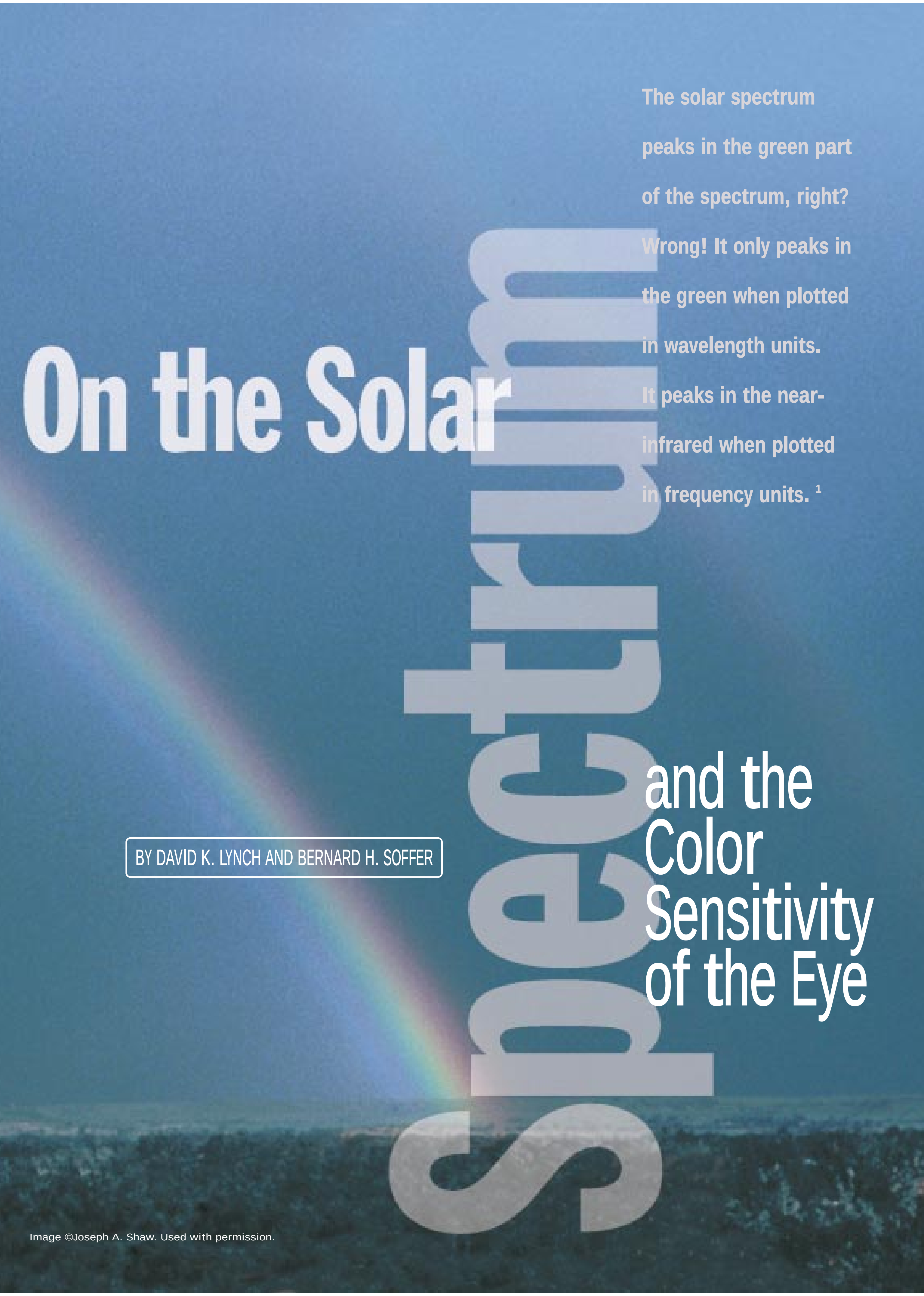

On the Solar Spectrum and the Color Sensitivity of the Eye

The solar spectrum peaks in the green part of the spectrum, right? Wrong! It only peaks in the green when plotted in wavelength units. It peaks in the near-infrared when plotted in frequency units. [1]

