vibration in the plane of the meridian. The discussion suffices, however, no matter what may be the initial direction of vibration. We need wait only until the trace of the pendulum started in the meridian shows the same azimuth as the one to be started in some other direction. Thereafter, if the second pendulum is started in motion at this time, the two pendulums will be indistinguishable, and the discussion of the motions of the one will apply unchanged to the motions of the other.

REFERENCES

[1] For the non-mathematical reader: dt and $d\theta$ do not indicate products of factors, but merely vanishingly small values of t and θ, respectively. Other readers will readily transform the discussion into a straightforward integration.

[2] The author expresses his thanks to Dr. F. C. Leonard for his critical reading of the manuscript.

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LOS ANGELES, CALIFORNIA, FEBRUARY 12, 1949.

Two New Families of Comets

By C. H. SCHUETTE

For many years it has very well been know that the periodic comets are divided into several families, which show an evident relationship with the great planets. This is most obvious when the comets are arranged according to the distances of their aphelions. The number of the discoveries, also of the periodic comets, has considerably increased in recent years, and several comets were found at their second apparition, so that at present we have knowledge of more than 40 comets which have been observed in at least two apparitions. Thus the number of well-known orbits has increased and, therefore, I was interested in a new discussion. After all, the family of the comets of Jupiter includes the majority of them and, with more than 50 members, is the best known.

We will divide all comets into two groups, *viz.*, those that have been observed in several apparitions, and those that have been observed in only one apparition. Thus we get the distribution shown in Table I with the mean values of the most important elements of their orbits.

The most important facts of the families agree perfectly in both groups and may be expressed as follows:

1. The distances of the aphelions group around the aphelion of the corresponding planet, so that in general the mean distance of the aphelions of each cometary family is somewhat greater than that of the planet. Only the family of Uranus shows a mean distance of the perihelion which seems to be a little smaller than that of the planet; but this is of no importance, the Uranus-family being the smallest.

2. The directions of the aphelions, at least in the mean of the Jupiter's family agree closely with the direction of the aphelion of the planet. In the first group we find the difference to be +9°, and in the

TABLE I

GROUP I. 39 COMETS WHICH HAVE BEEN OBSERVED IN MORE THAN ONE APPARITION

Family No.	Periods	Mean distances of aphelions	Mean perihelion of Comets —perihelion of planet	Mean inclin.	Mean eccentricity
— 1[1]	3ʸ.3	4.1	—	12°.6	0.85
Jupiter 26	4.9 — 8.5	5.54 (5.45)	+ 9°	14.1	0.58
Saturn 4	11.0 —18.0	10.4[3] (10.1)	—103	22.8	0.62
Uranus 2	27.9 —40.1	19.8 (20.1)	— 86	23.6	0.89
Neptune 6	49.1 —76.0	31.6 (30.3)	+ 75	35.2[2]	0.93
GROUP II. 31 COMETS WHICH HAVE BEEN OBSERVED IN ONLY ONE APPARITION					
Jupiter 26	4.5 —10.9	5.77 (5.45)	— 6°	11°.7	0.65
Saturn 2[4]	13.4 —16.4	11.4 (10.1)	+ 8	54[2]	0.83
Uranus 1	33	19.6 (20.1)	—130	17[2]	0.90
Neptune 2	75.7 —79.5	35.8 (30.3)	+ 80	54	0.95

The number in ( ) gives the distance of the aphelion of the corresponding planet.

Notes: [1]Comet Encke. [2]For retrograde orbits 180°—i is given. [3]If comet Schwassmann-Wachmann I is included, we obtain 9.6. [4]Here the uncertain comet 1916 II (Perrine) which was observed only one day is included.

second —6°. In all other families this peculiarity is not so evident, on account of the small number of their members. Figure 1 shows the distribution of the aphelion points of all 52 comets of both groups of Jupiter's family.

