THE POSSIBLE INFLUENCE OF INTERSTELLAR CLOUDS ON STELLAR VELOCITIES. II*

LYMAN SPITZER, JR., AND MARTIN SCHWARZSCHILD
Princeton University Observatory
Received February 26, 1953

ABSTRACT

The increase with time of random stellar velocities, as a result of gravitational encounters with interstellar cloud complexes, has been recomputed, taking into account the presence of differential galactic rotation. As a result of such nonuniform rotation, the clouds will have velocities relative to one another even if their random velocities are zero; the gravitational potential of these clouds will be a fluctuating function of time; and the stars will gain kinetic energy from the clouds.

To explain the increase of velocity dispersion with advancing spectral type along the main sequence, the mass of a typical cloud complex must be in the neighborhood of $10^6 m_\odot$, the value found previously; but the random velocity of a cloud complex, as a whole, is irrelevant and may be vanishingly small. Since inhomogeneities of density with the required scale of some 300 parsecs or more seem indicated by the extinction observations, it seems not unlikely that star-cloud encounters are, in fact, responsible for the greater velocity dispersion of the later-type, older stars of population type I.

I. INTRODUCTION

In a previous paper[1] (subsequently referred to as "Paper I"), it was suggested that gravitational encounters between stars and interstellar clouds might increase the velocity dispersion of the stars. Such a cause would naturally produce a greater effect on the older stars. The early-type stars, presumably formed recently from interstellar matter,[2] would not have had time to change appreciably their root-mean-square velocities, which would therefore equal more closely the velocities of the clouds from which they had formed. This mechanism provides a natural qualitative explanation of the fact, known for many years, that the root-mean-square random velocity for stars of each spectral type along the main sequence increases systematically with advancing spectral type, ranging from 10 km/sec for the O and early B stars to some 20 km/sec for the F stars.

The discussion in Paper I showed that small clouds, with radii of some 5 parsecs and masses of about $100 m_\odot$, would not produce any measurable effect on the velocities of the stars in times less than 10^{11} years. The same conclusion follows from the analysis by D. Osterbrock.[3] The larger complexes, of the type considered by Greenstein[4] and Bok,[5] have much greater masses, and it is these that must be primarily responsible for the effect in question. The detailed analysis in Paper I was based on conditions in an infinite homogeneous medium, and neglected galactic rotation. The results indicated that star-cloud encounters could, in fact, produce the results envisaged if:

a) Most of interstellar matter is gathered into cloud complexes with masses of the order of $10^6 m_\odot$.

b) The velocity dispersion of these massive cloud complexes is about 10 km/sec.

* This work was supported in part by funds of the Eugene Higgins Trust allocated to Princeton University.

[1] L. Spitzer, Jr., and M. Schwarzschild, *Ap. J.*, **114**, 385, 1951.

[2] For a survey of this theory see L. Spitzer, Jr., *J. Washington Acad. Sci.*, **41**, 309, 1951.

[3] *Ap. J.*, **116**, 164, 1952; *ibid.*, in press.

[4] *Harvard Ann.*, **105**, 359, 1937.

[5] *Centennial Symposia* (Cambridge, Mass.: Harvard College Obs., 1948).

The first of these requirements might seem rather difficult to meet, since the mass of a typical large cloud of the type observed seems to be more nearly $10^5 m_\odot$. However, the total mass of the Orion nebula may well exceed $10^6 m_\odot$, and such large masses cannot be excluded. Requirement *b* is even more difficult to meet. Although no extensive measurements are available on the systematic motion of very large clouds or groups of clouds, it would seem likely that the mean random velocity of such a very massive cloud complex would be substantially less than the velocity of the smaller clouds observed by Adams.[6]

In the present paper the analysis is extended to take into account the presence of differential galactic rotation. The results indicate that in this more realistic case requirement *b* is eliminated entirely, and the random motion of the clouds may be arbitrarily small; i.e., the star-cloud encounters will increase the stellar velocity dispersion even if each cloud is assumed to be moving with the circular velocity around the galactic center. Requirement *a* is formally not much changed—in fact, it becomes easier to meet, once requirement *b* is removed. Regions of excess density extending over several hundred parsecs and comprising masses of $10^6 m_\odot$ seem much more plausible than the relatively coherent dynamic units of this mass which were originally envisaged.

