ETERNALLY EXISTING SELF-REPRODUCING CHAOTIC INFLATIONARY UNIVERSE

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It is shown that the large-scale quantum fluctuations of the scalar field φ generated in the chaotic-inflation scenario lead to an infinite process of self-reproduction of inflationary mini-universes. A model of an eternally existing chaotic inflationary universe is suggested.

1. One of the most popular models of the evolution of the universe is the inflationary-universe scenario (for a review see ref. [1]). In our opinion, the most natural version of this scenario is the chaotic-inflation scenario [2–10]. This scenario can be implemented in a wide class of theories. In particular, it can be realized in the theories of scalar fields φ with polynomial effective potentials $V(\varphi) \sim \varphi^n$ [2–6], in GUTs [7], in the extended version [3] of the Starobinsky model [11], in $N=1$ supergravity [8], in some Kaluza–Klein and superstring theories [9,10], etc. The main aim of this paper is to investigate the global structure of the universe in the chaotic-inflation scenario.

2. We will consider the simplest version of this scenario based on the theory of a scalar field φ with the lagrangian $L = \frac{1}{2}\partial_\mu\varphi\partial^\mu\varphi - V(\varphi)$, where $V(\varphi) = \frac{1}{4}\lambda\varphi^4$ at $\varphi \gtrsim M_\mathrm{p}$ [2]. Here $M_\mathrm{p} \sim 10^{19}$ GeV is the Planck mass. If the classical field φ is sufficiently homogeneous (see below), its evolution in an expanding locally Friedmann universe with scale factor $a(t)$ is governed by equations

$$\ddot{\varphi} + 3H\dot{\varphi} = -\mathrm{d}V/\mathrm{d}\varphi, \tag{1}$$

where $H = \dot{a}/a$, and

$$H^2 + k/a^2 = \left(8\pi/3M_\mathrm{p}^2\right)\left[\tfrac{1}{2}\dot{\varphi}^2 + V(\varphi)\right]. \tag{2}$$

Here $k = \pm 1, 0$ for a closed, open and flat universe respectively. If the field φ initially is sufficiently large ($\varphi \gtrsim M_\mathrm{p}$), the behavior of $\varphi(t)$ and $a(t)$ very soon approach the asymptotic regime

$$\varphi(t) = \varphi_0 \exp\left[-\left(\sqrt{\lambda}/\sqrt{6\pi}\right)M_\mathrm{p}t\right], \tag{3}$$

$$a(t) = a_0 \exp\left[\left(\pi/M_\mathrm{p}^2\right)\left(\varphi_0^2 - \varphi^2(t)\right)\right]. \tag{4}$$

In this regime $k/a^2 \ll H^2$, $\dot{\varphi}^2 \ll V(\varphi)$, $\ddot{\varphi} \ll H\dot{\varphi}$, $\dot{H} \ll H^2$. The last inequality implies that during the typical time $\Delta t \sim H^{-1}$ the value of the Hubble parameter H remains almost unchanged, and the universe expands quasi-exponentially: $a(t+\Delta t) \approx a(t)\exp(H\Delta t)$. This regime of quasi-exponential expansion (inflation) occurs at $\varphi \gtrsim \frac{1}{3}M_\mathrm{p}$. In the region $\varphi \lesssim \frac{1}{3}M_\mathrm{p}$ the field φ rapidly oscillates, and its potential energy $V(\varphi \sim \frac{1}{3}M_\mathrm{p}) \sim \lambda M_\mathrm{p}^4$ transforms into heat.