3. The mean inclinations of orbits of the families increase with increasing distance from the sun.

4. The eccentricities increase also in the same manner.

Whatever may be the origin of these facts, there is no doubt of a relation between the families of the comets and the corresponding great planets. Moreover, the predominance of the Jupiter-family, caused by the dominating great mass of this planet, is very clear.

The last comet of the Neptune-family is the comet 1921 I (Dubiago) with a period of revolution of 79.51 years and a mean distance of 18.5 from the sun. Dubiago himself, who also calculated its orbit, pointed out that this comet is a new member of the Neptune-family (*Astron. Nachr.*, **222**, 111). Of all the above-mentioned comets this one has the greatest period and the greatest aphelion distance.

Besides the Neptune-family there is a great number of periodic comets with long periods of revolution which up to now have scarcely been discussed as to their families. The discovery of Pluto, the fact that two of these comets were identified with former comets (Comet 1939 VI, Rigollett, = 1788 II and Comet 1907 II, Grigg-Mellish, = 1742), and the fact that in 1931/32 there were discovered four new periodic comets with greater periods, whose orbits could be determined with relatively great accuracy, have led me to discuss these orbits once more on account of their families.

Considering all comets with great eccentricities and great periods of revolution, it may be noteworthy that after the last above-mentioned

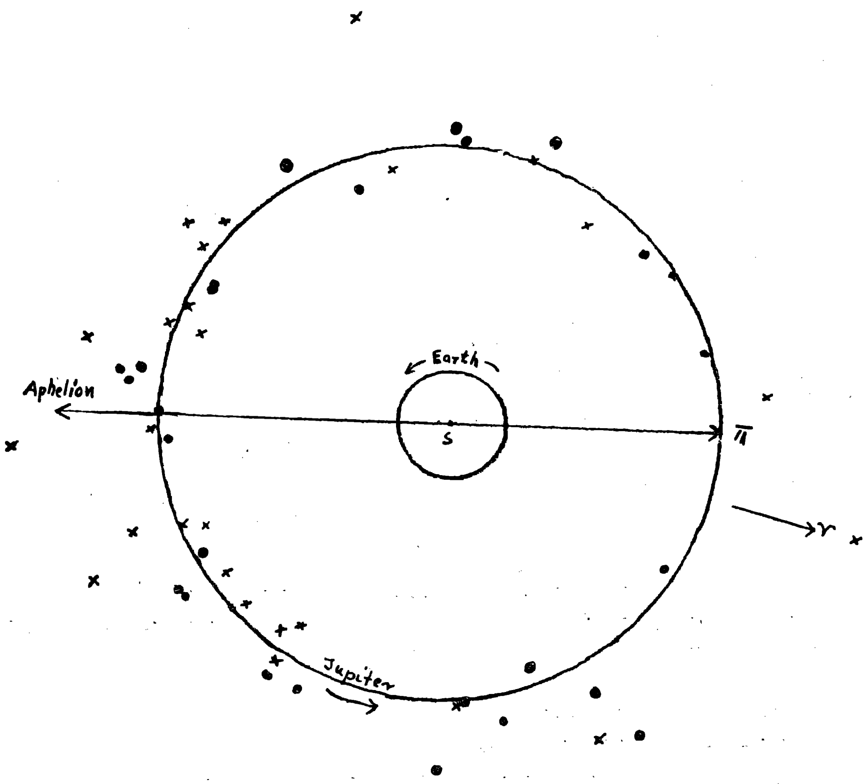

FIGURE 1
DISTRIBUTION OF THE APHELIONS OF 52 COMETS OF THE JUPITER FAMILY
(Circles denote comets observed at 2 apparitions or more;
crosses those observed at one apparition only.)

comet of the Neptune-family (Comet 1921 I, period 79.5 years) there is no comet up to a period of nearly 120 years, *i.e.*, we have no comets between the mean distances from 18.5 to 24. On the contrary, in the space from the mean distances of 24.2, to 30.0 we find five comets. From 30.0 to 38.0 there is a gap without any comets, but in the interval from 38.0 to 45.4 there are another eight comets. This seems to be a striking accumulation around these two regions; and it is very interesting to consider these two families that may be called the Pluto and Trans-pluto families. The most important elements of both these families may be seen from Table II.

