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Speaker that uses the non- linearity in air to create sound

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Abstract:

The aim of this work was to investigate theoretically and experimentally the fact that it is possible to create sound by using the nonlinearity in air. A speaker was built verifying the theory that if at least two high frequencies at high amplitude is propagating through a nonlinear medium, for example air, the difference frequency of these two is created in mid air. Measurements and simulations were performed on what the pressure distribution from a ultrasonic transducer looks like.

Keywords:

Parametric loudspeaker, Difference frequency generation, Biharmonic boundary condition, Nonlinear acoustics

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1 Notation

A	Constant
a	Radius or width (m)
b	Height (m)
c	Sound velocity (m/s)
f	Frequency (Hz) or function of outgoing wave
g	Function of incoming wave
i	Imaginary number
k	Wave number (rad/m)
P_0	Pressure amplitude (kg/ms ²)
p	Pressure (Pa)
Q	Volume flow (m ³ /s)
\dot{Q}	Source strength (m ³ /s ²)
\dot{q}_v	Source strength per unit volume (1/s ²)
\dot{q}_a	Source strength per unit area (m/s ²)
R	Distance (m)
R_0	Rayleigh distance (m)
r	Radial direction
S	Area (m ²)
t	Time (s)
u	Velocity (m/s)
\dot{u}_p	Piston acceleration (m/s ²)
x	Direction
y	Direction
z	Displacement (m)

<i>g</i>	Ratio of specific heat
<i>l</i>	Wavelength (m)
<i>x</i>	Displacement (m)
<i>r</i>	Density (kg/m ³)
<i>s</i>	Radial direction
<i>t</i>	Time coordinate system (s)
<i>f</i>	Velocity potential (m/s)
<i>y</i>	Angle (rad)
<i>w</i>	Angle velocity (rad/s)

2 Introduction

Often when one thinks about creating sound one thinks about a speaker that uses a piston movement to create the sound. This way of creating sound is not the only way. The audible sound can be created in midair by using ultrasound and the non-linearity in air.

If one sends out two frequencies with high frequency and amplitude into a nonlinear medium, in our case air, a third frequency can be heard, namely the difference frequency. In this thesis this phenomenon will be shown both theoretically and by measurement on a speaker, which was constructed within the project. There are many advantages of creating sound this way instead of the ordinary way. One of the advantages is that the sound that is created is very directed even of low frequencies. The directivity of sound is a problem in conventional speaker. The problem is that the low frequencies, is spreading in almost all directions in the room, and the higher frequencies are more directed. If one instead creates sound by sending out ultrasonic frequencies the nonlinearly created audible sound get the same directivity as the ultrasonic frequencies, which have a very narrow spreading beam. A simile can be done with a light bulb and a torch, the conventional speaker radiates the sound like a light bulb is radiates the light, the ultrasonic speaker is on the other hand radiating the sound more like a torch or a spotlight see figure 2.1.



Figure 2.1. Light bulb versus torch.

The directivity of a speaker that creates sound using ultrasound can be a huge advantage in for example a museum where you only want the people

in front of a particular painting to hear the information of that painting, and the rest of the visitors can concentrate on the other parts of the exhibition. Another interesting advantage is that the speaker can be built very thin and that can be of commercial interest in these days of flat screen TVs. For a comparison between the thicknesses see figure 2.2



Figure 2.2. Comparison of thickness of speaker.

In figure 2.2 the sound creating units of the speaker is compared. A conventional speaker does not produce sound very well without a speaker box and the speaker box need to be quite large. In our case with the speaker that uses the nonlinearity in air to create sound there is no need for any speaker box.

3 Vibrations and sound

A medium that is in complete rest is silent i.e. there is no sound. When for example people talk, vibrations occur to vocal cords. As a result of these vibrations sounds are created. This means that the vibrations are the source of sound. Other sources of sound are loudspeakers and all kinds of music instrument. But all vibrations are not audible, they have to be in the right frequency and amplitude. The audible sound for a human lies in the range of about 20 to 20,000 Hz with pressure range about $2 \cdot 10^{-5}$ to about 100 Pa. Usually sound pressure is given in decibels, dB. Decibel is not a unit in the sense that a meter or a gram, a decibel is a relationship between two values of power. In this case the relationship is often between the lowest hear able sound pressure namely about 10^{-5} Pa and the measured sound pressure [1].

The formula for decibels is:

$$dB = 10 \cdot \frac{PowerA}{PowerB} \quad (3.1)$$

Power of sound varies as the square of pressure and if a value is squared its logarithm is doubled, so the decibels formula becomes.

$$dB = 10 \cdot \log \left(\frac{Sound\ pressure\ A}{Sound\ pressure\ B} \right)^2 = 20 \cdot \log \frac{Sound\ pressure\ A}{Sound\ pressure\ B} \quad (3.2)$$

As mentioned previously sound pressure B is often the lowest sound pressure that is possible to hear, namely $2 \cdot 10^{-5}$ Pa. A sound pressure of 100 Pa is 133 dB (re 20 μ Pa).

$$20 \cdot \log \frac{100}{2 \cdot 10^{-5}} \approx 133\text{ dB (re } 20\mu\text{Pa)} \quad (3.3)$$

The pain threshold is about 120 dB (re 20 μ Pa).

Sound with frequencies lower than 20 Hz is often called infrasound and sound with higher frequencies than 20 kHz is called ultrasound. Ultrasound is going to be used a lot in this thesis.

Sound travels in an elastic medium in a form of wave motion. It is often called sound wave. Waves can be of various types the most common types are longitudinal waves and transverse wave. Sound waves propagating in air are longitudinal waves. The wave length depends on the frequency and the sound velocity.

$$I = c/f \tag{3.4}$$

Ultrasonic sound is often created with piezoceramic elements that transform an electric signal to sound. The sound beam the elements create can be divided in a couple of zones [2]. The zone nearest the element is called the near field or the Fresnel zone, in this zone the amplitude changes substantially. The amplitude changes is due to that the same wave is sent from each part of the element, the phase difference between all these waves affect the amplitude. On greater distance from the element these phase differences becomes less important. Therefore the wave gets more stable and spread more like a spherical wave in the so called far field or Fraunhofer zone. The length of the Fresnel zone depends both on the frequency and the size of the element, this will be shown in chapter 4.

4 Basics of nonlinear acoustics

The wave equations, which are usually used in solutions to acoustical problems, are valid only when the signal propagation is relatively small. In the derivation of these formulas the maximum displacement of the air particles is assumed to be small compared to the wavelength. This makes the density appear to be a linear function of the pressure. When these assumptions are no longer valid the wave will change shape as it propagates in the medium.

Each part of the wave travels with a velocity that is the sum of the signal velocity and the particle velocity [3]. In other words the peaks travel with a higher velocity than the rest of the wave. This makes the wave deforming as it propagates.

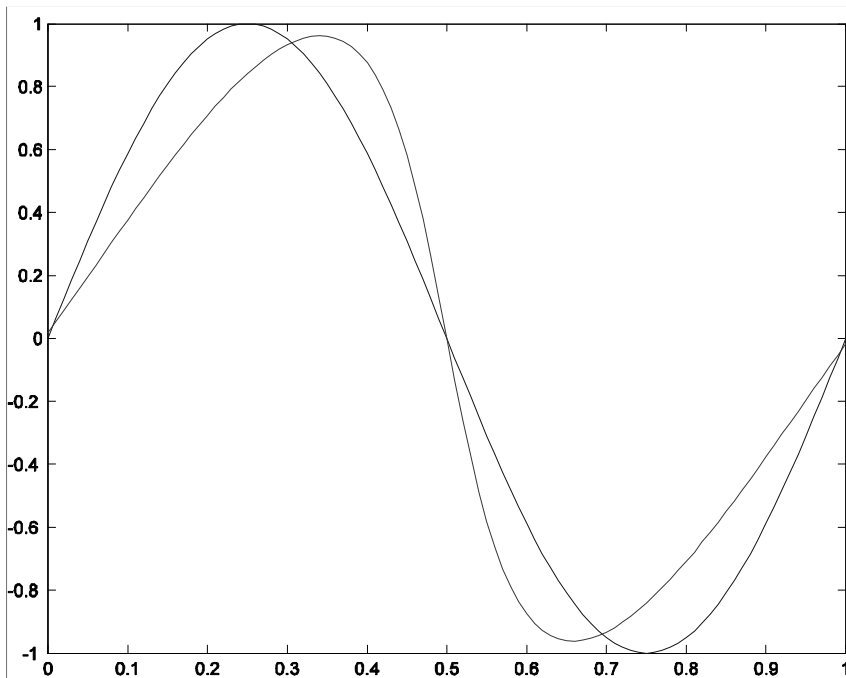


Figure 4.1 . Wave as it propagates through the medium

In this thesis the fact that the non-linearity in air creates new frequencies if at least two frequencies are sent in to air is very important. In the following example this fact will be shown.

4.1 Difference and Sum Frequency

We start with three equations of hydrodynamics for one-dimensional plane motion [4].

The first one is a generalization of the second Newton law with respect to a continuous medium.

$$\mathbf{r}_0 \frac{\partial^2 \mathbf{x}}{\partial t^2} = - \frac{\partial p}{\partial x} \quad (4.1)$$

Where the variable $\mathbf{x}(x, t)$ is the displacement of the medium particles from the initial position \mathbf{x}_0 .

The second one is the law of conservation of mass written down in differential form.

$$\mathbf{r}_0 = \mathbf{r} \left(1 + \frac{\partial \mathbf{x}}{\partial x} \right) \quad (4.2)$$

And the third one is the equation of state.

$$p = p(\mathbf{r}) = p_0 \left(\frac{\mathbf{r}}{\mathbf{r}_0} \right)^g \quad (4.3)$$

(4.2) can be rewritten.

$$\mathbf{r} = \frac{\mathbf{r}_0}{1 + \frac{\partial \mathbf{x}}{\partial x}} \quad (4.4)$$

Put (4.4) into (4.3)

$$p = p(\mathbf{r}) = p_0 \left(\frac{\mathbf{r}_0}{1 + \frac{\partial \mathbf{x}}{\partial x}} \right)^g \quad (4.5)$$

$$p = p_0 \left(\frac{\mathbf{r}}{\mathbf{r}_0} \right)^g = p_0 \left(\frac{\frac{\mathbf{r}_0}{1 + \frac{\partial \mathbf{x}}{\partial x}}}{\mathbf{r}_0} \right)^g = p_0 \left(1 + \frac{\partial \mathbf{x}}{\partial x} \right)^{-g} \quad (4.6)$$

(1.6) into (1.1)

$$\mathbf{r}_0 \frac{\partial^2 \mathbf{x}}{\partial t^2} = -\frac{\partial}{\partial x} \left(p_0 \left(1 + \frac{\partial \mathbf{x}}{\partial x} \right)^{-g} \right) \quad (4.7)$$

$$\mathbf{r}_0 \frac{\partial^2 \mathbf{x}}{\partial t^2} = p_0 \mathbf{g} \frac{\partial^2 \mathbf{x}}{\partial x^2} \left(1 + \frac{\partial \mathbf{x}}{\partial x} \right)^{-g-1}$$

rewriting

$$\frac{\partial^2 \mathbf{x}}{\partial t^2} = \frac{p_0 \mathbf{g}}{\mathbf{r}_0} \cdot \frac{\frac{\partial^2 \mathbf{x}}{\partial x^2}}{\left(1 + \frac{\partial \mathbf{x}}{\partial x} \right)^{g+1}} \quad (4.8)$$

This equation is the non-linear Earnshaw equation.