An important feature of inflation is that it occurs independently in any domain of a size exceeding the size of the event horizon $l \sim H^{-1}(\varphi)$ [12]. The degree of inflation is given by $\exp[(\pi/M_\mathrm{p}^2)\varphi_0^2]$ and it depends on the initial value φ_0 of the field φ. The only possible constraint on the amplitude of the field φ in the theory $\frac{1}{4}\lambda\varphi^4$ is $V(\varphi) \lesssim M_\mathrm{p}^4$, or $\varphi \lesssim \lambda^{-1/4}M_\mathrm{p}$. Therefore the most natural initial value of the field φ is $\varphi_0 \sim \lambda^{-1/4}M_\mathrm{p}$ [1,2]. Inflation occurs if $\partial_\mu\varphi\partial^\mu\varphi < V(\varphi) \sim M_\mathrm{p}^4$ at least in one domain of initial size $l \sim H^{-1} \sim M_\mathrm{p}^{-1}$. Since any classical description of space–time is possible only for $\partial_\mu\varphi\partial^\mu\varphi \lesssim M_\mathrm{p}^4$, $V(\varphi) \lesssim M_\mathrm{p}^4$, and only in domains of a size $l \gtrsim M_\mathrm{p}^{-1}$ are the above-mentioned conditions quite natural [1–4]. Any

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domain of initial size $O(M_p^{-1})$ containing the field $\varphi \sim \lambda^{-1/4} M_p$ after inflation becomes as large as $M_p^{-1} \exp(\pi/\sqrt{\lambda}) \sim 10^{10^6}$ cm for $\lambda \sim 10^{-12}$ (see below), which is much larger than the size of the observable part of the universe $\sim 10^{28}$ cm. After inflation the universe becomes *locally* flat, homogeneous and isotropic, and the density of all "undesirable" objects (monopoles, domain walls, etc.) created before or during inflation become exponentially small. This is the general scheme of chaotic inflation as it can be understood at the classical level [2]. Surprisingly enough, the *global* structure of the universe is formed due to quantum effects.

3. As is shown in refs. [13–15], inflation leads to the creation of long-wave inhomogeneities $\delta\varphi(x)$ of the classical scalar field φ. This effect occurs due to the fact that inflation leads to an increase of wavelengths of quantum fluctuations of the field φ. Perturbations with a wavelength $l \gtrsim H^{-1}$ do not oscillate [due to the large friction term $3H\dot{\varphi}$ in eq. (1)] and look like a frozen distribution of a classical field φ. The overall amplitude of these perturbations generated during a typical time $\Delta t = H^{-1}$ is given by

$$|\delta\varphi(x)| \sim H/\sqrt{2}\pi \sim (2/M_p)\sqrt{V(\varphi)/3\pi} = (\sqrt{\lambda}/\sqrt{3\pi})\varphi^2/M_p. \qquad (5)$$

Their typical wavelength is initially given by $H^{-1}(\varphi)$, but later it exponentially increases as $a(t)$, whereas their amplitude very slowly decreases, $\delta\varphi \sim \dot{\varphi}$ [16]. However, at the same time new perturbations of the field φ with a wavelength $O(H^{-1})$ are generated, etc. This process at a scale $O(H^{-1})$ looks like a brownian motion of the field φ. Inhomogeneities of the resulting distribution of the field φ lead to density perturbations $\delta\rho(x)$, which very slowly (logarithmically) grow with a growth of their wavelength [15,16]. On a galaxy scale $\delta\rho/\rho \sim 10^2\sqrt{\lambda}$, which at $\lambda \sim 10^{-12}$ gives the desirable value $\delta\rho/\rho \sim 10^{-4}$ necessary for galaxy formation [15,16,1]. However, at a much larger scale the perturbations $\delta\rho/\rho$ become very large. The estimates performed in our paper ref. [1] show that density perturbations formed at the moment when the classical scalar field was equal to φ have the amplitude

$$\delta\rho(\varphi)/\rho = C\rho\sqrt{V(\varphi)}/M_p^3 \sim \sqrt{\lambda}(\rho M_p)^3, \qquad (6)$$

where $C = O(1)$. This means that $\delta\rho/\rho \gtrsim 1$ for $\varphi \gtrsim \lambda^{-1/6} M_p$. Perturbations, which are formed at that moment, have at present the wavelength $l \gtrsim M_p^{-1} \exp(\lambda^{-1/3}) \sim 10^{10^4}$ cm for $\lambda \sim 10^{-12}$. This gives the size of a locally Friedmann part of the universe after inflation. Let us try to understand the origin of the large inhomogeneities formed at $\varphi \gtrsim \lambda^{-1/6} M_p$ and the global structure of the universe on scales larger than $M_p^{-1} \exp(\lambda^{-1/3})$.

