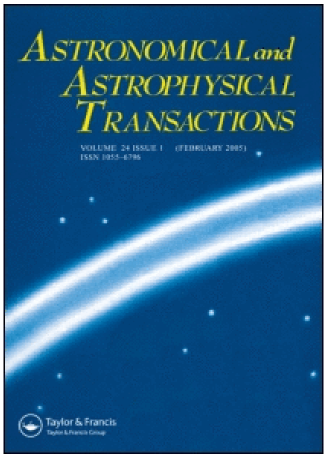

Astronomical & Astrophysical Transactions
The Journal of the Eurasian Astronomical Society
Publication details, including instructions for authors and subscription information:
http://www.informaworld.com/smpp/title~content=t713453505

Maxwellization of the einstein tetrad equations

R. F. Polishchuk [a]
[a] Lebedev Physical Institute, Moscow, Russia

Online Publication Date: 01 June 1996
To cite this Article: Polishchuk, R. F. (1996) 'Maxwellization of the einstein tetrad equations', Astronomical & Astrophysical Transactions, 10:1, 83 - 84
To link to this article: DOI: 10.1080/10556799608203249
URL: http://dx.doi.org/10.1080/10556799608203249

Astronomical and Astrophysical Transactions, 1996,
Vol. 10, pp. 83–84
Reprints available directly from the publisher
Photocopying permitted by license only

MAXWELLIZATION OF THE EINSTEIN TETRAD EQUATIONS

R. F. POLISHCHUK

Lebedev Physical Institute, Moscow, Russia

(Received June 20, 1995)

Deepening the analogy between the Maxwell and Einstein equations with the "Maxwell part" separated in the Einstein tensor provides a realistic approach to the energy problem solution in general relativity with tetrad potentials.

KEY WORDS **General relativity, tetrad field, conservation laws**

Weyl [1] discovered gradient invariance of the Maxwell equation and Lorentzian invariance of the Einstein equations. Transition from the metric to the tetrad permits one to demonstrate the gradient invariance of the Einstein equations.

In space time V the electromagnetic and gravitational fields may be described by 1-forms $A = A_\mu(x)\,dx^\mu$, $e_a = e_{a\mu}(x)\,dx^\mu$, respectively. Here $\mu = 0, i$ is the coordinate index, a is the Lorentzian one. Let $g_{\rm ab} := diag(-1,1,1,1)$. Riemannian metric $g_{\mu\nu} = e_{a\mu}e^a_\nu$ determines the Riemannian connection ∇_μ, Hodge operator $*$, codifferential δ, the Laplacian Δ [2], d'Alembertian $\Box = -\nabla^2$, as well as Riemannian $R_{\mu\nu\alpha\beta}$, Ricci R_a, and Einstein G_a tensors:

$$\delta = (-1)^p *^{-1} d* = (-1)^{pn+n+1}\mathrm{sgn}\,g * d*,\ \delta A = -\nabla^\mu A_\mu$$

$$**\alpha = (-1)^{p(n-p)}(\mathrm{sgn}\,g)\alpha,\ *1 = (-g)^{1/2}d^4x,\ *e_a =: |*e_a|d^3x$$

$$\Delta = (d+\delta)^2 = d\delta + \delta d,\ d^2 = \delta^2 = 0$$

$$R_a = R^\alpha_{\mu\alpha\nu}e^\mu_a\,dx^\nu = (\Delta - \Box)e_a,\ G_a = R_a - Re_a/2.$$

On the manifold V we have $g = \det\, g_{\mu\nu}$, $\mathrm{sgn}\,g = -1$, $n = 4$, $\delta = *d*$, p is the degree of the form α, $|*e_a|^2$ is the determinant of the 3-metric orthogonal e^μ_a. The Maxwell and the Einstein tetrad equations on V are the following:

$$\Delta\, A = 4\pi J,\ R_a = 8\pi(T_a - Te_a/2),$$

$$\Delta e_a = 8\pi S_a,\ S_a := T_a - Te_a/2 + \Box e_a/8\pi.$$

In the case of the Lorentzian gauge $\nabla^\mu A_\mu = \nabla^\mu e_{a\mu} = 0$ we have $\nabla^\mu J_\mu = \nabla^\mu S_{a\mu} = 0$. Let S_a be a tetrad current by definition as well as P_a be the total

4-momentum for a gravitating physical system on any spacelike hypersurface Σ on $V : P_a := \int_\Sigma *S_a = const.$

The energy density $S_{0\mu}e_0^\mu$ is the following:

$$8\pi S_{00} = 8\pi(T_{00} + T/2) + \nabla_\mu e_{0\nu}\nabla^\mu e_0^\nu.$$

For a semigeodesic ($g_{00} + 1 = g_{0i} = 0$) observer and for a semiharmonic ($g_{00} + \det g_{ij} = g_{0i} = 0$) one, $S_{00} \geq 0$. In a quasi-Newtonian field $S_{00} = -a^2/8\pi$ (a is the free fall acceleration), for weak flat gravitational waves along x^1 with little perturbations $h_{22} = -h_{33}$, h_{23} of constant metric we have $16\pi S_{00} = (\partial_0 h_{22})^2 + (\partial_0 h_{23})^2$. The energy-momentum pseudotensor is not required here.

The left parts of the equations

$$\delta dA = 4\pi J - d\delta A, \ \delta de_a = 8\pi S_a - d\delta e_a$$

are coclosed and are gradient invariants (for the transformations $A \rightarrow A + d\alpha$, $e_a \rightarrow e_a + d\alpha_a$). If $\delta e_a = 0$, one can assume $| * e_a| = 1$, the sum of the e_a-lines curvature vectors becomes zero (the quasiinertial tetrad). In quantum electrodynamics and quantum gravity, field states are admissible only with constraints for the quantum averages $\langle\delta A\rangle = \langle\delta e_a\rangle = 0$.

References

1. Weyl, H. (1929) *Zs.Phys.*, B. 56, S. 330.
2. de Rham, G. (1955) *Varietes Differentiables*, Hermann (ed.), Paris.