GEORGE SALMON 1819–1904
HIS MATHEMATICAL WORK AND INFLUENCE

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There are five Irish-born mathematicians who have been deemed worthy of inclusion in the *Biographical Dictionary of Mathematicians*, [5]: William Rowan Hamilton, Henry J. S. Smith, George Gabriel Stokes, Patrick D'Arcy, and George Salmon. (G. J. Stoney is also included, but he should really be considered a physicist. James MacCullagh is not included, although he was certainly a mathematical physicist.) Of these, it would generally be agreed that W. R. Hamilton (1805-65), whose discoveries are routinely used in several branches of mathematics, is the most illustrious. Stokes (1819-1903) and Smith (1826-1883) are held in high esteem internationally, but their creative work was done outside Ireland, Stokes working at Cambridge and Smith at Oxford. D'Arcy (1725-79) is probably not well known, and he lived most of his adult life in France.

In this article, we intend to discuss the remaining Irish mathematician in this list, George Salmon, who, without making any mathematical discoveries comparable with those of the three mathematicians mentioned above, exerted a great influence on mathematical research and teaching in Europe and America in the second half of the 19th century. This influence was derived from four textbooks that Salmon published in Dublin between 1848 and 1862. Each of these appeared in at least three editions, and photographic reprints of the final editions of each book, made in the 1960's by the Chelsea Publishing Company of New York, may still be available. Salmon's role as a disseminator and popularizer of contemporary research in algebra and geometry has been acknowledged by most historians of mathematics, although there is a tendency for the shorter histories to repeat the same basic assessment of his importance, and little else. We would like here to outline some details of his working life, concentrating on his mathematical activities. (It is important to point out that Salmon became Regius Professor of Divinity at Trinity College Dublin (TCD) in 1866 and essentially ceased to be involved in research level mathematics thereafter.) We will describe the contents of those books, indicating novel features which appealed to his readers, and discuss their shortcomings, seen from a later point of view. We will also provide contemporary evidence to illustrate the extent of the influence exercised by Salmon's textbooks.

1. Salmon's life and career at Trinity College Dublin

There are several sources giving details of Salmon's life. One is an obituary by Robert Stawell Ball, [1]. Ball had been a pupil of Salmon's at TCD in 1858-59. His obituary is perhaps a little too adulatory, although it contains useful information. Ball was a popular lecturer and writer on astronomy, as well as a professional astronomer, and he tended to be enthusiastic in his interests and opinions. Another obituary, signed C. J. J., was written by Charles Jasper Joly, [26]. Joly was also an astronomer, but he did original research in algebra, especially quaternions and Clifford algebras. There is another unsigned obituary in *Nature*, [14], perhaps also by Joly, since it contains details rather similar to those in [26]. The *Dictionary of National Biography* (DNB) has an article on Salmon by John H. Bernard, [4], who was then Bishop of Ossory and later became Archbishop of Dublin. Although Bernard studied mathematics at TCD, and his article shows appreciation and understanding of Salmon's mathematical work, his account concentrates on Salmon's theological and clerical work, which we will largely ignore here. We found the article on Salmon by A. J. McConnell in [31] disappointing, for it seemed to be largely a paraphrase of parts of Joly's obituary. As a Provost of TCD was writing about one of his predecessors as Provost, a good opportunity to give a more detailed assessment of Salmon's achievements and influence to a worldwide audience was missed. The history of TCD, [32], has several references to Salmon and gives instances of his biting wit, an interesting aspect of Salmon's personality. An earlier history of TCD, that of Constantia Maxwell, [33], also gives extracts from other appreciatory articles written at the time of Salmon's death. (Since writing this paper, our attention has been drawn to an article by T. D. Spearman, [57], but we have been unable to make use of the information contained therein.)

There are two competing birthplaces for Salmon: Cork and Dublin. What is certain is that he was born on 25 September, 1819. As far as we have been able to discover, any article written up to 1905 that gives information on Salmon's birthplace states that he was born in Dublin. Thus, for example, G. T. Stokes, writing on Salmon in "The Review of the Churches" for 15 June 1892 states

> Dr. Salmon was born in the city of Dublin, ... Though born in Dublin, he came of a Cork family and circle which has made its mark on this age of ours, for he is a cousin of the well-known Professor Edward Dowden, the critic, ...

(Edward Dowden (1843-1913) was Professor of English Literature at TCD from 1867 until his death. Salmon was first cousin of Dowden's mother.)

The obituary of Salmon in *The Times* of 23 January 1904 was written by Bernard, but it does not mention Salmon's birthplace. The later obituary, also by Bernard, in the *Proceedings of the British Academy* for 1904 states that Salmon was born in Dublin, but Bernard's DNB article, published in 1912, states that he was born in Cork. Bernard corresponded with Salmon's relatives and friends when compiling his DNB article and he may have found new information on the birthplace. Nonetheless, the issue remains perplexing, for while most recent articles on Salmon have given Cork as his birthplace, the standard source of information on graduates of TCD, [8], gives Dublin and the current Ordnance Survey of Ireland street guide to Dublin, in its section on famous scientific personages who have lived in Dublin, does the same. If Cork is the correct answer, Salmon made no attempt to correct erroneous articles published in his lifetime, and possibly he did not know the answer himself.

Salmon matriculated at TCD in 1833 and graduated as first

Senior Moderator in Mathematics and Physics in 1838, ahead of William Roberts and William Bindon Blood. The degree examinations were held in October, at the beginning of Michaelmas term, and thus Salmon was just nineteen years old when he graduated. As readers may not be familiar with the style of university examinations in mathematics in the nineteenth century, we present below some of the questions that were posed in 1838. These are taken from [15]. Six papers of ten questions each were set over three days, with one paper in the morning and one in the evening. The examiners were Mr Malet, Mr Sadleir, and Mr Luby. There was a strong bias towards mechanics in most of the questions posed, with calculus also prominent. The fourth paper, set by Mr Sadleir, contained several questions on the geometry of lines and surfaces, and this paper may well have appealed to Salmon, given his interest in analytic geometry.

Paper 4

1. If two right lines be constantly perpendicular to each other at a given point, and move in given planes, their planes envelope a cone.
2. Find the length and equations of the right line which is the shortest distance between two given right lines in space.
5. If three diameters of an ellipsoid cut at right angles, the sum of the squares of their reciprocals is constant.
6. What is Meusnier's theorem, and how is it proved?
7. Find the general equation of surfaces of revolution round any axis; and deduce the partial differential equation.
8. If the apex of a cone which touches a surface of the second order describe a surface of the same order, the plane of contact will envelope a surface of the second order.
10. Find the equation of the surface formed by right lines joining the points of intersection of two curves of double curvature made by a plane whose direction is given, and deduce the partial differential equation.

Salmon included Meusnier's theorem in his textbook [GTD], but he misspelled the name as Meunier.

Salmon sat the Fellowship Examination in 1840 and was elected a Fellow of the College in 1841. He was ordained deacon in 1844 and priest in the Church of Ireland in 1845. On his becoming a Fellow, Salmon received a college tutorship, which he retained until 1866. From 1845, he also served as a divinity lecturer. The duties of a tutor had changed following the reforms in the tutorial system introduced in the mid-1830's. It was the case that every student entering the college had to place himself under the tuition of one of the Junior Fellows who was a tutor. Up to 1834, tutors had lectured only to their own pupils, across a wide range of subjects, but, following the reforms, they admitted pupils of other tutors to their lectures and they lectured on a more limited range of subjects. Furthermore, remuneration no longer depended on the number of pupils under the tutor's guidance, but only on the tutor's seniority. Much of the stimulus for writing Salmon's first two textbooks must have come from the tutor's lectures that he gave during the 1840's. It is interesting to note that Salmon served as tutor to both of W. R. Hamilton's sons during the 1850's (they did not distinguish themselves academically).

In 1862, the Erasmus Smith's Professor of Mathematics at TCD, Charles Graves (1812-99), resigned the professorship. He had been appointed in 1843 and published twenty eight mathematical research papers in such areas as geometry, quaternions, and the calculus of operations, all popular topics for research at TCD during the 19th century. In 1841 he published a translation of two geometrical memoirs of Michel Chasles on cones of the second degree and spherical conics, together with notes and additions, and an appendix of his own work, [19]. Graves's monograph was probably influential on Salmon, and it is quoted occasionally in Salmon's textbooks (although Salmon mistakenly wrote that its date of publication was 1837).

It would have seemed, in view of Salmon's eminence in the mathematical world, an eminence based particularly on the reputation of his textbooks, that he was the worthiest successor to Graves. For whatever reason, however, Salmon must have already decided to concentrate his energies in theology, instead of mathematics, and he never offered himself as a candidate for the vacant mathematical professorship. Instead, a vacancy had occurred in the Archbishop King's Lectureship in Divinity. The office of Archbishop King's Lecturer commanded a salary of £700 per annum, equal to that of the Professorship of Mathematics, a considerable sum in the 1860's. According to Joly, [26, p.349], Salmon had been led to believe that a certain William Lee, who had been appointed Fellow of the College in 1839, would not apply for the Lectureship. If Lee were to apply for the position, he was certain to be preferred over Salmon by virtue of his seniority in the Fellowship. Unfortunately for Salmon, Lee changed his mind, after Michael Roberts had already been appointed Professor of Mathematics, and obtained the lectureship. Thus Salmon was frustrated in this promotion opportunity (Roberts retained the professorship until 1879, and Lee the lectureship until his death in 1883). Salmon had to wait a further four years before achieving the senior position he deserved, albeit in his second speciality of divinity. The circumstances of his elevation to the divinity professorship, following on from his decision not to offer himself as a candidate for the mathematical professorship, are also described in [32, note 19, p.543], although there is no mention of Lee's possible role:

> Erasmus Smith's Professorship of Mathematics fell vacant in 1862. Salmon for reasons which we have been unable to discover, did not offer himself as a candidate, although he was senior to the two Fellows who did compete (Michael Roberts and Townsend) and better qualified than either. He can scarcely have been reserving himself for the Regius Professorship of Divinity, as the occupant (Butcher) was only eight years older than Salmon, and his resignation at the age of fifty-seven to become Bishop of Meath could not have been predicted.

