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LETTERS

Measurement of the Relativistic Doppler Shift in Neon

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The relativistic Doppler shift is measured by the counting of the frequency difference between two cw dye lasers. One laser is locked to a two-photon transition in a fast beam of neon, and the other is locked to the same two-photon transition in thermal neon. The experimental result is compared to the prediction of special relativity. The result is in excellent agreement with this theory. An accuracy of 4×10^{-5} is obtained, which provides the most accurate direct verification of time dilatation to date.

Optical experiments have, historically, provided important tests for the theory of special relativity. In 1949, Robertson[1] presented a test theory for the kinematic aspects of special relativity and showed how the postulates of Einstein could be replaced by conclusions drawn from three optical experiments: the Michelson-Morley,[2] the Kennedy-Thorndike,[3] and the Ives-Stilwell[4] experiments. This exposition was expanded recently by Mansouri and Sexl.[5] With the development of highly stabilized lasers and vastly improved spectroscopic measuring techniques, an entire new generation of optical experiments for testing the fundamental postulates of physics has become possible. Already a laser version[6] of the Michelson-Morley experiment has demonstrated the isotropy in the round-trip speed of light to 2×10^{-15}. Other exciting possibilities are discussed at length by Hall.[7]

One of the basic kinematic results of special relativity is the prediction of the relativistic Doppler shift: A moving clock has its frequency modified by a factor $(1 - v^2/c^2)^{1/2}$ over the classical shift, where v is the velocity of the clock with respect to the observer. The presence of this time-dilatation factor gives a red shift of order v^2/c^2 in the frequency, even if there is no first-order term. This transverse Doppler shift was verified by Ives and Stilwell in 1938.[4] They measured

the difference in wavelengths of the fluorescence emitted by a 30-keV beam of H_2^+ ions when observed along and opposite to the direction of the ion beam. They found a second-order Doppler red shift which could be represented by $v^2/2c^2$, in agreement with special relativity. The accuracy was about 3%. Mössbauer experiments using a high-speed centrifuge have a comparable accuracy of 4%.[8] A first-generation laser test was reported by Snyder and Hall[9] in 1975. They used laser-saturation spectroscopy to transversely excite a beam of fast neon atoms. Although an accuracy of 0.5% was obtained, the transverse-excitation scheme has systematic errors which make its extension to higher accuracy extremely difficult. A related class of experiments on the relativistic time-dilatation effect measures the lifetime of elementary particles. A recent experiment at CERN on the muon lifetime achieved an accuracy of 1×10^{-3} on the time-dilatation factor.[10] Finally, a very precise test of general relativity has been made by use of a hydrogen maser launched vertically upwards to 10 000 km.[11] The combined measurement of the second-order Doppler shift and the gravitational red shift is in agreement with the predictions on the 7×10^{-5} level. If we assume the correctness of the usual expression for the gravitational red shift, this corresponds to an accuracy of

1.4×10^{-4} for determining the second-order Doppler shift.

In this paper, we report on our measurement of the relativistic Doppler shift, using two-photon absorption (TPA). In two-photon spectroscopy, the first-order Doppler shift is absent, and the second-order term becomes dominant. We measure the frequency difference between a two-photon transition in a fast neon-atom beam and in a cell. Two cw dye lasers are used, one stabilized to the fast-beam transition and the other to the cell. By direct measurement of the beat frequency between the two lasers, the second-order Doppler shift is determined to 4×10^{-5}, constituting the most precise direct measurement of the time-dilatation to date.

The Ne I $3s[3/2]_2^0-4d'[5/2]_3^0$ TPA transition was chosen, as it has been observed both in a cell[12] and in a fast atom beam.[13,14] Figure 1(a) shows the cell case, where several off-resonant intermediate states belonging to the $3p'$ manifold make this two-photon transition easily observable. However, in a fast atom beam with densities some 10^6 times smaller, this TPA process is observable only if a real intermediate level is realized as shown in Fig. 1(b). At a beam energy of $\simeq 120$ keV the high velocity of the absorbers allows Doppler tuning of one particular intermediate state, in this case the $3p'[3/2]_2$, into exact resonance in the rest frame of the atom. The resonant enhancement, in the order of 10^9, makes this TPA easily observable in a fast beam[13] without any ac Stark shifts at exact resonance.[14]

In a collinear geometry, the laser frequency measured in the rest frame of an atom is given by special relativity as $\nu_\pm = \gamma \nu_L (1 \pm v/c)$. Here $+$ $(-)$ refers to the laser beam traveling in the opposite (same) direction as the atom. ν_L is the laser frequency in the laboratory frame. $\gamma = (1 - v^2/c^2)^{-1/2}$ is the time-dilatation factor, and v is the velocity of the atom. If ν_1 is the frequency from the lower level of the TPA to

