APPENDIX 1

Hipparchus's Table of Chords

The construction of this table is based on the facts that the chords of 60° and 90° are known, that starting from chd θ we can calculate chd$(180° - \theta)$ as shown by Figure A1.1, and that from chd θ we can calculate chd $\frac{1}{2}\theta$. The calculation of chd $\frac{1}{2}\theta$ goes as follows; see Figure A1.2. Let the angle AOB be θ. Place F so that $CF = CB$, place D so that $DOA = \frac{1}{2}\theta$, and place E so that DE is perpendicular to AC. Then

$$ACD = \tfrac{1}{2}AOD = \tfrac{1}{2}BOD = DCB$$

making the triangles BCD and DCF congruent. Therefore $DF = BD = DA$, and so $EA = \frac{1}{2}AF$. But $CF = CB = \text{chd}(180° - \theta)$, so we can calculate CF, which gives us AF and EA. Triangles AED and ADC are similar; therefore $AD/AE = AC/DA$, which implies that $AD^2 = AE{\cdot}AC$ and enables us to calculate AD. AD is chd $\frac{1}{2}\theta$.

We can now find the chords of 30°, 15°, $7\frac{1}{2}$°, 45°, and $22\frac{1}{2}$°. This gives us the chords of 150°, 165°, etc., and eventually we have the chords of all

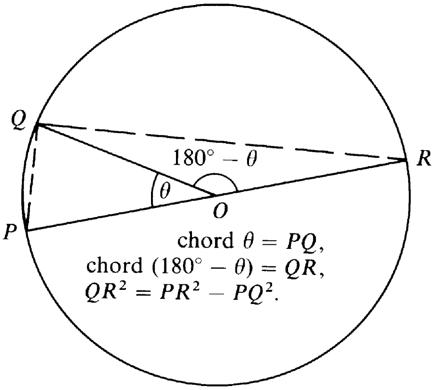

FIGURE A1.1.

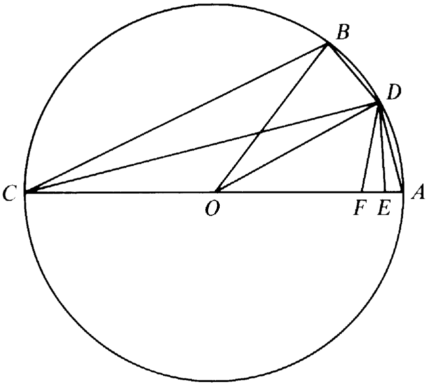

FIGURE A1.2.

multiples of $7\frac{1}{2}°$. The table starts:

θ	0°	$7\frac{1}{2}°$	15°	$22\frac{1}{2}°$	30°	$37\frac{1}{2}°$	45°	$52\frac{1}{2}°$
chd θ	0	450	897	1,341	1,779	2,210	2,631	3,041

We find the chords of angles not listed and angles whose chords are not listed by linear interpolation. For example, the angle whose chord is 2,852 is

$$\left(45 + \frac{2{,}852 - 2{,}631}{3{,}041 - 2{,}631} \times 7\tfrac{1}{2}\right)^{\circ} = 49^{\circ} \text{ approximately.}$$

APPENDIX 2

Calculation of the Eccentric-Quotient for the Sun, and the Longitude of its Apogee

This is Hipparchus's method as described by Ptolemy. However, Ptolemy used his own table of chords; I use the figures from Hipparchus's table as reconstructed by Toomer [103].

The basic data are that the interval from spring equinox to summer solstice is $94\frac{1}{2}$ days and the interval from then to autumn equinox is $92\frac{1}{2}$ days. In Figure A2.1, T is the earth, O is the center of the sun's orbit, H and L are the equinoxes, and J and K are the solstices.