TABLE II

Table of the elements of the orbits of 13 comets of the Pluto and Transpluto families.

No.	Name	T = Perihelion Passage in G.M.T.		Period in Years	Distances of Perih.	Distances of Aph.	Mean	Arc from Asc. Node to Perihelion	Longitude of Ascending Node	Inclination of Orbit to Ecliptic	Eccentricity	Reference
	Pluto Family											
1	1862 III Tuttle-Swift	Aug.	22.9	119.6	0.9626	47.6	24.28	152°76	137°45	113°57	0.96035	Yamamoto 294
2	1889 III Barnard	June	20.7	128.2	1.1024	49.8	25.44	60.14	270.97	31.21	0.95667	Yamamoto 406
3	1917 I Mellish	April	10.7	145.3	0.1902	55.0	27.62	121.30	87.53	32.68	0.99312	Yamamoto 544
4	1939 VI Rigollet[1]	Aug.	9.5	156.1	0.7485	57.2	28.99	29.30	355.13	64.20	0.97418	*VJS* **76**, 32
5	1907 II Grigg-Mellish[2]	Mar.	27.2	164.3	0.9233	59.0	29.96	328.43	189.23	109.84	0.96918	Yamamoto 496
	Transpluto Family											
1	1857 IV C. H. F. Peters	Aug.	24.0	234.6	0.7468	75.3	38.04	180.96	200.82	32.77	0.98037	Yamamoto 272
2	1932 X Dodwell-Forbes	Dec.	30.5	262.1	1.1308	80.8	40.96	327.34	77.67	24.50	0.97239	*VJS* **72**, 115
3	1931 III Nagata[3]	June	11.6	267.5	1.0449	82.0	41.50	319.96	191.32	42.33	0.97482	*VJS* **68**, 79
4	1885 III Brooks	Aug.	10.2	274.5	0.7491	83.7	42.23	42.86	204.76	59.11	0.98226	Yamamoto 382
5	1905 III Giacobini	April	4.1	297.2	1.1152	88.0	44.54	358.24	157.38	40.24	0.97496	Yamamoto 484
6	1932 I Houghton-Ensor	Feb.	28.9	302.0	1.2544	88.7	45.04	303.52	212.53	74.28	0.97215	*Nachr. Bl. Astr.* 7, 1.5
7	1932 V Peltier-Whipple	Sept.	1.9	302.5	1.0372	89.1	45.06	38.47	344.52	71.72	0.97698	*VJS* **69**, 121
8	1874 IV Coggia	July	17.7	306.0	1.6880	89.1	45.41	149.60	215.85	34.14	0.96283	Yamamoto 340

[1] L. E. Cunningham suggests identity with 1788 II.
[2] Probably identical with 1742.
[3] Crommelin suggests identity with 574 and 1092 (*M.N.*, **92**, 309).

The mean values of these elements, corresponding to those values of the groups I and II given in Table I are:

GROUP III. 13 COMETS OF THE PLUTO AND TRANSPLUTO FAMILIES

Family	No.	Periods y	Mean distances of aphelions	Mean Perihelion of Comets —perihelion of planet	Mean inclinat.	Mean eccentricity
Pluto	5	119.6—164.3	53.7 (49.3)	+51°	53°	0.9707
Transpluto	8	234.6—306.0	84.8 (?)	(?)	47	0.9746

There may be mentioned some remarkable facts.

Family of Pluto: The first and last comets have retrograde motion and all have very great inclinations of their orbits. The two last comets have been observed in two apparitions, but the others have not been observed after their discovery.

