ON BECOMING AN ACTUARY OF THE THIRD KIND

STEPHEN P. D'ARCY

Abstract

The growing importance of investment performance in insurance operations, the increasing volatility in financial markets and the emergence of investment-linked insurance contracts are creating the need for actuaries to develop new skills and a greater awareness of investment performance. Hans Bühlmann recently classified actuaries that work with the investment side of insurance as actuaries of the third kind. This paper describes the similarities and differences between actuarial science and financial economics, indicates the current issues in financial economics, and summarizes the major applications of financial economics to insurance.

1. INTRODUCTION

The purpose of this paper is to assist in the conversion of actuaries of the first kind or second kind into actuaries of the third kind. This actuarial classification system was recently proposed by Hans Bühlmann [15]. Actuaries of the first kind are life actuaries. According to Bühlmann, the primary methods of life actuaries involve deterministic calculations. Actuaries of the second kind, the casualty actuaries, develop probabilistic methods for dealing with risky situations. The actuaries of the third kind deal with the investment side of insurance and incorporate stochastic processes into actuarial calculations. I believe that all aspects of insurance product development and pricing will soon involve a combination of investment and insurance characteristics. This change will require all actuaries to become actuaries of the third kind.

The investment area falls into the academic dominion of the field called finance or financial economics. This area specializes in capital markets and the raising, spending, protecting and investing of money. The pricing of capital assets and the estimation of interest rates, two important functions of actuaries, attract a great deal of attention from financial economists. However, the basic concepts and perspectives of financial economists are, in some regards, alien to actuaries. Thus, the second section of this paper discusses how actuaries and financial economists each view some very basic common issues. The third section provides a synopsis of the leading issues in financial economics. The fourth section describes applications of financial economics to insurance. The final section draws some conclusions concerning the converging paths of actuarial science and financial economics and discusses likely future developments.

2. FINANCIAL ECONOMICS AND THE ACTUARY

Development

Actuaries and financial economists could be compared to distant cousins that would be surprised at discovering their degree of consanguinity. Both are mathematically inclined, address monetary issues and incorporate risk into their calculations. Both insurance and finance have ancient roots, and both have undergone dramatic transformations several times. The most notable transformations relevant for life actuarial science were the development of mortality tables, institution of nonforfeiture provisions and the recent connection of benefit levels to investment performance. For property-liability insurance the significant developments include the entrenchment of regulatory power, the elimination of traditional distinctions, initially leading to multiple line policies and eventually to full financial service firms, and the expansion of legal liability. Similar epochal developments for finance would be the development of central banks, organized stock exchanges, security regulation, modern portfolio theory and the development of markets for derivative securities such as options and futures.

Actuarial science and financial economics have developed tools to address the relevant issues for their disciplines independently. As in any profession, each has developed a specialized language to describe terms and techniques in the field. This specialized language, in some aspects similar to a secret code, serves as much to exclude outsiders as to facilitate communication within the field. However, now that insurance is moving into the investment domain, both in offering products tied to investment performance and in developing corporate investment strategies, the specialized languages are becoming a handicap, especially where similar terms have different meanings in the different disciplines. Financial economists are hindered in their analysis of insurance problems by the difficulty in understanding insurance terminology and practices. Actuaries are at a similar disadvantage in addressing issues in finance. This introduction will serve as a bridge between the areas of actuarial science and financial economics by discussing some very basic issues in these fields and illustrating the different approaches taken by the two specialties.

Risk

Risk is a central, if not the central, element in both insurance and finance. Individuals are assumed to be risk averse and thus would be willing to pay a premium over expected losses to reduce risk; initially, a similar assumption was often made about corporations, but more recent work has treated corporations as a web of contractual relationships (employer-employee, stockholder-bondholder-manager, supplier-consumer) that is itself risk neutral. Individuals purchase insurance because risk exists and they seek to minimize or avoid the financial consequences inherent in risk. In the area of finance, risk is involved in explaining the price level and required rate of return on different investments as well as the optimal investment strategies. However, how risk is considered in the two areas differs significantly.

In insurance, risk is generally defined as uncertainty concerning loss. A measure of risk is the expected deviation between actual and expected losses, generally scaled to the expected loss value. For an individual insured, the expected losses would commonly be a small value, representing the product of the loss frequency and the loss severity. Actual losses will generally be zero, but the possibility of a large value, representing some point on the loss distribution, must also be considered. For most lines of business, individual risks are assumed to be independent, so for an insurer the risk of a collection of policies will be less than the sum of the risk of the policies, or even the average risk level on the policies. Notable exceptions include financial guarantee, flood and earthquake coverages. In actuarial science, the law of large numbers dictates that the riskiness of a portfolio of independent risks will reduce as the size of the portfolio increases. In general, actuaries assume that the risk is eliminated from the point of view of the insurer as a result of writing a large number of policies. Thus, the riskiness of an individual insured is not relevant to the price of the policy. In most cases, only the expected value of the loss is used to establish the price level for an insured.

In investments, the potential wealth changes are not restricted to be zero or negative, as is the case for insurance policies, but can also be positive. Thus, the definition of risk is expanded to be the uncertainty concerning outcome. In general, the standard deviation of the return distribution is used as the measure of risk, although higher moments have also been used.

The key difference between actuarial science and finance in regard to risk is the effect of combining separate risks into a portfolio. The standard deviation is commonly used as a measure of risk. If R_p is used to denote the return on a portfolio in which the variance of each of n elements in the portfolio is denoted by σ^2 and the covariance between any two elements within the portfolio is $q\sigma^2$, then the risk of a portfolio can be calcualted as follows:

$$\text{Var}(R_p) = (\sigma^2/n)[1 + (n-1)q], \qquad (2.1)$$

where R_p = expected outcome (expected loss for an insurance policy or expected return for an asset) for the portfolio of elements;
σ = standard deviation of outcomes for the individual elements;
n = number of individual elements combined in the portfolio;
q = correlation coefficient between any two elements.