One may readily see why requirement *b* can be eliminated if differential galactic rotation is present. Let us suppose that the random motions of the clouds are rigorously zero. Then, if no differential rotation is present, the position of each cloud with respect to every other cloud is fixed in time. The potential energy of a star is then a simple function of position only, the sum of the potential and kinetic energies remains constant, and the velocity dispersion is unchanged by encounters. In the presence of differential rotation, the clouds nearer the galactic center have greater angular velocities than those farther out, the distances between different clouds will vary, and the potential energy of a star is no longer a simple function of position. In such a case, the stars form a nonconservative system, and their velocity dispersion will tend to increase as a result of the encounters.

To illustrate these effects by a specific example, let us consider a star in the neighborhood of the sun whose velocity relative to the local standard of rest is initially directed toward the galactic center and is numerically equal to 10 km/sec, a relatively low value. Such a star will move in an ellipse, with major and minor axes equal to 1400 and 700 parsecs, respectively, and with the minor axis pointing toward the galactic center. Over this range of distances from the galactic center the circular velocity of galactic rotation will change by some 30 km/sec. If the interstellar clouds are assumed to be moving with the local circular velocity, with no random velocities, the effective velocity spread of the clouds encountered by the star along its path will be ± 15 km/sec. While the detailed situation is somewhat more complicated than this simple example implies, this large effective spread of velocities greatly facilitates the transfer of energy from the inhomogeneous interstellar medium to the stars.

The energy gained by the stars must come, of course, from the interstellar matter, which will fall inward toward the galactic center by an inappreciable amount.

II. STATISTICS OF ENCOUNTERS IN A ROTATING SYSTEM

First, we shall consider the orbit of a single star, neglecting encounters. Then the effect of encounters can be considered, assuming that the clouds have negligible random velocities and utilizing in each segment of the star's orbit the results obtained in a uniform medium in the absence of external forces. An average over the orbit may next be taken, to find the mean change of the stellar velocity. Finally, an average over all stars in a particular group may be taken, to yield the rate of change of the velocity dispersion.

[6] *Ap. J.*, **109**, 354, 1949.

We write the equations of motion for a single star in the smoothed gravitational field of the Galaxy. Let x and y be the co-ordinates of a star relative to some point, P, which is at a fixed distance, R_p, from the galactic center but which is moving about the center at the angular velocity ω. The direction of positive x is taken to be the direction of increasing R, while y is taken to be positive in the direction of galactic rotation. Motions in the z direction, perpendicular to the galactic plane, do not affect motions in the xy plane, to a first approximation, and will be ignored. If Φ (R) is the gravitational potential per unit mass, then the familiar equations of motion in a frame of reference rotating with angular velocity ω become

$$\frac{d\,v_x}{dt} - 2\,\omega\,v_y = 4\,A\,\omega x\,, \tag{1}$$

$$\frac{d\,v_y}{dt} + 2\,\omega\,v_x = 0\,, \tag{2}$$

where A, the familiar Oort constant, may be written

$$A = \frac{\omega}{4}\left[1 - R_p\left(\frac{d^2\Phi/\,dR^2}{d\Phi/\,dR}\right)\right], \tag{3}$$

with the derivatives evaluated for R equal to R_p. In equations (1) and (2) terms of the order $(x/R)^2$ and $(y/R)^2$ have been ignored.

These equations yield the solution:

$$v_x = K \sin 2\,(-B\omega)^{1/2}t\,, \tag{4}$$

$$v_y = K\left(-\frac{\omega}{B}\right)^{1/2} \cos 2\,(-B\omega)^{1/2}t - 2\,A\,\bar{x}\,, \tag{5}$$

where K is a constant of integration and B is the second Oort constant, given by

$$B = A - \omega\,. \tag{6}$$

The term $\bar{x}$ in equation (5) will be ignored, since this term may be eliminated by a simple change of co-ordinate system, i.e., by a change of the point P about which the expansion is made. The values of x and y as functions of time are readily obtained by direct integration of equations (4) and (5).