BY DAVID K. LYNCH AND BERNARD H. SOFFER

Many people believe that the solar spectrum and the color sensitivity of the eye both peak at around 0.5 µm (500 nm) in the green.[2, 3] The notion is sometimes stated even more strongly, *i.e.,* that evolution has produced a human eye whose color sensitivity has been optimized to match the solar emission spectrum.[4, 5] But this apparent wavelength coincidence between the solar spectrum and the eyes' sensitivity is an artifact resulting from the units in which the solar spectrum is plotted. Comparing irradiance to sensitivity is like comparing apples to oranges: they are fundamentally different quantities and their shapes and peaks should not be likened to one another although they can legitimately be multiplied together.

Wavelengths and frequencies of peak emission

Figure 1 shows the spectrum of the Sun[6] at sea level for a mid-latitude climate along with a 5800 K Planck function scaled to approximately match the sunlight. The spectra are plotted in the units most commonly used for visible spectra, *i.e.*, irradiance per unit wavelength interval in W cm^{-2} μm^{-1} versus µm (wavelength units). In these units, the peak of the solar spectrum is near 0.5 µm (500 nm), unquestionably a wavelength that by itself would appear green and be so called. Also shown in Figure 1 is the luminous efficiency of the eye adapted from Judd and Wyszecki;[7] the peak of the luminous efficiency is also in the green.

The spectra can also be plotted per unit frequency interval in W cm^{-2} hz^{-1} as a function of frequency (see Fig. 2). In this case, the spectrum no longer peaks in the green, but rather in the IR, close to 0.88 µm (880 nm). Yet the peak of the luminous efficiency function remains in the green with no shift. Because frequency and wavelength are both equally valid descriptions of the state of affairs, we seem to be left with the question "where does the solar spectrum peak, in the green or in the IR?"

The answer is that it all depends on your choice of independent variable. To see this we will approximate the solar spectrum by a Planck function because it is analytic. In wavelength units the Planck function $B_\lambda(T)$ is

$$B_\lambda(T) = 2hc^2\lambda^{-5} /(e^{hc/\lambda kT} - 1) \qquad (1)$$

in units of W cm^{-3}, or power per unit area per unit wavelength interval. As Wien's law says, the wavelength of peak emission is .2897/T = 0.2897/5800 = 4.98 × 10^{-5} cm ≈ 500 nm, corresponding to a frequency ν of $\nu = c/\lambda = 6 \times 10^{14}$ Hz. This is in the green part of the spectrum and agrees with most people's idea of the shape and peak of the solar spectrum. Wien's law with the conventional constant, however, only works when the spectrum is plotted as irradiance per unit wavelength interval. When the same spectrum is plotted as irradiance per unit frequency interval (W cm^{-2} hz^{-1})

$$B_\nu(T) = 2h\nu^3 c^{-2} /(e^{h\nu/kT} - 1) \qquad (2)$$

it peaks at 3.4 × 10^{14} hz, corresponding to 8.8 × 10^{-5} cm (= 0.88 µm = 880 nm, see Figs. 1 and 2).

Although Equations 1 and 2 are equivalent, converting one to the other is not simply a matter of making the substitution $\nu = c/\lambda$ because the Planck function is a distribution function and is defined differentially such that

$$B_\lambda\, d\lambda = B_\nu\, d\nu. \qquad (3)$$

This is simply conservation of energy. Since $d\nu/d\lambda = -c/\lambda^2$ (and ignoring the minus sign), then

$$B_\lambda\, d\lambda = B_\nu\, c/\lambda^2\, d\lambda, \qquad (4)$$

and thus $B_\lambda = B_\nu c/\lambda^2$, or conversely $B\nu = B_\lambda \lambda^2/c$. The apparent "shift" in peak wavelength between B_λ and B_ν is simply due to the $1/\lambda^2$ Jacobian weighting factor, which is a result of the differential nature of the Planck function.