The denominator can be rewritten with the approximate relationship in form of the general type nonlinearity expansion in terms of power nonlinearities.

$$\left(1 + \frac{\partial \mathbf{x}}{\partial x} \right)^{-(g+1)} \approx 1 - (\mathbf{g} + 1) \frac{\partial \mathbf{x}}{\partial x} + \frac{1}{2} (\mathbf{g} + 1)(\mathbf{g} + 2) \left(\frac{\partial \mathbf{x}}{\partial x} \right)^2 \quad (4.9)$$

Substituting this expression in Earnshaws equation gives.

$$\frac{\partial^2 \mathbf{x}}{\partial t^2} = \frac{p_0 \mathbf{g}}{\mathbf{r}_0} \cdot \frac{\partial^2 \mathbf{x}}{\partial x^2} \left(1 - (\mathbf{g} + 1) \frac{\partial \mathbf{x}}{\partial x} + \frac{1}{2} (\mathbf{g} + 1)(\mathbf{g} + 2) \left(\frac{\partial \mathbf{x}}{\partial x} \right)^2 \right)$$

This can be written:

$$\frac{\partial^2 \mathbf{x}}{\partial t^2} = c_0^2 \cdot \frac{\partial^2 \mathbf{x}}{\partial x^2} \left(1 - (\mathbf{g} + 1) \frac{\partial \mathbf{x}}{\partial x} + \frac{1}{2} (\mathbf{g} + 1)(\mathbf{g} + 2) \left(\frac{\partial \mathbf{x}}{\partial x} \right)^2 \right) \quad (4.10)$$

where $c_0 = \left(\frac{p_0 \mathbf{g}}{r_0} \right)^{\frac{1}{2}}$ is the equilibrium sound velocity.

$$\frac{\partial^2 \mathbf{x}}{\partial t^2} \frac{1}{c_0^2} = \frac{\partial^2 \mathbf{x}}{\partial x^2} - \frac{\partial^2 \mathbf{x}}{\partial x^2} \cdot (\mathbf{g} + 1) \frac{\partial \mathbf{x}}{\partial x} + \frac{\partial^2 \mathbf{x}}{\partial x^2} \cdot \frac{1}{2} (\mathbf{g} + 1)(\mathbf{g} + 2) \left(\frac{\partial \mathbf{x}}{\partial x} \right)^2 \quad (4.11)$$

This can be rewritten to a nice nonlinear wave equation containing quadratic nonlinear and cubic nonlinear terms.

$$\frac{\partial^2 \mathbf{x}}{\partial x^2} - \frac{1}{c_0^2} \frac{\partial^2 \mathbf{x}}{\partial t^2} = (\mathbf{g} + 1) \frac{\partial \mathbf{x}}{\partial x} \cdot \frac{\partial^2 \mathbf{x}}{\partial x^2} - \frac{1}{2} \cdot (\mathbf{g} + 1)(\mathbf{g} + 2) \left(\frac{\partial \mathbf{x}}{\partial x} \right)^2 \frac{\partial^2 \mathbf{x}}{\partial x^2} \quad (4.12)$$

To investigate if any new frequencies are created when a couple of frequencies are propagating through air we are doing a simple calculation.

The calculations are made with the biharmonic boundary condition at $x=0$:

$$\mathbf{x} = A_1 \sin \mathbf{w}_1 t + A_2 \sin \mathbf{w}_2 t \quad (4.13)$$

This is a signal which has two frequencies before it has propagated in air.

where A_1 is the amplitude of the first frequency A_2 the amplitude for the second one. The frequencies \mathbf{w}_1 and \mathbf{w}_2 are in radians.

We start to investigate the quadratic non-linearity.

$$\frac{\partial^2 \mathbf{x}}{\partial x^2} - \frac{1}{c_0^2} \frac{\partial^2 \mathbf{x}}{\partial t^2} = (\mathbf{g} + 1) \frac{\partial \mathbf{x}}{\partial x} \cdot \frac{\partial^2 \mathbf{x}}{\partial x^2} \quad (4.14)$$

The left hand side is linear so it is the right hand side that is important in this case.

First of all we change the coordinate system to a coordinate system that travels with the wave with the sound velocity, $\mathbf{t} = \left(t - x/c_0 \right)$. The signal is written with the new coordinate system.

$$\mathbf{x} = A_1 \sin \mathbf{w}_1 \mathbf{t} + A_2 \sin \mathbf{w}_2 \mathbf{t} = A_1 \sin \mathbf{w}_1 \left(\mathbf{t} - x/c_0 \right) + A_2 \sin \mathbf{w}_2 \left(\mathbf{t} - x/c_0 \right)$$

Develop each part individually.

$$\begin{aligned} \frac{\partial \mathbf{x}}{\partial x} &= \frac{\partial}{\partial x} \left(A_1 \sin \mathbf{w}_1 \left(\mathbf{t} - x/c_0 \right) + A_2 \sin \mathbf{w}_2 \left(\mathbf{t} - x/c_0 \right) \right) = \\ &= -A_1 \cos \mathbf{w}_1 \left(\mathbf{t} - x/c_0 \right) \cdot \frac{\mathbf{w}_1}{c_0} - A_2 \cos \mathbf{w}_2 \left(\mathbf{t} - x/c_0 \right) \cdot \frac{\mathbf{w}_2}{c_0} \end{aligned} \quad (4.15)$$

$$\frac{\partial^2 \mathbf{x}}{\partial x^2} = -A_1 \sin \mathbf{w}_1 \left(\mathbf{t} - x/c_0 \right) \cdot \left(\frac{\mathbf{w}_1}{c_0} \right)^2 - A_2 \sin \mathbf{w}_2 \left(\mathbf{t} - x/c_0 \right) \cdot \left(\frac{\mathbf{w}_2}{c_0} \right)^2$$

All parts together becomes

$$\begin{aligned}
\frac{\partial^2 \mathbf{x}}{\partial x^2} - \frac{1}{c_0^2} \frac{\partial^2 \mathbf{x}}{\partial t^2} &= (\mathbf{g} + 1) \frac{\partial \mathbf{x}}{\partial x} \cdot \frac{\partial^2 \mathbf{x}}{\partial x^2} = \\
&= (\mathbf{g} + 1) \cdot \left(-A_1 \cos \mathbf{w}_1 \mathbf{t} \cdot \frac{\mathbf{w}_1}{c_0} - A_2 \cos \mathbf{w}_2 \mathbf{t} \cdot \frac{\mathbf{w}_2}{c_0} \right) \cdot \\
&\cdot \left(-A_1 \sin \mathbf{w}_1 \mathbf{t} \cdot \left(\frac{\mathbf{w}_1}{c_0} \right)^2 - A_2 \sin \mathbf{w}_2 \mathbf{t} \cdot \left(\frac{\mathbf{w}_2}{c_0} \right)^2 \right) = \\
&= (\mathbf{g} + 1) \cdot \left(A_1^2 \cos \mathbf{w}_1 \mathbf{t} \cdot \sin \mathbf{w}_1 \mathbf{t} \cdot \left(\frac{\mathbf{w}_1}{c_0} \right)^3 + \right. \\
&+ A_2 A_1 \cos \mathbf{w}_2 \mathbf{t} \cdot \sin \mathbf{w}_1 \mathbf{t} \cdot \left(\frac{\mathbf{w}_1}{c_0} \right)^2 \cdot \left(\frac{\mathbf{w}_2}{c_0} \right) + \\
&+ A_2 A_1 \cos \mathbf{w}_1 \mathbf{t} \cdot \sin \mathbf{w}_2 \mathbf{t} \cdot \left(\frac{\mathbf{w}_2}{c_0} \right)^2 \cdot \left(\frac{\mathbf{w}_1}{c_0} \right) + \\
&\left. + A_2^2 \cos \mathbf{w}_2 \mathbf{t} \cdot \sin \mathbf{w}_2 \mathbf{t} \cdot \left(\frac{\mathbf{w}_2}{c_0} \right)^3 \right)
\end{aligned}$$

If we rewrite this once more we can see that the first harmonic of the frequencies has been created.

$$\begin{aligned}
& \frac{\partial^2 \mathbf{x}}{\partial x^2} - \frac{1}{c_0^2} \frac{\partial^2 \mathbf{x}}{\partial t^2} = \\
& = (\mathbf{g} + 1) \cdot \left(\frac{A_1^2 \cdot \left(\frac{\mathbf{w}_1}{c_0} \right)^3 \sin 2\mathbf{w}_1 t + A_2^2 \cdot \left(\frac{\mathbf{w}_2}{c_0} \right)^3 \sin 2\mathbf{w}_2 t}{2} \right) + \\
& + (\mathbf{g} + 1) \cdot A_2 A_1 \left(\left(\frac{\mathbf{w}_2}{c_0} \right)^2 \cdot \left(\frac{\mathbf{w}_1}{c_0} \right) \cos \mathbf{w}_1 t \cdot \sin \mathbf{w}_2 t + \right. \\
& \left. + \left(\frac{\mathbf{w}_1}{c_0} \right)^2 \cdot \left(\frac{\mathbf{w}_2}{c_0} \right) \cos \mathbf{w}_2 t \cdot \sin \mathbf{w}_1 t \right)
\end{aligned} \tag{4.16}$$

The rest of the solution is treated separately.

$$\begin{aligned}
& (\mathbf{g}+1) \cdot A_2 A_1 \left(\left(\frac{\mathbf{w}_2}{c_0} \right)^2 \cdot \left(\frac{\mathbf{w}_1}{c_0} \right) \cos \mathbf{w}_1 t \cdot \sin \mathbf{w}_2 t + \right. \\
& \left. + \left(\frac{\mathbf{w}_1}{c_0} \right)^2 \cdot \left(\frac{\mathbf{w}_2}{c_0} \right) \cos \mathbf{w}_2 t \cdot \sin \mathbf{w}_1 t \right) = (\mathbf{g}+1) \cdot A_2 A_1 \cdot \\
& \left(\frac{\left(\frac{\mathbf{w}_2}{c_0} \right)^2 \cdot \left(\frac{\mathbf{w}_1}{c_0} \right) \cos \mathbf{w}_1 t \cdot \sin \mathbf{w}_2 t}{2} + \frac{\left(\frac{\mathbf{w}_2}{c_0} \right)^2 \cdot \left(\frac{\mathbf{w}_1}{c_0} \right) \cos \mathbf{w}_2 t \cdot \sin \mathbf{w}_1 t}{2} + \right. \\
& \left. + \frac{\left(\frac{\mathbf{w}_1}{c_0} \right)^2 \cdot \left(\frac{\mathbf{w}_2}{c_0} \right) \cos \mathbf{w}_2 t \cdot \sin \mathbf{w}_1 t}{2} + \frac{\left(\frac{\mathbf{w}_1}{c_0} \right)^2 \cdot \left(\frac{\mathbf{w}_2}{c_0} \right) \cos \mathbf{w}_1 t \cdot \sin \mathbf{w}_2 t}{2} + \right. \\
& \left. + \frac{\left(\frac{\mathbf{w}_2}{c_0} \right)^2 \cdot \left(\frac{\mathbf{w}_1}{c_0} \right) \cos \mathbf{w}_1 t \cdot \sin \mathbf{w}_2 t}{2} - \frac{\left(\frac{\mathbf{w}_2}{c_0} \right)^2 \cdot \left(\frac{\mathbf{w}_1}{c_0} \right) \cos \mathbf{w}_2 t \cdot \sin \mathbf{w}_1 t}{2} + \right. \\
& \left. + \frac{\left(\frac{\mathbf{w}_1}{c_0} \right)^2 \cdot \left(\frac{\mathbf{w}_2}{c_0} \right) \cos \mathbf{w}_2 t \cdot \sin \mathbf{w}_1 t}{2} - \frac{\left(\frac{\mathbf{w}_1}{c_0} \right)^2 \cdot \left(\frac{\mathbf{w}_2}{c_0} \right) \cos \mathbf{w}_1 t \cdot \sin \mathbf{w}_2 t}{2} \right) = \\
& = (\mathbf{g}+1) \cdot A_2 A_1 \cdot \left(\left(\frac{\left(\frac{\mathbf{w}_2}{c_0} \right)^2 \cdot \left(\frac{\mathbf{w}_1}{c_0} \right)}{2} + \frac{\left(\frac{\mathbf{w}_1}{c_0} \right)^2 \cdot \left(\frac{\mathbf{w}_2}{c_0} \right)}{2} \right) \sin(\mathbf{w}_1 + \mathbf{w}_2) + \right. \\
& \left. + \frac{\left(\frac{\mathbf{w}_2}{c_0} \right)^2 \cdot \left(\frac{\mathbf{w}_1}{c_0} \right)}{2} \sin(\mathbf{w}_2 - \mathbf{w}_1) + \frac{\left(\frac{\mathbf{w}_1}{c_0} \right)^2 \cdot \left(\frac{\mathbf{w}_2}{c_0} \right)}{2} \sin(\mathbf{w}_1 - \mathbf{w}_2) \right)
\end{aligned}$$