the intermediate state and ν_2 is the frequency from the intermediate state to the upper TPA level, then the condition for TPA is for ν_L to satisfy $\nu_1 + \nu_2 = \nu_+ + \nu_-$ $= 2\gamma\nu_L$. In a cell $\gamma - 1 \sim 5 \times 10^{-13}$, so that we may take the laser frequency at resonance to be simply $\nu_{L,c} = (\nu_1 + \nu_2)/2$. In a fast beam with $\gamma - 1$ $\sim 5 \times 10^{-6}$, the laser frequency for resonance is $\nu_{L,b} = (\nu_1 \nu_2)^{1/2}$, where we have introduced the conditions $\nu_- = \nu_1$ and $\nu_+ = \nu_2$ for the resonant intermediate state. Thus there is no need to measure the beam velocity. Using the frequencies $\nu_1 = 16\,816.666\,34(2)$ cm^{-1} [15] and $\nu_2 = 16\,937.3862(10)$ cm^{-1}, both measured in a cell, we obtain a theoretical prediction of $\nu_{L,c} - \nu_{L,b} = 3235.89 \pm 0.05$ MHz.

We now turn to the experiment, shown schematically in Fig. 2. Ne$^+$ ions produced in an rf ion source are accelerated to $\simeq 120$ keV. After magnetic separation, metastable Ne* atoms are produced by charge exchange of Ne$^+$ in Na vapor. A postacceleration voltage provides precise control of the velocity of the well collimated fast atom beam, the energy of which is long-term stable to 7×10^{-6}. The cw dye laser used for TPA in the fast beam is an actively stabilized Michelson laser with a bandwidth of 40 kHz. This Michelson laser consists of a traditional folded linear resonator, where the output coupler has been replaced by a double-arm Michelson interferometer for mode
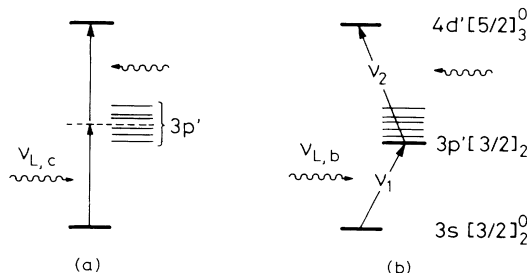

FIG. 1. Two different TPA schemes are used in this experiment. (a) The thermal-atom case in the cell, where the two-photon absorption is an off-resonant process. (b) The fast-beam case, where the Doppler effect allows one particular intermediate level to be tuned into exact resonance. The atom is viewed in its rest frame.
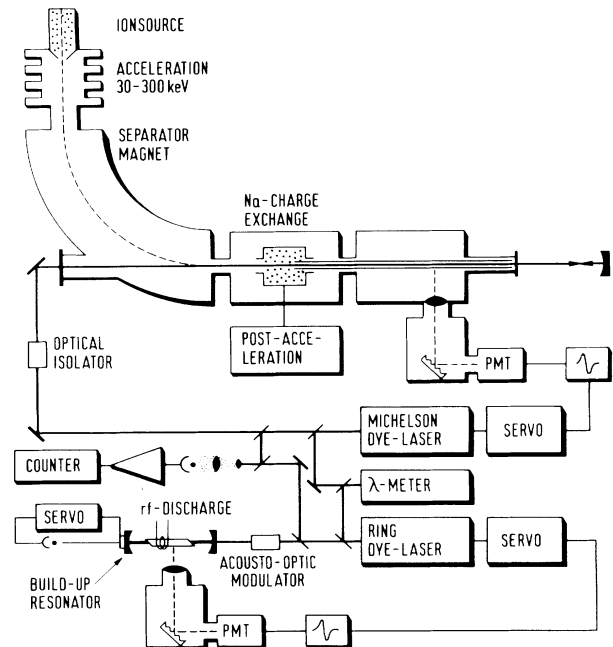

FIG. 2. The experimental apparatus used for measurement of the second-order Doppler shift consists of two actively stabilized dye lasers, an accelerator, an rf discharge cell, and frequency-counting equipment. A λ meter allows measurement of the laser wave numbers to 5×10^{-8}.

selection. The laser beam is matched to the atom beam via a telescope and is retroreflected with a curved mirror for mode matching of the copropagating and counterpropagating laser beams. The observed TPA resonance, with a natural width of 1.5 MHz, has a full width at half maximum (FWHM) of 3 MHz and is thus only weakly power broadened.

For our measurement, the long-term frequency stability of the Michelson laser is obtained by locking the laser to the center of the fast-beam TPA resonance. The Michelson laser is frequency modulated at 2.5 kHz, and a lock-in amplifier demodulates the detected fluorescence to provide the feedback signal for the laser.

The cell TPA experiment is performed in a 80-MHz rf flow discharge of neon contained in a cell with Brewster windows. The cell assembly is placed in a servo-controlled power-enhancement cavity. The circulating power inside the cavity (typically 1 kW/cm^2) is obtained from a ring dye laser (Coherent) with a bandwidth of 1 MHz. The TPA resonance in the cell has a FWHM of 3 MHz at the lowest pressure (0.1 Torr) used. Three pairs of Helmholtz coils surround the discharge cell. The interaction region in the fast beam is surrounded by Mumetal shields. In both cases, the residual magnetic field is below 50 mG.