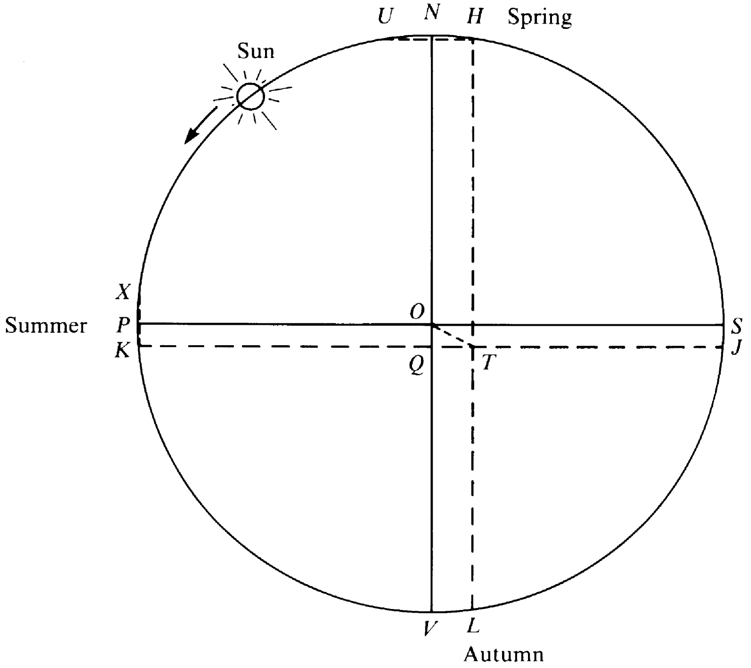

FIGURE A2.1.

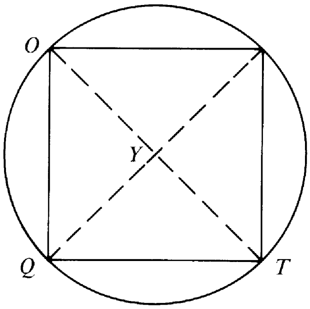

FIGURE A2.2.

The sun turns through the angle *HOL* in $92\frac{1}{2} + 94\frac{1}{2}$ days
$= 187$ days.

It turns through a whole circle in 365;14,48 days.
Therefore the angle *HOL* is $184°20'$,
and so $NOH + VOL = 4°20'$,
and so $UOH = 4°20'$.
Therefore, by linear interpolation, $HU = 260$.
The sun turns through the angle *HOK* in $94\frac{1}{2}$ days.
Therefore $HOK = 93°9'$.
But $NOH = \frac{1}{2}UOH$ and so $NOH = 2°10'$.
and so $POK = 59'$.
Then $KOX = 1°58'$.
Therefore, by linear interpolation, $KX = 118$,
and so $OQ = 59$.
But $TQ = \frac{1}{2}HU$
$= 130$,
and so, because $TO^2 = TQ^2 + OQ^2$, $TO = 143$.
Thus $TO/ON = 143/3438$
$= 1/24$ approximately.

This is the eccentric-quotient.
As above $OQ/OT = 59/143$
$= 2830/6876$
and so (see Figure A2.2) $OQ/OY = 2830/3438$.
Then by linear interpolation, $OYQ = 49°$,
and so $OTQ = 24\frac{1}{2}°$.

Therefore the apogee is $24\frac{1}{2}°$ west of the summer solstice, i.e., its longitude is $65\frac{1}{2}°$.

Appendix 3

Ptolemy's Table of Chords

Ptolemy's table of chords is much more sophisticated than the one that we think Hipparchus used (see page 128). The chords are in a circle of radius 60 instead of 3,438, which makes calculations much easier. The interval between entries is $\frac{1}{2}^\circ$ instead of $7\frac{1}{2}^\circ$; and the smaller the interval the smaller the errors introduced by linear interpolation. Attaining a smaller interval is not merely a question of subdividing more finely. Hipparchus could easily have produced a table with intervals of $3\frac{3}{4}^\circ$ or $1\frac{7}{8}^\circ$ or $\frac{15}{16}^\circ$ by the halving process, but such a table would have been awkward to use. Ptolemy stated and proved a theorem (usually known today, in fact, as Ptolemy's theorem) which enabled him to calculate the chords of $x + y$ and $x - y$ if the chords of x and y are known. He used Euclid's construction of a regular pentagon to find the chord of 36°, which, since he knew the chord of $37\frac{1}{2}^\circ$, enabled him to find the chord of $1\frac{1}{2}^\circ$.