Family of Transpluto: All comets are direct, and the inclinations of their orbits are great. This family has doubled by the discoveries of 1931 and 1932 and is divided into two groups with nearly the same longitude of perihelions (about 24° and 154°) and one individual comet. But having very different inclinations of orbits, any identity of the orbits of each group is not possible.

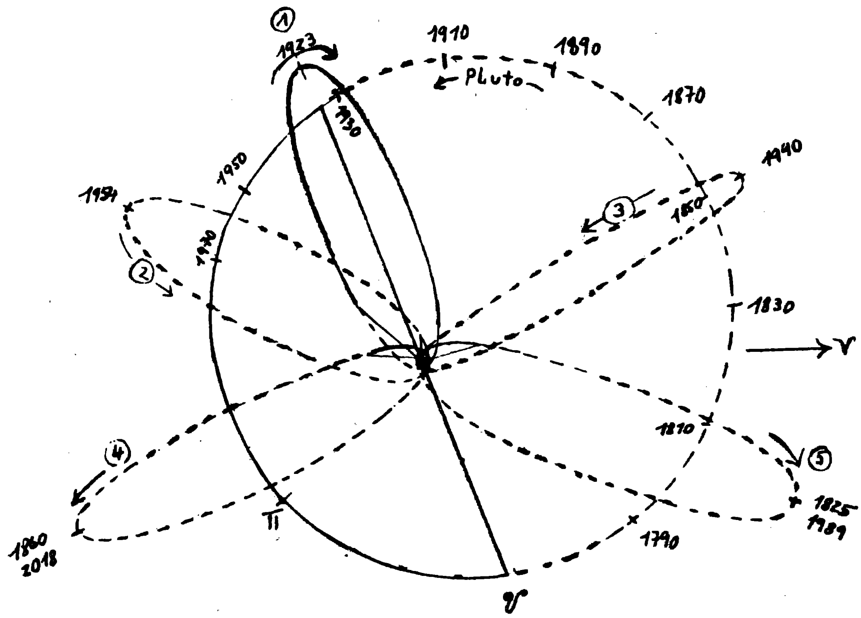

FIGURE 2

1 = Comet 1862 III (Tuttle-Swift); 2 = Comet 1889 III (Barnard); 3 = Comet 1917 I (Mellish); 4 = Comet 1939 VI (Rigollet) 1788 II; 5 = Comet 1907 II (Grigg-Mellish) 1742.