4. Evolution of the fluctuating field φ in any given domain can be described by its distribution function $P(\varphi)$ or in terms of its average value $\bar{\varphi}$ in this domain and by its dispersion $\Delta = \sqrt{\langle \delta\varphi^2 \rangle}$. However, one may obtain different results depending on the way of averaging: One can consider the distribution $P_c(\varphi)$ over the non-growing *coordinate* volume of the domain (i.e. over its volume at some initial moment of inflation), or the distribution $P_p(\varphi)$ over its *physical* (proper) volume, which exponentially grows at a different rate in different parts of the domain. It can be shown that the dispersion of the field φ in the coordinate volume Δ_c is much smaller than $\bar{\varphi}_c$ for $V(\bar{\varphi}_c) \ll M_p^4$. Therefore the evolution of the averaged field $\bar{\varphi}_c$ is described by eqs. (1)–(4). However, if one wishes to know the structure of the universe after (or during) inflation, it is more appropriate to take an average $\bar{\varphi}_p$ over the physical volume, and in some cases the behaviour of $\bar{\varphi}_p$ and Δ_p differs considerably from that of $\bar{\varphi}_c$ and Δ_c.

To illustrate this statement, let us note again that each domain of the inflationary universe of initial size exceeding the size of the event horizon in de Sitter space $O(H^{-1})$ evolves independently of what occurs outside it. Therefore, as a result of the generation of perturbations of the field φ with a wavelength $l \gtrsim H^{-1}(\varphi)$, the universe, during inflation, becomes effectively divided into many causally disconnected mini-universes of initial size $l \gtrsim H^{-1}(\varphi)$ with different values of the field φ inside each of them. The field inside a domain of a size $l = O(H^{-1})$ looks as being almost exactly homogeneous $[\partial_\mu(\delta\varphi)\partial^\mu(\delta\varphi) \sim H^4 \ll V(\varphi)$ for $V(\varphi) \ll M_p^4$ and for $\delta\varphi$ given by eq. (5)]. During a time $\Delta t = H^{-1}$ this field decreases by

$$\Delta\varphi \sim M_p^2/\varphi \qquad (7)$$

in accordance with eq. (3). The size of this domain during this time increases e times, and its physical volume grows e^3 times. As a result of the generation of the perturbations $\delta\varphi(x)$, (5), the value of the field φ in this domain becomes $\varphi - \Delta\varphi + \delta\varphi$. From (5), (7) it follows that $|\delta\varphi(x)| \gg \Delta\varphi$ for $\varphi \gg \lambda^{-1/6}M_p$. In such a case a domain of initial size $O(H^{-1})$ after expansion by e times becomes divided into $O(e^3)$ domains of a size $O(H^{-1})$, and almost half of these domains contain an *increased* field $\varphi + \delta\varphi(x) = \varphi + O(H)$. During the next interval $\Delta t = H^{-1}$ the total number of domains with a growing field φ increases again, etc. This means that the total physical volume of domains containing a permanently growing field $\varphi \gg \lambda^{-1/6}M_p$ increases as $\exp[(3-\ln 2)Ht]$, whereas the total physical volume of domains, in which the field does not decrease, grows almost as rapidly as $\frac{1}{2}\exp(3Ht)$. Since the value of $H(\varphi)$ increases with a growth of φ, the main part of the physical volume of the universe appears as a result of expansion of domains with a maximal possible field φ, i.e. with $\varphi \sim \lambda^{-1/4}M_p$, at which $V(\varphi) \sim M_p^4$. [Note, that at $\varphi \gg \lambda^{-1/4}M_p$, if it is possible to consider such domains at a classical level, the process of formation of inflationary mini-universes with a growing field φ becomes suppressed. Indeed, at $V(\varphi) \gg M_p^4$ a typical value of $\partial_\mu(\delta\varphi)\partial^\mu(\delta\varphi) \sim H^4 \gg V(\varphi)$, which does not lead to the creation of inflationary mini-universes with $\varphi \gg \lambda^{-1/4}M_p$.] Therefore, whereas the field φ averaged over the coordinate volume of any given domain (i.e. $\bar\varphi_c$) gradually decreases in accordance with eq. (3), the field φ averaged over the physical volume of a domain which initially contains the field $\varphi \gtrsim \lambda^{-1/6}M_p$, grows up to $\varphi \sim \lambda^{-1/4}M_p$.

