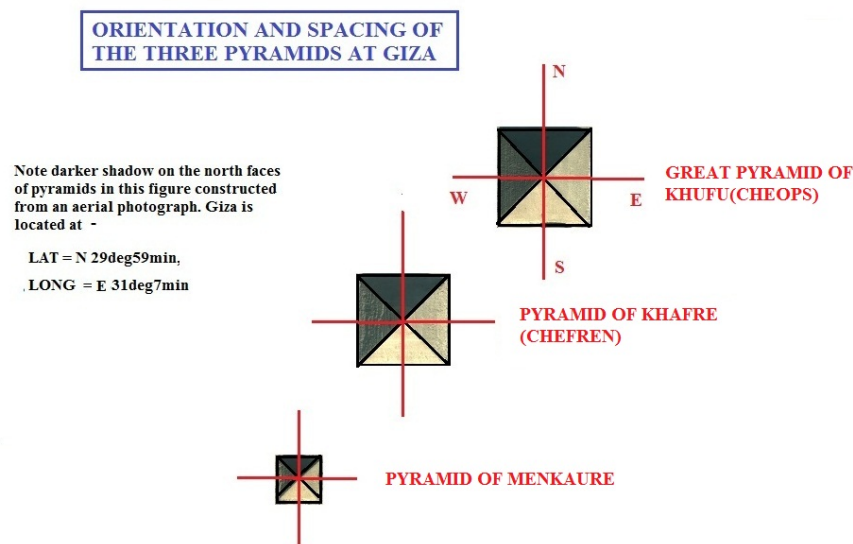# THE TECHNOLOGY BEHIND THE CONSTRUCTION OF THE PYRAMIDS OF GIZA

On a large plateau just west of Cairo one finds three huge square base pyramids constructed of mainly large limestone blocks. These pyramids were built during the time of the Old Egyptian Dynasty some 4500 years ago. The largest of the three pyramids, often referred to as the Great Pyramid , was commissioned by Pharaoh Khufu(2551-2528 BC) as his burial place and is the only one still existing of the seven wonders of the ancient world. People have speculated for years, starting with Herodotus , how these monuments could have been built by people who were not familiar with the wheel, worked only with copper tools, and had no idea about machinery such as cranes and/or block and tackle and had only limited knowledge of astronomy. Yet they did it using a large supply of human labor typically toiling some twenty years per pyramid. It is our purpose here to offer a more rational explanation of how this was accomplished without the need to introduce the use of technology which was not yet known at the time.
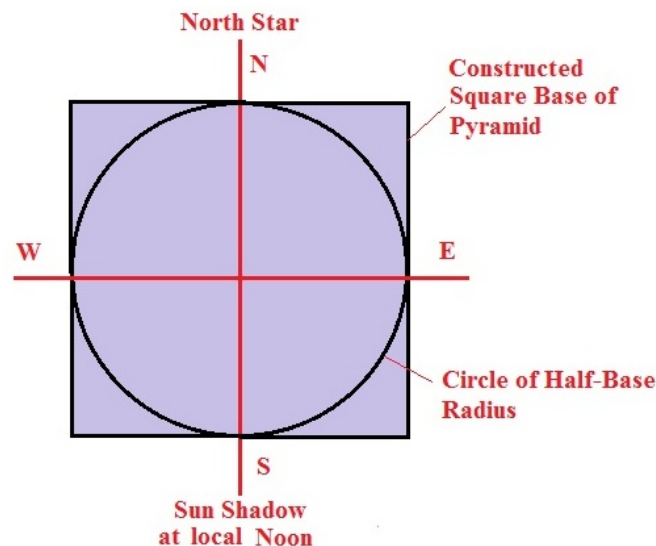
The driving force behind the building of the Egyptian pyramids was their religious belief that a deceased body must be preserved and protected to gain entrance into an afterlife. Mummification and large stable and impenetrable stone structures as a tomb in the form of pyramids met these criteria. Hence the ruling Pharaohs , with their almost unlimited powers and enormous wealth , commissioned these structures and supported research into superior embalming techniques. We want to focus here on the mathematics and technology used in constructing the best three of these pyramids, namely, the ones at the Giza Plateau.