In [1, pp.xxiii-xxiv], Ball describes lectures by Salmon that he attended in 1858-59:

> He was an admirable teacher. I particularly remember the course on conics. It was presumed that we had read or were reading his book, so the lectures often took the line of showing us the improvements or extensions which he was preparing for future editions. He would kindly encourage his pupils to make their comments, and even honour them by asking for their efforts to aid him in questions which were occupying him at the momemt. He used to warn us that the good things were mostly "threshed out," but still there were some leavings. . . . I also attended Salmon's lectures on "Elementary Theoretical Dynamics," and never shall I forget the delight with which we received his illustrations of the use of differential equations in dynamics.

During the tenure of his tutorship, Salmon had opportunities to lecture honours students and introduce them to the new mathematical theories and techniques being developed at that time. We may form some idea of the content of these lectures by looking at various examination papers that he wrote during the 1850's. In 1850, he examined for Bishop Law's Mathematical Premium and included the questions:

3. Find the condition that two conics should touch whose equations are given.

10. Find the multiple points of the curve

$$A^{1/2} + B^{1/2} + C^{1/2} + D^{1/2}.$$

In the Degree Examination of 1851, he set a paper on Algebra, Differential Calculus and Analytic Geometry, which included the question

2. Solve the equation $x^{17} = 1$.

We wonder whether he intended that the solution in terms of square roots, which arises in the theory of the construction of a regular polygon of 17 sides, should be given.

In the Degree Examination of 1854, he set questions on Physical Astronomy and he also examined in the same year on the Historical Books of the Old Testament for the Regius Professor of Divinity's Premium. In 1857, Salmon, then Donegal Lecturer in Mathematics, examined on the Theory of Curves, and Algebra for the Moderatorships in Mathematics and Mathematical Physics. Among his questions were:

4. Write down the equations of the tangents at the points of inflexion of the curve–

$$x^3 + y^3 + z^3 + 6mxyz = 0.$$

5. What is the value of the determinant

$$\begin{vmatrix} 1, & 1, & 1, & 1 \\ \alpha, & \beta, & \gamma, & \delta \\ \alpha^2, & \beta^2, & \gamma^2, & \delta^2 \\ \alpha^3, & \beta^3, & \gamma^3, & \delta^3 \end{vmatrix}$$

and find the determinant which is its square.

Salmon married Frances Anne Salvador in 1844, and the couple initially lived at 1 Carleton Terrace, Rathmines, Dublin. They had moved to 2 Heytesbury Terrace by 1848, remaining there until 1865 or 1866. They then moved to 81 Wellington Road (this house is probably still standing, as the houses in Wellington Road form a continuous terrace, dating from the Victorian era, with number 81 the second last house in the terrace). Finally, having been appointed Provost of Trinity College in 1888, Salmon took up residence in the Provost's House. There were six children of the marriage, four boys and two girls, but only two of the children survived Salmon, who died on 22 January, 1904. His wife had died in 1878. An *In Memoriam* notice, written by E. J. Gwynn and recalled in [33, p.217], describes Salmon's funeral procession:

> Our Provost was the first man in Ireland whose pre-eminence all acknowledged; the immense procession which on the 26th January stretched half–way from Chapel to cemetery was the outward and visible sign of a national grief.

Salmon is buried with eight family members in the family vault in Mount Jerome Cemetery, Dublin. The vault is located on the south side of the path known as the Long Walk. The unprepared visitor is unlikely to find the vault by chance, as there is no headstone, and the lid of the vault is covered in foliage. The words 'FAMILY VAULT OF GEORGE SALMON' are discernible with difficulty. Not more than 40 yards away, on the same side of the Long Walk as the Salmon vault, but set back further from the path, is the grave of Salmon's famous fellow mathematician William Rowan Hamilton. The headstone of Hamilton is still perfectly legible and is far easier to find than Salmon's vault.

Those who are interested in Salmon's career as Professor of Divinity, the theological books he published, and in his fifteen years as Provost of TCD, may read the articles and books that we mentioned at the beginning of this section. According to G. T. Stokes, Salmon enjoyed

> the most richly endowed academical position, if not in Europe, at least in the British Islands. For owing to the liberality of a deceased bene-

factor of the last [18th] century, the Provost has an estate of his own which makes his position worth more than £3,000 a year.

This income may account for why Salmon was able to make gifts of over £6,000 to TCD, a large sum of money in the 19th century, [32, p.292]. There is a seated marble statue of Salmon by John Hughes, unveiled on 14 June 1911, which stands near the Campanile in the main quadrangle of the Trinity College (the statue originally stood elsewhere).

While we are not attempting to describe Salmon's personality in this article, two anecdotes about him, told in [33, pp.217-218], may bear repetition here:

Many stories are told of the great man's absentmindedness. Standing carelessly dressed on the kerb-stone one day, and staring blankly at the sky, with his long white hair quivering in the breeze, he was mistaken for a blind beggar by a kind old lady who offered to lead him across the street.. . .

One Sunday he accidentally brought to St. Patrick's Cathedral the same charity sermon that he had preached there a year before. However, he went through with it to the bitter end, reasoning that half the congregation had not been there, that one quarter had heard and forgotten the sermon, and that the remaining quarter would be very glad to hear it again–a process of exhaustion worthy of Euclid himself.

2. Salmon's mathematical research

In the bibliography of this paper, we have given a list of Salmon's mathematical research papers. These are taken from the Royal Society's *Catalogue of Scientific Papers* vols. V and VIII, and we hope that the list is reasonably complete. It will be seen from the titles that almost all his papers are devoted to geometrical subjects. During the 19th century, many mathematicians connected with TCD published textbooks or research papers on geometry, often of the synthetic, rather than the analytic, variety (in this respect, Salmon was exceptional, as his bias was usually to the analytic side of geometry). Two textbook writers, active during Salmon's prime, were John Mulcahy, whose *Principles of Modern Geometry* (1852) acknowledged the influence of Graves's translation of Chasles and Salmon's *Conic Sections*, and

Richard Townsend, whose *Chapters on the Modern Geometry of the Point, Line, and Circle*, (two vols., 1863, 1865), sought to familiarize readers with the work of Michel Chasles, and to progress beyond Mulcahy's book. Those Fellows who wrote geometrical research papers included James MacCullagh, Charles Graves, William Rowan Hamilton (often employing quaternionic techniques), Andrew Searle Hart, Salmon, Richard Townsend, William Snow Burnside, John Kells Ingram, John William Stubbs. It is interesting to note that, in a letter to W. R. Hamilton, dated February 2, 1852, and published in [20, p.334], Augustus De Morgan wrote:

> How comes it that the geometrical extensions take such root in Ireland? Yourself, C. Graves, MacCullagh, Salmon, &C, are all *up* in them. Hardly a soul in England cares about them.

In his reply, dated February 9, 1852, [20, p.335], Hamilton wrote:

> I think that there *is* a greater, or at least a more general aptitude for pure geometry in Ireland than in England. The Fellows of T. C. D. are nearly all geometers, and some of them are extremely good ones, ...

Salmon's papers are for the most part quite short, often no more than research notes or announcements. Moreover, he published very little mathematical research after obtaining the professorship of divinity. Salmon was clearly influenced by the climate of geometric research that existed at TCD, but we cannot say for certain that Salmon was a follower of any particular person active at the College in the 1840's or 1850's (except possibly MacCullagh, some of whose work Salmon describes in his books). He seems to have been largely self-made, influenced by his reading of Poncelet's book, [39], and the research papers of such contemporaries as Cayley, Hermite and Sylvester. Regarding Salmon's own research, Joly wrote in [26, p.354]:

> It would be most unfair to Salmon to judge his contributions to mathematics by his papers alone. He had a great dislike to the physical trouble of writing; he modestly communicated his discoveries to friends, or reserved them for incorporation in his books, so that it is a matter of extreme difficulty to say how much is his. ...... Of Salmon's original contributions to science, the most worthy of notice are his solutions of

the problem of the degree of a surface reciprocal to a given surface; his researches in connection with surfaces subject to given conditions, analogous to those of Chasles in plane curves; his classification of curves of double curvature; his conditions for repeated roots of an equation; and his theorem of the constant anharmonic ratio of the four tangents from a point on a cubic curve.

Joly does not mention what is, in our opinion, Salmon's major achievement in mathematical research, namely, his work concerning the twenty seven lines on a cubic surface. In keeping with Joly's remark above, Salmon wrote very little in the research literature on this subject. To give some details, we recall that a cubic surface in the three dimensional complex projective space $P_3(\mathbf{C})$ is defined by an equation of the form

$$f(x_0, x_1, x_2, x_3) = 0,$$

where f is a homogeneous polynomial in the x_i of degree 3. Thus, the so-called Fermat surface

$$x_0^3 + x_1^3 + x_2^3 + x_3^3 = 0$$

is an example of a cubic surface. The surface is said to be *non-singular* if there is no point P on the surface for which

$$\frac{\partial f(P)}{\partial x_i} = 0$$

for all i. We may similarly define a surface of degree n using a homogeneous polynomial of degree n.

