthur Davies and Aldo Alvarez provide background information together with the cartographic theories of their respective authors. In addition, the website "Cartographic Images" contains a well-written introduction to the La Cosa map (http://www.henry-davis.com/MAPS/Ren/Ren1/305mono.html) with a list of further references in English.

2. Silió Cervera, *La carta de Juan de la Cosa*, 159.

3. Image taken from Arthur Davies, "The Date of Juan de la Cosa's World Map and Its Implications for American

Discovery," *The Geographical Journal* 1, no. 142 (1976): 111-116.

4. See discussion of La Cosa's sources in Silió Cervera, *La carta de Juan de la Cosa,* 74-94.

5. Jesús Varela Marcos, "Los reflejos políticos en la carta de Juan de la Cosa," in *Grandes viajes descubridores*, ed. Jesús Varela Marcos, 61-79, in *Descubrimientos y Cartografía IV: grandes viajes descubridores* (Valladolid: Instituto Interuniversitario de Estudios de Iberoamérica y Portugal, 2001), 61-79.

6. Hugo O'Donnell Duque de Estrada, "La carta de Juan de la Cosa: tradición y originalidad en sus aspectos decorativos," *Cuadernos monográficos del Instituto de Historia y Cultura Naval*, no. 35 (2000): 75-86. http://www.armada.mde.es/ArmadaPortal/ShowBinaryServlet?nodePath=/BEA%20Repository/Desktops/Portal/ArmadaEspannola/Pages/documentacion_revistas/04_cuadernosIH/35_cuaderno35_es/05_capitulo_355_es//archivo

7. Varela Marcos, "Los reflejos políticos."

8. Silió Cervera, *La carta de Juan de la Cosa,* 203.

9. Arthur Davies and Hugo O'Donnell are the two main proponents of this theory: Davies, "The 'English' Coasts on the Map of Juan de la Cosa," *Imago Mundi* 13:1 (1956): 26-29; O'Donnell, "El Mapamundi denominado Carta de Juan de la Cosa y su verdadera naturaleza," *Revista General de Marina* , special issue devoted to the 5th Centenary (Sept 1991): 161-181. But Fernando Silió disagrees—see Silió Cervera, *La carta de Juan de la Cosa,* 215.

10. Illaria Luzzana Caraci, "Algunas observaciones sobre la primitiva cartografía americana," in *Congreso de Historia del Descubrimiento: 1492-1556* (Madrid: Real Academia de la Historia, 1992), 179.

11. For instance in Davies. "The Date of Juan de la Cosa's World Map," 111-116; and in Ricardo Cerezo Martínez, "La Carta de Juan de la Cosa (II)," *Revista de Historia Naval* 42 (Q3 1993): 21-44.

12. In an earlier publication, the same author had calculated the apparent latitudes of a number of Atlantic toponyms, assuming a *plate carrée* projection: Ricardo Cerezo Martínez, "Aportación al estudio de la carta de Juan de la Cosa," in *Géographie du monde au Moyen Âge et à la Renaissance*, ed. Monique Pelletier (Paris: C.T.R.S., 1989),149-62.

13. Reproduced from Cerezo Martínez, "Aportación al estudio de la carta de Juan de la Cosa."

14. Silió Cervera, *La carta de Juan de la Cosa,* 228-229.

15. Ibid., 193.

16. Ibid., 197-201.

17. Reproduced from Ibid., 99.

18. Ángel Paladini Cuadrado, "Contribución al estudio de la carta de Juan de la Cosa," *Revista de Historia Naval* 47 (1994): 45-54.

19. Waldo R. Tobler, "Medieval Distortions: The Projections of Ancient Maps," *Annals of the Association of American Geographers* 56:2 (June 1966): 351-360.

20. Waldo R. Tobler, "Numerical Approaches to Map Projections," in *Studies in Theoretical Cartography*, ed. E. Kretschmer (Vienna: Deuticke, 1977), 51-64.

21. Joaquim Alves Gaspar, "Dead Reckoning and Magnetic Declination: Unveiling the Mystery of Portolan Charts," *e-Perimetron* 3:4 (2008): 191-203. http://www.e-perimetron.org/Vol_3_4/Gaspar.pdf

22. Chryssoula Boutoura and Evangelos Livieratos, “Some Fundamentals for the Study of the Geometry of Early Maps by Comparative Methods,” *e-Perimetron* 1:1 (2006): 60-70. http://www.e-perimetron.org/Vol_1_1/Boutoura_Livieratos/1_1_Boutoura_Livieratos.pdf

23. Silió Cervera, *La carta de Juan de la Cosa*, 73.

24. Ibid., 156.

25. Ibid., 188.

26. María Luisa Martín-Merás Verdejo, “La carta de Juan de la Cosa: interpretación e historia,” *Monte Buciero (Ayuntamiento de Santoña)*, 4 (2000): 71-86. http://dialnet.unirioja.es/servlet/fichero_articulo?codigo=206305&orden=72914

27. Aldo Alvarez. “Geomagnetism and the Cartography of Juan de la Cosa,” *Terrae Incognitae* 35 (2003): 1-15. http://www.sochistdisc.org/2003_articles/alvarez.htm. Also in Cerezo Martínez, “La Carta de Juan de la Cosa (II),” 21-44

28. Silió Cervera, *La carta de Juan de la Cosa*, 227-232

29. Johannes Keuning, “The History of Geographical Map Projections Until 1600,” *Imago Mundi* 12 (1955): 1-24.

30. Ignace-Gaston Pardies, *Globi Coelestis in Tabulas Planas Redacti Descriptio* (Paris, 1674). Image courtesy of the Internet Archive.

