A generative grammar approach to diatonic harmonic structure

Martin Rohrmeier
Centre for Music and Science, Faculty of Music, University of Cambridge, UK

***Abstract*— This paper aims to give a hierarchical, generative account of diatonic harmony progressions and proposes a generative phrase-structure grammar. The formalism accounts for structural properties of key, functional, scale and surface level. Being related to linguistic approaches in generative syntax and to the hierarchical account of tonality in the generative theory of tonal music (GTTM) [1], cadence-based harmony contexts and its elaborations are formalised. This approach covers cases of modulation, tonicisation and some aspects of large-scale harmonic form, and may be applied to large sets of diatonic compositions. Potential applications may rise in computational harmonic and corpus analysis, as well as in the music psychological investigation of tonal cognition.**[1]

I. Introduction

In music theory, harmony is a well researched area, and there are dozens of taxonomies for tonal harmony phenomena. Most theories concentrate on classifying chord types, and deducing complex phenomena from basic (diatonic) chords (such as explaining the pre-dominant Neapolitan chord from the altered fifth of a IV in IV^6 position). However, only few systems give a generative account of harmonic progressions. Piston [2] gives a table of "usual root progressions"(Tab.I). This table has been found to accord with harmony progressions in Bach's chorales in an empirical study (Fig.1). Similarly, a psychological study [3] has found agreement with Piston's table and subjects' probe-chord ratings.

Table of usual root progressions

	is followed by	sometimes by	less often by
I	IV or V,	VI,	II or III.
II	V,	IV, VI,	I, III.
III	VI,	IV,	I, II or V.
IV	V,	I or II,	III or VI.
V	I,	VI or IV,	III or II.
VI	II or V,	III, IV,	I
VII	III,	I.	

TABLE I

However, harmonic structure and chord progressions are not comprehensively explained by linear approaches. In compositional practice, harmonic progressions explore a large set of structural inter-connections, and may involve overarching long-term chord dependencies. Piston's characterisation of harmony transitions only takes a context of one (preceding) chord into account. Formally, it describes chord relationships in a way comparable to a (probabilistic) transition matrix, or a (probabilistic) finite-state grammar. For a more comprehensive view of harmony, it would be desirable to encompass larger contexts for complex phenomena (including modulations) and to distinguish between different generative origins of chord progressions such as prolongational or progressive relationships [5]. For instance, the II chord in the sequence I-II-V in major is more appropriately accounted for by being *dependent* on V than by being a consequent of I. This is not possible with a linear view of harmony. Steedman gives a hierarchical account of Jazz harmony using a context-free/categorial grammar [6]. However, these approaches are limited to the range of Jazz harmony.

This article aims to give a hierarchical generative account of diatonic harmony progressions from the common practice period. This work is still in progress and does not aim to give a comprehensive formalisation of functional harmony. It addresses cadence-based harmonic progressions which govern a major part of common practice harmony.

	I	II	III	IV	V	VI	VII
I		132	36	474	668	191	43
II	116		35	11	100	59	5
III	47	13		73	22	52	12
IV	351	63	31		138	29	45
V	1042	60	63	73		147	1
VI	106	72	62	64	159		14
VII	92	1	4	3	2	4	

Fig. 1. Absolute frequencies of diatonic chord progressions (columns as destinations) in major from Bach chorales (from [4]).

II. Formalisation

The proposed formalisation is based on main theories of harmony [2], [5], [7], [8]. The formalism distinguishes elements (chords) and their properties. It assumes the structural hierarchy outlined in Tab.II which reflects the fundamental difference between surface and (induced) structural levels (key, function, and scale degree) in music cognition (comp. [9]). Each chord event analysed embodies properties for all 3 levels. The *key level* refers to the 24 major and minor keys. At the *functional level*, theories of harmony generally distinguish the classes of tonic, dominant and predominant (or subdominant) harmonies which are represented here by the symbols t,d,s. The *scale degree level* refers to all 7 natural or altered scale degrees which can serve as roots of har-

[1]This research has been carried out under the funding of the Microsoft European PhD Scholarship Programme.

monies, and encompasses properties of chord inversion (none, first, or second inversion) and added tones (like a 7th). Scale degree level and functional level are assumed not to coincide.[2] The combination of all properties at the structural levels specifies a chord unambiguously on the surface level; however, chords on surface level may specify some structural properties but are generally not sufficient to specify structural properties unambiguously (comp. Tab.III).Hence, the effective problem for (any) harmonic analysis is to deduce structural properties from the given surface structure of chords.

	abstraction level	property	example
structure	key level	key:(tonic,mode)	key: tonic=E mode=major
	functional level	{t,d,s}	d
	scale degree level	{(♮,♯,♭)I,...,(♮,♯,♭)VII}	
		$el \subset \{1,\ldots,7\}$	$V_{el=7}$
		inv:$\{0,1,2\}$	
surface	chord level		B^7