It is known that in general a surface of degree n in $P_3(\mathbf{C})$ contains no lines lying entirely within the surface if $n > 3$. See, for example, [53, p.149]. Salmon sketches a 'generic' proof of this in [GTD, 1862, p.29]. A surface of degree 1 or 2 contains infinitely many lines. The case of degree 3 is especially interesting, as Cayley showed in 1849 that a non-singular cubic surface contains at most a finite number of lines. Furthermore, with some effort, one can show that a non-singular cubic surface always contains at least one line (see, for example, [42, Proposition 7.2]). In correspondence with Cayley during 1849, Salmon proved that there are exactly twenty seven lines on a non-singular cubic surface.

The first major article on the twenty seven lines is that of Cayley, [9]. Cayley ended his paper in the following manner:

...I may mention in conclusion that the whole subject of this memoir was developed in a correspondence with Mr. Salmon, and in particular, that I am indebted to him for the determination of the number of lines upon the surface and for the investigations connected with the representation of the twenty-seven lines by means of the letters a, c, e, b, d, f, as developed above.

The same volume of the *Cambridge and Dublin Mathematical Journal* contained a paper by Salmon, which presented supplementary notes to Cayley's paper.

Salmon devoted a few pages in the various editions of [GTD] to the subject of the twenty seven lines, and described his contribution in a footnote, as follows:

The theory of right lines on a cubical surface was first studied in the year 1849 in a correspondence between Mr. Cayley and me, the results of which were published, *Cambridge and Dublin Mathematical Journal*, vol. IV., pp. 118, 252. Mr. Cayley first observed that a definite number of right lines must lie on the surface; the determination of that number as above, and the discussions in Art. 458 were supplied by me.

It was largely left to other workers to elucidate the beautiful combinatorial and algebraic relations that are associated with the configuration of the twenty seven lines, and an extensive literature on the subject exists. There seems to be no elementary approach to writing down explicitly the algebraic form of the lines, even in the case of the Fermat surface mentioned above. There is a Galois theoretic problem, leading to an equation of degree 27 obtained by elimination techniques, that is attached to the question of determining the lines. A reasonably elementary approach to an explicit investigation may be found in [42, Section 7]. For a more abstract approach, one may consult [22, Chapter V, Section 4].

Salmon's last published research paper, that of 1873, is on the elementary theory of numbers, and it is a rather feeble effort.

Salmon began his paper as follows:

I wish to make a few remarks, ..., viz to find the number of figures in the period of a circulating decimal whose reciprocal is a prime number. This is evidently the same as to find what power of 10 is divisible by the prime with remainder 1; or to solve the equation $10^x \equiv 1$.

Salmon thus wished to find the order of 10 modulo a given prime. The subject was scarcely original at the time Salmon wrote, since it was treated in such university textbooks as that of Hymers, [25, p.265], but Salmon made no reference to any published sources, and his results are essentially trivial.

Although Salmon ceased to be involved in research level mathematics soon after obtaining his professorship, he still maintained an interest in mathematical questions at the recreational level, since his name appears in (virtually) every list of contributors to the publication *Mathematical Questions, with Their Solutions. From the "Educational Times"*. This serial was published between 1864 and 1918, and it contained, over the years, statements and solutions of more than 18,000 mathematical problems. We have not found many examples of problems or solutions contributed by Salmon that were actually published, but the following problem of his, [43], shows him working in an uncharacteristic area:

A teetotum of n sides is spun an indefinite number of times, and the numbers turning up are added together; what is the chance that a given number s will be actually arrived at?

(A teetotum is a small top inscribed with letters or numbers.)

3. Salmon's mathematical textbooks

Salmon's name would scarcely attract any attention among mathematicians so many years after his death if his reputation was based only on his research papers. As we stated in the introduction, Salmon's lasting fame lies in the influence exerted by four textbooks he wrote. These were: *A Treatise on Conic Sections*; *A Treatise on the Higher Plane Curves*; *Lessons Introductory to the Modern Higher Algebra*; *A Treatise on the Analytic Geometry of Three Dimensions*. We have given details of the various editions of these books in the bibliography of this article. The information is mainly extracted from the British Museum's *General Catalogue of Printed Books* (to 1955) and *The National Union Catalogue* (pre-1956 imprints), although the sources do not include all editions of Salmon's textbooks. We will consider each book in a separate section of this article. It will be seen that each of the four titles was translated into both French and German, and these translations often went through several editions (which we have not attempted to itemize). The German translations of the four main books, each by the German mathematician (Otto) Wilhelm Fiedler (1832-1912), who worked mainly in Zürich, were especially highly regarded. The historian of science, J. T. Merz, wrote as follows of these translations, [34, p.44] and their effect on the teaching of geometry in Germany in the second half of the 19th century:

> Germany indeed had not been wanting in original research, but the new ideas of Möbius, Steiner, Staudt, Plücker, and Grassmann in geometry found no adherents till, mainly through the translations of Salmon's text-books by Fiedler, a new spirit came over geometrical teaching.

Just before Salmon's death, Merz described Salmon's influence in [35, pp.684-685] in these words:

> The merit, however, of having brought together the new ideas which emanated from the school of Poncelet and Chasles in France, of Cayley and Sylvester in England, into a connected doctrine, and of having given the impetus to the fundamental remodelling of the text-books and school-books of algebra and geometry in this country [Britain] and in Germany, belongs undeniably to Dr Salmon of Dublin.

Felix Klein was also a great enthusiast for Salmon's books. He described them affectionately as follows:

> They are like refreshing and instructive walks through wood and field and cultivated gardens, where the guide draws attention now to this beauty, now to that strange appearance, without forcing everything into a rigid system of faultless perfection. . . and without digging out individual useful plants and transferring them to cultivated soil according to principles of intensive cultivation. In this flower garden we have all grown up, here we have gathered the foundation of knowledge on which we have to build.

The quote is taken from an English language version of part of

Klein's posthumously published lectures, [27, vol. I, p.165], found in [18, p.75]. (The reviewer of Klein's lectures in [56] notes that Salmon remarked, when Klein enquired why at Dublin there were only three faculties, theology, classics and mathematics, whereas at Cambridge all the newest subjects were taught–'Yes, but Cambridge is so ambitious!').

Klein was probably sympathetic to Salmon's style, since Klein was known not to be too interested in the intricate details of theories, and preferred broad pictures. He similarly praised the Salmon-Fiedler series of textbooks in his *Elementarmathematik von höheren Standpunkte aus*:

All that is needed in the geometry of invariant theory especially is to be found in the textbooks of G. Salmon, which have contributed most to spread the ideas which arise here. The German edition of Salmon's book by W. Fiedler has always enjoyed an unusually wide use.

The German version of Klein's book first appeared in 1908. The quote above is from the English translation of the third edition, [28, p.135].

Fiedler's versions of Salmon's books were more than just translations. The first editions were said to be adapted into German (Deutsch bearbeitet) and later ones were freely adapted into German (frei bearbeitet). Fiedler incorporated material of his own into the German versions.

Salmon was an early member of the London Mathematical Society (LMS), which he joined in 1866. In 1876, Henry Smith, then President of the LMS, gave an address to the Society, in which he noted how young British mathematicians often chose to study algebra and geometry through the influence of Salmon's textbooks, [45, p.26]:

Can we doubt that much of the preference for geometrical and algebraical speculation which we notice among our young mathematicians is due to the admirable works of Dr. Salmon; and can we doubt that, if other parts of mathematical science had been equally fortunate in finding an expositor, we should observe a wider interest in, and a juster appreciation of, the progress which has been achieved?

The *General Index* to each volume of the *Proceedings of the Lon-*

don Mathematical Society between 1866 and 1900 serves as a way of measuring how much practising mathematicians used Salmon's textbooks either for general reference or to motivate their own research, since citations given in the papers published in the *Proceedings* during this time are listed in this index. A rough count revealed 87 papers that cited Salmon one or more times. In particular, eight papers in vol. XIII (1881-82) and ten papers in vol. XIV (1882-83) cited textbooks by Salmon. Authors citing Salmon included: F. Brioschi, A. Cayley, P. A. MacMahon, A. R. Forsyth, J. Hammond, W. Spottiswoode, M. J. M. Hill, H. Lamb, E. J. Routh, W. K. Clifford, H. J. S. Smith, J. Larmor, L. J. Rogers, W. W. Rouse Ball, F. S. Macaulay, A. N. Whitehead, A. B. Kempe, A. G. Greenhill, H. G. Zeuthen, and A. Mannheim. This is an impressive list which fully indicates the extent of Salmon's influence. The references became fewer in the 1890's, perhaps as Salmon's contemporaries died or ceased to be active.

While we have catalogued favourable opinions of Salmon's textbooks, it seems to us that, on occasions, Salmon's writing was definitely unclear and badly expressed. In [7, p.39], Brock raised the question of whether Salmon's textbooks were really as clear and as popular with students as his obituarists claimed. He describes an annotated copy of [CS, 1879] in Leicester University Library which bears such comments as "Half the Book unfit for students but good for a Maths teacher of Analytical Geometry," "not easy, in fact very difficult," "badly expressed and explained," "little or no use–sheer rubbish," "I never remembered to have read a Chapter which required so much elucidation and received so little." In a copy of [HPC, 1852] that we have used, we found the comment "I find it impossible to make head or tail of this atrociously written ungrammatical nonsense. I do not know what is asserted in this attempt at a sentence." While students sometimes lay the blame for their own incomprehension at the feet of their lecturers or with their textbooks, these examples suggest that Salmon's writing was not always appreciated by students more interested in passing examinations than dwelling on the finer points of modern geometry.