All parts together gives.

$$\begin{aligned}
 \frac{\partial^2 \mathbf{x}}{\partial x^2} - \frac{1}{c_0^2} \frac{\partial^2 \mathbf{x}}{\partial t^2} = & (\mathbf{g} + 1) \cdot \left(\frac{A_1^2 \cdot \left(\frac{\mathbf{w}_1}{c_0}\right)^3 \sin 2\mathbf{w}_1 t + A_2^2 \cdot \left(\frac{\mathbf{w}_2}{c_0}\right)^3 \sin 2\mathbf{w}_2 t}{2} \right) + \\
 & + (\mathbf{g} + 1) \cdot A_2 A_1 \cdot \left(\left(\frac{\left(\frac{\mathbf{w}_2}{c_0}\right)^2 \cdot \left(\frac{\mathbf{w}_1}{c_0}\right)}{2} + \frac{\left(\frac{\mathbf{w}_1}{c_0}\right)^2 \cdot \left(\frac{\mathbf{w}_2}{c_0}\right)}{2} \right) \sin(\mathbf{w}_1 + \mathbf{w}_2) + \right. \\
 & \left. + \frac{\left(\frac{\mathbf{w}_2}{c_0}\right)^2 \cdot \left(\frac{\mathbf{w}_1}{c_0}\right)}{2} \sin(\mathbf{w}_2 - \mathbf{w}_1) + \frac{\left(\frac{\mathbf{w}_1}{c_0}\right)^2 \cdot \left(\frac{\mathbf{w}_2}{c_0}\right)}{2} \sin(\mathbf{w}_1 - \mathbf{w}_2) \right)
 \end{aligned}
 \tag{4.17}$$

Here we can see that the first harmonic of the two initial frequencies, the sum of the frequencies and the difference frequency is created when the signal is propagating through the medium.

4.1.1 Difference Frequency

The most important one of the created frequencies is, in this thesis, the difference frequency. This is the frequency that will be audible if the original frequencies lies less than 20 kHz from each other.

The difference frequency is created due to that the sound waves are propagating through the nonlinear air, and it therefore takes a while for the difference frequency to be created. The distance it takes for the difference frequency to be created depends among other things on the amplitudes of the original frequencies; higher amplitudes mean a shorter distance.

The amplitude of the difference frequency is only a fraction of the amplitude of the original frequency, due to the fact that the created sound is only a biproduct and the most energy is still in the original frequencies. Calculations can be done with Burgers' equation to look further in to these facts.

The results of such calculations are very hard to compare with measurements because we are not able to measure the amplitudes with the accuracy needed. For example we do not know the amplification of the mixer used in the measurements and we also do not know the sensitivity and frequency response of the microphone. The calculations with Burgers' equation is therefore left out in this thesis.

5 Radiation from a baffled piston

The difference frequency is radiating as the beam of the ultrasonic sound. This fact makes it interesting to know how the “ultrasonic beam” look likes, in order to understand how the audible sound radiates. There are many variables that affect how the beam of ultrasound looks like.

- Size of the element or the array of elements
- Frequency
- Shape of the element or the array of elements

All these variables will be investigated further below

A baffled piston, a piston surrounded of a rigid plane, radiates sound a little different from a piston which is not baffled. A baffled piston radiates only in the hemisphere in front of the piston not the whole spherical room.

5.1 Derivation of Rayleigh Integral

A baffled piston can be seen as a large number of point sources [2]. So first of all we will investigate how a point source radiates sound. The derivation does not need to consider nonlinearity, it is linear.

It can pretty easily be shown that the wave equation for spherical waves

$$\mathbf{f}_{rr} + \frac{2}{r}\mathbf{f}_r - \frac{1}{c_0^2}\mathbf{f}_{tt} = 0 \quad (5.1)$$

where \mathbf{f} is the velocity potential defined by $u = \nabla \mathbf{f}$

has the general solution

$$r\mathbf{f} = f(r - c_0t) + g(r + c_0t) \quad (5.2)$$

Where the first term is an outgoing spherical wave and the second term is an incoming spherical wave. In our case the incoming wave does not exist due to we have no reflection and things like that, the solution is conveniently expressed as

$$\mathbf{f} = \frac{f(t - r/c_0)}{r} \quad (5.3)$$

We now look at a small pulsating sphere. The volume flow of fluid is $Q(t)$ from the source.

$$Q(t) = 4\mathbf{p} \cdot a^2 \cdot u^{(r)}(a, t) \quad (5.4)$$

where a is the radius of the sphere and $u^{(r)}$ is the radial component of the velocity.

In the case with the baffled piston the radiation is restricted to the hemisphere in front of the piston not the whole spherical space. Therefore the factor $4\mathbf{p}$ in equation (5.4) is replaced by $2\mathbf{p}$, this gives the equation.

$$Q(t) = 2\mathbf{p} \cdot a^2 \cdot u^{(r)}(a, t) \quad (5.5)$$

The radial component of the velocity can be expressed with equation (5.3)

$$u^{(r)}(a, t) = \mathbf{f}_a = \frac{\partial}{\partial a} \left(\frac{f(t - a/c_0)}{a} \right) = -\frac{f(t - a/c_0)}{a^2} - \frac{f'(t - a/c_0)}{ac_0} \quad (5.6)$$

This expression is put into equation (5.5)

$$Q(t) = 2\mathbf{p} \cdot a^2 \cdot \left(-\frac{f(t - a/c_0)}{a^2} - \frac{f'(t - a/c_0)}{ac_0} \right)$$

$$Q(t) = 2\mathbf{p} \cdot \left(f(t - a/c_0) + \frac{a}{c_0} f'(t - a/c_0) \right) \quad (5.7)$$

To consider the sphere as a point source the radius a has to be very small therefore we take the limit as the radius $a \rightarrow 0$.

$$\lim_{a \rightarrow 0} Q(t) = -2\mathbf{p}f(t) \quad (5.8)$$

The pressure is

$$p = -\mathbf{r}_0 \mathbf{f}_t = -\mathbf{r}_0 \cdot \frac{f'(t - r/c_0)}{r} \quad (5.9)$$

Equation (5.8) gives

$$f(t) = -\frac{Q(t)}{2p} \quad (5.10)$$

and the derivative is

$$f'(t) = -\frac{\dot{Q}(t)}{2p} \quad (5.11)$$

Equation (5.9) gives the pressure

$$p = \frac{\mathbf{r}_0 \cdot \dot{Q}(t - r/c_0)}{2pr} \quad (5.12)$$

This is the pressure for one point source.

The quantity \dot{Q} is often called the source strength. If the element we are interested in is not small enough to be expressed as a point source we may divide it into small elements which individually act as point sources.

The radiation received at the field point (x, y, z) is the sum of the radiations from point sources.

$$p(x, y, z, t) = \mathbf{r}_0 \int_V \frac{\dot{q}_v(x', y', z'; t - R/c_0)}{2pR} dx' dy' dz' \quad (5.13)$$

R is the distance from the point on the source (x', y', z') to the field point (x, y, z) see figure 5.1.

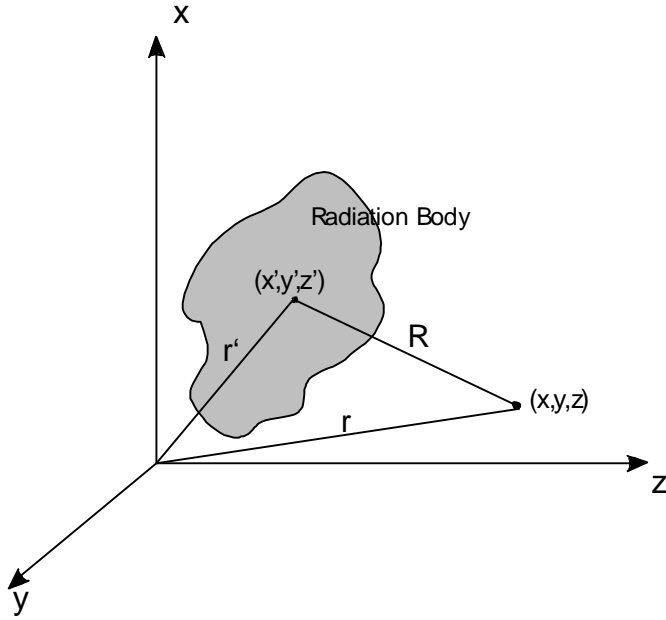


Figure 5.1. Radiation from body of arbitrary shape.

where \dot{q}_v is the source strength per unit volume. The time R/c_0 is the time it takes for the wave to travel from the source to the field point.

In most cases the sources are flat and represent an area instead of a volume, this gives the pressure

$$p(x, y, z, t) = \mathbf{r}_0 \int_s \frac{\dot{q}_a(x', y', t - R/c_0)}{2\rho R} dS \quad (5.14)$$

The piston is in the x,y plane so all z' is zero, see figure 5.2.

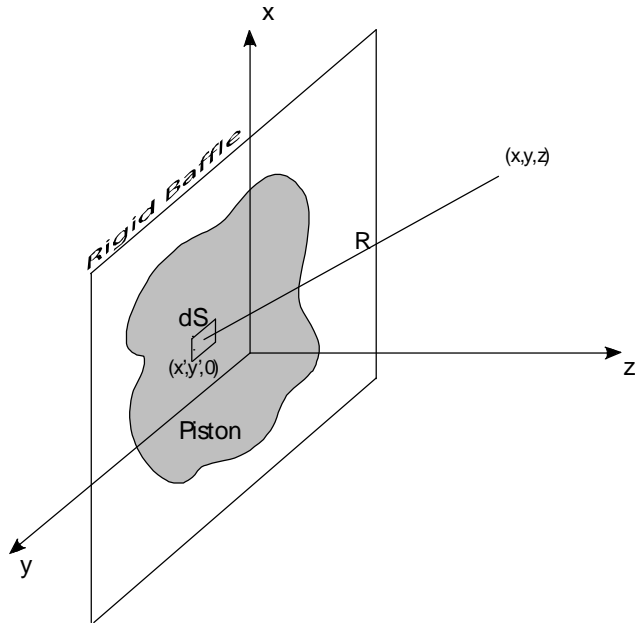


Figure 5.2. Radiation of flat piston of arbitrary shape.

In this case \dot{q}_a is the source strength per unit area or the volume acceleration per unit area, it is simply the piston acceleration \dot{u}_p .

$$p(x, y, z, t) = r_0 \int_S \frac{\dot{u}_p(x', y'; t - R/c_0)}{2\rho R} dS \quad (5.15)$$

This equation is often called the Rayleigh integral.

