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The Cardano - Tartaglia Dispute

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Introduction

As the Middle Ages came to a close, a rebirth of scientific inquiry occurred. Most large cities had universities, but these consisted primarily of lecturers, as there were few books beyond those of the classical authors. Hence the reputation of a university depended upon its ability to provide the best lecturers. But who was the best lecturer? One way to settle this was in a public debate with the winner gaining prestige and academic positions while the loser was ignored.

Since a scholar had to outperform all challengers to maintain his position, he needed every trick he knew. If someone knew something that no one else knew, his fame was insured. One assurance of success would be a knowledge of the general solution to the previously unsolved cubic equation. It was the search for this solution that led to the dispute between Gerolamo Cardano and Niccolo Tartaglia. But it is impossible to discuss the debate without first giving brief biographies of the antagonists to show why they acted as they did.

The Life of Tartaglia

Niccolo Tartaglia—his actual family name was Fontana—was born in Brescia, Italy, about the turn of the sixteenth century.[1] When the French sacked Brescia in 1512, the youthful Fontana suffered severe saber cuts about the face and mouth which caused a speech impediment. Because of this he was nicknamed Tartaglia, "the stutterer." He later wore a long beard to hide the scars, but he could never overcome the stuttering.

His education was very meager. In fact he tells, in one of his books, that his mother had accumulated a small amount of money so that he might be tutored by a writing master. The money ran out, so he says, when the instruction reached the letter *K* and his education stopped before he could write his own initials. But being an enterprising youth, he stole the master's lecture notes and completed the course on his own. In times of extreme poverty he even used tombstones in place of writing slates.

His mathematical knowledge was completely self-taught, but his attempts to teach himself Latin resulted in failure and he was forced to write his treatises in Italian instead of the accepted Latin. He taught primarily in Venice, and, once established, lived quite comfortably on his wages and the wagers he won in public disputes and challenges.

The Life of Cardano

The other participant, Gerolamo Cardano,[2] was born in Milan in 1501. His father, a lawyer and lecturer in geometry, married Cardano's mother, who came from a "socially unacceptable" family, a few years after Cardano's birth. This led to later complications.

He started his studies at the University of Pavia, but transferred to the University of Padua when war broke out. At college he supplemented his

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meager finances by constant gambling, at which he became very proficient. He wrote a treatise, *Liber de ludo aleae,*[3] which not only introduced the idea of probability as we use it today, but also included ways to cheat in these games.

At twenty-five, he graduated as a doctor of his chosen profession, medicine. Due to his illegitimate birth, he was not allowed to practice in Milan. Later he was recognized as a doctor and proved his ability by becoming the second most renowned medical expert in Europe at that time. Most of his 412 written works[4] were in the field of medicine, popular science, and astrology. One of these was an autobiography in which he describes himself as follows: "living from day to day, outspoken, despising religion, mindful of injuries caused by others, envious, melancholy, a spy, a betrayer of trusts, a deviner, a caster of charms, subject to frequent failures, hateful, given to shameful lust, solitary, unpleasant, strict, foretelling the future willingly, jealous, obscene, lascivious, abusive in speech, inconsistent, two-faced, dishonest, a slanderer, entirely unparalleled in vices, and by nature incompatible even with those with whom he converses daily."[5]

His sons were "chips off the old block," as one robbed his father and the other married and later murdered an immoral girl.

In 1570, Cardano was arrested and jailed on a charge of heresy. The charge could be substantiated by a horoscope of Christ, and by a book which Cardano had published praising Nero, the Roman emperor well known for his persecution of the early Christians. After he was convicted, he was forced to admit and renounce his heresies. As a punishment he was denied the right to lecture publicly and was ordered to refrain from writing and publishing books. Heartbroken, he accompanied one of his students to Rome, where he received an invitation to become a consultant to the College of Physicians. A pension was received from the Pope and soon, possibly insane,[6] he died.

A student of Cardano's who received fame as a mathematician was Ludovico Ferrari. He entered Cardano's house as a servant, soon was elevated to secretary, and became a public lecturer before he was twenty years old. His major contribution to mathematics was a general solution to the biquadratic equation $x^4 + ax^2 + b = cx$. Unfortunately his career was cut short by premature death when in his early forties.

The Dispute[7]

About 1510, Scipione del Ferro found a general solution to $x^3 + ax = b$, but he died before he could publish his discovery. His student, Antonio Maria Fiore, knew the solution and attempted to gain a reputation by exploiting his master's discovery. He challenged Tartaglia with thirty questions, all of which reduced to the solution of $x^3 + ax = b$. Tartaglia had the general solution to $x^3 + ax^2 = b$, so he responded with thirty questions of a more general theoretical nature, although some resolved to this equation. Besides the prestige to be gained, the winner and his friends were to receive thirty banquets from the loser. Just before the time limit elapsed, Tartaglia found general solutions to both $x^3 + ax = b$ and $x^3 = ax + b$.[8] With these, Tartaglia solved all of Fiore's problems, but Fiore was unable to solve any of the questions proposed by Tartaglia and so was vanquished. The banquets were not collected.

At this time Cardano was writing the *Practica arithmeticae generalis,* which would encompass arithmetic, geometry, and algebra. Inasmuch as Fra Luca Pacioli had earlier stated that there could not be a general solution to the cubic, Cardano had ignored this topic. Upon hearing that Tartaglia had a solution for $x^3 + ax = b$, he tried to find one. Failing, he asked Tartaglia for the solution so that he might publish it in a special section of the *Practica arithmeticae generalis* under Tartaglia's name. Tartaglia refused, stating he would publish the solution himself at a later date. This prompted Cardano to label Tartaglia as greedy and unwilling to help mankind.