If the elements are not correlated ($q = 0$), then the portfolio risk converges to zero as n approaches infinity. This is, for insurers, the law of large numbers. However, if the elements are correlated, then the portfolio risk does not converge to zero, but to some value dependent on the degree of correlation. This relationship is the key aspect of portfolio theory in investment analysis. Individual investments are not independent of each other. Thus, the risk of a portfolio will not reduce to zero by combining a large number of different investments. This residual risk is a central concern to financial economists. Financial economists classify investment risk on an individual security into two components, diversifiable and systematic risk. Diversifiable risk is the degree of fluctuation that is uncorrelated with other securities. This risk does cancel out in a portfolio, similar to the effect of the law of large numbers on insurance policies. Also similar to insurance, this form of risk is ignored in most asset pricing models. As an investor can eliminate this type of risk from his or her portfolio by diversifying, diversifiable risk is assumed to be irrelevant in pricing capital assets.

The remaining risk inherent in individual investments is termed systematic risk. This risk does not cancel out in a portfolio, because it is common to all risky investments. As the investor cannot eliminate this form of risk, it becomes important in pricing the capital asset. A high level of systematic risk requires a greater rate of return.

Thus, an actuary views risk as a component of an individual insured that cancels out at the level of the insurer due to the law of large numbers. The financial economist views risk as a combination of two factors, diversifiable risk that is irrelevant for pricing assets and systematic risk that enters into the asset pricing determination.

Interest Rates

Although casualty actuaries have ignored interest rates in pricing insurance until recently, life actuaries have traditionally included an interest rate factor in the determination of rates. The interest rate used to price policies has generally been a conservative level that the actuary feels certain can be achieved by the company under almost any economic conditions. Through the early 1970s in the United States, rates of three or four percent were used in setting rate levels. The interest rate levels chosen to price guaranteed rate life insurance policies were not current market rates and were not historic levels earned by the insurer, but instead, worst case scenario types of values. Actuaries tended to view interest rates as a one dimensional value and inherently assumed that they would be constant over the policy period. This attitude is changing only gradually.

For financial economists, interest rates have multiple dimensions. Initially, all rates of return, including interest rates, are classified as ex ante, those expected to occur in a future period, or ex post, actual realized returns. Ex post results can be viewed as a sample drawn from the ex ante distribution and, thus, provide only limited information about the true return distribution. Interest rates are then categorized as "real" or "nominal." Nominal interest rates are the full rates earned on investments. These rates vary over time and have been extremely volatile in recent years. Real interest rates have inflation (or inflationary expectations) factored out so that they represent the purchasing power effect of interest. This relationship between interest rates and inflation is known as the Fisher Effect based on work by Irving Fisher [35]. As interest rates tend to move in line with inflation, the real interest rate is much less volatile than nominal interest rates (Ibbotson and Sinquefeld [40]). If a life insurance policy were providing a benefit that were indexed to inflation, then the real interest rate would be relevant for pricing the policy. For traditional fixed benefit policies, the nominal interest rate is the proper one to use. Similarly, if loss reserves are to be discounted, the real interest rate should be used if unpaid losses will be affected by future inflation. If the values are unaffected by inflation, then the nominal interest rate is appropriate.

Another dimension to interest rates recognized by financial economists is termed the yield curve and represents the different interest rates available on similar bonds of different maturities. Often short term bonds have the lowest interest rate, with the interest rate increasing as the time to maturity increases. This occurs because the prices of longer term bonds are more volatile, creating greater risk for the long term bond holder. An alternative explanation for the normal slope of the yield curve is termed a liquidity premium, as money is tied up longer in long term bonds. For whatever reason, the normal yield curve is continually upward sloping. Occasionally an inverted yield curve occurs in which short term interest rates are higher than longer term rates. This tends to occur when inflation increases, but the general expectation is that it will reduce in the future. Other expectations about future economic conditions can lead to mountain shaped yield curves or even flat yield curves.

A third dimension of interest rates reflects differences between similar maturity bonds that are issued by different guarantors. This difference, termed a risk premium, reflects the different levels of risk inherent in different debtors. Frequently bonds issued by major industrial nations are considered risk free in their own currency, although this is an overly optimistic view under any long term historical perspective. Bonds issued by corporations would pay an interest rate that exceeds the national debt rate by varying amounts depending on the perceived riskiness of the issuer.

Another interest rate distinction considered important by financial economists is whether the interest rate is a market rate or a historical rate. Market rates are those interest rates available in the financial markets when the analysis is being performed, basically the current interest rates. Historical rates can be mean values for interest rates of a given risk classification and maturity over a known period of time, or achieved interest rates on a portfolio over a recent time period. Any measure of past performance, though, is a historical rate that does not necessarily reflect current market conditions. A standard consideration in applications of financial economics to pricing is that the market rate be used rather than historical rates. The current market conditions, not prior, perhaps unavailable rates, influence prices of financial instruments.

Related to the distinction between market and historical interest rates is another major difference between how actuaries and financial economists view interest rates. Most actuaries consider interest rates to be deterministic, or unchanging. An interest rate used as an actuarial assumption is considered to be at that level over the duration of the contract. Financial economists now are tending to view interest rates as stochastic, or essentially a random variable. Interest rates are expected to fluctuate over any future period. A number of different models have been developed to forecast interest rate movements, with differing degrees of success. No universally accepted stochastic interest rate model has yet been developed. However, these models tend to explain actual interest rate levels much more effectively than the deterministic models.