31. The underlying image of the La Cosa map is courtesy of Wikimedia Commons.

32. “Note, Without the knowledge of the true Quantity of the Obliquity or Latitude of that Great Circle which will pass directly over the Places propounded, there can be no compleat Demonstration, much less Arithmetical Calculation, of things pertaining thereunto [i.e. to the calculation of great-circle navigation] .” Samuel Sturmy, *The Mariners Magazine; or Sturmy's Mathematical and Practical Arts* (London: E. Cotes, 1669), Book IV, chapter XI, 181.

33. José Antonio Hurtado García, “La ‘longitud del occidente’ y la ‘latitud del equinoccial’: un sistema de coordenadas geográficas, ortogonal, inédito,” in the p roceedings of the International Congress *"V Centenario de la Muerte del Almirante,"* ed. Jesús Varela Marcos and María Montserrat León Guerrero (Valladolid, 2006) , 1:545-564.

34. See for instance, Rolando Laguarda Trias, “El enigma de las latitudes de Colón,” *Cuadernos Colombinos* 4 (1974): 1-67.

35. Alves Gaspar, “Dead Reckoning.”

36. Arthur Davies explained how the information from Cabot voyages reached La Cosa and proposed an identification for that coastline. Davies “The 'English' coasts,” 26-29. Fernando Silió disagrees completely with Davies’ identification: Silió Cervera, *La carta de Juan de la Cosa*, , p.215.

37. Silió Cervera, *La carta de Juan de la Cosa*, 221. See also Alvarez, “Geomagnetism ,” 1-15.

38. Ángel Paladini Cuadrado, “Sobre la génesis de los portulanos,” *Boletín de información del Servicio Geográfico del Ejército* 76 (1993): 7-18.

39. Joaquim Alves Gaspar, “The Myth of the Square Chart,” *e-Perimetron* 2: 2 (2007): 66-79. http://www.e-perimetron.org/Vol_2_2/Gaspar.pdf

40. Ibid., 68.

41. Ibid., 74-75

42. Keuning, “The History of Geographical Map Projections until 1600,” 1-24.

43. *Real provisión* dated in Burgos, July 24, 1512. Written by Lope Conchillos, secretary of Queen Juana, under orders of regent Ferdinand of Aragon (“por mandado del rey su padre”) and validated by bishop Fonseca. Manuscript preserved at the Archivo de Indias, Contratación, leg.5784, fol.15 . Transcription in Juan Manzano Manzano, *Los Pinzones y el Descubrimiento de América*, vol. 3 (Madrid: Ediciones de Cultura Hispánica, 1988), 344-346. Excerpt from the original text: “...e porque yo he sabido que ay muchos padrones de cartas fechos de diversas maneras e por diversos maestros (...) muy diferentes las unas de las otras asy en la derrota como en el assentamiento de las tierras (...), es mi merced e voluntad que se haga un Padrón General (...) confiando de vos Juan de Solís, nuestro Piloto Mayor, e de vos miçer Juan Vespuchi, nuestro piloto (...) hagays juntar todos los más pilotos que ser pudiere e que mas supieren en las navegaciones y astrolabios e alturas e compases (...); e después que todos ayan dicho sus paresçeres, (...) se haga por anbos a dos vosotros un padrón general que se llame el Padrón Real, en pergamino, e que esté puesto públicamente en la dicha Casa de la Contrataçión, por el qual todos los pilotos se ayan de regir e governar e hazer sus viajes, e que para que todos lo tengan en su poder e se rijan por ellos, vos el dicho micer Juan Vespuchi los podais hazer e hagays todos los treslados del dicho Padrón Real e no otro ninguno, (...) e que ningund piloto use de otro ningund padrón sino del que vos le dierdes”

44. Eric H. Ash,“Navigation Techniques and Practice in the Renaissance,” in *Cartography in the European Renaissance*, ed. David Woodward, vol. 3, *The History of Cartography* (Chicago: University Of Chicago Press, 2007), 515.

45. W.G. Randles, “La crise de la cartographie au XVIe siècle,” in *Géographie du monde au Moyen Âge et à la Renaissance*, ed. Monique Pelletier (Paris: C.T.R.S., 1989).

46. Ash, “Navigation Techniques,” 524.

47. Felipe Fernández-Armesto, “Maps and Exploration in the Sixteenth and Early Seventeenth Centuries,” in *Cartography in the European Renaissance*, ed. David Woodward, vol. 3, *The History of Cartography* (Chicago: University Of Chicago Press, 2007), 746.