TABLE II

surface level	given chord:	$d^{\emptyset 7}$	$\frac{F}{3}$
structural level	poss. key	c-minor	F,$B_\flat$,$b_\flat$,C,d,a
	poss. function	s	{t,d,s}
	poss. scale degree	$II_{el=7}$	I, V, IV, III, VI $inv = 1$

TABLE III

Building on the outlined element structure, it is possible to propose the following (provisional) context-free/phrase-structure grammar formalisation based on the literature above.All rules capture fundamental harmonic relationships, and refer to manipulations on the structural levels outlined above.

$$t \rightarrow t_{key=x} \quad (1)$$
$$t \rightarrow t \;\; t \quad (2)$$
$$t \rightarrow t_{\llcorner} \; d \quad (3)$$
$$t \rightarrow d \; {}_{\lrcorner}t \quad (4)$$
$$d \rightarrow s \; {}_{\lrcorner}d \quad (5)$$

$$t \rightarrow I \quad (6)$$
$$t \rightarrow tp \quad (7)$$
$$tp \rightarrow \begin{cases} VI & \text{if key is major} \\ \{VI, III\} & \text{if key is minor} \end{cases} \quad (8)$$
$$d \rightarrow \begin{cases} \{V, VII\} & \text{if key is major} \\ \{V, \natural VII\} & \text{if key is minor} \end{cases} \quad (9)$$
$$s \rightarrow \{IV, II, VI, \flat II(\text{in minor})\} \quad (10)$$

These rules define a general (diatonic) cadential framework. Rule 2 is an important rule for the prolongation of tonic harmony which turns out to be of general importance for the generation of larger harmonic contexts.

[2]This refers to a music theoretical discussion which cannot be dealt with here.

Any generation can begin with a simple tonic harmony t (rule 1); it specifies that the head of the phrase has to specify the key property, which realises the key property for the entire phrase to be analysed. Rule 8 captures the replacement function of tonic parallels, which is, for instance, important for deceptive cadences. The '$\llcorner$','$\lrcorner$'-symbols denote the dependency of the generated harmony from the generating harmony. Beginning from a single t symbol and specifying a key, these rewrite rules can be recursively applied to generate basic tonal phrases. Some additional rules are required to cover particular phenomena:

$$I \rightarrow I \; IV \; {}_{\lrcorner}I \quad (11)$$
$$x \rightarrow V(x) \; {}_{\lrcorner}x \quad (12)$$
$$x \rightarrow x_{\llcorner} \; V(x) \quad (13)$$
$$IV \rightarrow III \; {}_{\lrcorner}IV \quad (14)$$

Rule 11 accounts for the case of tonic prolongations by an intermediate subdominant harmony. Rule 12 accounts for the frequent case of secondary dominants, dominant sequences and falling fifth sequences. Rising fifth sequences are covered by rule 13. Rule 14 covers a relatively rare use of scale degree III (as at the beginning of Schubert's song "Im Frühling"). Functional symbols are used in order to distinguish elementary relations in key areas from the realisation of different scale degrees within given key areas. This is necessary to account for harmonic events which fall out of the diatonic frame work (such as secondary dominants), tonicisations and modulations. These cases are captured by the constraint that every structural property is carried down through all rewriting steps unless specified otherwise. The sample analysis (Fig.2) illustrates this fact. In the case of tonicisation, the key feature of an element is changed which may result in a change of the other elements: $III_{key=t} \rightarrow t, I_{key=iii}$ (denotes a tonicisation of scale degree III to the new key, resulting in the fact that the same event locally becomes the new t and I.)

On the scale degree level, a number of *adjustment rules* may be postulated to cover the various surface structures of chords, and dissonance phenomena, e.g. V^{6-5}_{4-3} which translates into $V \rightarrow V_{el=4,6} \; V_{el=3,5}$. The description of the details of these (stylistic) rules will be a matter of fine-grained stylistic analysis or computational corpus analysis.

The analysis completes with simple *transformational rules* which transform a scale degree into a chord on surface level, given its key property. These rules follow the standard definition of scale degrees in a straightforward way, for instance: $V_{el=7,key=E_\flat maj} \rightarrow B^7_\flat$.[3]

III. Sample Analyses

In practice, it turns out that few rules suffice to cover a large number of cases. One example (Fig.2) shows

[3]A full account of these *transformational rules* cannot be given here due to space limitations.

the general application of the rules for the parsing of a phrase, and involves special cases of modulation and a deceptive cadence. The diagram reads like common linguistic parsing trees. As the example shows, simple prolongational and progressive phenomena, as well as simple cadences, modulations and deceptive cadences can be accounted for. An advantage of the proposed