5.2 Time Harmonic Signals

The vibrations in a piston like element are often time harmonic [2], that is

$$u_p = u_0 e^{i\omega t} \quad (5.16)$$

The acceleration term in equation (5.15) becomes

$$\dot{u}_p(x', y'; t - R/c_0) = \frac{\partial}{\partial t} (u_0 e^{i\omega(t-R/c_0)}) = i \cdot \mathbf{w} \cdot u_0 \cdot e^{i\omega(t-R/c_0)} \quad (5.17)$$

A commonly used expression is the wave number that is defined as

$$k = \frac{\mathbf{w}}{c_0} \quad (5.18)$$

The angle velocity can then be expressed as

$$\mathbf{w} = k \cdot c_0 \quad (5.19)$$

The acceleration term can be expressed as

$$\dot{u}_p(x', y'; t - R/c_0) = i \cdot \mathbf{w} \cdot u_0 \cdot e^{i\omega t - ikR} = i \cdot k \cdot c_0 \cdot u_0 \cdot e^{i\omega t - ikR} \quad (5.20)$$

The pressure then becomes

$$p(x, y, z, t) = \mathbf{r}_0 \int_S \frac{i \cdot k \cdot c_0 \cdot u_0 \cdot e^{i\omega t - ikR}}{2\mathbf{p}R} dS \quad (5.21)$$

We now move all the constant terms outside the integral.

$$p(x, y, z, t) = \frac{i \cdot \mathbf{r}_0 \cdot k \cdot c_0 \cdot u_0 \cdot e^{i\omega t}}{2\mathbf{p}} \cdot \int_S \frac{e^{-ikR}}{R} dS \quad (5.22)$$

The pressure amplitude $\mathbf{r}_0 \cdot c_0 \cdot u_0$ is denoted P_0

$$p(x, y, z, t) = \frac{i \cdot k \cdot P_0 \cdot e^{i\omega t}}{2\mathbf{p}} \cdot \int_S \frac{e^{-ikR}}{R} dS \quad (5.23)$$

5.3 Circular elements

The elements that are used in the construction of the loudspeaker in this thesis are circular. It is therefore interesting to investigate how the radiation looks like from a circular piston. In this case the surface integral can be changed to a double integral over the radius of the piston and the angle \mathbf{y} see fig 5.3.

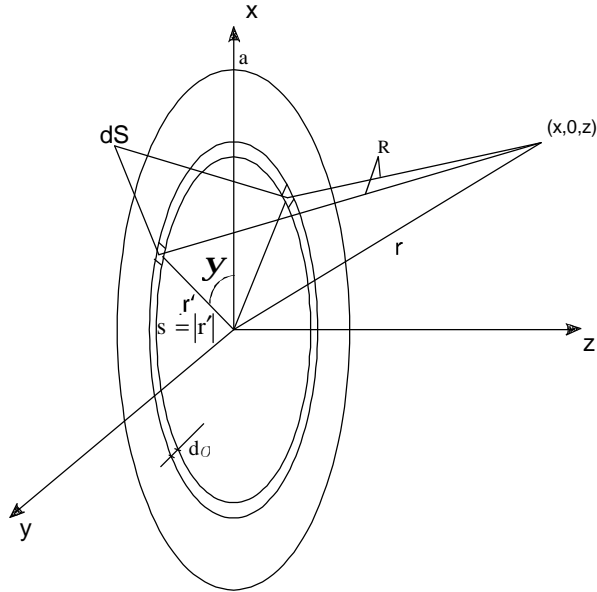


Figure 5.3. Radiation from circular piston.

Due to symmetry we can locate the field point in the x,z plane. The symmetry also makes it possible to locate two identical elements on the source, one when \mathbf{y} goes in the positive direction and one when it goes in the negative direction. These two elements have the same distribution to the pressure in each field point so instead of calculating the distribution from each of the two elements the pressure is calculated for one of the elements and then doubled to get the total pressure. The two elements have the combined area

$$dS = 2 \cdot \mathbf{s} \cdot d\mathbf{s} \cdot d\mathbf{y} \quad (5.24)$$

The pressure radiation from a circular piston becomes

$$p(x, y, z, t) = \frac{i \cdot k \cdot P_0 \cdot e^{i\omega t}}{2\rho} \cdot \int_0^a \int_0^a \frac{e^{-ikR}}{R} 2 \cdot \mathbf{s} \cdot d\mathbf{s} \, dy \quad (5.25)$$

The distance R is still the distance from the source point to the field point. In this case it can be calculated from the two vectors r and r'.

$$R = |r - r'| \quad (5.26)$$

Often the Rayleigh distance is mentioned, it is the distance which roughly marks the end of the near field. Rayleigh distance is denoted [2]

$$R_0 = \frac{S}{I} \quad (5.27)$$

Where S is the area of the element. For a circular piston this becomes.

$$R_0 = \frac{S}{I} = \frac{\mathbf{p} \cdot a^2}{I} = \frac{k \cdot a^2}{2} \quad (5.28)$$

Vector r is the vector from the origin to the field point.

$$r = ix + j0 + kz \quad (5.29)$$

Note that i is not the imaginary unit in this case.

Vector r' is the vector from the origin to the source point.

$$r' = i \cdot \mathbf{s} \cos \mathbf{y} + j \cdot \mathbf{s} \sin \mathbf{y} + k \cdot 0 \quad (5.30)$$

The distance R then becomes.

$$R = |r - r'| = \sqrt{(x - \mathbf{s} \cos \mathbf{y})^2 + \mathbf{s}^2 \sin^2 \mathbf{y} + z^2} \quad (5.31)$$

The pressure is calculated with use of MATLAB in a couple of cases.

5.3.1 Sound field on axis

As told in chapter 3 the field in front of the element can be divided in the near field and the far field. The near field is the part with heavily varying amplitudes, in this case with circular elements the amplitude has nulls along the distance see figure 5.4. The number of nulls depends on how long the wavelength is compared to the radius of the element. If the wavelength is half the radius there are two nulls and so on, see figure 5.4 and figure 5.5. The special case on axis can be solved analytically. [2]

The infinitesimal element can be seen as a ring around the centre of the piston. The area of the element is

$$dS = 2 \cdot \mathbf{p} \cdot \mathbf{s} \cdot d\mathbf{s} \quad (5.32)$$

The distance from the source point to the field point is simply

$$R = \sqrt{r^2 + \mathbf{s}^2} \quad (5.33)$$

The Rayleigh integral can now be rewritten

$$p(x, y, z, t) = i \cdot k \cdot P_0 \cdot e^{i\omega t} \cdot \int_0^a \frac{e^{-ikR}}{R} \cdot \mathbf{s} \cdot d\mathbf{s} \quad (5.34)$$

The integral can now be changed from one over \mathbf{s} to one over R with the fact that $\mathbf{s}d\mathbf{s} = RdR$ [2]. The integral is now

$$p(x, y, z, t) = i \cdot k \cdot P_0 \cdot e^{i\omega t} \cdot \int_r^{r_{\max}} e^{-ikR} \cdot dR = P_0 \left[e^{i(\omega t - kr)} - e^{i(\omega t - kr_{\max})} \right] \quad (5.35)$$

$$\text{where } r_{\max} = \sqrt{r^2 + a^2} \quad (5.36)$$

This integral is solved for every point on the axis in a MATLAB script that can be seen in appendix 9.3.

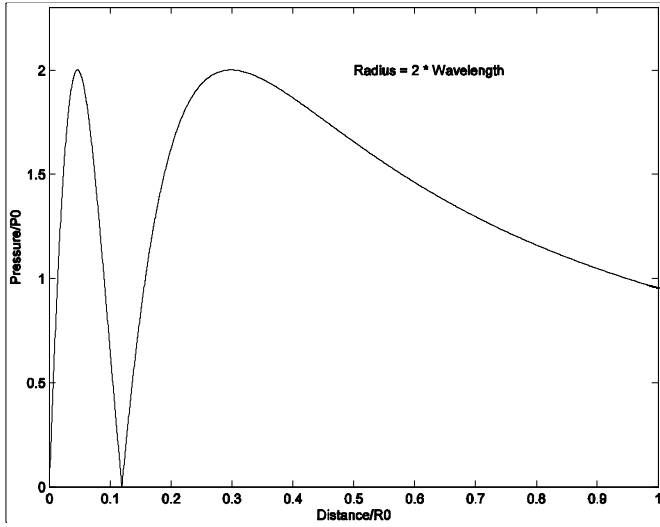


Figure 5.4. Pressure distribution on axis.

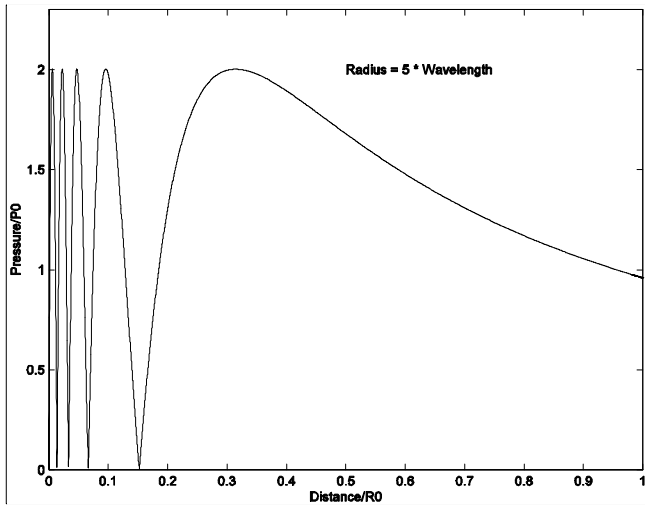


Figure 5.5. Pressure distribution on axis.

5.3.2 Sound field off axis

One thing that can not be seen on the axis is how directed the sound beam is, therefore the pressure distribution off axis is calculated.

Off axis the pressure is calculated from equation (5.25) in a little more complex MATLAB script that can be seen in appendix 9.4. The result can be seen in figure 5.6. The result is of course the same on the axis. A very interesting and useful thing to notice is that when the wavelength decreases, the sound beam gets more directed. When the wavelength is relatively large compared to the piston radius the sound is radiating spherically, but when the wavelength get smaller the sound is radiating more like a beam.

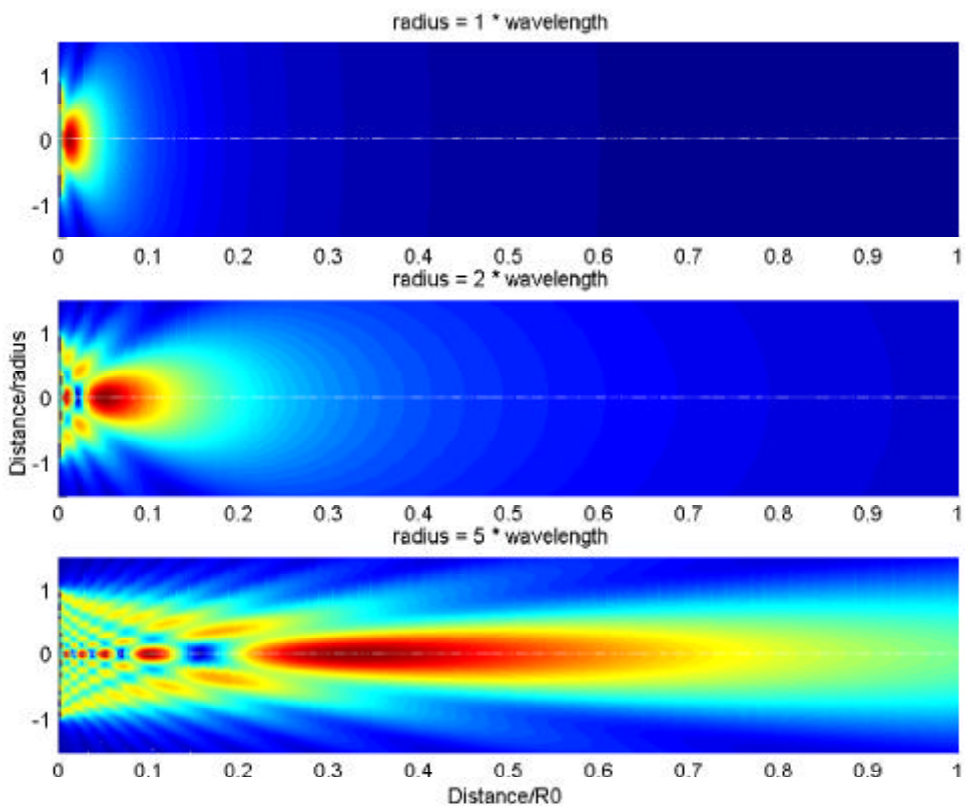


Figure 5.6. Pressure distribution off axis.