Profitability

Actuaries, especially casualty actuaries, tend to use a profit margin as the measure of profitability. The difference between premiums (plus investment income in some cases) and losses plus expenses is divided by the premiums to determine the profit as a percent of premium income. Target profit margins are established and actual performance is compared with these goals.

Financial economists tend to ignore profit margins, on the assumption that excess profits would be competed away, and concentrate on rates of return and, where appropriate, risk adjustments. The rate of return is determined by dividing the profit achieved by the investment made in order to earn the profit. For insurance the profit remaining after deducting losses and expenses from premiums and investment income is calculated, but this profit is divided by the investment necessary to initiate the insurance contract, generally the surplus of the insurer, rather than the premium income. Rates of return can be calculated for an insurance firm in aggregate, but adjustments must be made to statutory values in order to get a reasonable estimate of the true economic value of the initial investment. Allocating the investment amount, as well as many of the expense components, on a more specific level is increasingly difficult. Thus, at the current time, rates of return for insurance are generally determined only for the insurer in aggregate, and not by line or policy type.

Valuation

When providing a valuation of the assets and liabilities of an insurer, actuaries need to be aware that adjustments to the statutory (also known as "book") values are necessary. Statutory values are the ones recognized by insurance regulatory authorities and are considered to be conservative values. These values do not represent the market value of various assets or liabilities. For example, bond investments are valued at the amortized value, which is determined by gradually adjusting any difference between the purchase price and the maturity value of a bond over the remaining life of the bond. As the market value of a bond fluctuates inversely with interest rate changes, the amortized value of a bond can deviate significantly from the market value. In times of rising interest rates, amortized values of bonds exceed market values, which is not a conservative valuation. In times of falling interest rates, the market values exceed amortized values, which would impart a degree of conservatism depending on the speed and amount of the interest rate reduction.

Statutory values for liabilities are also generally set at conservative values, although the degree of conservatism is not constant. For casualty insurance the largest liability is the loss reserve. In the United States the loss reserves are not discounted to reflect the time value of money, except for fixed periodic payments or specific regulatory exceptions. For life insurance the statutory value of reserves for future benefit payments are established based on conservative mortality and interest rate values. However, some future liabilities are not recognized. For example, in the United States no reserve for future taxes on unrealized capital gains is established, despite the inclusion of equity investments at their market value which could exceed the purchase price.

Financial economists place great faith in the ability of competitive markets to price assets accurately. Therefore, the market value of specific assets and liabilities would be used in any valuation determination. For insurer assets this would be relatively easy, as most assets are in types of investments for which market prices could be readily determined. Real estate investments could present one problem in determining market value, but appraisals of the property value could provide usable values. Similar problems exist in evaluating private placement bonds and mortgages. In general, though, the liabilities of insurers are more difficult to calculate a market value for, as these liabilities are rarely traded, and when they are, through a reinsurance contract, the price is not publicly available.

Empirical studies of the insurance industry performed by financial economists are generally restricted to the few pure insurers, not part of a conglomerate, for which equity is publicly traded. As these studies are forced to exclude mutual insurers, a major force in both life and property-liability insurance markets, as well as financial service firms that own insurance companies, the conclusions from such data are limited. Financial economists are hampered in attempting to estimate market values of assets and liabilities not publicly traded by a lack of understanding of the composition of these components. Actuaries, who understand what the figures consist of, are also hampered in this regard, but for actuaries the handicap is derived from a professional tendency towards conservatism and statutory valuation. Hopefully, the third kind of actuary will be able to overcome such prejudices and arrive at a more market-oriented valuation of assets and liabilities.

Summary

Actuaries and financial economists are kindred spirits with a wide divergence in terminology and techniques separating their respective specialties. Volatile financial markets, higher nominal interest rates and the connection of benefit levels with investment performance will require a closer working relationship between the two groups. Such basic concepts as risk, interest rates, profitability and valuation are viewed differently by the two areas. Actuaries must recognize the viewpoint of financial economists in order to cope with the expanding actuarial horizons.

3. CURRENT STATE OF FINANCIAL ECONOMICS

Valuation

Before beginning to present what financial economists do know, or at least claim to know, about financial markets, a brief discussion of what is not known is in order. Financial economists do not know what the price of a stock will be at any future date. In the early years of this specialty, much attention was given to determining the value of an individual stock (Reilly [58]). Valuation models were developed that purported to indicate the intrinsic value of a stock. Investments made in stocks that were underpriced were expected to yield abnormally high profits. Numerous valuation models have been proposed and some claim to have worked over numerous investment cycles. Unfortunately, valuation models do not explain why prices diverge from the intrinsic value, thus producing opportunities for excessive profits, or how long it will take for prices to return to this benchmark level. More recently, most research in finance has adopted the efficient market hypothesis that states that the current price of a stock accurately reflects all publicly available information. Based on this hypothesis, the market price cannot diverge from the intrinsic value, negating much of the valuation theory research. It should be easy to understand that, when frustrated by not being able to explain what a stock price should be, claiming that whatever price exists is, by definition, the proper price, is an understandable approach.

Asset Pricing Models

After shifting away from attempting to explain price levels for stocks, attention moved to explaining the rate of return on different investments. The Capital Asset Pricing Model (CAPM) was developed to explain the rate of return on specific investments (Lintner [45], Mossin [51] and Sharpe [63]). The CAPM is explained and analyzed in such texts as Brealey and Myers [8], Ross and Westerfield [61] and Haugen [38]. The formulation of the CAPM is:

$$E(R_i) = R_f + \beta_i(E(R_m) - R_f), \tag{3.1}$$

where

R_i = return on a specific security;

R_f = risk free rate of return;

R_m = return on the market portfolio;
E = expected value operator;
β_i = $\mathrm{Cov}(R_i, R_m)/\sigma_m^2$ = systematic risk.