4. A Treatise on Conic Sections

The early publishing history of Salmon's most famous and enduring textbook, *A Treatise on Conic Sections*, is a little complicated and does not seem to have been commented upon before by anyone writing about Salmon. We will try to unravel some of the mystery that surrounds the first appearance of this book.

In the 1840's, Salmon had planned to write a treatise in two volumes on analytic geometry, intended for the use of undergraduates at TCD. The first volume was to be divided into two parts, Part I consisting of elementary analytic geometry, and Part II of the geometry of conic sections, with special emphasis on modern developments in the subject. The second volume (Part III of the treatise) was to be devoted to higher plane curves. This plan was essentially fulfilled, with *Conic Sections* of 1848 forming the first volume and *Higher Plane Curves* of 1852 the second. What is surprising to us is that, in 1847, Salmon decided to issue each of Parts I and II separately, entitled *A Treatise on Analytic Geometry. Part I. The Right Line and Circle*, which appeared late in 1847, and *A Treatise on Analytic Geometry and on the Theory of Plane Curves. Part II. Conic Sections*, which was to appear in January 1848. Copies of each separate part are very rare, and neither is listed in the British Museum's *General Catalogue of Printed Books* or *The National Union Catalogue*. The Trinity College Library has copies of Parts I and II bound together in a single volume, entitled on the spine *Salmon's Geometry I–II*. This copy was presented to the Library by Salmon himself in 1896. There is also a copy of Part I only in the National Library of Ireland.

The preface of [AGI], which we include below, makes it clear that Salmon brought out the first part of the treatise as a separate volume to provide an elementary introduction to analytic geometry suitable for first and second year undergraduates at TCD.

> It was not my original intention to publish an independent treatise on Analytic Geometry, but rather a supplement to the ordinary elementary works on that subject, in which I meant to give an account of the principal additions made by modern geometric and algebraic methods to the theory of Curves and Surfaces. In attempting to execute this

design I found some inconvenience from the fact, that there was no single work in general use among the students of this College, to which I could refer for elementary information. The late Dr. Lloyd's treatise had been for some years out of print, and no other work has taken its place as a recognised text-book for the use of our junior students. This deficiency I have attempted to supply in the following pages, which have been drawn up with an especial reference to the course of study pursued in this college. As the class now commencing the study of this science are not required, during the approaching term, to proceed beyond the properties of the Right Line and Circle, it has been thought advisable to publish, without delay, that portion of the present work which is required for their immediate use. I have, for the same reason, discussed these properties at greater length, I fear, than most readers will approve. The additions which I have made to the matter usually contained in elementary treatises consist chiefly in a tolerably copious collection of examples (Chaps. III. and VII.), and in an account of those methods of abridged notation (Chaps. IV. and VIII.) which have lately become so much used, and which the Tutors' Lectures have, during the last few years, made generally known among the candidates for honors.

The Second Part of this Treatise, containing the theory of Conic Sections, is intended to be published at the beginning of next Hilary Term, and the remainder of the work as soon after as is found practicable.

I have to return thanks to Mr. Townsend for many useful suggestions, and for much kind help in superintending the progress of these pages through the Press; and to the board of Trinity College for the liberal assistance which they have afforded to the present publication.

GEORGE SALMON

TRINITY COLLEGE,
October 20th, 1847

The advertisements at the beginning of the *Dublin University Calendar* for 1848, [48], announced the appearance of Salmon's *A Treatise on Analytic Geometry. Part I*, at a cost of 5s., and stated that Part II was to be published immediately. The back cover of the copy of [AGI] in the TCD Library advertised that Part II, containing the theory of conic sections, was to be published in January [1848]. At least one copy of Part II was printed and bound, but we are not certain if any copies were ever offered for sale, as we have seen no advertisements relating to Part II. *The London*

Catalogue of Books, 1816-51, which lists books published in England between 1816 and 1851, has entries for both *A Treatise on Analytic Geometry. Part I* and *Conic Sections*, which were both published in London by Whittaker and Co., but makes no mention of *A Treatise on Analytic Geometry and on the Theory of Plane Curves. Part II. Conic Sections*. What is certain is that Parts I and II were combined to form the first edition of *Conic Sections*, which was published in 1848. Salmon's obituarists Bernard and Joly give a publication date of 1847 for the first edition of *Conic Sections*, whereas the correct date is 1848. They must have confused the incomplete [AGI], published in 1847, with the complete [CS], published in 1848. The 108 pages of Part I form the first eight chapters and the 192 pages of Part II the last six chapters of *Conic Sections*. Part II contained the following advertisement:

> PART III. will be published as speedily as the Author's other engagements will permit, and will contain an account of the properties of the higher Plane Curves, together with a General Index, and Notes on the history of the more remarkable Methods and Theorems.

The first and second editions of Salmon's *Conic Sections* contained no author's preface, which is a pity in view of the confusion caused by the earlier publication of *A Treatise on Analytic Geometry.* The first edition did, however, include a frontispiece advertisement, the first part of which we display below (the second part of the advertisement is just the advertisement which had already appeared in [AGII]):

> The following pages form Parts I. and II. of a Treatise on ANALYTIC GEOMETRY, and on the THEORY OF PLANE CURVES, intended for the use of Undergraduate Students of the University of Dublin.

We will devote the rest of this section to consideration of the various editions of the complete *Conic Sections*. The fourteen chapters of [CS, 1848] are as follows:

Chapter I.	The point
Chapter II.	The right line
Chapter III.	Examples on the right line
Chapter IV.	The right line continued
Chapter V.	Equations above the first degree

	representing right lines
Chapter VI.	The circle
Chapter VII.	Examples on the circle
Chapter VIII.	The circle continued
Chapter IX.	Properties common to all curves of the second degree, deduced from the general equation
Chapter X.	Equations of the second degree referred to the centre as origin
Chapter XI.	The parabola
Chapter XII.	Examples and miscellaneous properties of the conic sections
Chapter XIII.	Methods of abridged notation
Chapter XIV.	Geometrical methods

The first twelve chapters, which occupy 199 pages, constitute a fairly complete introduction to the study of analytic geometry in the plane, up to and including conic sections, suitable for undergraduates. In [AGII], Salmon indicated that the more advanced Chapters XIII and XIV might be omitted by readers commencing the study of conic sections. The work was popular for teaching purposes, partly on account of the extensive treatment, with plentiful examples, of this elementary material. The thirteenth chapter was on the methods of abridged notation, a topic continued in all subsequent editions, although it has fallen out of favour nowadays. For professional mathematicians, it was the final chapter, occupying pp.239-300 in the first edition, on geometrical methods that was of greatest interest. We will briefly describe its contents.

The chapter begins with the principle of duality and the method of reciprocal polars. Of the method of reciprocal polars, which concerns a duality between points and lines, Salmon wrote in a footnote to [CS, 1848]:

> This beautiful method was introduced by M. Poncelet, whose account will be found at the commencement to the fourth volume of Crelle's Journal.

There then follows a section on harmonic and anharmonic properties of conics. This section was influenced by the work of Michel Chasles. It is succeeded by the method of infinitesimals. The final section is the method of projections, which Salmon described as follows:

> The method is the invention of M. Poncelet. See his *Traité*, published in the year 1822. I shall be glad if the slight sketch here given induces any reader to study a work, from which I have perhaps derived more information than from any other in the theory of curves.

This footnote was modified in the fourth edition, as follows:

> ... See his *Traité des Propriétés Projectives*, published in the year 1822, a work which, I believe, may be regarded as the foundation of Modern Geometry. In it were taught the principles, that theorems concerning infinitely distant points may be extended to finite points on a right line; that theorems concerning systems of circles may be extended to conics having two points common; and that theorems concerning imaginary points and lines may be extended to real points and lines.

The final section of the footnote given above, concerning imaginary points and lines, was referred to as the *principle of continuity*. It was a contentious issue when introduced by Poncelet, as it involved the use of complex methods in real geometry, and it was strongly opposed by Cauchy, who served as a referee for one of Poncelet's related papers. The article by René Taton on Poncelet, [47], provides information on his work and the problems it encountered. We also note how Salmon refers to the subject of 'Modern Geometry' in this footnote. Modern geometry, as developed in the late 18th and early 19th century by Monge, Poncelet, Chasles, and others, represented the first major extension of pure geometry (as opposed to analytic geometry) beyond the state in which the Greeks had left it. The two geometry textbooks published in Dublin that we referred to in Section 2, those of Mulcahy and Townsend, contained the phrase 'Modern Geometry' in their titles.

The fourth and later editions of [CS] contained separate chapters on each of the topics of reciprocal polars, harmonic and anharmonic properties of conics, the method of projections and the method of infinitesimals. A further chapter, on invariants and covariants of systems of conics, was adjoined. This was clearly influenced by Salmon's newer work on invariant theory.

Title page of
A Treatise on Analytic Geometry, Part I

Title page of
A Treatise on Analytic Geometry, Part II

A feature of [CS, 1848] is the inclusion of extensive footnotes, many of which did not survive into later editions. Some of these footnotes contain curious admissions of forgetfulness or lack of knowledge of the literature. For example, on p.298 of [CS, 1848], he wrote:

I forget where I met with the following method of obtaining the properties of angles subtended at the focus from those of small circles on a sphere.. . .

but in later editions, he decided that John Mulcahy had given this method.