5.4 Rectangular elements

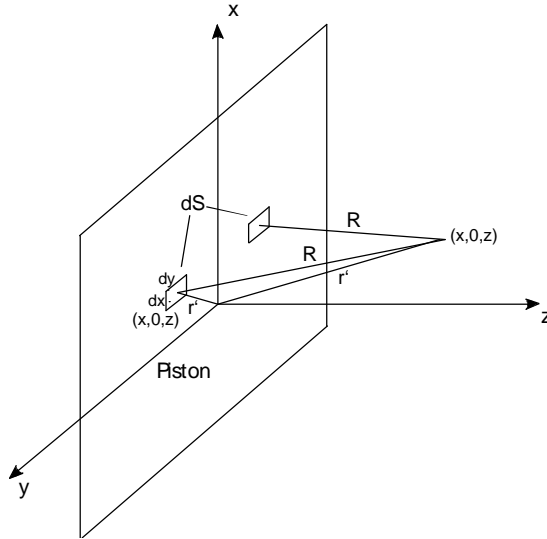


Figure 5.7. Radiation from rectangular piston.

We start with the Rayleigh integral [2].

$$p(x, y, z, t) = \mathbf{r}_0 \int_S \frac{\dot{u}_p(x', y'; t - R/c_0)}{2pR} dS \quad (5.37)$$

We use the same expression for the acceleration term as before and in this case the two elements have the combined area

$$dS = 2 \cdot dx \cdot dy \quad (5.38)$$

We get the pressure equation

$$p(x, y, z, t) = \frac{i \cdot k \cdot P_0 \cdot e^{i\omega t}}{\mathbf{p}} \cdot \int_0^a \int_0^a \frac{e^{-ikR}}{R} dx \cdot dy \quad (5.39)$$

The distance from the source point to the field point is in this case described as

$$R = \sqrt{(x - x')^2 + (y - y')^2 + z^2} \quad (5.40)$$

Rayleigh distance for a rectangular piston is

$$R_0 = \frac{S}{\lambda} = \frac{a \cdot b}{\lambda} \quad (5.41)$$

where a and b is the height and width of the element.

5.4.1 Sound field on axis

To calculate the sound field on axis for a rectangular piston the Rayleigh integral is used. There is no simplification done, as in the case of circular piston. To make it easier to compare with the result from the circular piston the outer dimension is the same as for the circular piston, and the piston is square. The result from the calculations can be seen in figure 5.8 and figure 5.9.

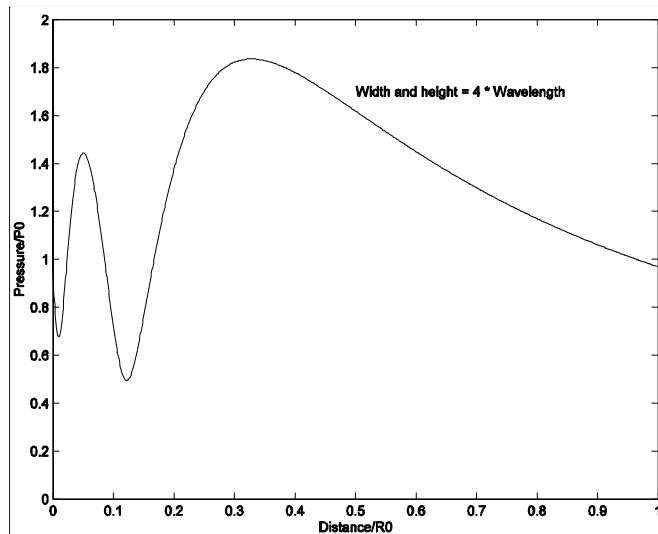


Figure 5.8. Pressure distribution on axis.

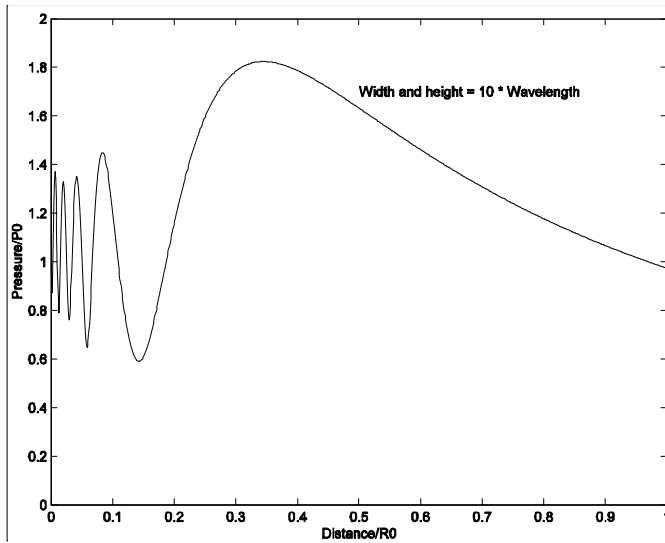


Figure 5.9. Pressure distribution on axis.

One significant difference from the circular piston is that the nulls, that where present in the case with circular elements, is now only dips in the amplitude. The near field is therefore much smoother for a rectangular piston than for a circular piston.

5.4.2 Sound field off axis

To calculate the pressure distribution off axis for the rectangular element the same MATLAB script was used as for rectangular on axis. The results from the calculations are similar to the ones from circular element off axis. The greatest difference can be seen on the near field, see figure 5.10. The same conclusions about that a smaller wavelength makes a narrower sound beam can be made here.

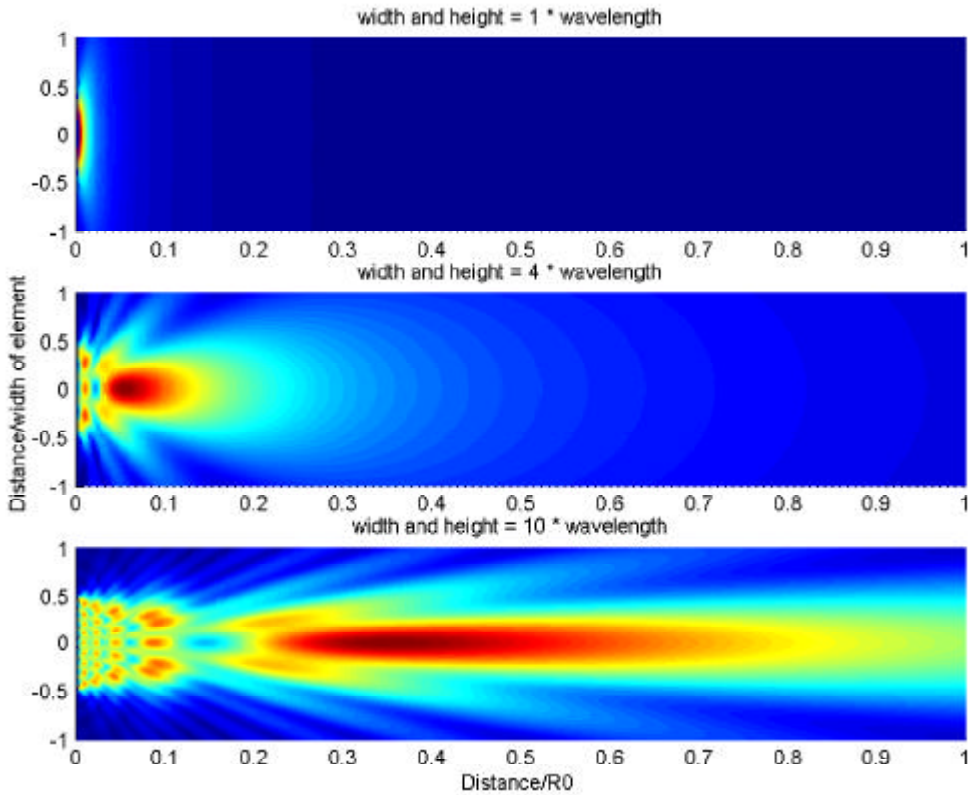


Figure 5.10. Pressure distribution off axis for rectangular element.

5.5 Array of elements

An array is when many elements are put in a pattern. The way this pattern is designed determines how the sound field will look like. An advantage of using an array is that a higher output level is possible than with a single element. Because of the larger dimensions the directivity of an array is higher than for a single element. A disadvantage is that the near field, due to the larger dimensions, is more heavily changing amplitude. One way to get a smoother near field is to place the elements in a rectangular pattern, this has the same effect on the near field as for a single element that is rectangular instead of circular. To calculate the pressure distribution from an array the pressure distribution from many single elements are simply summed in each point that are coinciding.

A calculation of a five element line array has been done to compare the near and far field of an array with the same outer dimension as a large element. The frequency is 40 kHz and the elements in the array have the radius five millimetres, the distance between the elements is ten millimetres. The single element that the comparison is made with is five times fifty millimetres. The MATLAB scripts for this calculation can be seen in appendix 9.6 to 9.8.

In figure 5.11 and figure 5.12 the results from the calculations are presented. The most obvious difference is that the near field is not the same for the two calculations. The plots of the pressure distributions on axis may be misleading, because in the array the whole area is not producing pressure so the pressure looks pretty much the same on axis but if we look at the pressure between two elements it is far from the same. This means that an array cannot be simplified as one element when it is interesting what happens in the near field. In some cases the directivity in the far field can be calculated with the simplification with one single element, but only when only the far field is of interest. The simplification is only valid when the distance between the elements is small. If the distance is so small that the elements in the array cover the whole area, the simplification is then of course valid in both the near field and the far field.

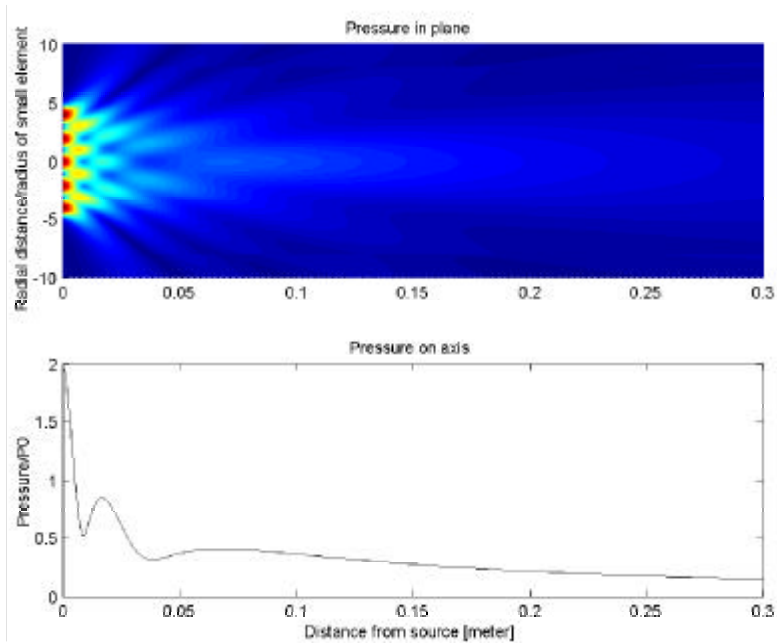


Figure 5.11. Pressure distribution from 5 element line array.

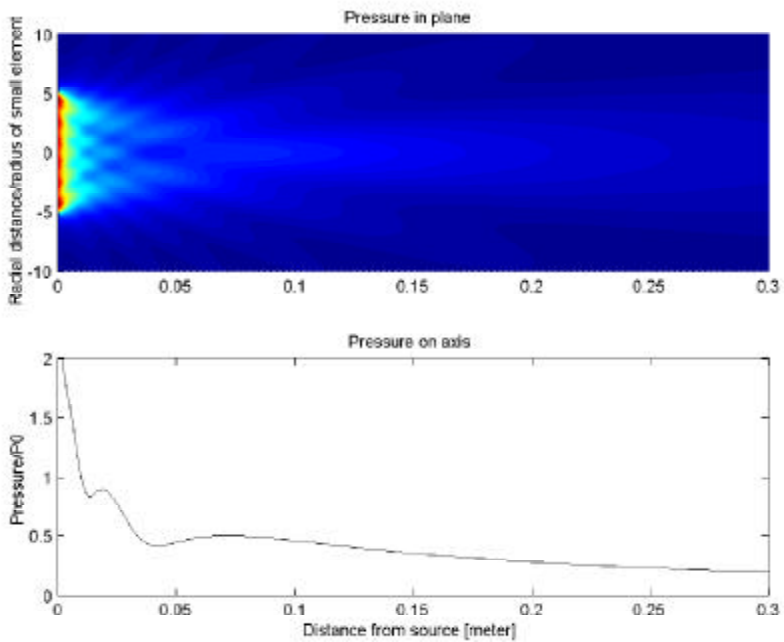


Figure 5.12. Pressure distribution from large piston.