The term systematic risk, the β in equation (3.1), was introduced to describe the covariability of a specific investment's return with the market return. This single relationship replaced all the covariances between individual securities in the portfolio and greatly simplified the determination of portfolio risk. Unsystematic risk, the variation of returns on an investment that is independent of the market fluctuations, was assumed to cancel out in a diversified portfolio and therefore was considered irrelevant in pricing a given investment. The systematic risk level of an investment indicated the required rate of return on an investment and therefore determined the current price. The expected return on any individual asset was determined by multiplying the β times a value representing the market return in excess of the risk free rate and adding the product to the risk free rate. The value for the excess market return is generally assumed to be a constant and has been estimated in the range of 7 to 8.5 percent. If the (nominal) risk free rate were 6 percent and the excess market return were 8 percent, then a security with a β of 1.5 would have an expected return of 18 percent (6 + 1.5(8)).

Thus, based on the CAPM, the total variability of a stock price was not important for determining the rate of return on an investment. Only the systematic risk was important in determining the expected rate of return for an individual security.

Empirical tests of the CAPM tended to support the theory, but notable exceptions surfaced. Seasonal factors, size factors and some economic factors appeared to influence the achieved rates of return in addition to the systematic risk level. Additionally, the systematic risk factors were found to vary over time, in many cases tending to revert to the mean value, or one. Eventually, researchers recognized that tests of the CAPM were essentially joint tests of the CAPM and the proxy used for the market (Roll [59]). A more general asset pricing model, termed the Arbitrage Pricing Model (APM), of which the CAPM is simply a specialized application, has been introduced and is being widely tested (Ross [60]). The APM is explained and evaluated in such texts as Ross and Westerfield [61] and Haugen [38]. The formulation of the APM is:

$$E(R_i) = R_f' + \sum_{j=1}^{n} b_{ij}\lambda_j, \tag{3.2}$$

where

R_f' = expected return on the zero systematic risk portfolio;
b_{ij} = sensitivity of asset's return to a specific index;
λ_j = excess return in a portfolio with only one unit of systematic risk of that factor and no other systematic risk.

One major limitation of the APM is its failure to specify the number of factors that are expected to impact on security prices or what those factors should be. The CAPM is a special case of the APM under which one factor, the market performance in excess of the risk free rate, is assumed to be the only relevant factor. In this case, R_f' would be equal to R_f, b would equal β, and λ would be the excess market return.

The reliance of APM tests on the data used for the test, and the constantly changing investment environment, make tests of this model difficult to judge. In general, financial economists cannot say what the price of a stock should be, or exactly what rate of return should be expected on an investment. However, another, possibly more fruitful, area of pricing has developed.

Option Pricing Models

Although failing, to date, to explain security prices or rates of return, financial theory has moved in the direction of trying to explain the prices of derivative securities, those dependent on the price of another security. Many types of options, where an option is defined as a security that derives its value based on an underlying stock's price, are now traded on different exchanges. Perhaps coincidentally, the Option Pricing Model was developed only slightly prior to the explosive growth of the options market. An option gives the owner the right, but not the obligation, to trade a given security at a predetermined price either at a specific future date (European options) or any time up to a specific date (American options). A call option confers the right to buy a security, and a put option gives the owner the right to sell a security. The purchaser of the option has control over whether or not the future transaction is undertaken. The seller of the option commits to enter into the future transaction at the choice of the purchaser.

Options on major common stocks and stock indices are now widely traded. An option is described by its striking, or exercise, price, which is the price per share of stock at which the potential future transaction will be made, and the expiration date, which is the date at which, or by which, the transaction must be effected. The price of the option (which is often termed the premium) is the cost of buying the option, which does not include the price of the underlying security on which the option is written. Options and option pricing models are presented in detail in such books as Cox and Rubinstein [18], Jarrow and Rudd [42], Haugen [38], Brealey and Myers [8] and Ross and Westerfield [61], and in a paper by Wilkie [74].

The Black-Scholes [7] Option Pricing Model determines a value for the option based on the total variability, not just the systematic risk, of the underlying asset. This model takes the form:

$$P_c = P_s N(d_1) - Xe^{-rt} N(d_2), \tag{3.3}$$

where

P_c = price of a call option of the European type when no dividend is paid;
P_s = current asset price;
X = exercise price;
d_1 = $(\ln(P_s/X) + (r+\sigma^2/2)t)/\sigma t^{1/2}$;
d_2 = $d_1 - \sigma t^{1/2}$;
r = continuously compounded risk-free interest rate;
t = time to expiration of the option;
σ = annualized standard deviation of the returns of the underlying asset;
N = normal distribution function.

For example, the value of a one year call option with an exercise price of \$100 for a stock with a current price is \$90 and a standard deviation of 30 percent per year if the risk free interest rate is 10 percent is:

$d_1 = (\ln(90/100) + .1 + .3^2/2)/.3 = .132$;
$d_2 = .132 - .3 = -.168$;
$N(.132) = .5525$;
$N(-.168) = .4333$;
$P_c = 90(.5525) - 90.48(.4333) = 10.52$.

The assumption that security returns are lognormally distributed is essential to this model. As option markets developed, historical returns on the options themselves were not available to help participants establish price levels. The Black-Scholes model, despite its initial apparent complexity, was actually quite easy to use once the practitioner became familiar with it. The required inputs for the model were readily available, except for the measure of the underlying security's variability. This value could be estimated from historical data or backed out of the market price for other derivative securities. The popularity of the Black-Scholes model was such that some dealers circulated the price level determined by the model to traders as a recommended value for an option. Thus, the model was being used to influence price levels almost from the start of stock option trading.