5. It is useful to look at the same problem from another point of view. The brownian motion of the field φ at $M_p \lesssim \varphi \lesssim \lambda^{-1/4}M_p$ can be described (for not too large changes of the field φ) by the diffusion equation

$$\partial P_c/\partial t = (\partial/\partial\varphi)[\partial(\mathscr{D}P_c)/\partial\varphi + (1/3H)P_c\,\partial V/\partial\varphi] \qquad (8)$$

where $\mathscr{D}$ is the coefficient of diffusion, $\mathscr{D} = H^3/8\pi^2$ [17], see also refs. [18,19]. A detailed discussion of the solutions of eq. (8) for the chaotic-inflation scenario will be contained in a separate publication [20]. Here we will represent some of the main results of ref. [20].

A stationary solution of eq. (8) $(\partial P_c/\partial t = 0)$ would be

$$P_c \sim \exp(3M_p^4/8V(\varphi)), \qquad (9)$$

which would resemble the square of the Hartle–Hawking wavefunction of the universe [21,5]. However, eq. (8) actually has no normalizable stationary solutions, the field φ does not flow upwards from the region $\varphi \lesssim M_p$, where eq. (8) is not valid, and the average field $\bar\varphi_c$ at $\varphi \gtrsim M_p$ decreases according to eq. (3). To obtain a nonstationary solution of eq. (8) let us consider a domain of initial size $l \sim H^{-1}$, which is filled with a sufficiently homogeneous field $\varphi = \varphi_0 \ll \lambda^{-1/4}M_p$. There are two main stages of evolution of the field φ inside this expanding domain [20]. The first stage has a duration $\Delta t \sim [\sqrt{\lambda}M_p)^{-1}$. During this time the average field $\bar\varphi_c$ remains approximately equal to φ_0(3), and the dispersion $\Delta_c^2 = \langle\delta\varphi^2\rangle$ grows as $(H^3/4\pi^2)\,\Delta t$ [13–15] up to the value $\Delta_c^2 \sim \lambda\varphi_c^6/M_p^4$. At the next stage the field $\bar\varphi_c$ exponentially decreases (3). Fluctuations $\delta\varphi$ are also generated at that stage, but they have much smaller amplitude and dispersion, and the resulting value of $P_c(\varphi)$ is determined by fluctuations generated at the first stage. Since the amplitude of perturbations produced at this stage decreases as $\dot{\bar\varphi}_c$ during inflation [16], dispersion also decreases as $\dot{\bar\varphi}_c \sim \sqrt{\lambda}M_p\bar\varphi_c$ (3), $\Delta_c^2(t) \sim C\,\lambda\bar\varphi_c^2\varphi_0^4/M_p^4$, where $C = O(1)$, and

$$P_c(\varphi, t) \sim \exp[-(\varphi-\bar\varphi_c)^2/2\Delta_c^2]$$
$$\sim \exp[-(\varphi-\bar\varphi_c)^2M_p^4/2C\lambda\bar\varphi_c^2\varphi_0^4] \qquad (10)$$

Note, that for $\bar\varphi_c \approx \varphi_0$, $\varphi - \bar\varphi_c = O(\bar\varphi_c)$; this result is in a qualitative agreement with the result of

Hawking and Moss [22] concerning the probability of "tunneling" to a state with $\varphi > \bar{\varphi}_c$. (This agreement is complete for $\bar{\varphi}_c = \text{const}$ [17], "tunneling" here being just another word for brownian motion.)