[CS, 1879] was the last to contain any changes. All subsequent editions have been reissues of the sixth edition. They have continued to appear, as Chelsea Publishing Company reprints, to the end of the twentieth century. *Conic Sections* was Salmon's most popular textbook in terms of sales and probably the one for which he is best known. Books on conic sections appeared fairly regularly during the 18th and 19th centuries, and the subject formed a key part of a university mathematics course. Some textbooks, such as that of Todhunter, [52], also ran to several editions but none seems have attracted the affection that was reserved for Salmon's book. It was perhaps its mixture of elementary methods and introduction to the modern geometry of the 19th century which appealed to readers. One may compare Salmon's book with that of Hymers, [24], which was used at Cambridge before Salmon's book appeared. Hymers's book is little more than the first eight chapters of Salmon's, with none of the more exciting modern material. Todhunter's 1855 treatise dealt with some more advanced topics, such as the method of abridged notation, but again it is mainly devoted to elementary material.

In 1948, on the hundredth anniversary of the first appearance of [CS, 1848], the *Mathematical Gazette* contained a short celebratory note by E. H. Neville, [37], entitled 'Salmon,' together with a frontispiece photograph of the title page of [CS, 1848]. The sentiments expressed in the article seem to have been felt by many people interested in the study and teaching of geometry. We quote the article in full:

The learner of one generation is the teacher of the next, and as a rule it is only for a decade or so that a textbook or treatise plays its part in the development of its subject directly; what is original in matter or method is absorbed by readers and permeates the books which they in their turn write. Chrystal's *Algebra* and Hobson's *Trigonometry* are fountain-heads, but the young mathematician does not go to them now; he imbibes their wisdom through channels fashioned in his own lifetime.

To the rule there is one outrageous incredible glorious exception: 'Salmon'. The name stands not for an unknown provost and professor of divinity, nor generically for the four mathematical treatises of which three have suffered the common fate. 'Salmon' is 'A Treatise on Conic Sections containing an account of the most important modern algebraic and geometric methods' and nothing else. And our frontispiece shows that this book, which schoolboys and undergraduates of today are urged to read as a matter of course, has in sober fact reached its centenary. It is the moment for our salute.

The third edition of [CS] was published in London, rather than in Dublin, and this may have ensured that it enjoyed a wider circulation than the first two editions. Unlike the first and second editions, it contained an author's preface, which did not explicitly link the book to [HPC], and it was no longer stated that it was intended for the use of students of the University of Dublin. Its sales may also have been improved by the appearance in December 1855 of a favourable book review in a widely read scientific journal, [23]. According to [7, p.33], this review was written by Thomas Archer Hirst (1830-1892). Hirst was one of the first British mathematicians to study for a Ph. D. in mathematics, which he gained in 1852 at the University of Marburg in Germany. His thesis topic was the geometry of ellipsoids and his mathematical research was in the area of geometrical transformations. Hirst became Professor of Pure Mathematics at University College, London in 1866 and later was Director of Studies at the Royal Naval College, Greenwich. He thought highly of Salmon, as he expressed the following opinion of him in his journal, [7, p.34]:

...For my part the more I know him the more I respect him; by nature he is a single-minded, modest man, but he is endowed with an intellectual power and a capability of work such as we rarely meet.

Hirst was an influential administrator in mathematical and scientific societies, and he was instrumental in securing Salmon's election as a Fellow of the Royal Society of London in 1863, [7, p.35]. His review is quite detailed and perspicuous, and it pinpoints several reasons why Salmon's book was so popular. We present the whole of the review here, as it gives a good contemporary assessment of Salmon's masterpiece.

It is a source of considerable satisfaction to find, amongst the crowd of very imperfect educational books which are daily issued from our press, a treatise so truly valuable as the present; and it is also cheering to learn that the public has so far recognized its merits as to demand a third edition. The book is now sufficiently well known, otherwise its title might mislead many; for although it is true that the greater part of it is devoted to an examination of the properties of the conic sections, yet this being done from an analytical point of view chiefly, it was necessary to prepare the reader by a similar investigation of the properties of the line and circle; so that, in fact, the whole constitutes a very efficient treatise on the elements of analytic geometry, such as may with advantage be placed in the hands of every student who has mastered the elements of Euclid, plane trigonometry and algebra. We venture to assert, that amongst students already acquainted with its merits, this is one of their favourite text-books; for the treatment throughout is admirably clear, strict and elegant,–in fact, such as can be achieved only by one who, besides that perfect mastery of the subject which can only be acquired by original research, possesses also that unacquirable talent of lucid exposition, and is guided by that knowledge of the difficulties usually encountered by students, which experience only can give.

This third edition is revised and enlarged, and contains many improvements upon the former two, with respect to its type, arrangement, and its well-chosen and numerous selection of examples. The student will do well to work all these conscientiously, and to pay particular attention to those interesting chapters on "abridged notation." The last chapter contains a short but clear account of the most beautiful modern geometric methods, amongst which is that beautiful method of reciprocal polars first introduced by Poncelet, an acquaintance with which may be said to form an epoch in the history of every young mathematician.

The importance of these methods may be estimated from the fact, that in the hands of Steiner, with scarcely any help from algebra, they have become the most powerful instruments of discovery, and have given a wonderful insight into the nature and properties of curves; an insight which is, perhaps, more thorough and direct than any attainable by one who is accustomed to call in the more mechanical aid of algebraic calculation.

We do not wish here to revive the old and useless discussion on the comparative merits of the algebraic and geometric methods; both have undoubtedly their advantages, and both are indispensable. In the greater part of the present treatise, however, the former of these methods is adopted; and in drawing attention to the importance of the last chapter, we would merely remark, that if his object be to obtain a thorough knowlege of the properties of conics, the student will do well to combine both methods to a greater extent than is done here; for the fact cannot be disputed, that the very facility with which results can be obtained algebraically, may indirectly prevent that intimate acquaintance with the properties of curves which a rigid geometrical investigation also secures.

In view of Salmon's admitted indebtedness to Poncelet's pioneering work, especially that contained in [39], it may be informative to take into account some of Poncelet's published opinions on Salmon's work, in particular [CS, 1855]. Jean–Victor Poncelet (1788-1867) was the inventor of many of the techniques of projective geometry that were so popular in the 19th century, including the methods of duality, reciprocal polars, projection and complex methods. His mathematical career was blighted by priority disputes and criticism of his 'principle of continuity.' His comments in his later works, [40] and [41], show that he was particularly sensitive to the question of priority of discovery in mathematical research and he took such mathematicians as Hamilton and Cayley to task for failing to admit his (Poncelet's) priority in such matters as the problem of inscribing or circumscribing polygons in or around conic sections. (For more on the work of Poncelet, Cayley, and Salmon on this subject, see the article by J. A. Todd, [50].) Recalling Joly's remark, included in Section 2, that it is matter of extreme difficulty to say how much of the material in his textbooks was original to Salmon, Poncelet's comments at the end of the quotation below, taken from [41, pp.211-212], are understandable. Nonetheless, as the quotation indicates, Poncelet acknowledged that Salmon's *Conic Sections* helped to rescue the methods of his own *Traité des Propriétés projectives des figures*, [39], from relative oblivion.

Ces diverses propositions sur les polygones d'ordre pair et impair, inscrits ou circonscrits aux sections coniques, se retrouvent démontrées, ainsi que beaucoup d'autres, par la voie géométrique, dans le Chap. II, Sect. IV, de mon *Traité des Propriétés projectives des figures* (1822). Elles ont, depuis quelques années seulement, attiré l'attention de plusieurs éminents géomètres étrangers: MM. Moebius et Göpel (*Journal de Crelle*, 1848), sir William Hamilton (*Lectures on quaternions*, 1853), Rev. George Salmon (*Treatise on conic sections*, 1855, p.282)...

[There follows a description of the work of Moebius, Göpel, and Hamilton on inscribing and circumscribing polygons in or around polygons.]

Ces théorèmes [those of Moebius, Göpel, and Hamilton] ont une analogie évidente avec ceux des n^{os} 547 à 561 de ce Traité [*Propriétés projectives*], dont ils sont la simple extension au cas de l'espace, comme le remarque lui-même M. Hamilton, qui ne connaissait d'ailleurs mes propres recherches que par les citations de l'excellent *Traité des sections coniques* du Rev. D^{r} Salmon, postérieur au mien de plus de trente années.

A l'égard de ce dernier et savant ouvrage, la publication de 1855 jouit, en France, d'une juste célébrité, et n'a pas peu contribué à relever les méthodes de démonstration et de recherches géométriques du Traité des Propriétés projectives, de l'espèce de discrédit où elles étaient tombées depuis 1826, par suite de fâcheuses discussions de priorité et de préventions aussi peu justifiées que mal déguisées. Néanmoins, on doit regretter que, dans le n^{o} 338 de son remarquable Traité élémentaire, M. Salmon n'ait point accordé plus d'attention à l'importante et délicate théorie de l'inscription des polygones aux coniques, théorie que, à l'exemple de M. Townsend, il rattache à la considération, plutôt synthétique qu'analytique, bien que symbolique et abréviative, des *faisceaux projectifs* de droites convergentes, dont M. Göpel s'était également servi dans son Mémoire allemand, de 1848. Peut-être même, serait on en droit de lui reprocher, comme à tant d'autres, de n'avoir pas toujours tenu un compte suffisament exact de la différence, à mon sens capitale, entre découvrir et démontrer–Sir Hamilton me paraît avoir mieux saisi l'esprit et la portée de la question dont il s'agit.

Despite the criticism, Poncelet had a high opinion of Salmon, since in [40, p.405], he described him as

le loyale et docte Salmon, cet interprète bienveillant et lucide des nouvelles doctrines algébrico-géométriques,...