48. Alves Gaspar, “The Myth of the Square Chart,” 66-79.

49. Randles, “La crise de la cartographie."

50. Douglas T. Peck, “The History of Early Dead Reckoning and Celestial Navigation: Empirical Reality Versus Theory,” *New World Explorers*, 2002. http://www.newworldexplorersinc.org/EarlyNavigation.pdf

51. W.G. Randles, “Portuguese and Spanish Attempts to Measure Longitude in the 16th Century,” *Vistas in Astronomy* 28 (1985): 235-241.

52. Svat Soucek, “Islamic Charting in the Mediterranean,” in *Cartography in the Traditional Islamic and South Asian Societies*, ed. David Woodward and John Brian Harley, vol 2, bk 1, *The History of Cartography* (Chicago: University Of Chicago Press, 1992), 263-292. See also, Fuat Sezgin, *Mathematical Geography and Cartography in Islam and their Continuation in the Occident* (Frankfurt am Main: Institute for the History of Arabic-Islamic Science, 2005), part I, 155-160.

53. See for instance, Pedro de Medina, *Regimiento de navegación* (Sevilla, 1563), bk 1, xi – xiiii.

54. Tara Adamek, K. Penkalski, and G. Valentine. “The History Of Trigonometry,” 2005. http://math.rutgers.edu/~mjraman/History_Of_Trig.pdf ; Morris Kline. *Mathematical Thought from Ancient to Modern Times* (New York: Oxford University Press, 1972), vol. 1, 126.

55. Carl B. Boyer and Uta C. Merzbach, *A History of Mathematics* (2nd ed.; New York: Wiley, 1991), .238.

56. S. Kamal Abdali, “The Correct Qibla,” 1997. http://nurlu.narod.ru/qibla.pdf. Waldo R. Tobler, “Qibla, and Related, Map Projections,” *Cartography and Geographic Information Science* 29:1 (2002): 17-23.

57. Ibn Muad's "Book on the unknowns of the arcs of the sphere" was first published in MariaVictoria Villuendas, *La trigonometria europea en el siglo XI. Estudio de la obra de Ibn Mu'ad, El Kitab mayhulat* (Barcelona: Instituto de Historia de la Ciencia de la Real Academia de Buenas Letras, 1979). A summary of this part of Ibn Muad's work can be found in pages 142-143 of the main reference today on science in Muslim Spain: Julio Samsó. *Las ciencias de los antiguos en al-Andalus* . (Madrid: Mapfre, 1992).

58. Julio Samsó . "La astronomía magrebí en tiempos de Ramon Llull," *Quaderns de la Mediterània* 9 (2001): 324-327. http://www.iemed.org/publicacions/quaderns/9/q9_324.pdf

59. Julio Samsó and Juan Vernet, "El saber científico y técnico (1086-1492)," in *El reino nazarí de Granada (1232-1492)*, 293-298, vol. 4 t. 8, of *Historia de Espan~a Mene´ndez Pidal*, ed. María Jesús Viguera et. al (Madrid : Espasa Calpe, 2000), 293-298.

60. Daniel Eisenberg, "Cisneros y la quema de los manuscritos granadinos," *Journal of Hispanic Philology* 16 (1992): 107-124. http://users.ipfw.edu/jehle/DEISENBE/Other_Hispanic_Topics/Cisneros_y_la_quema_de_los_manuscritos_granadinos.htm

61. Julio Samsó, "Las ciencias exactas en Castilla durante la Edad Media," in *Historia de una cultura*, ed. Agustín García Simón et. al. (4 v.; [Valladolid] : Junta de Castilla y Leo´n-Consejeri´a de cultura y turismo, 1995-1996) , 2:661-689.

62. Elena Ausejo Martínez, "Trigonometría y astronomía en el Tratado del Cuadrante Sennero (c. 1280)." *Dynamis* 4 (1984): 7-22. http://www.raco.cat/index.php/Dynamis/article/viewFile/105858/149399

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65. Elly Dekker, "The Copernican Globe: A Delayed Conception," *Annals of Science*, 53 (1996): 541-566 (footnote 11). An image of a portion of the Behaim Globe showing the ecliptic can be found in figure 6 of Lionel Dorffner, "Der digitale Behaim-Globus —Visualisierung und Vermessung des historisch wertvollen Originals." *Cartographica Helvetica* 14:96 (1996): 20-24. http://www.ipf.tuwien.ac.at/publications/ld_ch96/ld_ch96.html

66. See image in Svat Soucek, "Islamic Charting in the Mediterranean," 282.

67. Bernhardus Varenius, *A Compleat System of General Geography: Explaining the Nature and Properties of the Earth*. Trans. DUGDALE. (2nd ed; London: Stephen Austen, 1734) , 2:531. http://books.google.com/books?id=oAkTAAAAYAAJ&client=firefox-a&pg=PR1#v=onepage&q=&f=false

68. Hendrik Hondius, "Noua totius terrarum orbis geographica ac hydrographica tabula," 1630. Map reproduction courtesy of the Norman B. Leventhal Map Center at the Boston Public Library.