Eq. (10) shows that the coordinate volume occupied by a large field φ is exponentially small. However, during the time $\Delta t \sim (\sqrt{\lambda}\, M_p)^{-1}$, at which the distribution (10) remains unchanged, domains of a large field φ expand $\exp(B\, \varphi^2/M_p^2)$ times, $B = O(1)$. This gives the distribution $P_p(\varphi)$ over the *physical* volume

$$P_p(\varphi) \sim \exp\left[-(\varphi - \bar{\varphi}_c)^2 M_p^4 / 2C\lambda\bar{\varphi}_c^2\varphi_0^4 + 3B\varphi^2/M_p^2\right]. \tag{11}$$

At $\bar{\varphi}_c$, $\varphi_0 \gtrsim \lambda^{-1/6} M_p$ the distribution $P_p(\varphi)$ *grows* with an increase of φ, in agreement with our previous results. A similar conclusion is valid at large φ in all versions of the chaotic-inflation scenario.

For completeness we will mention here another solution of eq. (8). If the initial value of the field φ is very large, $\varphi_0 \gtrsim \lambda^{-1/4} M_p$, i.e. if one starts with the space–time foam with $V(\varphi) \gtrsim M_p^4$, then it can be shown that the distribution of the field φ, formed during the time $\Delta t \sim (\sqrt{\lambda}\, M_p)^{-1}$, at $\varphi \lesssim \lambda^{-1/4} M_p$ is given by

$$P_c(\varphi) \sim \exp\left(-C M_p^4 / V(\varphi)\right), \tag{12}$$

where $C = O(1)$, and this distribution later evolves just as in the case considered above. This result is in agreement with the previous estimates of the probability of quantum creation of the universe [6,23–25]. However, in our case there is no need to talk about "creation of everything from nothing": Creation of an inflationary (mini-)universe may look either as a classical motion or as a diffusion (= tunneling) from the space–time foam with $V(\varphi) \gtrsim M_p^4$, which fills the main part of the physical volume of the universe and plays the role of the unstable (but regenerating) gravitational vacuum [1,6].

6. From our results it follows that the inflationary universe, which contains at least one domain with $\varphi \gtrsim \lambda^{-1/6} M_p$, infinitely reproduces new and new mini-universes with $\varphi \gtrsim \lambda^{-1/6} M_p$, and its global geometry has nothing in common neither with the geometry of an open or flat homogeneous universe with a gradually decreasing energy density, nor with the geometry of a closed universe, which is created at some initial moment $t = 0$ and which disappears as a whole at some other moment $t = t_{max}$. In our case the universe infinitely regenerates itself, and there is no global "end of time". Moreover, it is not necessary to assume that the universe as a whole was created at some initial moment $t = 0$. The process of formation of each new mini-universe with $\varphi \gtrsim \lambda^{-1/6} M_p$ occurs independently of the pre-history of the universe, it depends only on the value of the scalar field φ inside a domain of a size $O(H^{-1})$ and not on the moment of the mini-universe formation. Therefore the whole process can be considered as an infinite chain reaction which has no end and which may have no beginning.

In the eternally existing chaotic self-reproducing inflationary universe the main part of the physical volume always contains a large scalar field φ with $V(\varphi) \gtrsim M_p^4$. In this sense the universe is effectively singular. It is important, that these "singularities" in our model do not form a global spacelike singular hypersurface, which would imply the existence of the "beginning of time" for the whole universe. The existence of such a hypersurface does not follow from general topological theorems concerning singularities in general relativity [26], but it is usually assumed that our universe looks like a slightly inhomogeneous Friedmann universe, in which such a hypersurface does exist. However, as we have emphasized, the *global* geometry of the inflationary universe differs crucially from the geometry of the Friedmann universe; the global spacelike singular hypersurface in the inflationary universe does not exist in the future and may well not exist in the past.