Before the theory of encounters can be applied, we must know u_x and u_y, the components of the star's velocity at each point relative to the local standard of rest. Since the circular velocity, in this co-ordinate system, is $-2Ax$, parallel to the y axis, we have

$$u_x = v_x\,, \tag{7}$$

$$u_y = v_y + 2\,A\,x\,. \tag{8}$$

If we insert in equation (8) the value of x found by integration of equation (4), and substitute equation (5) for v_y, we find

$$u_y = K\left(-\frac{B}{\omega}\right)^{1/2} \cos 2\,(-B\omega)^{1/2}t\,. \tag{9}$$

It may be remarked that in an inverse-square field of force, B equals $-\omega/4$. Then v_y at its greatest is twice as great as the maximum value of v_x, while u_y at maximum is only half the greatest value of u_x.

We now assume that the star encounters clouds in its orbit. The problem will be treated on the assumption that each encounter takes place in a region small compared to the size of the stellar orbit about the point P. Thus the effect of the encounter may be computed as though the star were moving in a straight line. Each cloud will be assumed motionless relative to the local standard of rest. All deflections will be assumed small, a legitimate approximation for inverse-square encounters in general, and particularly appropriate in the present case, where the finite radius of the clouds reduces the deflections produced by the closer encounters. Under these assumptions, the effect of each encounter is to deflect the star by some angle $\Delta\theta$ in the xy plane, without changing its local velocity u. Thus the changes of u_x and u_y are given, to the first order, by

$$\Delta u_x = -u_y \Delta\theta , \qquad \Delta u_y = u_x \Delta\theta . \tag{10}$$

It is readily verified that the mean square local velocity $u_x^2 + u_y^2$ is unchanged by this transformation.

Such a deflection will, in general, change the velocity v, measured relative to the point P. Equations (4), (7), and (9) may be combined to determine K in terms of u_x and u_y, yielding

$$K^2 = u_x^2 - \frac{\omega}{B} u_y^2 . \tag{11}$$

If transformation (10) is now used to find the change of K resulting from a deflection $\Delta\theta$, we have

$$K \Delta K = -\frac{A}{B} u_x u_y \Delta\theta . \tag{12}$$

To relate our results to observable quantities, we express K in terms of V^2, defined as half the mean square value of u, averaged around the star's orbit. Evidently

$$V^2 = \tfrac{1}{2} (\overline{u_x^2} + \overline{u_y^2}) = \tfrac{1}{4} K^2 \left(1 - \frac{B}{\omega}\right). \tag{13}$$

Hence

$$V \Delta V = \frac{A}{4B} \left(1 - \frac{B}{\omega}\right) u_x u_y \Delta\theta . \tag{14}$$

The mean value of ΔV in one revolution about the point P is zero. However, the mean square value of ΔV is not zero. If we let V_n be the value of V after n deflections, then

$$\overline{V_n^2} = \overline{(V_0 + \Delta V_1 + \Delta V_2 + \ldots + \Delta V_n)^2} , \tag{15}$$

$$= V_0^2 + \sum_{i=1}^{n} \overline{(\Delta V_i)^2} , \tag{16}$$

since the terms not involving squares of ΔV cancel out on averaging. If we denote by $\langle(\Delta V)^2\rangle$ the mean sum of all the separate values $(\Delta V)^2$ within a unit time, then

$$\frac{d\overline{V^2}}{dt} = \langle (\Delta V)^2 \rangle . \tag{17}$$

The quantity $\langle(\Delta V)^2\rangle$, a diffusion coefficient for the change in velocity, is equal to

$$\langle (\Delta V)^2 \rangle = \frac{1}{16 V^2} \left[\frac{A}{B}\left(1 - \frac{B}{\omega}\right)\right]^2 \overline{u_x^2 u_y^2 \langle (\Delta\theta)^2 \rangle} , \tag{18}$$

where the horizontal bar denotes an average around the star's orbit. The quantity $\langle(\Delta\theta)^2\rangle$, the sum of all the mean square deflections per second, may be taken from Chandrasekhar,[7] and we have

$$\langle (\Delta \theta)^2 \rangle = \frac{4\pi G^2 n_c m_c^2 \ln a}{u^3}, \tag{19}$$

where n_c and m_c are, respectively, the number of clouds per unit volume and the mass of each cloud.