Despite the bias introduced by the model's being used to set prices of options, subsequent empirical tests of the Black-Scholes OPM found that it worked only fairly well. The model tended not to explain the prices of options that had striking prices far from the current market price of the underlying security, that were on securities with volatility measures that were considerably above or below standard volatility measures, or that had a very long time to expiration (Black and Scholes [6], Chiras and Manaster [16], Galai [36], Rubinstein [62] and Whaley [70]). Despite these limitations, the option pricing approach became very popular for addressing other issues in finance, including capital structure, valuation, capital budgeting and insurance pricing (Firth and Keane [34], Smith [67] and Smith [65]).

Diffusion Processes

Diffusion processes are the more general type of models from which option pricing models are derived. Diffusion processes are stochastic processes with continuous paths. The first noted application of a diffusion process was documented by Robert Brown in 1827 in describing the path of minute particles suspended in liquid, and the term Brownian motion recognizes his contribution to this area. The mathematics of Brownian motion were presented by Albert Einstein [31] and enhanced by Norbert Wiener (1923). The term Wiener process is often used to mean diffusion models, but technically this term is restricted to a specific diffusion model with an initial value of zero, a mean of zero and a variance of 1.

The attraction of Brownian motion for mathematicians is that the probability distribution for the path of particles after a period of time is normally distributed, or, if the particles are subject to an absorbing barrier that affects the amount of movement as the particle approaches the barrier, then lognormally distributed. The models can be extended by including a drift factor, allowing the variance to change over time and even including a jump factor, usually a Poisson process, that introduces a discontinuity in the process. Financial economics focused on these processes for describing security prices (Ingersoll [41] and Malliaris and Brock [47]). Individual security prices were assumed to be subject to random movements over time, generally with an upward drift. The attraction of a lognormal distribution was the fact that a security cannot have a negative price and, once attaining a level of zero, cannot be allowed to have a positive price in any future period or else an individual could buy a security for nothing and have the possibility of a positive price at some future time, violating the no arbitrage condition required for efficient prices. The jump processes accounted for exogenous changes in the market.

Diffusion models have been widely, and very successfully, applied in such divergent fields as physics, biology, engineering and risk theory in insurance. An early application of diffusion processes to investments was presented by Bachelier [1] which attempted to explain movements in the French stock market by use of a Markov process. In the insurance area, Lundberg [46] applied a diffusion model in developing collective risk theory. Both of these researchers were working independently of Einstein but arrived at very similar conclusions.

A Markov process is defined as a stochastic process in which only the current value of the random variable is relevant in forecasting future values. Past values, other than the latest one, do not affect future values. The Black-Scholes Option Pricing Model and all other option pricing models are also based on the assumption that security prices follow Markov processes.

The assumption that a random variable has no "memory" of prior values seems a reasonable one when describing particle movements, transmission of genetic characteristics, production line defects and insurance claim activity. However, when this lack of memory is applied to prices of financial assets, which are set by individuals who do have a memory of past prices, this assumption may introduce an unacceptable amount of error. Individuals do relate current price levels to past levels, base decisions on whether a stock price is increasing or decreasing, and on how rapidly a price is changing. Assuming that these individual tendencies cancel out in aggregate may be inaccurate. Empirical studies indicate that over short trading periods, stock prices do approximate diffusion processes. However, over longer periods (for example, several years or longer), autoregressive tendencies become apparent. An extensive study of the characteristics of investment performance is included in Wilkie ([71], [72], [73]). The issue of whether the diffusion models can be used to explain security returns is not yet settled.

Hedging

Arranging one's financial affairs such that one cannot suffer adverse consequences from future developments is termed hedging. In many regards hedging in finance is similar to hedging bets by taking offsetting positions so, regardless of the outcome of the contingent event, the economic effect is assured. Insureds typically hedge when they purchase insurance, thus offsetting the financial risk of loss. Financial institutions can also hedge by allocating their assets in such a way that any event affecting their liabilities has a similar but offsetting effect on their assets. Numerous hedging strategies for firms have been developed, varying in degrees of complexity, practicality and expense. A recent hedging strategy involving a combination of equity investments and derivative securities, termed portfolio insurance, has been proposed that adjusts the distribution of investments depending on equity price movements (Leland [44]). This strategy has received extensive publicity, mostly unfavorable, as a consequence of the October, 1987 market decline (Sloan and Stern [64]).

The simplest way, in principle, for a financial institution to hedge its known future obligations perfectly is to invest in instruments that pay off exactly when the obligation matures. For banks that typically offer certificates of deposit (CD) for periods of no more than ten years, this strategy is at least possible. To match a CD maturing in seven years, a zero coupon bond with the same maturity can be purchased. The institution has assured itself, subject only to risk of default, that the funds needed to satisfy the liability will be available. Interim interest rate fluctuations will not affect the availability of funds to discharge the liability. However, for life insurers that accept obligations to make payments as far as a lifetime in advance, or even longer for annuitized benefits, the financial instruments that could match these payout patterns exactly simply do not exist. Alternative approaches to hedge a set of liabilities without exact asset-liability matching are based on a concept known as duration.