Thus, there exist two main versions of the chaotic-inflation scenario:

(i) There may exist an initial global singular spacelike hypersurface. In this case the universe as a whole emerges from a state with a Planck density $\rho \sim M_p^4$ at some moment $t = t_p$, at which it becomes possible to speak about the universe in terms of classical space–time. A most natural initial value of the field φ at the Planck time is $\varphi \sim \lambda^{-1/4} M_p$. Then the universe infinitely reproduces itself due to generation of long-wave fluctuations of the field φ. This process occurs in domains with $\lambda^{-1/6} M_p \lesssim \varphi \lesssim \lambda^{-1/4} M_p$ (with $100 \times M_p \lesssim \varphi \lesssim 1000 M_p$, for $\lambda \sim 10^{-12}$). In domains with $\varphi \lesssim \lambda^{-1/6} M_p$ this process becomes inefficient, and each such domain after inflation looks like a Friedmann mini-universe of a size $M_p^{-1} \times \exp(\lambda^{-1/3}) \sim 10^{10^4}$ cm. Formation of each mini-universe can be described either by eqs. (10, (11) or by eq. (12). In this model the universe has a beginning but no end.

(ii) The possibility that the universe has a global singular space-like hypersurface seems rather improbable unless the universe is compact and its initial size is $O(M_p^{-1})$: There is no reason for different causally disconnected regions of the universe to start their expansion simultaneously. If the universe is not compact, there should be no global beginning of its evolution. A model which illustrates this possibility was suggested above: The inflationary universe may infinitely reproduce itself, and it may have no beginning and no end. Note, that the main problem of the standard Friedmann cosmology was "the existence of time when there was no space–time at all". In our case such a problem may not appear. For a more detailed discussion of this possibility and of the associated problems see ref. [27].

It is worth noting that the process of self-reproduction of the universe occurs not only at the Planck density, but at much smaller densities as well, e.g. at $V(\varphi) \gtrsim \lambda^{1/3} M_p^4 \sim 10^{-4} M_p^4$ for $\lambda \sim 10^{-12}$. Therefore to prove the very existence of the regime of self-reproduction of inflationary mini-universes in our scenario there is no need to appeal to unknown physical processes at $\rho \gtrsim M_p^4$.

On the other hand, it is very important that independently of the origin of the universe in our scenario (either the universe was created as a whole at $t = t_p$ or it exists eternally) it now contains an exponentially large (or even infinite) number of mini-universes, and a considerable part of these mini-universes was created when the field φ was $O(\lambda^{-1/4} M_p)$ and its energy density was $O(M_p^4)$. (Note, that this is true in the chaotic-inflation scenario only, in which inflation may occur even at $V(\varphi) \sim M_p^4$.) At such densities, fluctuations of all fields and fluctuations of metric are very large at a typical scale $\sim H^{-1} \sim M_p^{-1}$. This may lead to the generation of different classical scalar fields Φ_i, corresponding to different local minima of $V(\Phi_i, \varphi)$ in different domains of the universe and to processes of compactification or decompactification which occur *independently* in each of the causally disconnected mini-universes of initial size $l \gtrsim H^{-1} \sim M_p^{-1}$. As a result, our universe at present should contain an exponentially large number of mini-universes with *all* possible types of compactification and in *all* possible (metastable) vacuum states consistent with the existence of the earlier stage of inflation. If our universe would consist of one domain only (as it was believed several years ago), it would be necessary to understand why Nature has chosen just this one type of compactification, just this type of symmetry breaking, etc. At present it seems absolutely improbable that all domains contained in our exponentially large universe are *of the same type*. On the contrary, *all* types of mini-universes in which inflation is possible should be produced during the expansion of the universe, and it is unreasonable to expect that our domain is the only possible one or the best one. From this point of view, an enormously large number of possible types of compactification which exist e.g. in the theories of superstrings should be considered not as a difficulty but as a virtue of these theories, since it increases the probability of the existence of mini-universes in which life of our type may appear. The old question why our universe is the only possible one is now replaced by the question in which theories the existence of mini-universes of our type is possible. This question is still very difficult, but it is much easier than the previous one. In our opinion, the modification of the point of view on the global structure of the universe and on our place in the world is one of the most important consequences of the development of the inflationary-universe scenario.