To compare the results on axis further the plots from the two calculations are put in the same figure, see figure 5.13. Here it can be seen that the pressures follow each other very well. The amplitude difference is due to that the single large element has a larger area than the array.

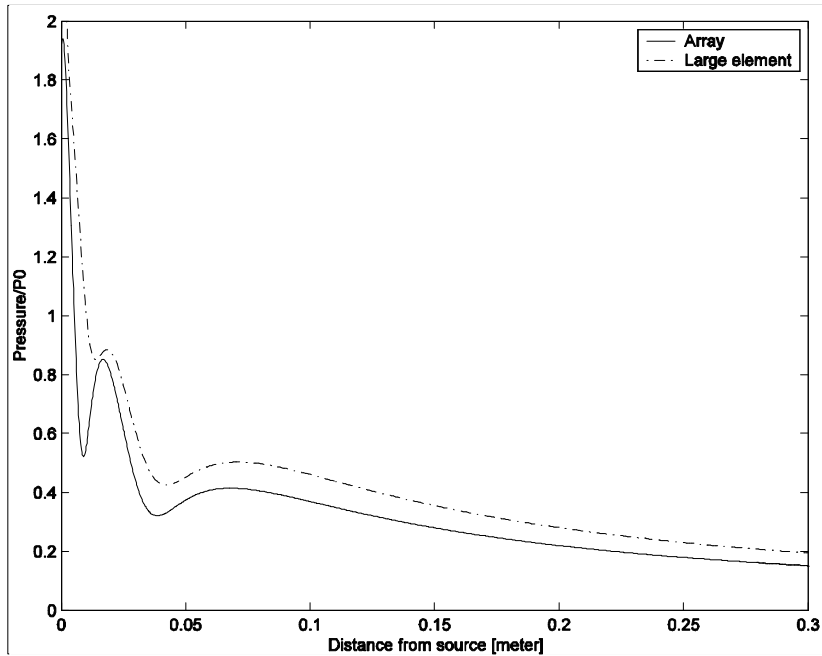


Figure 5.13. Pressure distribution on axis in the two cases.

6 The Speaker

6.1 Building the speaker

There are a couple of parameters to think about when a speaker of this kind should be built.

- Size

As large as possible to get a narrow beam, but not too big because of a larger element gives a longer near field

- Shape

A rectangular array makes the near field less jumpy

- Distance between elements

If the distance is great the phase difference is large.

- Manageable number of elements in the array

It takes some time to solder all elements

- Two fields

It is not possible in this case to play both frequencies in all of the elements. If this is done the nonlinearity arise in the element instead of in the air in front of the speaker.

- If possible; four fields

Four fields can make it possible to make experiments on “Disappearing Sound”.

The speaker was built before all calculations were made, so some assumptions made before the calculations may be incorrect. The speaker is built with 196 piezo ceramic elements which are connected in parallel in two groups. The reason to divide the elements into two groups is as said in the list above that it is a great risk that the difference frequency is created in the elements due to nonlinearity in these instead of in the air. It is possible by a simple operation to divide the elements into four groups. This can be used to make experiments on “Disappearing sound”. “Disappearing sound” is when two pairs of high frequency at high amplitude are creating the same

difference frequency but one frequency in one of the pairs is in another phase. This makes the difference frequency audible only in a limited area. The finished speaker can be seen in figure 6.1.

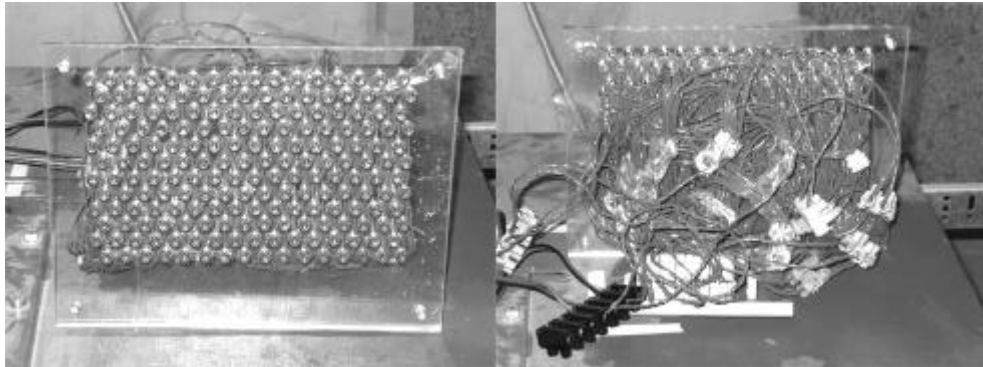


Figure 6.1. Front and back of the speaker.

6.2 Creating the Signal

It is of great interest that the speaker can be used as a demonstration unit, to show that it is really possible to create sound by using the nonlinearity in air. To make the speaker more interesting as a demonstration unit it has to be able to change frequency, because it is getting very annoying to listen to a single frequency for a long time. This is done by programming the signal generator with the program LABVIEW in a computer.

One of the two frequencies is the same at all times and the other frequency is changed up and down randomly in the chromatic scale. It is not really interesting to make a more detailed explanation of how the signal generator was programmed, but a screenshot from the program is presented in figure 6.2.

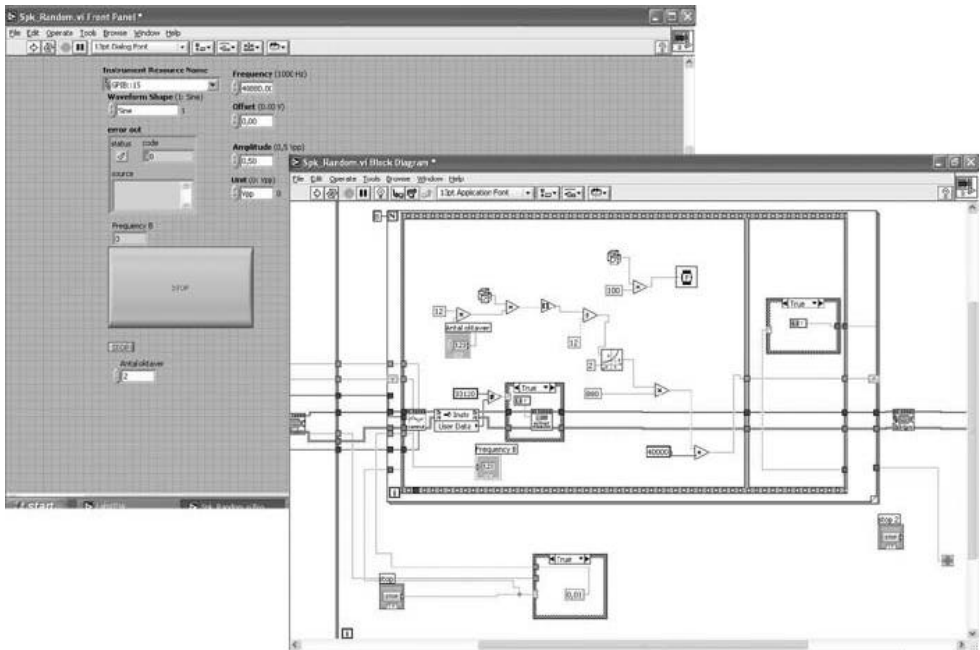


Figure 6.2 Screenshot from LABVIEW.

6.3 Measuring the speaker

Some measurements are made of the speaker. The biggest reason for this is to ensure that it really is the difference frequency that appears. One interesting thing to measure is how the frequencies change on different distances from the speaker. A couple of different measurement set ups where tried, the hardest task was to move the microphone in a straight line in front of the speaker. The final solution was to use a robot that holds a microphone and makes a perfect sweep over the measurement domain. The signal from the microphone first ran through a mixer that amplified the signal, and then the signal was filtered and sent to a lock in amplifier that gave the amplitude and phase of the selected frequency. LABVIEW was used to collect the data.

A schematic picture of the measurement set up can be seen in figure 6.3.

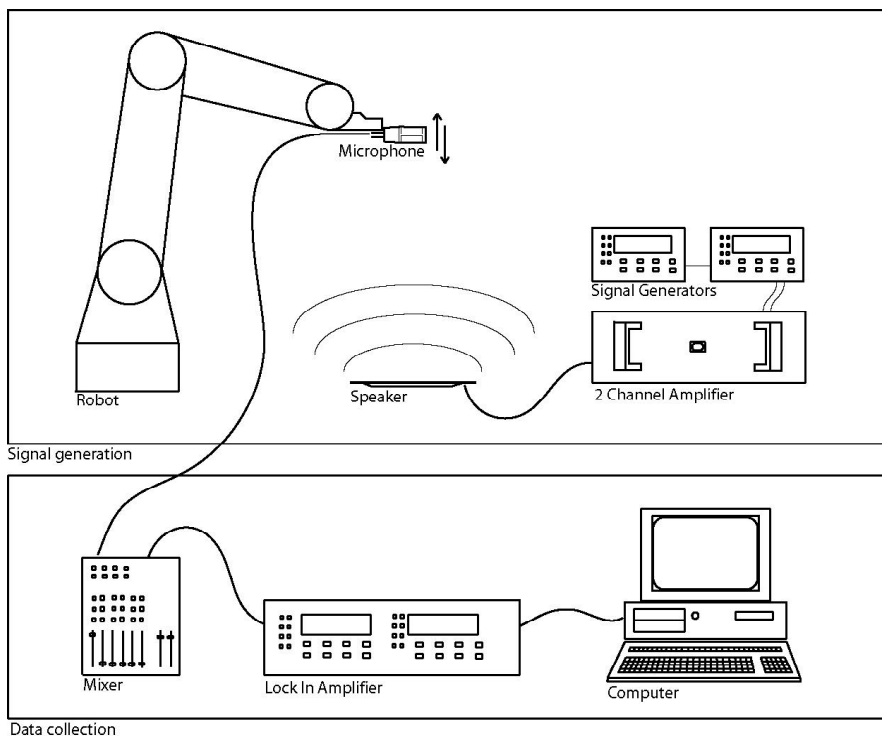


Figure 6.3 Schematic picture of measurement set up.

Three measurement results are presented in figure 6.4 to figure 6.6. The figures show the amplitude at the axis for the speaker both for the difference frequency (1 kHz) and for the two initial frequencies (40 and 41 kHz).

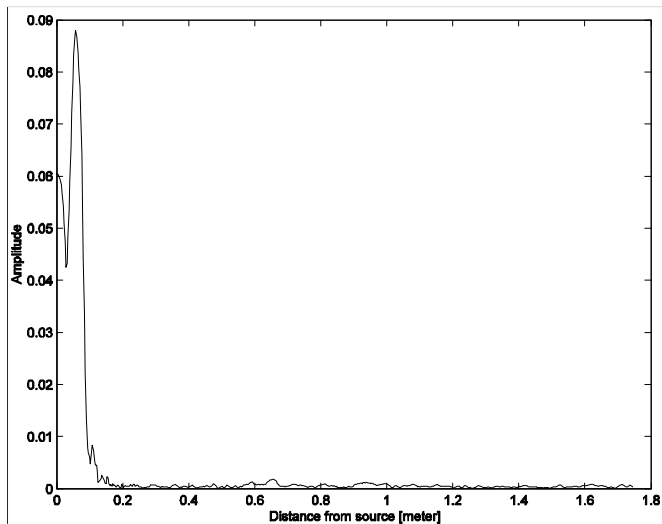


Figure 6.4. 40 kHz.