It is not possible to trisect an angle of $1\frac{1}{2}^\circ$ by Euclidean methods, but it is possible to find a good enough approximation to the chord of $\frac{1}{2}^\circ$ by using the result that

$$\text{if} \quad x > y, \quad \text{then} \quad \frac{\text{chd}\, x}{\text{chd}\, y} < \frac{x}{y}.$$

Taking $x = 1\frac{1}{2}^\circ$ and $y = 1^\circ$, we have

$$\begin{aligned} \text{chd}\, 1^\circ &> \tfrac{2}{3}\, \text{chd}\, 1\tfrac{1}{2}^\circ \\ &> \tfrac{2}{3} \times 1;34,14,41 > 1;2,49,47. \end{aligned}$$

Similaly, taking $x = 1^\circ$ and $y = \frac{3}{4}^\circ$, we can show that

$$\text{chd}\, 1^\circ < 1;2,49,55.$$

Between them these show that chd 1° = 1;2,50 correct to two sexagesimal places. It is now easy to calculate the chord of $\frac{1}{2}^\circ$ to two places and to complete the table. (Ptolemy's own explanation of his calculations was a trifle careless. He worked to only two sexagesimal places, and stated that the chord of 1° was both greater than 1;2,50 and less than 1;2,50.)

APPENDIX 4

Calculating the Radius of the Moon's Epicycle

On page 133 we saw how Hipparchus (or Ptolemy) could calculate the radius of the moon's epicycle from data obtained by observing three eclipses. Here are the details of one such calculation carried out by Ptolemy using eclipses observed by the Babylonians in the first and second years of the reign of Marduk-apal-iddina, about 720 B.C. The time intervals between the eclipses, reduced to mean solar time, were 354 days, 2 hours, 34 minutes from the first to the second, and 176 days, 20 hours, 12 minutes from the second to the third. From the anomalistic period Ptolemy calculated how far round the epicycle the moon traveled in these two intervals. If the positions of the moon on the epicycle at the times of the eclipses are P_1, P_2, and P_3, respectively, then, measured clockwise

$$\text{arc } P_1P_2 = 306°25', \qquad \text{arc } P_2P_3 = 150°26'. \tag{1}$$

From the times of the eclipses, converted to Alexandria time, Ptolemy found the longitudes of the sun and hence of the moon. From these, as described on page 133, he found (see Figure A4.1, in which T denotes the earth)

$$\text{angle } P_2TP_1 = 3°24', \qquad \text{angle } P_2TP_3 = 0°37'. \tag{2}$$

(1) and (2) are the numerical data for the calculation.

Ptolemy several times used the table of chords to find the proportions of a right-angled triangle. This is how it is done. Let ABC be a triangle with a right angle at B (see Figure A4.2). Suppose that the angle ACB is $\frac{1}{2}x$ and we want to find AB/AC. If O is the midpoint of AC, then $AOB = x$. If we look up x in the table of chords and find that chd $x = y$, this means that $AB = y$ on a scale in which $AO = 60$. Thus

$$AB/AC = y/120.$$

This is the reason for such items as $\frac{1}{2} \times 6°48'$ or $\frac{1}{2} \times 1°14'$ in various steps of the calculation.

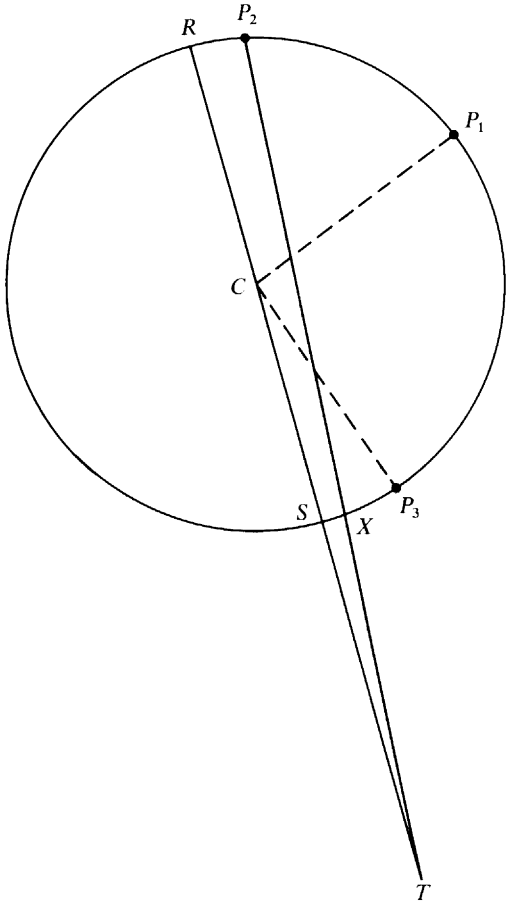

FIGURE A4.1.