The concept of duration was developed by Macaulay [48], and more recently discussed by Ferguson [33] and Tilley [69], to combine the size and timing of coupon payments with the time to maturity. Duration is the weighted average length of time prior to full recovery of principal and periodic payments. Each payment is weighted by its present value. Equivalently, the duration is the negative of the derivative of the present value of a stream of cash flows with respect to the interest rate divided by the present value of the stream of cash flows. The formulae for calculating duration are:

$$D = \frac{\sum_{t=1}^{n} C_t(t)/(1+r_t)^t}{\sum_{t=1}^{n} C_t/(1+r_t)^t}, \tag{3.4}$$

where

D = duration
C_t = interest or principal payment at time t;
(t) = length of time to payment;
n = length of time to maturity;
r_t = yield per period for an asset maturing at time t;

or

$$D = -(d\mathrm{PV}(C)/dr)/\mathrm{PV}(C), \tag{3.5}$$

where

d = partial derivative operator;
$\mathrm{PV}(C)$ = present value of a stream of cash flows;
r = current interest rate.

The denominator of equation (3.4) is the present value of the fixed income investment. The numerator is the present value of the payments weighted by the length of time until they are received. The higher the duration, the longer into the future the payments will, on average, be received. In many cases, the r_t's are assumed to be equal, implying a flat yield curve. As this is rarely the case in practice, equation (3.4) allows for interest rates to vary by the length of time to maturity. In equation (3.5) the duration is shown to be the negative of the effect of change in interest rates on the present value of the cash flows in relation to the present value of the cash flows. This equation will hold for any shape yield curve.

The effect of interest rate changes on bond prices is proportional to the duration of the bond. This suggests a strategy of hedging, or immunizing, a portfolio by matching the duration of the assets and liabilities, without the necessity of exactly matching the terms of each. Thus, by applying the concept of duration, an alternative hedging strategy can be developed.

A complication that arises in measuring the duration of a bond is that the duration value depends on the structure of interest rates. Under deterministic interest rates, which are assumed not to change over the life of the bond, one measure of duration is determined. If interest rates are allowed to be stochastic, or random variables, then different duration values result. Several researchers have compared the duration measures based on different interest rate structures (Bierwag ([5] and [4]) and Boyle ([12] and [13])). In general, the duration measure is lower under stochastic interest rates than under deterministic interest rates. Thus, to immunize a given set of liabilities a financial institution would have to invest in more long term bonds under fluctuating interest rates.

4. APPLICATIONS OF FINANCIAL ECONOMICS TO INSURANCE

Introduction

The increasing interrelationship between insurance and financial economics has been recognized by both financial economists and insurance specialists. Smith [66] analyzes the convergence of the fields of insurance and finance, but indicates that few researchers combine an understanding of the mechanics of insurance with a knowledge of the analytical tools of finance. Thus, sophisticated financial research tends to apply insurance inappropriately whereas more accurate models of the insurance industry tend to lack the rigorous technical approach. Garven [37] also describes applications of finance to insurance issues. Borch [11] explains the reluctance of actuaries to adopt financial models and proposes a solution to some of the drawbacks of financial models.

Initial applications of financial economics to insurance issues covered pensions and life insurance. More recently, extensive applications of financial economics to property-liability issues have been developed. While this paper will concentrate on property-liability applications, a review of the major directions of research in the other insurance areas will serve as an introduction.

Pensions

As a result of the Employee Retirement Income Security Act of 1974 (ERISA), pension plan assets became a major aspect of corporate finance. Finance academics began to look into how pension fund management affected firm value. Such issues were addressed as whether firm value is affected by the pension plan investment strategy, how pension assets should be invested optimally, and whether under or over funding of pension plans is reflected in the market value of the firm. Actuarial science and financial economics converged on the valuation issue, as financial economists examined the effect of funding on firm value but relied on actuarial science to produce estimates of future liabilities. In many cases, the dichotomy described by Smith [66] led to inaccurate assumptions by financial economists. The results of these efforts are described by D'Arcy and Chen [22]. In general, the findings support the effectiveness of the market to evaluate liabilities correctly.

Life Insurance

New forms of life insurance policies, introduced in the last decade under the names of maturity guarantee contracts or variable life or universal life, provide a benefit level that fluctuates with the performance of some investment index. Additionally, many of these policies include guarantees that assure the policyholder of some minimum benefit level. Thus, the benefit provided under those contracts with a guarantee is equal to:

$$B = \text{Max}[M, S(t_m)], \tag{4.1}$$

where

B = benefit level;
M = guaranteed minimum amount;
$S(t)$ = investment index value at time t;
t_m = time of maturity of the contract.

The similarity of the payment formulation of this policy and that of an option was quickly noted and addressed. Various models were developed to determine the optimal investment strategy for the insurer offering this type of contract. The conventional strategy expounded by Benjamin [3] suggested investing an amount sufficient to provide the variable investment in the variable asset, with any residual assets invested in fixed interest investments. With this strategy the insurer is at risk in case the terminal value of the variable investment is less than the guarantee by more than the terminal value of the fixed interest investment.

An alternative approach to investing assets for a maturity guarantee contract, developed by Brennan and Schwartz ([9], [10]), is to vary the allocation of the investment portfolio between the variable assets underlying the guarantee and cash depending on the likelihood of the final value of the variable investment being less than or greater than the guarantee. The likelihood of the variable investment exceeding the guarantee is determined based on the Black-Scholes OPM, with the current value of the variable asset, the guarantee, the time to expiration and the volatility all affecting this likelihood. Collins [17] tested the two strategies on the period 1930 through 1978 and found that the conventional strategy worked better. The primary reason for this performance related to the sharp increase in prices following the 1974 market decline. A similar effect occurred more recently. The dramatic market decline on October 19, 1987, followed five years of unusually high rates of return. The diffusion process upon which the option pricing model rests does not anticipate such a reaction. The autoregressive tendency documented by Wilkie ([72], [73]) explains this behavior. The option pricing methodology greatly reduced the holding of variable investments in 1974 as the value of the market declined. Thus, this strategy was underinvested when the sharp price increase occurred. Conversely, this strategy generated a greatly increased holding of variable investments as the market increased up through 1987.