Another French geometer whose work influenced Salmon was Michel Chasles (1793-1880). Chasles himself wrote a treatise on conic sections, [12], but only the first part ever appeared. We found only one brief reference to Salmon in [12, p.321], which we include here:

M. Salmon, dans son excellent *Traité des Sections coniques* (4^{e} édition, page 291), est parvenu à une équation fort simple du lieu cherché.

5. A Treatise on the Higher Plane Curves

Salmon's second mathematical textbook was a *A Treatise on the Higher Plane Curves*, with the subtitle *intended as a sequel to A Treatise on Conic Sections*, published in 1852. As we mentioned in the previous section, this was the second volume of a treatise on the analytic geometry of space of two dimensions. It is aimed at a more advanced mathematical audience than [CS], and was suitable only for senior undergraduates or researchers. In the preface Salmon acknowledges the influence of Poncelet, [39], Chasles, [11], and Plücker on his work.

The book has seven chapters, dealing with the following subjects: tangential coordinates; general properties of algebraic curves; curves of the third degree; curves of the fourth degree; transcendental curves; general methods; applications of the integral calculus. There are also twenty six pages of notes. The notes are revealing, as they deal in part with determinants and elimination theory, and indicate the beginnings of Salmon's interest in invariant theory, the subject of his next book in 1859. There is also a short discussion of the meaning of complex number methods in geometry, including a mention of quaternions, then a subject of much research in Dublin.

The style of writing in [HPC] is looser than that in [CS] and at times Salmon displays a nonchalance with regard to his background reading, a trait we drew attention to in the previous section. In the preface he writes:

Considered as a book for advanced readers, I regret that I have not had the leisure for the reading necessary to make this work as complete as it ought to be. My knowledge of the older writers on Geometry is, for the most part, either second-hand or superficial; and I cannot even claim acquaintance with all the most remarkable works and memoirs which of late years have been published on the subject.

On p.239 he writes vaguely:

I ascribe the method of transformation to M. Chasles, because I know that I was familiar with it, and believed it to be his, long before the publication of any other papers on the subject cited below. The only writing of M. Chasles I can now refer to is note xxi. p.352, of the Aperçu Historique, which, however only treats of the case where $n = \frac{1}{2}$. I cannot tell whether I was led to the general method as an obvious extension of this, or whether some paper on the subject may not be found among M. Chasles's many contributions to scientific periodicals.

Then, as a footnote on p.269, he writes:

I find that I was mistaken in supposing (p.239) that M. Chasles had published anything on this subject beyond the note to the Aperçu Historique already referred to;...

This correction to his earlier surmisal is possibly explained by the usual method of printing a textbook, in which sections of manuscript were set into type by the University Press, while others were still being written.

On p.209, he shows a disregard for researching his sources, as he writes:

Mr. J. S. Mill (in a passage in his Logic, on which I cannot lay my hand) erroneously represents Galileo's experiments to have been successful,...

Salmon's slack style of writing did not meet with the approval of the more painstaking T. A. Hirst, who wrote as follows in his personal journal in 1859, [7, p.33]:

I have been writing French all week preparing my memoir [on derived surfaces] for *Tortolini's Journal*. It is a laborious task. There are paragraphs that I have written three or four times over before I could satisfy myself. I question whether many mathematicians of this day take equal pains to be lucid in their demonstrations... Salmon in his *Higher Plane Curves* has devoted a few pages to the same subject. I wish he had taken as much trouble to be clear in his demonstrations as I have done...

Ball wrote the following of [HPC] in [1, xxiv]:

I must, however, add, though I am perhaps expressing my own opinion, when I say that the *Higher Plane Curves* is the least interesting of Salmon's four great works [*sic*]. It lacks the magic of the *Conics*. It has not the importance in nature possessed by the *Quadrics*, which form so large part of the *Geometry of Three Dimensions*. Nor does it lead to such splendid theories and conceptions as do the lessons in *Modern Higher Algebra*.

[HPC] seems to have been the least successful in sales terms of his textbooks, since it only ran to three editions, the second not being issued until twenty one years after the first. We quote from [1, xxv] on the circumstances of its writing.

By the time a second edition of *Higher Plane Curves* was called for, Salmon had become a Professor of Divinity, and had thus made engagements which, in his own words, "left me no leisure to make acquaintance with recent mathematical discoveries, or even to keep up any memory of what I previously had known." In this emergency he applied to Professor Cayley to obtain advice as to the choice of some younger mathematician, who would undertake the editing of the work. To his agreeable surprise, Professor Cayley offered to do this himself. This offer was gratefully accepted, and the second edition was the joint work of Salmon and Cayley. ...

Cayley contributed the first chapter, ten pages long, on coordinates, as well as several articles to the new edition. Salmon also incorporated into the text material from various unpublished manuscripts by Cayley, including one on the theory of envelopes. The main changes in the second edition were the introduction of two new chapters on envelopes and on metrical methods, and the omission of one on applications of integral calculus.

We have the feeling that, initially at least, Salmon probably did not put as much effort into his writing of [HPC] as he did into his other textbooks (although the second edition was a considerable improvement on the first). While the book certainly had its adherents, and was essentially the only English language textbook on its subject for many years, it had less impact on mathematical teaching and research than his other books.

6. Lessons Introductory to the Modern Higher Algebra

The subject matter of [MHA] is largely invariant theory, or as Salmon called it, the algebra of linear transformations. The book grew out of what was originally intended to be an appendix for the then forthcoming book [GTD, 1862]. We observed in Section 5 that a similar short appendix had already appeared in [HPC, 1852]. Salmon's aim in writing [MHA] was to introduce his readers to the study of invariant theory, which had been developed in the 1840's and 1850's. The exact nature of invariant theory is difficult to explain, but it may be seen as a combination of linear algebra and algebraic geometry. Nowadays, it is partly subsumed in the theory of the representations of classical groups on tensor algebras. David Lewis has given a description of invariant theory and its methods in a previous edition of this *Bulletin*, [29].

Salmon traced the origin of invariant theory to a paper of Boole, [6], of 1841, and he acknowledged this paper for having introduced him to the study of linear transformations. Cayley had extended Boole's work to a much greater class of functions than those considered by Boole. As Salmon explained in the preface to [MHA, 1859]:

> The geometrical importance of this theory [invariant theory] is now manifest. When we are given the equation of any curve or surface, the theory of linear transformations at once presents us with equations representing other curves and surfaces, and possessing permanent relations to the given one, which will be unaffected by any change of the axes of co-ordinates. And in like manner the same theory presents us with certain functions of the given equation, the vanishing of which must express a property of the given curve or surface, wholly independent of the choice of axes. Besides these geometrical applications, the theory has other important uses, which I shall not stop to enumerate.

In rather similar vein, Salmon wrote in a footnote of [MHA, 1859, p.52]:

> ...invariants, then, are functions of the coefficients expressing certain fixed properties of the curve or surface which are independent of our

choice of axes; such as the condition that a curve or surface should have a double point, &C. Covariants represent certain other curves or surfaces having a fixed relation to the given one, independent of our choice of axes.

There are seventeen lessons in [MHA, 1859], but we will not attempt to describe them in any detail. The topics dealt with include determinants, elimination theory, discriminants, quantics, invariants and covariants, canonical forms, Sturm's theorem, and orthogonal transformations. A proof of Borchardt's theorem, that the eigenvalues of a real symmetric matrix are real, is begun on p.127. This proof, using Sturm remainders, is considerably more complicated than modern proofs, which exploit eigenvectors in conjunction with the associated quadratic form.

Like the third edition of [CS], the first edition of [MHA] was the subject of a review in the *Philosophical Magazine* (**18** (1859), 67-68). We quote part of the review, which makes a poor attempt at being humorous, largely at the expense of the exotic names which had been introduced to the subject by Sylvester. Reading the whole review, most of which consists of quotations from the preface of [MHA], we can only conclude that the reviewer scarcely read beyond the preface.

Within the last eighteen years the old and well-trodden field of Algebra has been invaded by a host of new and strange intruders, with the odd sounding names of 'Determinants,' 'Hyperdeterminants,' 'Discriminants,' 'Emanants,' 'Invariants,' 'Evectants,' 'Bezoutiants,' 'Hessians' (having no connexion, however, with either 'Boots' or 'Crucibles'), 'Canonizants' (of no religion), 'Dialytics,' and 'Quantics.' Many a reader of the Cambridge Mathematical Journal, the Philosophical Magazine, Philosophical Transactions, &c, has wondered what it all meant–wondered sometimes, indeed, whether there was *any meaning at all* in these new expressions and symbols. Very few even of the best mathematicians of the day have paid much attention to the subject as yet; but they are beginning to do so, finding that there is really something like a new branch growing out of their old tree–nay, more, that the young off–shoot is already bearing fruit. ...

In view of Cayley's importance to the development of invariant theory, it may be worthwhile to note an article entitled *Recent terminology in mathematics* that Cayley wrote in 1859 or 1860, [10]. The subject matter is algebra, especially invariant theory. It touches upon groups (including the symmetric group S_3), determinants, matrices, linear transformations, invariants, covariants, canonical forms, among other things. One of the most prescient of Cayley's articles describes matrices, which he had brought to the public's attention in a paper of 1858. Although not strictly relevant to Salmon's work, for Salmon speaks exclusively of determinants, we cannot forbear from quoting a little of Cayley's writing:

Moreover, in a system of simple equations, if the coefficients in the natural square order are considered apart by themselves, this leads to the theory of *matrices*, a theory which indeed might have preceded that of determinants; the matrix is, so to speak, the matter of a determinant;...