The relation between B_λ and B_ν is illustrated in Figures 3 and 4 (see page 30). Figure 3 shows a 5800 K Planck function in wavelength units and Figure 4 shows

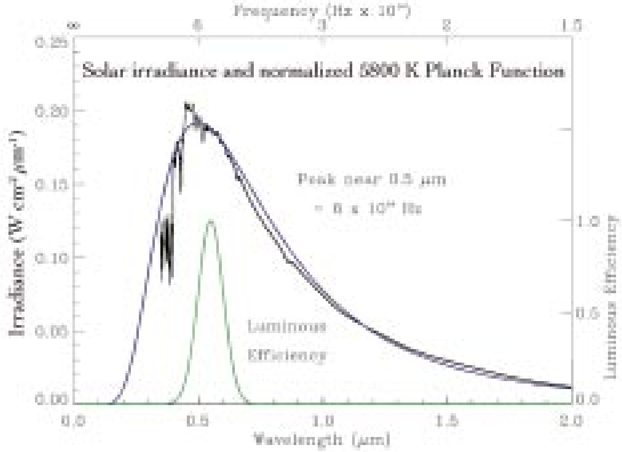

Figure 1. The solar spectrum (black) plotted in wavelength units peaks near 0.5 µm. Also shown is an approximate fit of a 5800 K Planck function (purple), which shows that the solar spectrum is roughly Planckian in the optical part of the spectrum. The luminous efficiency of the eye (green) peaks at 0.56 µm. All three curves appear to peak near 0.50–0.55 µm, a wavelength region generally perceived as being green.

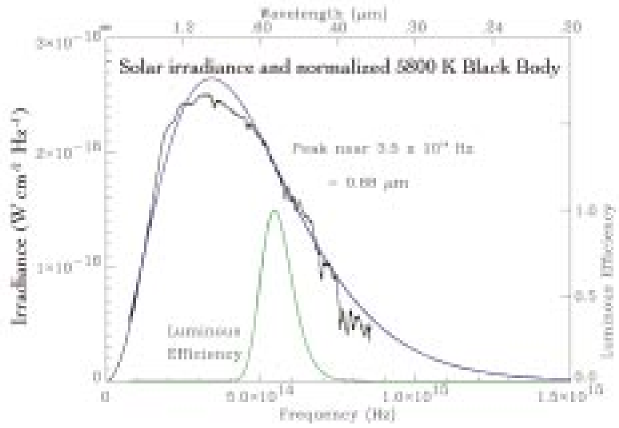

Figure 2. The same data as shown in Figure 1 except plotted in frequency units. Here the Sun and Planck functions peak near 0.88 µm in the near-IR while the luminous efficiency curve still peaks at 0.56 µm. The solar irradiance and Planck function transform differently than the luminous efficiency and therefore their peaks are shifted by different amounts.

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the same function in frequency units. Figure 3 is divided into equal intervals of wavelength in the amount of 0.1 µm. The same intervals are marked in frequency units in Figure 4, where they are clearly unequally spaced because there are more wavelengths per unit frequency at longer wavelengths than at shorter ones. Conversely, there are more frequencies per unit wavelength at shorter wavelengths. Clearly then, a plot of irradiance per unit frequency would skew the curve to longer wavelengths, which is exactly what happened in Figures 1 and 2.

Spectral responsivity and filter transmission

Spectral sensitivity of any detector including the eye is expressed in units of amps/watt, volt/watt, or, in the case of the eye, lumens/watt, each at a given wavelength. Filter transmission is similarly simple, being a unitless ratio between zero and one at each wavelength. This is fundamentally different than irradiance, which by virtue of being a density distribution function, is expressed as a *value per unit wavelength interval.* Consequently sensitivities possess no differential weighting factor, *i.e.*, when transforming the eye's sensitivity from wavelength to frequency interval. One need only use the substitution $\nu = c/\lambda$. This is why the peak in the luminous efficiency function remains at the same frequency (and wavelength) when plotted in either frequency or wavelength units (see Figs. 1 and 2).