We begin with a figure we have constructed from an aerial view of these pyramids. Here is the figure showing both their orientation with respect to the cardinal points and their relative spacing-

ORIENTATION AND SPACING OF THE THREE PYRAMIDS AT GIZA

Note darker shadow on the north faces of pyramids in this figure constructed from an aerial photograph. Giza is located at -

LAT = N 29deg59min,

LONG = E 31deg7min

GREAT PYRAMID OF KHUFU(CHEOPS)

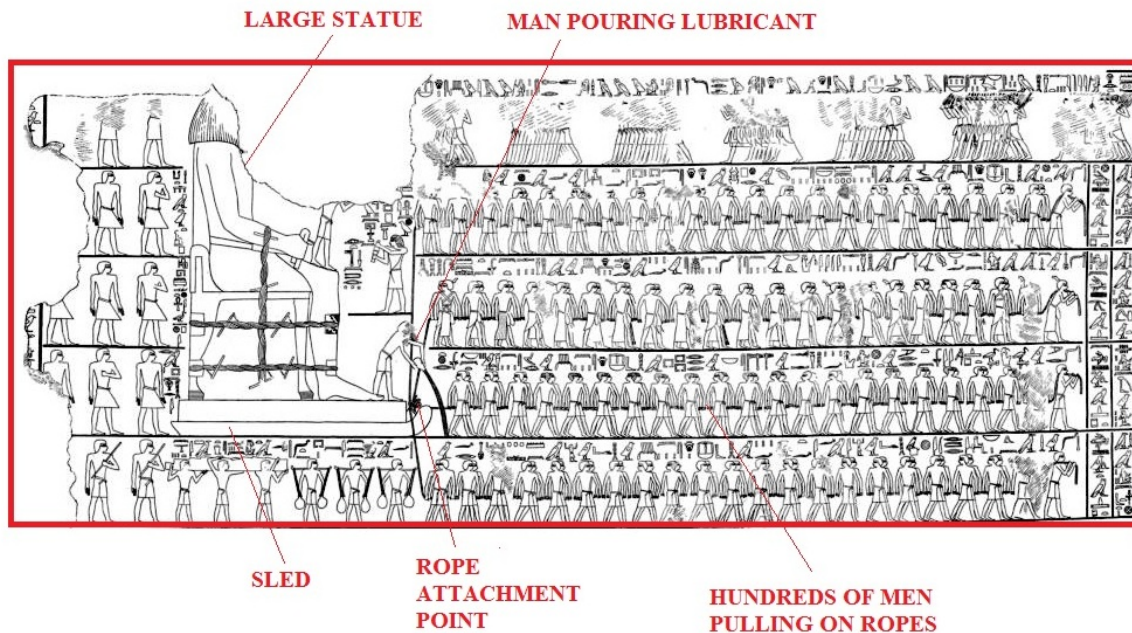PYRAMID OF KHAFRE (CHEFREN)

PYRAMID OF MENKAURE

The first thing one notices is that the pyramids are each oriented almost perfectly in the direction of the cardinal points and that the views in the N-E-S-W directions are unobstructed from the center of each pyramid base. The actual departure from the true north-south line for the Pyramid of Khufu is about 4 minutes of arc (or 1/15th degree). Clearly this orientation was important and was probably accomplished as follows. In 2500BC the Egyptian astronomers would have been familiar with the location of the north star and also knew the direction in which a vertical stick casts its shadow at local noon. Hence connecting these observation points, an accurate north-south line could be drawn as the accompanying figure indicates-

CONSTRUCTING A PYRAMID BASE WITH NORTH-SOUTH ORIENTATION

North Star

N

Constructed Square Base of Pyramid

W   E

Circle of Half-Base Radius

S

Sun Shadow at local Noon

Next the center of the pyramid is placed at a point on this N-S line. Then a circle is drawn around the point using a rope of length equal to one half the intended pyramid base. Next a pair of north-south lines parallel to the original north-south line are drawn such that they are just tangent to the circle. Then an east-west line (E-W) is drawn through these two tangent points. Finally a pair of lines parallel to the east-west line are drawn which just touch the circle at the north-south line intersections. The result is a perfect square oriented in the N-E-S-W directions.