Because of these insults, a correspondence developed between the two mathematicians which resulted in Tartaglia visiting Cardano in Milan. During this visit Tartaglia relented and offered Cardano a cryptic poem containing a solution to $x^3 + ax = b$, provided Cardano swore an oath that he would never reveal the solution. Cardano accepted the terms, but was unable to decipher the code, so he asked for and received the necessary clue from Tartaglia. Using the solution to $x^3 + ax = b$, Cardano and Ferrari found solutions for $x^3 + ax^2 = b$, $x^3 = ax^2 + b$, and $x^3 + b = ax^2$ by employing substitutions which reduced them to the known case.

In 1545, Cardano published the *Ars magna,* which contained Tartaglia's solution of the cubic with a statement that del Ferro and Tartaglia had each found solutions by independent research. Cardano also included some of his own discoveries, including the idea that every cubic should have three roots. Cardano also published Ferrari's solution to the biquadratic equation here, with due credit to Ferrari.

When he had seen the *Ars magna,* Tartaglia publicly denounced Cardano for breaking an oath sworn on the Gospels, and he ridiculed Cardano's mathematical ability.

Cardano disdained to refute the slur, but Ferrari attacked Tartaglia, charging that Tartaglia had built up his reputation by defaming others, had stolen one proof in his new book without giving credit, and in addition had at least one thousand errors in the text. Ferrari ended his published response by challenging Tartaglia to a public debate on mathematics and all related subjects.

Tartaglia answered with further insults and refused the debate on the grounds that Cardano knew the men who would be the judges. Perhaps he really feared a public debate because he stammered.

After a further exchange of insults, each proposed thirty-one questions which were exchanged, answered, and returned. However, no decision was reached, because each tore the other's answers to shreds.

Then for no given reason, Tartaglia accepted a debate to be held in Milan, Cardano's stronghold.[9]

On August 10th, 1548, Tartaglia and Ferrari met in combat, Cardano having left town.

Very little is known about the actual debate, but it appears to have degenerated into an invective match, with Tartaglia doing most of the shouting. Tartaglia left after the first day, claiming to have won, although it seems Ferrari won by default.

An indication of Ferrari's triumph is that Tartaglia lost his teaching post in Brescia, and Ferrari was invited to lecture in Venice, Tartaglia's stronghold.

Tartaglia died in 1557 without publishing his solution to the cubic, and when an attempt was made to publish his unpublished papers, none could be found which even mentioned the solutions to the cubic.

With Cardano's death in 1576, one of the most interesting and colorful episodes in the history of mathematics ended.

NOTES

1. The actual year is in doubt; Oystein Ore lists it as 1499 in *Cardano, the Gambling Scholar,* and D. E. Smith gives 1506 in his *History of Mathematics.*

2. Hieronymus Cardanus in Latin; also translated into English as Jerome Cardan.

3. *A Book on Games of Chance.*

4. At the time of his death, 131 of his works had been published, 111 existed in manuscript, and he claimed to have burned 170 which he found unsatisfactory.

5. A quote taken from the introduction by Gabriel Naude in Cardano's autobiography, *Liber de propria vita,* Amsterdam, 1654.

6. Charles W. Burr, *Jerome Cardan as Seen by an Alienist,* in University of Pennsylvania: University Lectures Delivered by Members of the Faculty in the Free Public Lecture Course, 1916–1917, vol. 4.

7. The two main sources for the dispute are Oystein Ore, *Cardano, the Gambling Scholar,* Princeton:

(Princeton University Press, 1953) and M. A. Nordgaard, "Sidelights on the Cardan-Tartaglia Dispute," *National Mathematics Magazine,* XII (1937), 327–346.

8. Due to the limited use of symbols and a lack of understanding of negative quantities, these equations had to be treated separately. For example, Cardano wrote $x^3 + 6x = 20$ as cub^s p: 6 reb^s aeqlis 20. Other methods of writing equations during this period can be seen in D. E. Smith's *History of Mathematics,* II, pp. 427–431.

9. Perhaps Brescia, Tartaglia's city, demanded it because of civic pride.

HISTORICAL EXHIBIT 6.1

Cardano's Technique for the Solution of a Reduced Cubic Equation

A solution is sought for an equation of the form

$$x^3 + ax = b,\ a > 0,\ b > 0.$$

It is known that

$$(p-q)^3 + 3pq(p-q) = p^3 - q^3;$$

therefore, if we let

$$x = (p-q) \text{ then } a = 3pq \text{ and } b = p^3 - q^3.$$

It follows then that:

$$p = \frac{a}{3q} \text{ and } b = \left(\frac{a}{3q}\right)^3 - q^3 \text{ or}$$

$$27b\ q^3 + a^3 - 27\left(q^3\right)^2 \text{ which can be rewritten as}$$

$$27\left(q^3\right)^2 + 27b\ q^3 - a^3 = 0.$$

This last equation is a biquadratic for which use of the existing quadratic solution scheme could supply a solution for q^3, that is,

$$q^3 = \frac{-b \pm \sqrt{b^2 + \frac{4a^3}{27}}}{2} \quad \text{and}$$

q is found to be

$$q = \sqrt[3]{-\frac{b}{2} + \sqrt{\frac{b^2}{4} + \frac{a^3}{27}}}\ .$$

In a similar manner, the value for p is also obtained:

$$p = \sqrt[3]{\frac{b}{2} + \sqrt{\frac{b^2}{4} + \frac{a^3}{27}}},$$

and finally

$$x = p - q = {}^3\sqrt[3]{\frac{b}{2} + \sqrt{\frac{b^2}{4} + \frac{a^3}{27}}} - \sqrt[3]{\frac{-b}{2} + \sqrt{\frac{b^2}{4} + \frac{a^3}{27}}}\ .$$