One problem faced by life insurers in applying option pricing models to maturity guarantee contracts is that the contracts are usually multiple payment contracts; so, at any given point in time, future income will be received by the insurer. The Black-Scholes model is essentially a single payment contract. However, an extension of the OPM by Merton ([49]), which was derived to allow for dividend payments on the underlying security, can be utilized to apply to multiple payment life insurance contracts. The future payments on the contract are considered negative dividends, thus payments in rather than payouts.

Another area of application of financial economics to life insurance addressed the issue of asset-liability matching. This area is also applicable to property-liability insurance, but the initial insurance applications focussed on life insurance for several reasons. Life insurers were more adversely affected by the interest rate volatility of the late 1970s and early 1980s, have longer term contracts and have fixed dollar contracts.

Life insurers contract to make future payments to policyholders or beneficiaries. Although the timing of these payments on an individual contract is a random variable, the independence of most risks tends to generate a fairly predictable payment schedule. Thus, mortality risk is ignored in most liability determinations. The payment schedule on liabilities runs for the maximum lifespan of existing insureds, plus additional maximum potential lifespans of any beneficiaries who elect to receive the policy proceeds in the form of a life annuity. As a result, the liability composition of life insurers can stretch for over a century.

If a life insurer invested the assets intended to cover these liabilities for a shorter term than that of the liabilities, then the proceeds from these investments would have to be reinvested at an uncertain interest rate level at the maturity of the investment. The insurer could not be sure of the interest rate to be earned on the assets intended to cover the liabilities. In this case, the insurer faces interest rate risk.

Even if the insurer invested the assets in a fixed interest rate investment that matures when the liability is to be paid, the insurer still faces interest rate risk on the coupon payments that will be received on the investment prior to the need for funds. These interim receipts will be received periodically and reinvested until the liability is to be paid. The only way to avoid this interest rate risk is to invest in zero coupon bonds that mature at the time needed to satisfy the liability. If this strategy of exactly matching assets and liabilities were adopted, the insurer would not be exposed to any interest rate risk. However, the risk of the liability payout pattern differing from the projected rate, which has been assumed away, does still exist. Unfortunately for life insurers, zero coupon bonds, or even any coupon bonds, with maturities running for as long as a century do not exist. This situation has led researchers to recommend that life insurers use duration as a means of avoiding interest rate risk.

As long as the duration of the assets and liabilities is equal, then the insurer would be protected from interest rate fluctuations, as any loss (gain) in the reinvestment rate is expected to be offset by capital gains (losses) on the value of existing holdings. Redington [57], one of the pioneers in developing such a strategy, based his analysis on life insurance contracts.

The early work on duration was based on deterministic interest rates. More recent research, including Bierwag ([5] and [4]) and Boyle ([12] and [13]), demonstrate the effect of stochastic interest rates on duration. In general, life insurers would have to extend the maturity of investments if interest rates are assumed to be stochastic rather than deterministic, as the mean reverting tendencies of the typical interest rate models assume long term interest rates will be less volatile than short term rates.

Property-Liability Insurance

A typical property-liability insurance contract involves exchanging a fixed, or, if variable, bounded, sum of money (premium) for the agreement to pay a variable sum depending on the outcome of particular uncertain events (claims). Standard ratemaking procedures through the middle of the 1970s involved adding the expected losses and expenses to a proportional profit margin to determine the premium. The effect of the time value of money on the lag between the receipt of premium and the payment of claims was recognized in theoretical works at the beginning of that decade (Haugen and Kroncke [39] and Quirin and Waters [56]). As documented in Derrig [27], the first regulatory application of financial economics to insurance pricing occurred in Massachusetts for private passenger automobile insurance rates in 1978. The CAPM was invoked in a manner described by Fairley [32] to determine the allowable underwriting profit margin as follows:

$$p = -k[R_f + \beta_L[E(R_m) - R_f]] + R_f t/(1-t)s, \qquad (4.2)$$

where

p	= underwriting profit margin;
k	= funds generating coefficient representing average lag between receipt of premium and payment of claim;
R_f	= risk free rate of return;
β_L	= underwriting profit beta;

$E(R_m)-R_f$ = market risk premium;
t = effective federal tax rate;
s = premium to surplus ratio.

Based on equation (4.2), a value, k, representing the average holding period of a dollar of premium, is multiplied by the risk adjusted rate of return determined from the CAPM. If the underwriting beta is negative, as it often is when calculated empirically, then this k is multiplied by a rate below the risk free rate. The negative of this expression is used to indicate that investment income offsets underwriting income on a total return basis. If the insurer were not subject to taxation, this would be the relationship, and the indicated underwriting profit margin would be the negative of the risk adjusted (based on the covariance between underwriting returns and the return on the market) rate of return on investments. However, as the insurer is subject to taxation on investment income and underwriting profits, then the last term of equation (4.2) indicates that the underwriting profit margin has to be increased by a value proportional to the leverage of the insurer to account for this taxation.

The most controversial result of this application of the CAPM to insurance pricing was that, when interest rates were high, as they were in the late 1970s, and when the time lag between premium payment and claim payment was sizeable, then the indicated underwriting profit margin could be negative. Application of this model to bodily injury liability coverage produced just such a result, indicating a −4 percent underwriting profit margin for 1978, −8 percent for 1979 and −13 percent for 1980.