Next we come to the manufacturing and transportation of cut rectangular stones to the pyramid site. The blocks were mostly limestone of density 2.6 gm/cm^3 some cut in a quarry near Cairo but most coming directly from the Giza Plateau. The material is relatively soft and so can be worked with hardened copper chisels existing at the time. Once cut, levers were used to loosen the stones. The transport was most likely by multiple men pulling on ropes attached to a sled carrying a typical block of limestone of about one cubic meter volume. Tomb paintings indicate the use of such sleds before the invention of the spoked wheel which did not arrive in Egypt until some 500 years later. Here is a sketch of the sled transportation method-

LARGE STATUE   MAN POURING LUBRICANT

SLED   ROPE ATTACHMENT POINT   HUNDREDS OF MEN PULLING ON ROPES

The block used in pyramid construction at Giza ranged in weight from a fraction of a ton to several tens of tons. The heaviest , of which there were only a few, were granite imported from Aswan. The average block was about 1meter cubed and so weighed around 2600kg=5720lb =2.86 tons each. Assuming a sled transport to the construction site similar to what is depicted in the above wall painting one will have a friction force of (μ W) resisting motion on a horizontal surface. Here W is the stone plus sled weight and μ the coefficient of friction between the sled runner and the horizontal ground. When moving uphill at angle θ the force F required to move the block up the incline will be-

$$F \geq W[\sin(\theta)+\mu\cos(\theta)\}$$

When no force F is applied the sled and attached block will slide backwards unless $\mu>\tan(\theta)$. Now the Egyptians were smart enough to realize that water-oil mixtures poured onto the sled runners will reduce the friction coefficient considerably and thus require less man power to move a given block. With proper lubrication between hard oak sled runners (with possible copper cladding) against a smooth stone surface can probably bring the dynamic friction coefficient μ down to 0.1. Thus if twenty men are assigned to pull a three ton block sitting on a sled with lubricated runners, it will take an effort of just 30lb each to move the block on a horizontal surface for a sled - block combination weighing 6000lb. They might, however, need some extra help when going up a 30deg incline for then each would need to

pull with a force of 175 lb . Doubling the man power per sled should solve the problem until arrival at the pyramid site.

The pyramids envisioned by the architecs at Giza have a square base bxb and height H .The volume of such a pyramid is just $b^2H/3$. For Khufu's Great Pyramid where b=230.4 meters and H=146.5 meters, this means a volume of 2.59 million cubic meters and so requiring the delivery of some three million blocks assuming some damage along the way. Working at the rate of 10 hours per day every day of the year for twenty years means that cut stones had to arrive at the rate of about 35 blocks per hour. Under equilibrium conditions this also implies that 35 stones are being quarried per hour and that 350 cubic meter stones are being lifted up the sides of the pyramid each day. Wow! Quite an effort, probably involving about one hundred thousand workers working in three month shifts as the Greek historian Herodotus (484-425BC) claims.