Here is his description of linear transformations, leading to the subject of invariance:

LINEAR TRANSFORMATIONS.– In this theory the variables of a function are supposed to be respectively linear functions of a new set of variables, so that the function is transformed into a similar function of these new variables, with of course altered values of the coefficients, and the question was to find the relations which existed between the original and new coefficients and the coefficients of the linear equations. The determinant composed of the coefficients of the linear equations is said to be the *modulus of transformation*, and when this determinant is unity the transformation is said to be *unimodular*. It was observed that a certain function of the coefficients, namely the discriminant, possessed a remarkable property, found afterwards to belong to it as one of a class of functions called originally hyperdeterminants, but now *invariants*, and it was in this manner that the problem of linear transformation led to the general theory of covariants.

Likewise, here is his definition of invariant:

INVARIANT.– An invariant is a function of the coefficients of a rational and integral homogeneous function or quantic, the characteristic property whereof is as follows: namely, if a linear transformation is effected on the quantic, then the new value of the invariant is to a factor *près* equal to the original value; the factor in question (or quotient of the two values) being a power of the modulus of transformation, and the two values being thus equal when the transformation is unimodular....

After listing the principal textbooks on the subjects of the article (by Spottiswoode, Brioschi, Baltzer, Faà di Bruno), Cayley concluded his article by recommending Salmon's new textbook:

And extending to nearly all the subjects: Salmon, 'Lessons introductory to the modern higher algebra,' 8vo, Dublin, 1859.

When the second edition of [MHA] appeared in 1866, it was double the size of the first edition, and Salmon apologized in the preface that its title was no longer appropriate, as the work constituted more of a treatise than a set of lessons. The new edition was noteworthy for an extensive number of explicit calculations of invariants. Lesson XVII (Applications to binary quantics) contains various vast formulae, each occupying several pages. The book may be said to be notorious for one particular calculation, that of a skew invariant E of degree fifteen attached to a binary sextic. The result of the calculation contains 1367 terms and it occupies thirteen pages. Ball thus described Salmon's work on E in his obituary, [1, xxvii]:

It is in this book [MHA, 1866] that Salmon has undertaken some of the most formidable calculations that any pure mathematician has faced. The portentous invariant E alone requires several pages for its expression. He has told us that, when the work was done, he proceeded to check it by testing whether the sum of the coefficients was zero [as must be the case for a skew invariant]. He was dismayed by finding a balance of several thousands, but, divining a possible error, he halved this balance, and finding this was exactly one of the two coefficients, he immediately surmised a wrong sign, which proved to be the case, and the verification was complete.

The two subsequent editions of [MHA] omitted this spectacular formula. Salmon also alludes on p.190 of [MHA, 1866] to a formula of almost 900 terms, which he had calculated and which appears in a paper of Cayley, [Phil. Trans. 1858, p.455]. He spared his readers the details.

The inclusion of these calculations, which was hardly vital to the object of introducing readers to the latest researches on invariant theory, indicates an aspect of Salmon's mathematical personality–a tendency to be involved in calculation for calculation's sake. Joly disapproved of this tendency, since he wrote, [26, p.355]:

It is, however, curious that the fascination with arithmetical work should have detained Salmon on calculations such as that of E [as above] at a time when Boole's great conception was pushing on the mathematical world to feverish haste in new discovery.

T. A. Hirst also observed the love of calculation, [7, p.33]:

He is a great calculator, fond of calculating for its own sake. I do class him among the high mathematicians however. The more ready-reckoning element is too prominent in him.

Although a fourth edition of [MHA] appeared in 1885, Salmon admitted in its preface that he had not contributed to it. He wrote:

The pressure of other engagements having prevented me from taking any part in the preparation of this new edition of my *Modern Higher Algebra*, I have to express my obligations to the good offices of my friend Mr. Cathcart in revising the work and superintending its passage through the press. ...

The contribution to the spread of knowledge about invariant theory made by the various editions of [MHA] and Fiedler's German translations is highlighted in the following extract from a report, [36], made to the Deutsche Mathematiker-Vereinigung in 1892 by W. F. Meyer (1856-1934), himself a specialist in invariant theory. We quote from the English translation given by Merz in [35, p.685]:

Recognising how the special results in this domain [invariant theory] gradually acquired a great bulk, we must the more gratefully acknowledge the work of Salmon–who had already, in the direction of algebra as well as geometry, furnished valuable contributions of his own–in undertaking the labour of collecting the widely-scattered material in a concise monograph. For the promulgation in Germany we have to thank Fiedler both for his edition of Salmon, and for having already given an independent introduction to the subject, ...

As David Lewis's article [29] makes clear, interest in the subject of invariant theory waned by the start of the 20th century, partly because Hilbert's abstract methods had solved some of the central finiteness problems of the subject. Invariant theory began to be reinterpreted in the 20th century using the tools of group representation theory. One may compare Salmon's treatment of Hermite's law of reciprocity for the invariants of binary quantics, [MHA, 1859, p.82], with that of Speiser in 1937, [46, §73]. Speiser makes use of the Clebsch-Gordan formula for the representations of the group $\mathrm{GL}_2(\mathbf{C})$. The noted group theorist Philip Hall drew attention to the effectiveness of more modern representation theory in his review of Speiser's book in [21]:

> The fundamental nature of the connection between the classical theory of forms and representation theory is even today not so widely appreciated as it deserves to be. That after only seven pages Speiser is already completing his second proof of Hermite's reciprocity theorem for binary forms is an indication both of the usefulness of group-theoretic methods and also the smoothness of the present treatment.

7. A Treatise on the Analytic Geometry of Three Dimensions

Salmon's last mathematical book was *A Treatise on the Analytic Geometry of Three Dimensions*, first published in 1862. This was the largest of Salmon's works, 465 pages in length, and the later editions were even longer. The first edition contained fifteen chapters, on the following subjects: the point; interpretation of equations; the plane; properties of quadrics in general; classification of quadrics; central surfaces; methods of abridged notation; confocal surfaces; cones and sphero-conics; general theory of surfaces; curves and developables; families of surfaces; surfaces derived from quadrics; surfaces of third degree; general theory of surfaces. Chapter XIII (Surfaces derived from quadrics) begins with a discussion of the wave surface, introduced by Fresnel in his study of light. The wave surface had been studied by W. R. Hamilton, who had predicted the phenomenon of conical refraction on the basis of his analysis of cusps on the surface. The surface had also been studied by the Trinity College mathematical physicist, James MacCullagh. Chapter XIV (Surfaces of the third degree) contained the theory of the twenty seven lines on a cubic surface, which we have already described in Section 2.

An interesting aspect of [GTD, 1862] is an article in the appendix on the calculus of quaternions, applied to geometrical investigations. Hamilton had invented the quaternions in 1843 and both Salmon and Cayley had attended his lectures in TCD on quaternions in June 1848. Hamilton was anxious to see the acceptance of quaternionic methods as standard research tools, especially in geometry, and R. P. Graves records a correspondence in 1857 between Hamilton and Salmon on the subject of quaternions in [20, pp.86-92]. While Salmon seems to have maintained doubts as to whether quaternions could be used to prove anything more conveniently than by traditional methods, he nonetheless admitted the beauty of quaternionic methods. Hamilton entertained high hopes that Salmon would become as great a devotee of quaternions as he himself was, and he encouraged Salmon to apply quaternions with his own geometric insight, as the following portion of a letter, [20, p.91], from Hamilton to Salmon, dated 15 September 1857, makes clear:

> As you seem to have withdrawn, for the present, from the subject [quaternions], let me at least ask you to allow me to thank you for the kind attention which you have given lately to it.
>
> And do not imagine it to be flattery, if I add that I consider you to be *very likely to excel myself*, even in *this*, which (of these late years) may be regarded as my *own* department, if you shall ever be induced to pursue it...... You possess *a far larger stock of geometrical knowledge*, although I have always had something of a geometrical *taste*, and carried away all the honors of my division in geometry, when I was, very long ago, an undergraduate in our University. If you shall ever seriously take up the Quaternions, your *geometry* will enable you to go far beyond *me* in that subject.

Salmon's interest in quaternions seems to have diminished from this time and Hamilton's hopes for a high-profile disciple at TCD were never fulfilled. By the time of the third edition of [GTD], the appendix on quaternions had disappeared, as Salmon considered that the subject was now adequately handled in the textbook of Kelland and Tait (1873).

Title page of
Lessons introductory to the Modern Higher Algebra

Just as he had assisted Salmon in the preparation of a new edition of [HPC], so also did Cayley assist him in writing the third edition of [GTD], which appeared in 1874. Cayley's contribution was less than his previous one, but he nonetheless provided several new articles, and influenced Salmon's writing of other articles, as Salmon explains in the preface to [GTD, 1874]. When the fourth edition of [GTD] was being prepared, Salmon took no part in the revision, leaving this to his colleague Mr Cathcart, who likewise revised the fourth edition of [MHA] in 1885, as described in Section 6.

[GTD] was the only one of Salmon's mathematical books to undergo a substantial revision after his death, for a fifth edition appeared in two volumes, published in 1912 and 1915. The editor of the new work was Reginald A. P. Rogers, a fellow of TCD. The following review of the second volume, by M. Long, [30], show that enthusiasm for Salmon's writing style and his choice of content still remained high ten years after his death.

It is interesting to note how little change it has been necessary to make in Salmon's own work. A word here and there has been changed, but otherwise the original text is almost intact, and the style and character of it have been carefully followed by the editor and those who have helped him...... It is not the least of Salmon's many virtues as a writer of text-books that what he says may be added to by later editors, but need rarely be displaced to make way for newer methods of treatment. Salmon once said of Cayley that he "both discovered the 'diggin's' and got out some of the biggest nuggets." It is often difficult to sort out Salmon's own "nuggets," but there can be no doubt of his genius for cleaning up and arranging those already found, and for pointing out the "diggin's" which would repay further work. Certainly many of the "nuggets" contained in this new volume are the direct product of the indications he gave of directions in which further progress might be made.