The quantity u^2 is again the local velocity, equal to $u_x^2 + u_y^2$, while, as in Paper I, the term $\ln a$ may be set equal to 3. We may substitute from equations (4), (7), and (9) for u_x and u_y, and express $\bar{K}$ in terms of V by means of equation (13). If we also assume that, for all the stars in a group, V has a Maxwellian distribution, with root-mean-square value V_m (neglecting u_z again), we obtain

$$\langle (\Delta V_m)^2 \rangle = \frac{\pi^{3/2} G^2 n_c m_c^2 \ln a}{2 V_m} F\left(\frac{B}{\omega}\right), \tag{20}$$

where

$$F\left(\frac{B}{\omega}\right) = -\frac{\omega}{B}\left(1-\frac{B^2}{\omega^2}\right)^2 \left(1-\frac{B}{\omega}\right)^{-1/2} \times \frac{2}{\pi}\int_0^{\pi/2} \frac{\sin^2\theta \cos^2\theta\, d\theta}{(\sin^2\theta - B\cos^2\theta/\omega)^{3/2}}. \tag{21}$$

The integral in equation (21) may be evaluated directly, if the denominator is expanded as a series in $1 + B/\omega$, and we obtain

$$F\left(\frac{B}{\omega}\right) = -\frac{\omega}{8B}\left(1-\frac{B^2}{\omega^2}\right)^2 \left(1-\frac{B}{\omega}\right)^{-1/2} G\left(1+\frac{B}{\omega}\right), \tag{22}$$

where

$$G(x) = 1 + \frac{3}{4}x + \frac{75}{128}x^2 + \frac{245}{512}x^3 + \frac{6615}{16{,}384}x^4 + \ldots . \tag{23}$$

In the extreme case of an inverse-square force field, $1 + B/\omega$ is 0.75, and the series in equation (23) converges very slowly. A numerical integration in this case gives

$$G(0.75) = 2.48 . \tag{24}$$

If equation (20) is now substituted in equation (17), averaged over all stars in the group, we may integrate at once. If $V_m(0)$ denotes the value of V_m at time zero, we have

$$V_m = V_m(0)\left(1 + \frac{t}{t_E}\right)^{1/3}, \tag{25}$$

where

$$t_E = \frac{4 V_m^3(0)}{3\pi^{3/2} G^2 n_c m_c^2 \ln a\, F(B/\omega)}. \tag{26}$$

Equation (25) is clearly only approximate, since the distribution of stellar velocities will not remain exactly Maxwellian even if it is so initially; more accurately, one should compute the distribution function $f(V)$ as in Paper I. However, the results of Paper I indicate that results obtained on the assumption of a Maxwellian distribution are remarkably close to the truth.

[7] *Principles of Stellar Dynamics* (Chicago: University of Chicago Press, 1942), eq. (5.724). This equation must be divided by 2, since deflections in the z direction are not considered here. The quantity $H(x_0)$ is unity, since the clouds are stationary and both j and x_0 are, therefore, infinite.

III. APPLICATION TO OBSERVED VELOCITIES

The observations of V for stars near the sun are summarized in Table 4 of Paper I. We discuss the extent to which the theory in the preceding section may be used to explain these observations.

In the neighborhood of the sun, the values of A and B are[8] about 0.021 km/sec psc, and -0.007 km/sec psc. Thus ω is about 0.028 km/sec psc, or 0.9×10^{-15} sec^{-1}, while $-B/\omega$ is 0.25, the value corresponding to an inverse-square field of force. Combining equations (22) and (24), we see that the corresponding value of $F(B/\omega)$ is 0.98.

We have seen in Paper I that, if the increase of velocity along the main sequence is to be explained as an age effect, the value of V for the F stars—about 20 km/sec—must be attained in half the age of the universe, or about 1.5×10^9 years. If $V_m(0)$ is assumed to be 10 km/sec, we compute from equation (25) that t_E must equal about 2×10^8 years. If this value of t_E is inserted in equation (26) and the product $n_c m_c$ is set equal to 3×10^{-24} gm/cm^3, we find

$$m_c = 4.9 \times 10^5 m_{\odot} , \tag{27}$$

or about one-half the value found in Paper I. If we assume that only half the interstellar medium is concentrated in such clouds, with the rest distributed somewhat more uniformly, m_c will increase to $9.8 \times 10^5 m_{\odot}$. Evidently, m_c must be about $10^6 m_{\odot}$ if star-cloud encounters are to have an appreciable effect. If the density in such a massive cloud, or cloud complex, corresponds to 15 H atoms/cm^3, the radius[9] of such a cloud will be about 100 parsecs. The mean distance between centers of neighboring clouds, if the interstellar medium were infinite and homogeneous, would be about 350 parsecs.