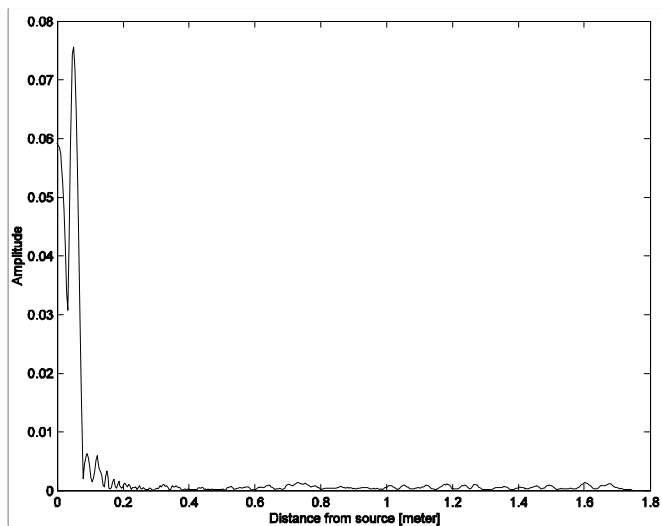


Figure 6.5. 41 kHz.

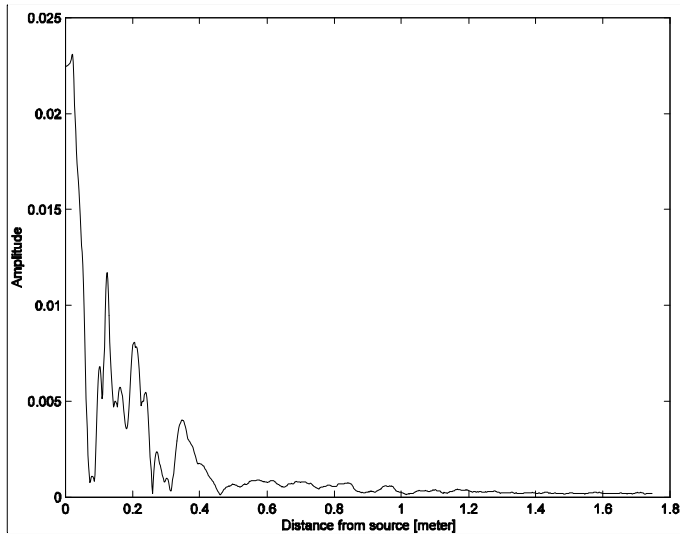


Figure 6.6. 1 kHz.

As can be seen in figure 6.4 and 6.5 the ultrasonic frequencies are damped out quite quickly. It is in the high peak in the ultrasonic field that the hearable 1 kHz tone is created. The audible tone in figure 6.6 seems to die out quite fast as well, but this can be a little misleading. If one listens in front of the speaker the difference frequency can be heard at great distances, because ears are extremely sensitive detectors. The ears sensitivity for sound amplitude is logarithmic. For the same reason the sound level is often measured in decibels [1]. It is a good idea to look at the result of the difference frequency in logarithmic scale on the amplitude axis. Figure 6.7 shows the same as figure 6.6 but the amplitude is presented in logarithmic scale.

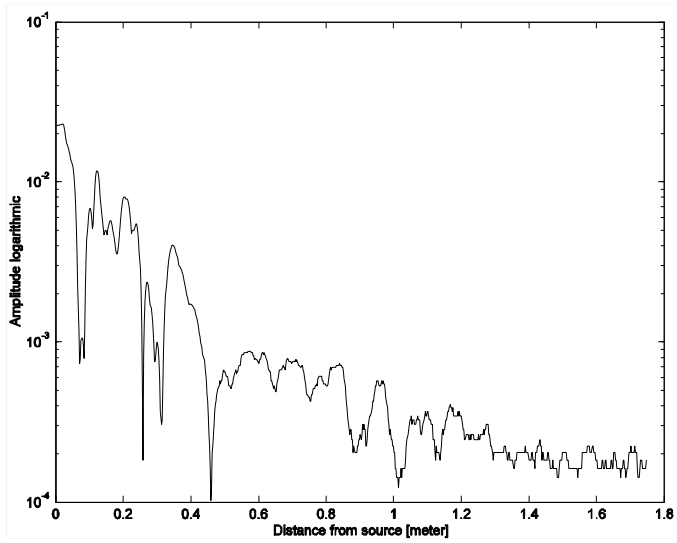


Figure 6.7 1 kHz, logarithmic amplitude scale.

7 Discussion and conclusion

The most interesting thing with the speaker is not any calculations or measurement results, it is to actually listen to the speaker. It is very clear that the sound is very directed. You do not hear the sound if you are not in the beam of sound and if there are no reflections.

The speaker has already been used at a demonstration at a local school playing simple one tone melodies, and the students all seemed very amazed by the fact that the sound they heard did not come from the speaker but from the wall that reflected the sound. I take that reaction as a proof that I have built an interesting demonstration unit.

It is possible to move the point on the wall that reflects the sound by simply redirect the speaker in another direction. This can be used to fool the listener that the sound comes from an object that can not produce sound itself.

To be used commercially the speaker has to be able to play any sound, not only simple sinusoidal tones. This can be done with some signal processing and some additional theory. Due to lack of certain equipment, such as digital to analogue converters that can manage very high frequencies, and also due to some lack of time, no attempts on creating more complex sound has been done.

8 References

1. Kinsler L.E., Frey A.R., Coppens A.B. & Sanders J.V., (2000), *Fundamentals Of Acoustics, fourth edition*, John Wiley & Sons, New York.
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5. Enflo B.O. & Hedberg C.M., (2002), *Theory Of Non-linear Acoustics In Fluids*, Kluwer Academic Publishers, Dordrecht.
6. Rudenko O.V. & Soluyan S.I., (1977), *Theoretical Foundations Of Nonlinear Acoustics*, Consultants Bureau, New York

9 Appendix MATLAB scripts

There are a couple of MATLAB functions that are used in almost every script in this thesis. These two functions shows the status of the progress in the scripts, the reason is that the scripts can take a while to run.

9.1 Statusbar.m

```
% STATUSBAR - statusbar.m makes a figure that shows the status of o
process
% open a figure for progress bar
% Ex: [figHndl statusHndl] = statusbar('Calculating LS Per Tone ...');
% Ex #2 [figHndl statusHndl] = statusbar('Calculating LS Per Tone ...',2);
if it is the second window in the same file
% to update the status bar write for example this in your loop that you want
to know the status of%
% Ex: for j=1:T
%     updatestatusbar(statusHndl,j,T);
%     end
function [figHndl, statusHndl] = statusbar(namestring,n)

if nargin==1
    n=1;
end
% open figure
figHndl = figure('Name',namestring,...
                'IntegerHandle','off',...
                'NumberTitle','off',...
                'MenuBar','none',...
                'position',[330 500-(n-1)*70 400 40]);

% setup axes
statusHndl=axes( ...
    'Units','normalized', ...
    'Position',[0.05 0.1 0.9 0.6], ...
    'Box','on', ...
    'UserData',0, ...
    'Visible','on', ...
```

```
'XTick',[],'YTick',[], ...  
'XLim',[0 1],'YLim',[0 1]);
```

9.2 Updatestatusbar.m

```
% Used in statusbar to update  
function updatestatusbar(statusHndl,status,maxstatus)  
  
% make the axes with the handle statusHndl active  
axes(statusHndl);  
% fill the bar one additional step  
xpatch=[0 status status 0]/(maxstatus+1);  
ypatch=[0 0 1 1];  
patch(xpatch,ypatch,'r','EdgeColor','none','EraseMode','none');  
view(2);  
% save the current status of the bar  
set(statusHndl,'UserData',status);  
drawnow;
```

9.3 Pressureaxis_2o5.m

```
%% Script that plots pressure distribution on axis for two cases
%% one case with radius of 2 wavelength and one with radius of 5
wavelength
clear all; clc; close all;
```

```
global dsigma k
tic
```

```
u0=0.005;
f=40000;
c0=343;
lambda=c0/f;
rho0=1.21;
a=lambda*2; %Radius of element
```

```
% On the axis on different distance
```

```
w=2*pi*f;
k=w/c0;
R0=(k*a^2)/2; %Rayleigh distance.
rsteps=10000;% steps along axis
r=[1e-7:R0/rsteps:R0];
r1=sqrt(r.^2+a^2);
t=1;
P0=rho0*c0*u0;
```

```
%% a = lambda * 2 %%%
```

```
p(1,:)=P0*(exp(i*(w*t-k*r))-exp(i*(w*t-k*r1)));
rgR0=r/R0;
Pplot=abs(p)/P0;
```

```
figure;
plot(rgR0,Pplot,'k'); axis([0 1 0 2.3]);
xlabel('Distance/R0');
ylabel('Pressure/P0');
text(0.5,2,'Radius = 2 * Wavelength');
```

```
%% a = lambda * 5 %%%
```

```

a=lambda*5; %Radius of element
% On the axis on different distance
w=2*pi*f;
k=w/c0;
R0=(k*a^2)/2; %Rayleigh distance.
rsteps=10000;% steps along axis
r=[1e-7:R0/rsteps:R0];
r1=sqrt(r.^2+a^2);
    p(1,:)=P0*(exp(i*(w*t-k*r))-exp(i*(w*t-k*r1)));
rgR0=r/R0;

Pplot=abs(p)/P0;

figure;
plot(rgR0,Pplot,'k'); axis([0 1 0 2.3]);
xlabel('Distance/R0');
ylabel('Pressure/P0');
text(0.5,2,'Radius = 5 * Wavelength');

```

9.4 Pressureofaxis_2o5.m

```
%% Script that plots pressure distribution off axis for two cases
%% one case with width and height of 2 wavelength and one with width
and height of 5
clear all; clc; close all;
global dsigma k
tic
u0=0.005;
f=40000;
c0=343;
lambda=c0/f;
rho0=1.21;
a=1*lambda; %% Radius of element for case with radius 1 * wavelength
%a=2*lambda; %% Radius of element for case with radius 2 * wavelength
%a=5*lambda; %% Radius of element for case with radius 5 * wavelength

% On the axis on different distance
w=2*pi*f;
k=w/c0;
R0=(k*a^2)/2; % Rayleigh distance.
rsteps=500;% steps along axis
z=[1e-7:0.6735/rsteps:0.6735];
%r1=sqrt(r.^2+a^2);
t=1; %time cant vary in this program. Its only the amplitude thats
interesting so it doesn't matter
P0=rho0*c0*u0;

astep=50;
dsigma=a/astep;
sigma=0:dsigma:a;
xistep=150;
dxi=pi/xistep;
xi=0:dxi:pi;
xtot=3*a;
xstep=200;
dx=xtot/xstep;
x=0:dx:xtot;
```

```

[fig status]=statusbar('Calculating Pressure for each z');

for n=1:length(z);

updatestatusbar(status,n,length(z));
[fig2 status2]=statusbar('Calculating Pressure over x for each z',2);
    for m=1:length(x);
        %tic
            updatestatusbar(status2,m,length(x));
            R2=feval('R3',z(n),x(m),xi,sigma);
            p1=sum(R2,2);
            p(n,m)=i*k*P0/pi*exp(j*w*t)*sum(p1*dxi,1);
        %    toc
    end
close('Calculating Pressure over x for each z')

end
close('Calculating Pressure for each z')
% figure;
% plot(z/R0,abs(p)/P0); axis([0 1 0 2]);
% toc
%save rekt300g200;

P(:,xstep+2:2*xstep+2)=p;
P(:,1:xstep+1)=flipdim(p,2);
X(:,xstep+2:2*xstep+2)=x;
X(:,1:xstep+1)=-flipdim(x,2);

mesh(z(:,2:rsteps)/0.6735,X/a,abs(P(2:rsteps,:))'/P0); view([0 90])
axis equal
axis([0 1 -1 1])
%text(0.06,0.04,'radius = 1 * wavelength') %% For the case with radius 1
* wavelength
%text(0.06,0.04,'radius = 2 * wavelength') %% For the case with radius 2
* wavelength
%text(0.25,0.1,'radius = 5 * wavelength') %% For the case with radius 5 *
wavelength