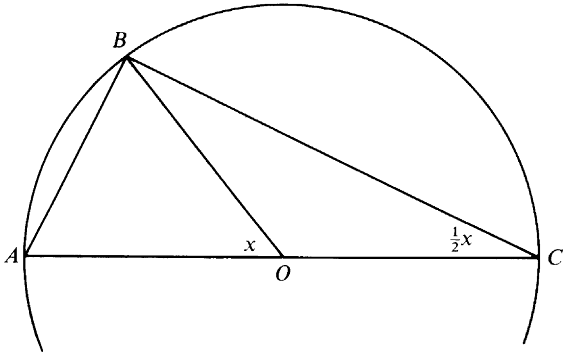

FIGURE A4.2.

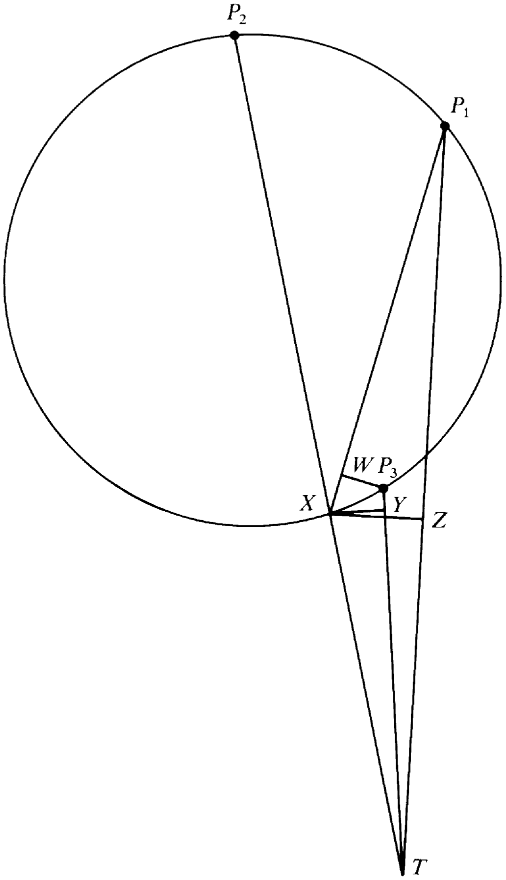

FIGURE A4.3.

Let P_2T cut the epicycle at X (Figure A4.3). Drop perpendiculars XY and XZ to TP_3 and TP_1. Drop a perpendicular P_3W to P_1X. Choose a scale in which $XT = 120$. Then:

	XTZ	$= \frac{1}{2} \times 6^\circ 48'$	from (2)	
Therefore	XZ	= 7;7	from tables	(3)
	arc P_2P_1	$= 360^\circ -$ arc P_1P_2		
		$= 53^\circ 35'$	from (1)	
Therefore	P_2XP_1	$= \frac{1}{2} \times 53^\circ 35'$	by the angle-at-the-circumference theorem	
	P_2TP_1	$= \frac{1}{2} \times 6^\circ 48'$	from (2)	
Therefore	XP_1T	$= \frac{1}{2} \times 46^\circ 47'$	by subtraction	
Therefore	XZ/P_1X	= 47;38,30/120	from tables	
Therefore	P_1X	= 17;55,32	from (3)	(4)
Again,	XTY	$= \frac{1}{2} \times 1^\circ 14'$	from (2)	
Therefore	XY	= 1;17,30	from tables	(5)
	P_2XP_3	$= \frac{1}{2} \times 150^\circ 26'$	from (1) and the angle-at-the-circumference	
	P_2TP_3	$= \frac{1}{2} \times 1^\circ 14'$	theorem from (2)	
Therefore	XP_3T	$= \frac{1}{2} \times 149^\circ 12'$	by subtraction	
Therefore	$XY : P_3X$	= 115;41,21/120	from tables	
Therefore	P_3X	= 1;20,23	from (5)	(6)
	arc P_1P_3	= arc P_2P_3 − arc P_2P_1		
		$= 96^\circ 51'$	from (1)	
Therefore	P_3XW	$= \frac{1}{2} \times 96^\circ 51'$	from the angle-at-the-circumference theorem	
and	WP_3X	$= \frac{1}{2} \times 83^\circ 9'$	being $90^\circ - P_3XW$	
Therefore	P_3W/P_3X	= 0;44,53,7		
and	XW/P_3X	= 0;39,48,57,30	from tables	
Therefore	P_3W	= 1;0,8 and XW = 0;53,21	from (6)	(7)
Then	P_1W	$= P_1X - XW$		
		= 17;2,11	from (4) and (7)	(8)
Then	$P_1P_3^2$	$= P_1W^2 + P_3W^2$		
		= 290;14,19 + 1;0,7	from (7) and (8)	
		= 291;14,36		
Therefore	P_1P_3	= 17;3,57		(9)
But	arc P_1P_3	$= 96^\circ 51'$	from (1) as above	
Therefore (see Figure 10.5, in which C is the center of the epicycle)				
	P_1P_3/CR	= 1;29,46,14	from tables	
Therefore	P_3X/CR	= 1;29,46,14 × 1;20,23/17;3,57	from (6) and (9)	
		= 0;7,2,50		(10)
Therefore	arc P_3X	$= 6^\circ 44' 1''$	from tables	
Therefore	arc P_2X	$= 157^\circ 10' 1''$	from (1)	
Therefore	P_2X/CR	= 1;57,37,32	from tables	
But	XT/CR	= 0;7,2,50 × 120/1;20,23	from (6) and (10)	
		= 10;31,13,48		
Therefore	P_2T/CR	= 12;28,51,20	by addition	
Then	$P_2T \cdot XT/CR^2$	= 131;18,20,5,32	by multiplication	
But	$P_2T \cdot XT$	$= RT \cdot ST$	by a theorem in geometry	
and	TC^2	$= RT \cdot ST + CR^2$	by another.	
Therefore	TC^2/CR^2	= 132;18,20,5,32		
and so	TC/CR	= 11;30,8,42		
giving	CR/TC	= 0;5,13.		

APPENDIX 5

The Eccentric-Quotient and Apogee of Mars

As pointed out on page 166, Ptolemy could calculate the eccentric-quotient and the direction of apogee of Mars if he knew the angles marked Z_1TZ_2, Z_2TZ_3, Z_1EZ_2, and Z_2EZ_3 in Figure 6.32. This is by no means obvious, so let us follow the method in some detail.

In Figure A5.1, the points Z_1, Z_2, Z_3, E, and T are as in Figure 6.32, K is the point where Z_3T cuts the circle $Z_1Z_2Z_3$ again, and KF, KG, Z_1H, and EN are perpendicular to Z_1T, Z_2T, Z_2K, and Z_3K, respectively, A is the apogee.

Knowing Z_2TZ_3, we know the angles KTG and TKG, and KG/TK. (i)
We know Z_2KZ_3 (it is $\frac{1}{2}Z_2EZ_3$), and so by (i) we know Z_2KG and KG/Z_2K. (ii)

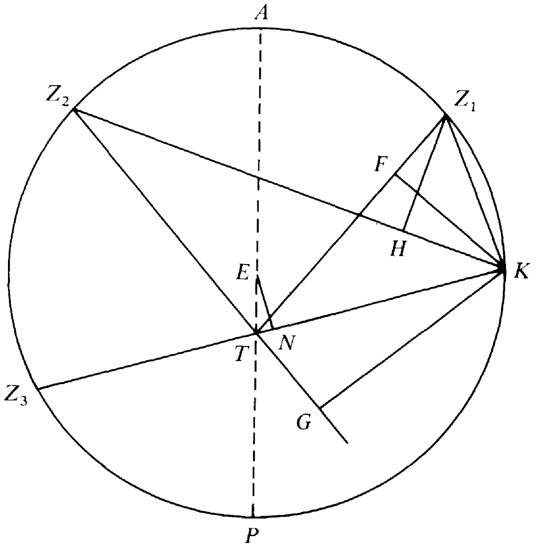

FIGURE A5.1.