In order to get a clear view of both these families Figures 2 and 3

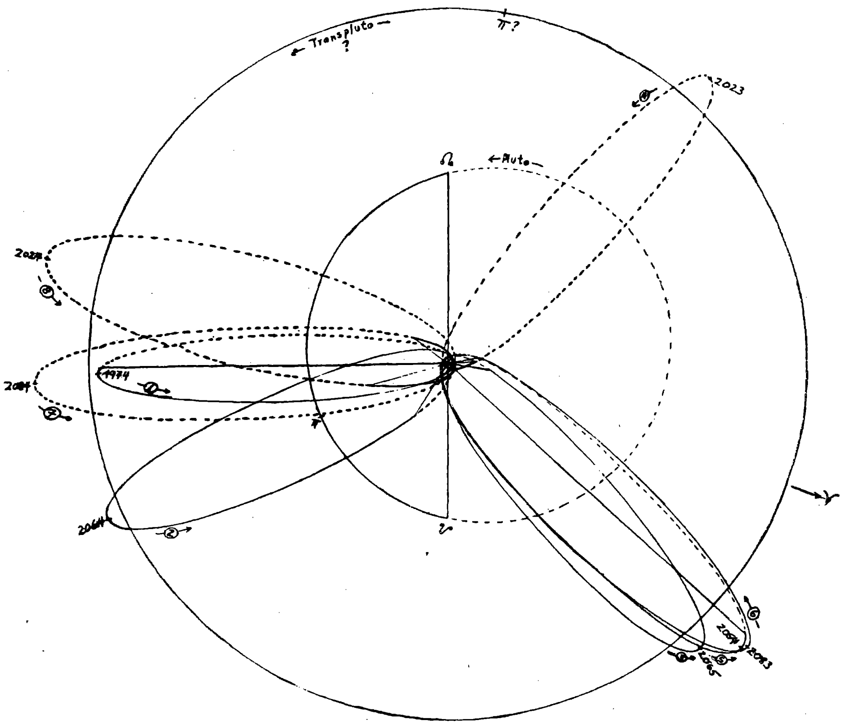

FIGURE 3

EIGHT COMETS OF THE TRANSPLUTO FAMILY

1 = Comet 1851 IV (C. H. F. Peters); 2 = Comet 1932 X (Dodwell-Forbes); 3 = Comet 1937 III (Nagata); 4 = Comet 1885 III (Brooks); 5 = Comet 1905 III (Giacobini); 6 = Comet 1932 I (Houghton-Ensor); 7 = Comet 1932 V (Peltier-Whipple); 8 = Comet 1874 IV (Coggia).

are drawn. No regard for inclination of the orbits has been taken, but those parts of them that are south of the ecliptic are dotted.

What are the consequences of the existence of both these families? There is no doubt that the Pluto family confirms anew the fact of families under the influence of a great planet. The existence of a Transneptunian planet was suspected by several astronomers on account of the Comets 1862 III and 1889 III. Although the five comets of this family were known before the discovery of Pluto, nobody seems to have had the idea that this great family announced very positively the planet Pluto.

If we consider all the families as caused by the action of the great planets, then we should do the same with the eight comets of the Transpluto family. There must be a new planet, a Transpluto, that has created this family. After the well-known law of Titius-Bode the new planet has probably a mean distance of 77.2 astronomical units from the sun. Therefore a circle with the radius of 77.2 units is drawn in Figure 3, and we see that the mean distances of the aphelions of the comets of this group are in agreement, in that they are somewhat greater than the circle, in the same manner as has been pointed out for the other families. It is very remarkable that the planes of the orbits of this family arise above the ecliptic in the same direction, so that the orbit of Transpluto may probably also have a great inclination; the longitude of its perihelion may be near 100°, if we suppose the difference between the mean of the cometary aphelions and that of the planet to be 0°.

Whether it is possible to determine the orbit of Transpluto by the aid of its perturbations of Uranus, Neptune, or Pluto may be very doubtful. Soon after the discovery of Pluto, E. W. Brown and others maintained that the outstanding discordances of Uranus on which the prediction of Lowell are based cannot be attributed to the perturbations of that planet. The new planet Transpluto will probably be so faint that its discovery may only happen by chance.

Investigating the orbits of the further comets at greater mean distances from the sun, we find about 40 comets with periods of revolutions up to 10,000 years, but there is no sign of any family.

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The Triple Conjunctions of Jupiter and Saturn

By J. STEIN S.J.

On the occasion of the 1940 triple conjunction of Jupiter and Saturn, the writer published a short paper in POPULAR ASTRONOMY (November, 1940), calling attention to the remarkable conjunction of the two giant planets in the year 1563. This note will now be amplified and supplemented.

Riccioli in his *Almagestum Novum* (I, 676) erroneously contends that the conjunction of 1563 was a *triple* conjunction, that is, a conjunction of Jupiter and Saturn which occurs three times in the same year. Calculating the epoch of conjunction by means of Neugebauer's chronological tables, we find that heliocentric conjunction occurred on November 1, 1563, the heliocentric longitude of the two planets[1] then being $L = 117°$ and the heliocentric longitude of the earth being $E = 49°$. According to the well-known criterion, however, a triple conjunction is possible only when $L - E < 30°$, and consequently such a conjunction did *not* take place in the year 1563. As a matter of fact, no triple conjunction of Jupiter and Saturn occurred during the whole interval from 1425 to 1682-1683.

[1] Incorrectly said to be the longitude of the *sun*, in line 9 of the original paper, POPULAR ASTRONOMY, **48**, 481.