```

```

save bild1
%save bild2
%save bild3

```

Functions for Pressureofaxis_2o5.m

R3.m

```

function R2=R3(Z,X,xi,sigma);
global k dsigma

%[figHndl statusHndl] = statusbar('Calculating the R matrix over r');

R=zeros(length(sigma),length(xi));%R=zeros(length(r),length(teta),length(s
igma),length(xi));

%for k=1:length(r);
% updatestatusbar(statusHndl,k,length(r));
%[figHndl2 statusHndl2] = statusbar('Calculating the R matrix over
teta',2);
%for n=1:length(teta);
%updatestatusbar(statusHndl2,n,length(teta));
for m=1:length(xi);

for c=1:length(sigma);
R1(m,c)=sqrt((X-
sigma(c)*cos(xi(m)))^2+sigma(c)^2*(sin(xi(m)))^2+Z^2);
R2(m,c)=sigma(c)*exp(-i*k*R1(m,c))/R1(m,c)*dsigma;
end
end
%end
%close ('Calculating the R matrix over teta');
%end
%close('Calculating the R matrix over r');

```

9.5 Pressure_rect_4o10.m

```
%% Script that plots pressure distribution for two cases
%% one case with width and height of 4 wavelength and one with width
and height of 10

clear all; clc; close all;
global da k

u0=0.005;
f=40000;
c0=343;
lambda=c0/f;
rho0=1.21;

%a=lambda*1;      %Height of element in case a = lambda*1
%a=lambda*4;     %Height of element in case a = lambda*4
%a=lambda*10;    %Height of element in case a = lambda*10
b=a;             %Width of element

w=2*pi*f;
k=w/c0;
R0=(a*b)/lambda; %Rayleigh distance.

zstep=500; % Steps along z-axis
%ztot=R0; % Size of room in z direction in the case on axis
ztot=0.8575; % Size of room in z direction in the case off axis where R0 is
0.8575 in the last case
z=[1e-7:ztot/zstep:ztot];

xstep=200; % Steps along x-axis
xtot=1*a; % Size of room in x direction
x=[-xtot:xtot/xstep:xtot]; % In the case off axis
%x=0; % In the case On axis

t=1; %time cant vary in this program. Its only the amplitude thats
interesting so it doesn't matter
```



```
P0=rho0*c0*u0;
```

```
astep=70;
```

```
da=a/astep
```

```
A=-a/2:da:a/2;
```

```
bstep=70;
```

```
db=b/bstep;
```

```
B=-b/2:db:b/2;
```

```
[fig status]=statusbar('Calculating Pressure for each z');
```

```
for n=1:length(z);
```

```
    updatestatusbar(status,n,length(z));
```

```
    [fig2 status2]=statusbar('Calculating Pressure over x for each z',2);
```

```
        for m=1:length(x);
```

```
            updatestatusbar(status2,m,length(x));
```

```
            R2=feval('R3',z(n),x(m),B,A);
```

```
            p1=sum(R2,2);
```

```
            p(n,m)=i*k*P0/pi*exp(i*w*t)*sum(p1*db,1);
```

```
        end
```

```
    close('Calculating Pressure over x for each z')
```

```
end
```

```
close('Calculating Pressure for each z')
```

```
figure;
```

```
% plot(z/R0,abs(p)/P0,'k'); axis([0 1 0 2]); % in the case on axis
```

```
% xlabel('Distance/R0'); % in the case on axis
```

```
% ylabel('Pressure/P0'); % in the case on axis
```

```
mesh(z(:,2:zstep)/0.8575,x/a,abs(p(2:zstep,:))'/P0); view([0 90])
```

```
%text(0.5,1.7,'Width and height = 1 * Wavelength'); %in the case a =  
lambda*1
```

```
%text(0.5,1.7,'Width and height = 4 * Wavelength'); %in the case a =  
lambda*4
```

```

%text(0.5,1.7,'Width and height = 10 * Wavelength'); %in the case a =
lambda*10

%save bild1
%save bild2
%save bild3

```

Functions for Pressureaxis_rect_4o10.m

R3.m

```

function R2=R3(Z,X,A,B);
global k da

for s=1:length(B);
for c=1:length(A);
    R1(s,c)=sqrt((X-A(c))^2+(B(s))^2+Z^2);
    R2(s,c)=exp(-i*k*R1(s,c))/R1(s,c)*da;
end
end

```

9.6 Comparison.m

To get any results from this script it is necessary to run the two next coming scripts first, Large_element.m and Small_element.m

```

%% Script that reads and plots results to compare Array with large element.
%% The first operation is to make the sumation to an array
clear all; clc; close all;

load rektWithFarfield2;

P(:,302:602)=p;
P(:,1:301)=flipdim(p,2);
X(:,302:602)=x;
X(:,1:301)=-flipdim(x,2);

distance=0.010%0.0125
MT=distance/dx;

```

```

X1=-dx*(602+4*MT)/2:dx:dx*(602+4*MT)/2-dx;
S(:,1)=P;
S(:,1+MT:602+MT,2)=P;
S(:,1+2*MT:602+2*MT,3)=P;
S(:,1+3*MT:602+3*MT,4)=P;
S(:,1+4*MT:602+4*MT,5)=P;
S=sum(S,3);
figure(1);
subplot(2,1,1);
mesh(z(:,2:1500),X1/0.005,abs(S(2:1500,:)/P0)); view([0 90]); axis([0 0.3 -
10 10]);
title('Pressure in plane'); ylabel('Radial distance/radius of small element')

```

```

figure(1);
subplot(2,1,2);
plot(z(2:1500),abs(S(2:1500,(602+4*MT)/2)/P0),'k'); axis([0 0.3 0 2]);
title('Pressure on axis'); xlabel('Distance from source [meter]');
ylabel('Pressure/P0')

```

```

figure(3)
plot(z(2:1500),abs(S(2:1500,(602+4*MT)/2)/P0),'k'); axis([0 0.3 0 2]);
xlabel('Distance from source [meter]'); ylabel('Pressure/P0')

```

```
hold on;
```

```
clear all;
```

```
load rektangular1000g300
```

```
figure(2)
```

```
subplot(2,1,2)
```

```
plot(z(2:1000),abs(p(2:1000,300)/P0),'k'); axis([0 0.3 0 2]);
```

```
title('Pressure on axis'); xlabel('Distance from source [meter]');
```

```
ylabel('Pressure/P0')
```

```
figure(2);
```

```
subplot(2,1,1);
```

```
mesh(z(2:1000),x/0.005,abs(p(2:1000,:)/P0)); view([0 90]);
```

```
axis([0 0.3 -10 10]); title('Pressure in plane');
```

```
ylabel('Radial distance/radius of small element')
```

```
% z(2:100)/R0,x
```

```
figure(3)
```

```
plot(z(2:1000),abs(p(2:1000,300)/P0),'k-.'); axis([0 0.3 0 2]);
```

```
legend('Array','Large element')
```

9.7 Large_element.m

```
%% Script that calculates pressure distribution from a rectangular element,
%% with the dimensions a and b
clear all; clc; close all;
global da k
tic
u0=0.005;
f=40000;
c0=343;
lambda=c0/f;
rho0=1.21;
a=0.005;    %Height of element
b=0.0505;   %Width of element

w=2*pi*f;
k=w/c0;
R0=(k*a^2)/2; %Rayleigh distance.

zstep=1000; % Steps along z-axis
ztot=0.32036; % Size of room in z direction
z=[1e-7:ztot/zstep:ztot];

xstep=300; % Steps along x-axis
xtot=0.05; % Size of room in x direction
x=[-xtot:xtot/xstep:xtot];

t=1; %time cant vary in this program. Its only the amplitude thats
interesting so it doesn't matter
P0=rho0*c0*u0;

astep=70;
da=a/astep
A=-a/2:da:a/2;
bstep=100;
db=b/bstep;
B=-b/2:db:b/2;
```

```

[fig status]=statusbar('Calculating Pressure for each z');

for n=1:length(z);
%tic
updatestatusbar(status,n,length(z));
[fig2 status2]=statusbar('Calculating Pressure over x for each z',2);
    for m=1:length(x);
        %tic
            updatestatusbar(status2,m,length(x));
            R2=feval('R3',z(n),x(m),B,A);
            p1=sum(R2,2);
            p(n,m)=i*k*P0/pi*exp(i*w*t)*sum(p1*db,1);
            %toc
        end
    end
    %toc
close('Calculating Pressure over x for each z')

end
close('Calculating Pressure for each z')
toc
save rektangular1000g300;

```

Functions for Large_element.m

R3.m

```

function R2=R3(Z,X,A,B);
global k da

for s=1:length(B);
for c=1:length(A);
    R1(s,c)=sqrt((X-A(c))^2+(B(s))^2+Z^2);
    R2(s,c)=exp(-i*k*R1(s,c))/R1(s,c)*da;
end
end

```

9.8 Small_element.m

```
%% Script that calculates pressure distribution from a circular element,
%% with the radius a
clear all; clc; close all;
global dsigma k
tic
u0=0.005;
f=40000;
c0=343;
lambda=c0/f;
rho0=1.21;
a=0.005;%0.005; %Radius of element

% On the axis on different distance
w=2*pi*f;
k=w/c0;
R0=(k*a^2)/2; %Rayleigh distance.
rsteps=1500;% steps along axis
z=[1e-7:35*R0/rsteps:35*R0];
%r1=sqrt(r.^2+a^2);
t=1; %time cant vary in this program. Its only the amplitude thats
interesting so it doesn't matter
P0=rho0*c0*u0;

astep=100;
dsigma=a/astep
sigma=0:dsigma:a;
xistep=250;
dxi=pi/xistep;
xi=0:dxi:pi;
xtot=12*a;
xstep=300;
dx=xtot/xstep;
x=0:dx:xtot;

[fig status]=statusbar('Calculating Pressure for each z');
```

```

for n=1:length(z);

updatestatusbar(status,n,length(z));
%tic
[fig2 status2]=statusbar('Calculating Pressure over x for each z',2);

for m=1:length(x);
    %tic
        updatestatusbar(status2,m,length(x));
        R2=feval('R3',z(n),x(m),xi,sigma);
        p1=sum(R2,2);
        p(n,m)=i*k*P0/pi*exp(j*w*t)*sum(p1*dxi,1);
        % toc
    end
close('Calculating Pressure over x for each z')

%toc
end
close('Calculating Pressure for each z')
%figure;
%plot(z/R0,abs(p)/P0); axis([0 1 0 2]);
toc
save rektWithFarfield2;

```

Functions for Small_element.m

R3.m

```

function R2=R3(Z,X,xi,sigma);
global k dsigma

%[figHndl statusHndl] = statusbar('Calculating the R matrix over r');

R=zeros(length(sigma),length(xi));%R=zeros(length(r),length(teta),length(s
igma),length(xi));

%for k=1:length(r);
    % updatestatusbar(statusHndl,k,length(r));
    %[figHndl2 statusHndl2] = statusbar('Calculating the R matrix over
teta',2);

```

```

%for n=1:length(teta);
    %updatestatusbar(statusHndl2,n,length(teta));
for m=1:length(xi);

for c=1:length(sigma);
    R1(m,c)=sqrt((X-
sigma(c)*cos(xi(m)))^2+sigma(c)^2*(sin(xi(m)))^2+Z^2);
    R2(m,c)=sigma(c)*exp(-i*k*R1(m,c))/R1(m,c)*dsigma;
end
end
%end
%close ('Calculating the R matrix over teta');
%end
%close('Calculating the R matrix over r');

```




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