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References

[1] A.D. Linde, Rep. Prog. Phys. 47 (1984) 925.
[2] A.D. Linde, Phys. Lett. B 129 (1983) 177.
[3] L.A. Kofman, A.D. Linde and A.A. Starobinsky, Phys. Lett. B 157 (1985) 361;
V.A. Belinsky, L.P. Grishchuk, I.M. Khalatnikov and Ya.B. Zeldovich, Zh. Eksp. Teor. Fiz 89 (1985) 346.
[4] A.D. Linde, Phys. Lett. B 162 (1985) 281; Suppl. Prog. Theor. Phys., to be published.
[5] S.W. Hawking, Nucl. Phys. B 239 (1984) 257.
[6] A.D. Linde, JETP 60 (1984) 211; Lett. Nuovo Cimento 39 (1984) 401.
[7] A.D. Linde, in: Composite models of quarks and leptons (University of Tokyo, Yamada P.C., Tokyo 1985), p. 415.
[8] A.S. Goncharov and A.D. Linde, Class. Quant. Grav. 1 (1984) L75.
[9] Q. Shafi and C. Wetterich, Phys. Lett. B 152 (1985) 51.
[10] P. Oh, Phys. Lett. B 166 (1986) 292.
[11] A.A. Starobinsky, JETP Lett. 30 (1979) 682; Phys. Lett. B 91 (1980) 99.
[12] G.W. Gibbons and S.W. Hawking, Phys. Rev. D 15 (1977) 2738.
[13] A. Vilenkin and L. Ford, Phys. Rev. D 26 (1982) 1231.
[14] A.D. Linde, Phys. Lett. B 116 (1982) 335.
[15] A.A. Starobinsky, Phys. Lett. B 117 (1982) 175.
[16] V.F. Mukhanov and G.V. Chibisov, JETP Lett. 33 (1981) 532;
S.W. Hawking, Phys. Lett. B 115 (1982) 295;
A.H. Guth and S.-Y. Pi, Phys. Rev. Lett. 49 (1982) 1110;
J. Bardeen, P.J. Steinhardt and M.S. Turner, Phys. Rev. D 28 (1983) 679.
[17] A.A. Starobinsky, in: Fundamental interactions (MGPI Press, Moscow, 1984), p. 55.
[18] A.S. Goncharov and A.D. Linde, Phys. Elem. Part. At. Nucl. 17 (1986), to be published.
[19] A. Vilenkin, Phys. Rev. D 27 (1983) 2848.
[20] A.S. Goncharov and A.D. Linde, to be published.
[21] J.B. Hartle and S.W. Hawking, Phys. Rev. D 28 (1983) 2960.
[22] S.W. Hawking and I.G. Moss, Phys. Lett. B 110 (1982) 35.
[23] Ya.B. Zeldovich and A.A. Starobinsky, Pis'ma Astron. Zh. 10 (1984) 323.
[24] V.A. Rubakov, Phys. Lett. B 148 (1984) 280.
[25] A. Vilenkin, Phys. Rev. D 30 (1984) 509.
[26] S.W. Hawking and G. Ellis, The large-scale structure of space–time (Cambridge U.P., London, 1973).
[27] A.D. Linde, Non-singular regenerating inflationary universe, Cambridge University preprint (1982); Lebedev Phys. Inst. preprint No 106 (1986) (extended version), in: Proc. Nobel Symposium on Unification of fundamental interactions (1986), to be published; and preprint, submitted to Lett. J. Mod. Phys. A.