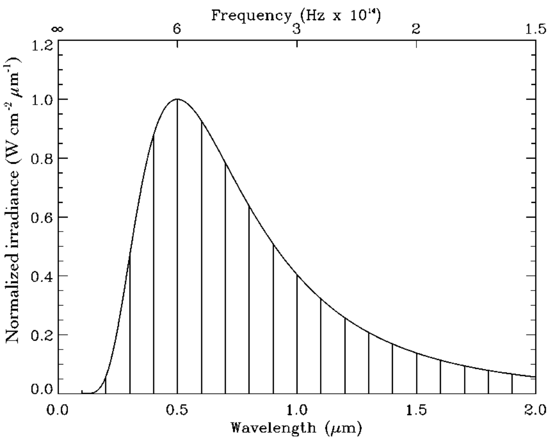

Figure 3. A 5800 K Planck function divided into equal 0.1 µm wavelength intervals.

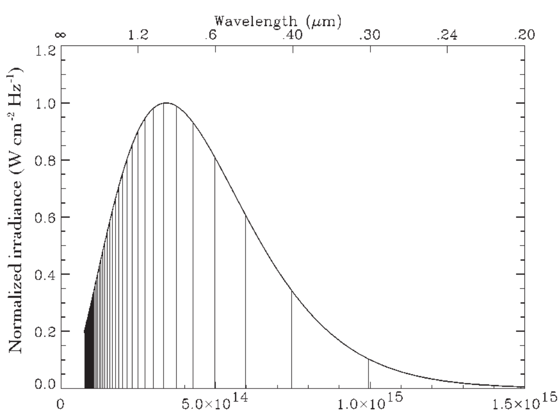

Figure 4. The same Planck function and wavelength intervals as Figure 3 transformed into frequency units. Note that the frequency intervals are not equal.

This explanation may leave some people feeling a little uneasy. After all, if you are building a detector of broadband light and you want to know what wavelength to position a filter of a fixed width to transmit the most power, your intuition says "put it at the peak of the source's spectrum and multiply them together." Yet we seem to be saying that this method of mentally sliding the filter back and forth to maximize power near the peak does not work. What's going on here?

The answer is that for a fixed filter width in wavelength, maximizing in wavelength space is not the same as maximizing in frequency space. What may not be apparent is that sliding the wavelength filter back and forth in an attempt to maximize the power causes the width of the filter in frequency space to change. Thus, a filter (like the eye) whose FWHM is 100 nm centered at 520 nm just happens to be near the maximum in the wavelength curve. The same filter is nowhere near the peak in frequency space, nor should it be. If we were to take the filter in frequency space with the same fixed wavelength band that was used to optimize in wavelength space, and move it to smaller frequencies to maximize the signal, we would find that its width in wavelength space had increased. We would find a new maximum greater that the one previously found in wavelength space—greater only because the filter is wider. This is a result of the relation $d\lambda = -c\, d\nu/\nu^2$. Since $d\nu$ is held constant, as ν decreases, $d\lambda$ increases. Of course, this mental exercise of sliding the filter back and forth could not be done physically because a real filter has fixed properties.

Summary and conclusions

By plotting the spectra, we have shown that the peak wavelength of the solar spectrum and the peak sensitivity of the eye are not coincident as commonly believed. The fact that in wavelength units the solar spectrum roughly agrees with the peak sensitivity of the eye is merely a conceptual artifact involving the units in which the spectrum is plotted. Computing it in frequency units is just as valid and results in a peak near 880 nm, well away from the peak sensitivity of the eye. Other units result in yet other apparent peak positions. There is, however, no paradox or inconsistency. While we firmly believe in the modern theory of evolution, we wish to warn others about the dangers of glibly assigning Darwinian significance to an artificial wavelength coincidence. We develop these ideas and the evolutionary story further in a forthcoming publication.[8]

Acknowledgment

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David K. Lynch (david.k.lynch@aero.org) is a senior scientist at The Aerospace Corp., Los Angeles, CA and Bernard H. Soffer (bhsoffer@alum.mit.edu) resides in Los Angeles and is a consultant to industry and academia.