The ring dye laser is locked to the center of the cell TPA resonance to keep its long-term center frequency to within 40 kHz. For stabilization, the ring laser is frequency modulated at 30 kHz, and a lock-in amplifier demodulates the detected TPA fluorescence to provide the feedback signal to the laser. An acousto-optic crystal isolates the dye laser from the enhancement
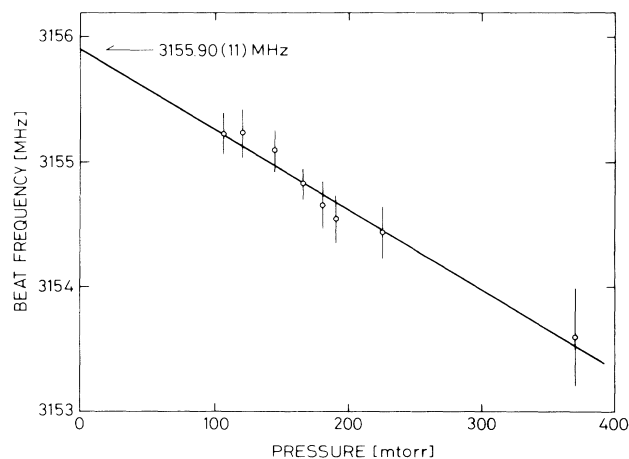
cavity. The modulator shifts the laser frequency by 80 MHz.

The third part of the experiment involves measuring the frequency difference between the two lasers, each locked to its respective TPA resonance. Approximately 1 mW of each laser is sent to a fast photodiode (ITL model TF1850) to produce the beat frequency. The two laser beams are mode matched, overlapped, and expanded by a telescope to 1.5 cm in diameter for uniform illumination of the diode. The output beat frequency from the diode, ~ 3 GHz at a level of -70 dBm, is amplified by two low-noise amplifiers to a level of -25 dBm. The beat frequency is then counted directly with a microwave-frequency counter (EIP 545A).

With the cell TPA resonance being pressure shifted and broadened, the beat frequency between the two dye lasers is measured as a function of neon pressure in the discharge cell. The result is shown in Fig. 3, and it yields a pressure shift of -6.44 ± 0.60 MHz/Torr and a beat frequency of 3155.90 ± 0.11 MHz at zero neon pressure. Each datum point consists of typically twenty frequency readings, and each reading is an average over a period of 3 sec. The uncertainty in the zero-pressure beta frequency includes statistical errors in the frequency readings and the pressure extrapolation.

The ac Stark effect is absent in the fast beam,[14] and in the cell this effect has been calculated, with the contributions from possible intermediate states and the Gaussian laser beam profile in the power-enhancement cavity taken into account. We obtain a shift of -19 ± 10 kHz/W. A correction of 42 ± 20 kHz is added to the beat frequency to account for the 2.2-W power level in the cell experiments. The ac Stark effect due to the rf discharge gives a shift of 1 Hz and is completely negligible.

In the fast-beam case, a slight angular misalignment θ of the two counter-running laser beams with each other and with the atom beam will give frequency shifts proportional to $v\theta^2/c$. With $v/c \simeq 4 \times 10^{-3}$ and

FIG. 3. The beat frequency $v_{L,c} - v_{L,b}$ is dependent on the pressure of neon in the discharge cell. We find the pressure shift to be -6.44 ± 0.60 MHz/Torr, in agreement with Ref. 12, which finds -6 ± 2 MHz/Torr. A pressure broadening of 16 ± 4 MHz/Torr makes the ring-laser lock less tight at high pressures.

TABLE I. The second-order Doppler shift in neon.

Measured beat frequency:	
Cell $-$ beam (zero pressure)	3155.90 $\pm$ 0.11 MHz
Corrections:	
Acousto-optic modulator	80.00 $\pm$ 0.00 MHz
ac Stark shift in cell	0.04 $\pm$ 0.02
Laser locks (each laser)	$\pm$ 0.04
Beam-crossing uncertainty	$\pm$ 0.02
Residual magnetic field	$\pm$ 0.02
High-voltage setting for TPA	$\pm$ 0.05
Experimental Doppler shift:	3235.94 $\pm$ 0.14 MHz
Theory:	3235.89 $\pm$ 0.05 MHz

$\theta \leq 2 \times 10^{-4}$, these terms result in an uncertainty of ± 20 kHz.

Finally, to realize the resonant intermediate state, the beam energy is located to within ± 2 eV, resulting in an uncertainty of ± 50 kHz on the beat frequency.

Our final result is shown in Table I, with the statistical and all systematic errors added in quadrature. We obtain an experimental result for the second-order shift of 3235.94 ± 0.14 MHz. This is in excellent agreement with the prediction of special relativity. The accuracy of our experiment is 4×10^{-5}, which represents, by a factor of 3, the most accurate direct verification of the time dilatation due to special relativity.

The present experiment is not easily improved, but with the advent of heavy-ion storage rings, highly velocity-stable ion beams will become available. With a transit-time–limited optical resolution, a $\times 1000$ improvement is envisaged in the determination of the time dilatation. This, however, will also require an improvement in the absolute value of the transition frequencies ν_1 and ν_2 so that a meaningful comparison with theory can be made.

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