So far it looks that everything involving pyramid construction we have discussed is possible with enough people working and use of the technology of the day. But now we come to the most difficult aspect of the construction process, namely, how does one get the stone blocks up the sides of the pyramid at the estimated rate. It must be remembered that in 2500 BC the Egyptians had available the lever and the inclined plane but had no idea about the use of rollers or the wheel or more complicated machines such as block and tackle, pulleys and the screw. Also their supply of wood was very limited so that this material had to be used sparingly. People have suggested that the use of the shadoof (A lever device used for lifting water from the Nile) or wood bearings capable of redirecting force along a rope as possible devices which could be used for lifting process. The difficulty with these ideas is that there is absolutely no evidence of their existence in 2500BC. The solution clearly lies with the use of ropes , inclined planes , and massive manpower. The angle the three pyramids at Giza make with respect to the horizontal is arctan(2H/b). This equals θ=57.6deg for the Khufu Pyramid. Some have suggested that this angle is too steep to move a rectangular block up the side of the pyramid without first building a massive ramp to reduce the inclination angle. Such a suggestion is unrealistic as the ramp would have to be hundreds of meters long for stone block transfer to the higher levels of the pyramid. We suggest, rather that the 210 layers of the Khufu pyramid were built up layer by layer as a step pyramid with steps about h=0.70 meter high and w=0.55 meter horizontal. The tangent of the angle which the corners of such steps make with the horizontal is 1.27 and thus agrees with the θ=57.6 deg given above. The ancient Egyptians measured their angles in terms of sekels which are essentially the tangents of an angle. According to their measurement techniques the slope of the Khufu Pyramid is 1cubit divided by 5 palms and 2 digits. This gives a shekel measure of 7x4/(4x5+2] =1.27 . The Egyptian measure of length was the royal cubit(20.7") with one cubit equal to 7 palms and one palm equal to 4 digits. These measures are based on the human dimension with the width of four fingers equal to the width of a palm. The seven palm widths from the human elbow to the tip of the middle finger produce the cubit.

Now getting to what seems to us the most logical method for raising blocks up the side of a pyramid using only 2500BC technology and minds. I think it involved something as simple as constructing four several meter wide causeways along the four pyramid faces by inserting triangular cros-section blocks between steps and then pulling the large stone blocks mounted securely to a small sled up the causeway by brute force via two groups at the top level pulling on two ropes attached to the sled. The

geometry of the causeways would have been not unlike what I saw back in 1983 at the Castilio Pyramid in the Yucatan. Here is a photo-

CHICHEN-ITZA-1983

Note the causeways on the sides of the pyramid. We envision these causeways in the Giza pyramids to have not been built-up as at Chichen Itza but rather to be flush with the remaining pyramid steps.

To pull a limestone block up along such a steep inclined causeway (by the way, one on which one could not remain standing without the presence of steps), we have two crews of men standing on the latest finished pyramid level and pulling on two ropes attached to the sled and the securely mounted block. It may also have been that no sleds were used in this raising step since the friction term becomes small compared to the downward weight when a block is pulled up such a steep incline. If the angle between the two ropes is α degrees the force which needs to be exerted by each crew will be-

$$F = \left(\frac{W}{2}\right)\left(\frac{\sin(\theta) + \mu\cos(\theta)}{\cos(\alpha)}\right)$$

Sliding a 6000lb block up such a causeway when μ=0.1, θ=57.6deg, and α=30deg will require each crew pull with 2855 lb. With 40 men in each grew this load reduces to 71.4 lb each. It should be remembered that the other three causeways are also in use simultaneously. It still requires considerable effort among these crews to match the nine blocks per hour which each set of crews must slide up to the construction level. A fortunate property of pyramids is that most of the required stones go to the bottom portion so that by the time the level is at half the final height, fully seven-eights of the total stone supply has already been incorporated into the structure. Nevertheless, it was the last portion of pyramid construction which must have been the most difficult and is the reason the stone blocks become smaller near the top. After the highest level was reached and a capstone placed, they then finished the entire

stepped surface of the pyramid with a brilliant white limestone cladding most of which was removed many years later to help build mosques in Cairo. If you look at a picture of the Chefren Pyramid or actually have the opportunity to visit in person, you can still see the cladding near its top. I remember being inside this pyramid back in the 1980s. It was a somewhat un-impressive sight as it led to an empty tomb in which was located an open and empty sarcophagus plus the name of Belzoni (an Italian explorer of the early 18 hundreds of dubious reputation but also the discoverer of the Ramses tomb at Abu Simbel) written in large letters on the wall. No hieroglyphics but plenty of heat and humidity.

We have shown that , with the rather primitive technology available to the ancient Egyptians, the construction of the pyramids was quite feasible without outside help and was accomplished with the use of only ropes, copper tools, sleds, levers and inclined planes and of course an almost unlimited supply of Egyptian labor. There is no need to postulate the need for help from Atlantians or other nonsense such as aid from extra-terrestrials.