Knowing Z_1TZ_3, we know FTK and KF/TK. (iii)
We know Z_1KT (it is $\frac{1}{2}Z_1EZ_3$) and Z_1TK (by iii) and
therefore TZ_1K and KF/Z_1K. (iv)
We know Z_1KH (it is $\frac{1}{2}Z_1EZ_2$), so we know Z_1H/KH and Z_1K/KH. (v)

Now in terms of KG we know Z_2K (by ii), TK (by i), KF (by iii), Z_1K (by iv), and KH (by v). Therefore we know Z_2H. We also know Z_1H (by v), so we know Z_1Z_2 (in terms of KG). But we also know Z_1Z_2 in terms of the radius r of the circle $Z_1Z_2Z_3$, because we know the angle Z_1EZ_2. Therefore, we know all these lengths in terms of r. In particular, we know Z_1K, therefore Z_1EK, therefore Z_3EK, therefore KZ_3.

Knowing Z_3K and TK, we know Z_3T. Since $AT \cdot TP = Z_3T \cdot TK$, we know $AT \cdot TP$. But $AT \cdot TP + TE^2 = r^2$, so we know TE (in terms of r) —we have found the eccentric-quotient TE/r.

We know Z_3N (it is $\frac{1}{2}Z_3K$) and Z_3T. Therefore we know NT. We know also TE, so we know the angle NTE. This gives us the direction of TE (the direction of apogee) in term of the observed direction TZ_3.

Appendix 6

Reversed Epicycles

In Figure 6.9, let T be the earth, let C be the center of the epicycle of a planet revolving about T anticlockwise in a circle of radius 60, and let P be the planet revolving clockwise round C in an epicycle of radius r. The minimum velocity of P as seen from T (counting anticlockwise velocities as positive) occurs when P is at the point A beyond C on the line TC.

Let the sidereal period of the planet be x years and its synodic period y years. Then the (angular) velocity of the line TCA about T is $1/x$ revolutions per year. The distance T is $60 + r$ and so the linear velocity of A is $2\pi(60 + r)/x$. The angular velocity of the line CP relative to CT is $1/y$ revolutions per year, and so the linear velocity of P, when it is at A, relative to A is $2\pi r/y$. Thus the condition that the planet should retrogress is $2\pi r/y > 2\pi(60 + r)/x$, i.e., $y < rx/(60 + r)$.

Figures from the *Almagest* are as follows:

	r	x	$rx/(60 + r)$	y
Mercury	$22\frac{1}{2}$	1	0.03	0.3
Venus	$43\frac{1}{3}$	1	0.4	1.6
Mars	$39\frac{1}{2}$	1.9	0.7	2.1
Jupiter	$11\frac{1}{2}$	11.9	1.9	1.1
Saturn	$6\frac{1}{2}$	29.4	2.9	1.0

This shows that the first three planets will not retrogress. This conclusion would not be reversed if we made the orbit of C eccentric and introduced an equant.

Besides this, it is possible that if Ptolemy went through the detailed calculations to find the parameters of the planets' orbits using clockwise epicycles, his data would not yield coherent results. And, for the outer planets, making it part of his theory that CP points toward the mean sun, coupled with the fact that the synodic periods are greater than a year, requires the epicycle to rotate anticlockwise.

If the motion of the sun is presented as epicyclic motion (see Figure 7.1) then, because the line joining the mean sun to the sun is in a fixed direction in space, the sun must move clockwise round its epicycle, like someone walking down an up-escalator at precisely the speed of the escalator. It is possible that the clockwise epicycle for the moon was copied from the theory for the sun. In spite of all this, there is evidence that some early Greek astronomers did use clockwise epicycles [154].

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Answer to the question on page 197: The next day is 2 Ik 5 Pop.

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