After a string of defeats in Massachusetts for the insurance industry in proposing rate filings and contesting the decisions in court, the industry supported an alternative financial economics approach to insurance pricing termed the discounted cash flow (DCF) model. This methodology, documented in Myers and Cohn [52], established an equality between the present value of premiums and the present value of losses and expenses plus the present value of taxes incurred on investments and underwriting. Mathematically this model is:

$$\mathrm{PV}(P) = \mathrm{PV}(L) + \mathrm{PV}(UWPT) + \mathrm{PV}(IBT), \qquad (4.3)$$

where

PV = present value operator;
P = premiums;
L = losses, loss adjustment expenses and expenses;
$UWPT$ = tax generated on underwriting income;
IBT = tax generated on income from the investment balance.

The present values are determined based on different discount rates, depending on the perceived risk of each cash flow. Premiums and the tax on investment income are discounted at the risk free rate. Losses and expenses and the tax on the underwriting profit margin were discounted based on the risk adjusted rate as determined by the CAPM. In general this discounted cash flow model produced higher underwriting profit margins (although still negative) for bodily injury, but slightly lower values for property damage and physical damage.

Kraus and Ross [43] applied the arbitrage pricing model (APM) to property-liability insurance pricing and determined that changes in nominal interest rates should not affect the competitive rate of return on insurance contracts, but changes in real interest rates should have an inverse effect on insurance prices. The complexity of applying the APM to actual data has limited the application of this model in pricing techniques.

The Option Pricing Model (OPM) has also been applied to property-liability insurance pricing. Doherty [28] and Doherty and Garven [30] test the OPM for pricing reinsurance as well as primary policies and demonstrate that realistic values can be derived. In this work insurance contracts are viewed as contingent claims by policyholders, tax authorities and the owners of the insurance company. The equity holders have to be assured a competitive rate of return, given the recognition that their claim is residual to the other claimants. This model is extremely sensitive to the applicable tax rate and the variability of investment performance and claim costs.

The applications of the CAPM, APM, OPM and DCF models for property-liability insurance pricing, as well as the drawbacks of each technique, are described in D'Arcy and Doherty [23]. The primary problem with the various approaches involves obtaining accurate values for the various parameters used in the models. D'Arcy and Garven [24] test the CAPM, DCF and OPM, as well as the more traditional target underwriting profit margin and total rate of return techniques over the period 1926 through 1985 and find that the total rate of return model and the option pricing model tend to perform best over this period. This study also demonstrates the sensitivity of the results to parameter estimates, indicating the importance of utilizing accurate measures of the various input parameters.

Historically, the issue of insurance solvency has been addressed by actuaries using such tools as risk theory and ruin theory (Beard, Pentikäinen and Pesonen [2], Bühlmann [14], Pentikäinen [55]). These techniques do not consider the covariance between underwriting performance and investment results or the effect of competitive markets on prices. Financial economists have begun to address the insurance solvency area. Doherty [29] analyzes the optimal leverage for an insurer and determines that surplus should be the minimum allowed by regulators, or zero if no regulatory restrictions apply. Derrig [26] applies financial theory to determine optimal risk loadings in premiums. Cummins [19] develops risk based insurance guaranty fund premiums based on stochastic processes for assets and liabilities. Diffusion processes are used to describe asset and liability movements, with a jump process added to the liabilities to allow for catastrophes. In aggregate, the risk based premiums are in line with actual insolvency assessments.

The Working Party on Solvency of the General Insurance Study Group for the Institute of Actuaries summarizes the major issues involved in solvency determinations and integrates ruin theory with financial economics (Daykin, et al. [25]). This study uses a simulation approach to combine underwriting and investment risk. The recommendations of this Working Party include specific solvency margins to recognize different levels of riskiness, rather than the traditional fixed premium to surplus level.

Asset-liability matching for property-liability insurers involves additional considerations for those used for life insurance and other financial institutions. As the liabilities of property-liability insurers are not fixed value items, the effect of inflation on loss reserves and future losses on the unearned premium reserve must be considered. D'Arcy [21], Noris [53] and Panning [54] indicate how this distinction affects asset-liability matching for property-liability insurers.

A final application of financial economics to property-liability insurance relates to valuation of a firm for such purposes as merger, acquisition or conversion from a mutual to a stock ownership form. Sturgis [68] and Miccolis [50] address this issue. Such considerations as valuing future renewals and reputation enter into this determination. In these situations, statutory valuation is inappropriate. Statutory valuation centers on an insurer going out of business, whereas valuation for merger purposes considers an insurer an on-going concern.

5. CONCLUSION

Financial economists have developed a number of tools to aid in understanding financial markets. A number of pricing models have been proposed and, although none is accepted as being a perfect explanation of prices or rates of return, the CAPM, APM and OPM provide useful insights into the workings of financial markets. As life insurers offer products tied to investment performance, as property-liability insurers guarantee financial instruments, and as both life and property-liability insurers seek to manage their own investment portfolios more effectively, knowledge of the tools and models of financial economics is becoming more important for actuaries. Thus, all actuaries may need to become, in the not-too-distant future, actuaries of the third kind.

Future insurance related research by financial economists and actuaries of the third kind is likely to be directed at developing improved estimates of the input parameters for the various pricing, hedging and solvency models. All models are sensitive to parameter estimation, and many prior estimated values have been derived from the limited publicly available data. More extensive testing will require the cooperation of insurers in providing data. Greater actuarial involvement in the direction and application of future studies may encourage increased cooperation. Additionally, the long term nature of insurance contracts, as opposed to the fairly short expiration periods of most traded options, may require the development of security price models that are not Markov processes but include some autoregressive tendencies.

The convergence of financial economics and insurance suggests that future insurance based research will focus on financial economic issues. When this research is conducted by actuaries, or other insurance experienced individuals, it should have the joint advantages of being aimed at the key insurance issues, be documented in terminology familiar to insurance practitioners and incorporate previously unavailable empirical data.

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