The mining analogy which Salmon applied to Cayley's work in invariant theory was made in Salmon's appreciatory article, [44]. Ball continued the analogy (although in the context of stone quarrying, rather than gold mining) when he wrote in [2, pp.268-269]:

Dr. Salmon writes about Conic Sections–and his work has been a quarry in which all other writers at home and abroad have ever since mined without even exhausting its resources.

In a paper, [17], published as late as 1980, W. L. Edge pointed out that all five editions of [GTD], as well as its French and German translations, contained an error in the calculation of the discriminant of a cubic surface in three dimensional projective space. Edge provided corrected formulae and, despite the error, he also testified to the clarity of Salmon's writing.

8. Salmon's legacy and continuing influence

We have provided documentation in this article, from varied sources, to suggest that Salmon was one of the most influential textbook writers of the 19th century. His gift, as others saw it, was to draw together subjects of current research, display them to the reader, and illustrate the theory by well-chosen examples. Inevitably, a mathematician of a later century who reads Salmon's books will find them lacking in precision and rigour, but his style of exegesis was typical for the time, when the definition, lemma, theorem, proof mode of presentation found in current mathematical textbooks did not exist.

The subject matter of two of Salmon's books, [HPC] and [GTD], would be classified as algebraic geometry, a subject well known for the difficulties it presents. Until the 1940's, most books on algebraic geometry were imprecise and often appealed to intuition or generic arguments that excluded degenerate or troublesome cases. One has only to examine textbooks of, say, J. L. Coolidge, to observe this tendency. Salmon himself was not exempt from criticism of the inadequacy of his treatment of certain topics, as the following review by F. Bath of H. Hilton's *Plane Algebraic Curves* (1932), [3], shows:

Unfortunately the theorem regarding the postulation of an r-ic through certain of the intersections of an n-ic and an N-ic and its proof have been left in this chapter as Salmon left them, incomplete and misleading. It is to be regretted that the true theorem embracing all cases has not yet been made available in an English text-book, particularly as the essential result was known to Jacobi, ...

As the arguments of algebraic geometry were clarified and made rigorous by such mathematicians as O. Zariski, B. van der Waerden, A. Weil, and others, so the subject became less intuitive and perhaps less accessible to the outsider who did not wish to be burdened by notions of intersection multiplicities, divisor classes, holomorphic differentials, transcendence degree, and so on. However, as Zariski said at the International Congress of Mathematicians in 1950, "Our aim is not to prove that our fathers were wrong, but that they were right!". Salmon's non-rigorous arguments have been replaced by rigorous, if longer arguments. To see this in practice, a more modern proof of a theorem of Salmon on cubic curves may be found in [54, p.194], and an updated version of a favourite topic of Salmon, systems of conics and quadrics, may be studied in [51, pp.174-270].

While modern research requires a different set of textbooks for its basic references, compared with those used in 19th century research, nonetheless Salmon's textbooks continue to be quoted in the research literature, as we have found by browsing through a few journals. To take a random example, in a collection of copies of *Mathematical Proceedings of the Cambridge Philosophical Society* in our office, dated between 1975 and 1981, we found three references to Salmon's treatises, [16], [38], and [55], which is a good citation record for books that were nearly 100 years old at the time they were cited. At the time of this writing, the most recent citation to Salmon's work that we have seen is [13], of 1996.

Salmon's place in the history of mathematics appears to be assured, as his textbooks gained an unusually high degree of recognition for at least 50 years, and in the case of *Conic Sections*, for more than 100 years. Recognition occurred both at the elementary level and at the research level. Of university-level mathematics books written or published in Ireland, only Casey's *Sequel to Euclid* or Burnside and Panton's *Theory of Equations* can have been as successful as Salmon's works, and as scientific publishing is virtually non-existent now in Ireland, it is unlikely that any other scientific book published in Ireland will ever be as successful as Salmon's.

Further progress on Salmon's career and his relationships with contemporary mathematicians will require study of his correspondence. According to [49], there are family papers and letters of Salmon in the TCD library, and we have already noted that the Hamilton correspondence contains several letters from Salmon to Hamilton on the subject of quaternions, dating from 1857. In addition, the library of University College London has 18 letters from Salmon to T. A. Hirst dated between 1858 and 1873.

It is perhaps difficult now for us to appreciate how highly Salmon, an academic and theologian, was regarded during the latter part of his life. The many appreciations of Salmon that we have read, written both before and after he died, all portray a man admired to an extraordinary extent. As it states in [32, p.292]:

> More, perhaps, than any member of the College throughout its history he left his contemporaries the impression of a great man, and posterity has seen no reason to reverse this verdict.

English Language Editions of Salmon's Books

[AGI] A Treatise on Analytic Geometry. Part I. The Right Line and Circle. 8vo, iv, 108p. Hodges and Smith: Dublin, 1847.

[AGII] A Treatise on Analytic Geometry and the Theory of Plane Curves. Part II. Conic Sections. 8vo, i, [1, errata], 192p. Hodges and Smith: Dublin, 1848.

[CS] A Treatise on Conic Sections. 8vo, ii, [1, errata], 300p. Hodges and Smith: Dublin, 1848.

2nd ed. 8vo, iv, 343p. Hodges and Smith: Dublin, 1850.

3rd ed. 8vo, xv, 324p. Longman, Brown, Green, and Longmans: London, 1855.

4th ed. 8vo, xvi, 362p. Longman, Brown, Green, Longman, and Roberts: London, 1863.

5th ed. 8vo, xv, 377p. Longmans, Green, Ryder, and Dyer: London, 1869.

6th ed. 8vo, xv, 399p. Longmans, Green, and Co.: London, 1879.

[HPC] A Treatise on the Higher Plane Curves. 8vo, xii, 316p. Hodges and Smith: Dublin, 1852.

2nd ed. 8vo, xix, 379p. Hodges, Foster, and Co.: Dublin, 1873.

3rd ed. 8vo, xix, 395p. Hodges, Foster, and Figgis: Dublin, 1879.

[MHA] Lessons Introductory to the Modern Higher Algebra. ix, 147p. Hodges, Smith, and Co.: Dublin, 1859.

2nd ed. 8vo, xv, 296p. Hodges, Smith, and Co.: Dublin, 1866.

3rd ed. 8vo, xix, 318p. Hodges, Foster, and Co.: Dublin, 1876.

4th ed. 8vo, xv, 360p. Hodges, Figgis, and Co.: Dublin, 1885.

[GTD] A Treatise on the Analytic Geometry of Three Dimensions. 8vo, xv, [1, errata], 465p. Hodges, Smith, and Co.: Dublin, 1862.

2nd ed. 8vo, xv, 520p. Hodges, Smith, and Co.: Dublin, 1865.

3rd ed. 8vo, xvii, [1, errata], 583p. Hodges, Foster, and Co.: Dublin, 1874.

4th ed. 8vo, xix, [1, errata], 612p. Hodges, Figgis, and Co.: Dublin, 1882.

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Translations of Salmon's Books

(a) Analytische Geometrie der Kegelschnitte, mit besonderer Berücksichtigung der neueren Methoden von G. Salmon. Unter Mitwirkung des Verfassers. Deutsch bearbeitet von Dr Wilhelm Fiedler. Teubner: Leipzig, 1860.

(b) Analytische Geometrie des Raumes von George Salmon. Deutsch bearbeitet von Dr Wilhelm Fiedler (2 vols.). Teubner: Leipzig, 1863.

(c) Vorlesungen zur Einführung in die Algebra der linearer Transformationen von George Salmon. Deutsch bearbeitet von Dr Wilhelm Fiedler. viii, 271p. Teubner: Leipzig, 1863.

(d) Analytische Geometrie der höheren ebenen Curven von George Salmon. Deutsch bearbeitet von Dr Wilhelm Fiedler. xvi, 471p. Teubner: Leipzig, 1873.

(e) Leçons d'algèbre supérieur. Traduit de l'anglais par M. Bazin. Augmenté de notes par M. Hermite. xii, 274p. Gauthier-Villars: Paris, 1868.

(f) Traité de géométrie à deux dimensions (sections coniques). Traduit de l'anglais sur la cinquième édition par H. Resal et V. Vaucheret. Gauthier-Villars: Paris, 1870.

(g) Traité de géométrie analytique à trois dimensions. Ouvrage traduit de l'anglais sur la quatrième édition par O. Chemin. Paris, 1882-92.

(h) Traité de géométrie analytique (courbes planes). Ouvrage traduit de l'anglais par O. Chemin et suivi d'une étude sur les points singuliers par G. Halphen. xxiii, 667p. Gauthier-Villars: Paris, 1884.

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6. *On the number of normals which can be drawn from a given point to a given surface*, Camb. and Dubl. Math. J. **3** (1848), 46-47; Nouv. Ann. Math. **9** (1850), 274-276.
7. *Note on a result of elimination*, Camb. and Dubl. Math. J. **3** (1848), 169-173.
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9. *On the triple tangent planes to a surface of the third order*, Camb. and Dubl. Math. J. **4** (1849), 252-260.
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17. *Exercises in the hyperdeterminant calculus*, Camb. and Dubl. Math. J. **9** (1854), 19-33.
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20. *Geometrical notes*, Quart. J. Math. **1** (1857), 237-241.
21. *On the order of certain systems of equations*, Quart. J. Math. **1** (1857), 246-257.
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