It is evident that with such large cloud complexes the analysis in Section II is not exactly applicable for two reasons. In the first place, the spacing between the clouds is about equal to the thickness of the Galaxy. In the second place, the size of the clouds and the spacing between them are not so very much smaller than the size of the star orbits. As we have already seen, if K is 10 km/sec, corresponding to a value of 5.6 km/sec for V, then the major and minor axes of the orbit, given by $-K/B$ and $K/(-B\omega)^{1/2}$, are equal to 1400 and 700 parsecs, respectively. We investigate in a rough way the modifications required by these two effects.

One may readily compute $(\Delta\theta)^2$ for a star moving in a plane, encountering clouds which are all situated in the same plane. If n_c' is the density of such clouds per unit area and a is the radius of a cloud, then, if we ignore encounters in which the star passes through the cloud (for which the distance of closest approach is less than a), we find

$$\langle (\Delta\theta)^2 \rangle = \frac{8G^2 n_c' m_c^2}{a u^3} . \tag{28}$$

The ratio of equation (28) to equation (19) is simply $2n_c'/\pi a n_c \ln \alpha$. Since $n_c'/a n_c$ is somewhere between 2 and 5, this ratio is not far from unity. Hence the difference between equations (28) and (19) does not have a major effect on the increase of stellar velocity in the galactic plane, although this difference must be considered in a more precise analysis.

If deflections perpendicular to the plane are considered, a quite different result is found. In the idealized two-dimensional picture, all deflections are in the galactic plane, and the velocities in the z direction, perpendicular to the galactic plane, are not affected by the star-cloud encounters. A more realistic analysis would yield some increase of

[8] J. H. Oort, *Ap. J.*, **116**, 233, 1952.

[9] Paper I incorrectly gives 100 parsecs for the diameter, instead of the radius, of a cloud complex with a mass of $10^6 m_{\odot}$.

these z velocities, but certainly less than is found in the simple three-dimensional picture, where velocities parallel and perpendicular to the plane are affected equally. Hence we see that the theory yields a simple qualitative explanation of the persistence of low z velocities in the older stars.

The second effect to be considered is the large size of the cloud complexes relative to the stellar orbits. If the minimum distance of closest approach to be considered is 100 parsecs, the curvature of the star's orbit is certainly not negligible during an encounter. A detailed analysis, taking into account the motion of the star in an elliptical orbit, would be complicated. It would seem that such a more realistic analysis would give a somewhat more rapid rate of deflection of a star. This result follows, in part, from the fact that for clouds outside the star's orbit the differential galactic rotation will reduce the relative velocity between star and cloud, in the y direction, and thus lengthen the time during which the star is accelerated by a single cloud. However, it seems unlikely that these effects would decrease m_c to much less than half the value in equation (27), and we shall neglect this change.

IV. CONCLUSION

The results in Paper I were somewhat speculative, since it was uncertain whether such massive cloud complexes could behave as single dynamical entities, with the assumed root-mean-square velocity of some 10 km/sec. The present analysis indicates that the presence of differential galactic rotation makes the random velocities of the cloud complexes irrelevant. Acceleration of type I stars during times of 3×10^9 years will be important if density fluctuations exist in the interstellar medium with a scale of not less than about 300 parsecs and with an amplitude of fluctuation equal to an appreciable fraction of the mean density. Such density fluctuations need have no coherent structure and need not even be stable.

The large fluctuations of obscuration observed in the Milky Way over relatively large regions strongly suggest that inhomogeneities of the type required by the present theory are, in fact, present. The authors have taken as a working hypothesis the picture that type I stars, or "cloud stars," are forming continuously from interstellar matter and that the mean age of all type I stars later than spectral type F is about 1.5×10^9 years. Even if this hypothesis should not be substantiated in detail, it now appears likely that large-scale fluctuations in density of the interstellar medium are responsible, at least in part, for the increase of velocity dispersion with advancing spectral type along the main sequence.