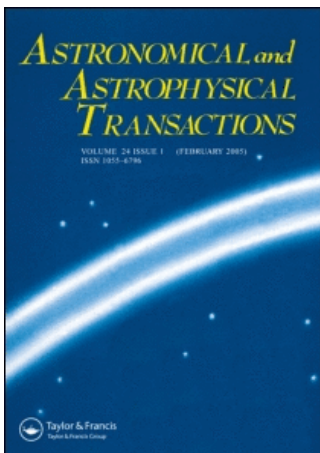


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Maxwellization of the einstein tetrad equations

R. F. Polishchuk ^a

^a Lebedev Physical Institute, Moscow, Russia

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MAXWELLIZATION OF THE EINSTEIN TETRAD EQUATIONS

R. F. POLISHCHUK

Lebedev Physical Institute, Moscow, Russia

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Deepening the analogy between the Maxwell and Einstein equations with the “Maxwell part” separated in the Einstein tensor provides a realistic approach to the energy problem solution in general relativity with tetrad potentials.

KEY WORDS General relativity, tetrad field, conservation laws

Weyl [1] discovered gradient invariance of the Maxwell equation and Lorentzian invariance of the Einstein equations. Transition from the metric to the tetrad permits one to demonstrate the gradient invariance of the Einstein equations.

In space time V the electromagnetic and gravitational fields may be described by 1-forms $A = A_\mu(x) dx^\mu$, $e_a = e_{a\mu}(x) dx^\mu$, respectively. Here $\mu = 0, i$ is the coordinate index, a is the Lorentzian one. Let $g_{ab} := \text{diag}(-1, 1, 1, 1)$. Riemannian metric $g_{\mu\nu} = e_{a\mu} e_a^\nu$ determines the Riemannian connection ∇_μ , Hodge operator $*$, codifferential δ , the Laplacian Δ [2], d’Alembertian $\square = -\nabla^2$, as well as Riemannian $R_{\mu\nu\alpha\beta}$, Ricci R_a , and Einstein G_a tensors:

$$\begin{aligned} \delta &= (-1)^p *^{-1} d* = (-1)^{pn+n+1} \text{sgn } g * d*, \quad \delta A = -\nabla^\mu A_\mu \\ **\alpha &= (-1)^{p(n-p)} (\text{sgn } g)\alpha, \quad *1 = (-g)^{1/2} d^4x, \quad *e_a := |*e_a| d^3x \\ \Delta &= (d + \delta)^2 = d\delta + \delta d, \quad d^2 = \delta^2 = 0 \\ R_a &= R_{\mu\alpha\nu}^\alpha e_a^\mu dx^\nu = (\Delta - \square)e_a, \quad G_a = R_a - R e_a/2. \end{aligned}$$

On the manifold V we have $g = \det g_{\mu\nu}$, $\text{sgn } g = -1$, $n = 4$, $\delta = *d*$, p is the degree of the form α , $|*e_a|^2$ is the determinant of the 3-metric orthogonal e_a^μ . The Maxwell and the Einstein tetrad equations on V are the following:

$$\begin{aligned} \Delta A &= 4\pi J, \quad R_a = 8\pi(T_a - T e_a/2), \\ \Delta e_a &= 8\pi S_a, \quad S_a := T_a - T e_a/2 + \square e_a/8\pi. \end{aligned}$$

In the case of the Lorentzian gauge $\nabla^\mu A_\mu = \nabla^\mu e_{a\mu} = 0$ we have $\nabla^\mu J_\mu = \nabla^\mu S_{a\mu} = 0$. Let S_a be a tetrad current by definition as well as P_a be the total

4-momentum for a gravitating physical system on any spacelike hypersurface Σ on $V : P_a := \int_{\Sigma} *S_a = \text{const.}$

The energy density $S_{0\mu}e_0^\mu$ is the following:

$$8\pi S_{00} = 8\pi(T_{00} + T/2) + \nabla_\mu e_{0\nu} \nabla^\mu e_0^\nu.$$

For a semigeodesic ($g_{00} + 1 = g_{0i} = 0$) observer and for a semiharmonic ($g_{00} + \det g_{ij} = g_{0i} = 0$) one, $S_{00} \geq 0$. In a quasi-Newtonian field $S_{00} = -a^2/8\pi$ (a is the free fall acceleration), for weak flat gravitational waves along x^1 with little perturbations $h_{22} = -h_{33}$, h_{23} of constant metric we have $16\pi S_{00} = (\partial_0 h_{22})^2 + (\partial_0 h_{23})^2$. The energy-momentum pseudotensor is not required here.

The left parts of the equations

$$\delta dA = 4\pi J - d\delta A, \quad \delta de_a = 8\pi S_a - d\delta e_a$$

are coclosed and are gradient invariants (for the transformations $A \rightarrow A + d\alpha$, $e_a \rightarrow e_a + d\alpha_a$). If $\delta e_a = 0$, one can assume $|*e_a| = 1$, the sum of the e_a -lines curvature vectors becomes zero (the quasiinertial tetrad). In quantum electrodynamics and quantum gravity, field states are admissible only with constraints for the quantum averages $\langle \delta A \rangle = \langle \delta e_a \rangle = 0$.

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