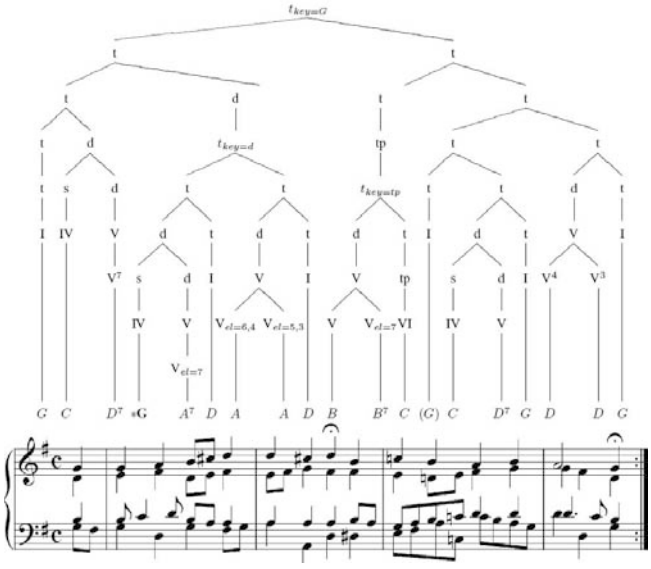

Fig. 2. Analysis of Bach's chorale "Ermuntre Dich, mein schwacher Geist",mm.1-4

method of analysis is that it further allows to account for adjacencies of structurally not closely related chords, such as the progressions F-D^7,G-E^7,a-$F^0_\sharp$ in example 2(which cannot be explained from a linear framework like Tab.I). Note that the application of changed key and functional properties do not intend to imply that each segment is modulating. Rather, it signifies a very brief passing tonicisation which accounts for the occurrence of the chords C^7, D^7, E^7 and $F^0_\sharp$ which fall out of the diatonic framework of C major.

IV. Special cases

In order to work towards a more comprehensive account of complex harmonic phenomena beyond diatonic harmony, several extensions would need to be incorporated to cover chromaticisms, altered sixth and other complex chords. Another open aspect is the modelling of the role of dual-function pivot chords which happen in modulations (such as $*$ in Fig.2).Many theorists propose that those pivot chords technically belong to both keys. It is unclear how a formalisation of harmony should reflect this fact, since double generation of events is outruled by generative grammars. One viable possibility may be to allow for the exceptional double generation of the same pivot element(s) from both adjacent subtrees, by including a structural *pivot constraint* which demands the surface chord as specified from both different structure levels to be identical.

Some examples from compositional practice suggest the application of another complex rule. Whereas the

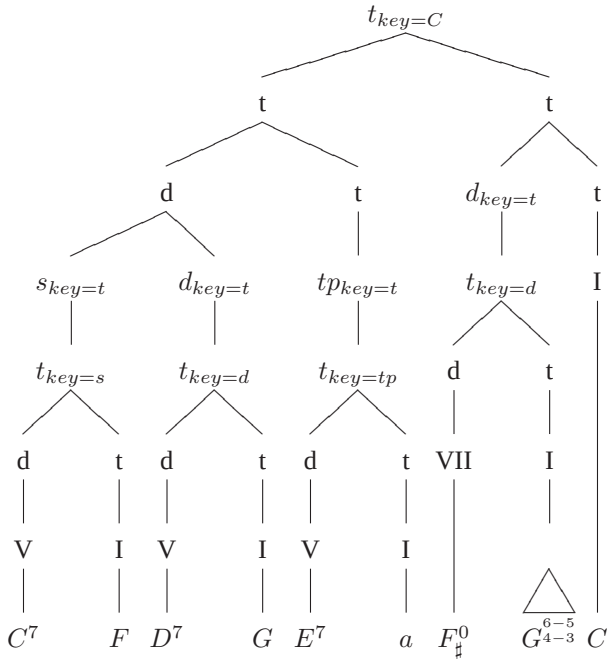

Fig. 3. Analysis of Bortnianski's 'Tibje Pajom' from Fig.5

harmony sequence in Fig.4 cannot be explained by the rules above, an extension helps to add sufficient context.

$$d\ t \rightarrow d\ [t] \tag{15}$$

$$[X] \rightarrow \epsilon \tag{16}$$

Here, the brackets denote the structurally significant element which, however, is not realised on surface level (rule 16, here ϵ denotes the empty terminal symbol). This rule serves to deal with complex harmonic phenomena which may not be analyseable otherwise. However, it is not unproblematic since it may result in the grammar not being expressible as context-free grammar any more. Therefore, it might be replaced by a different way of writing the rule:

$$t \rightarrow s\ \lrcorner d\ \lrcorner [t] \tag{17}$$

Another important fact is illustrated by this example (Fig.4): Here, harmonic coherence is the result of two factors: the overarching structural relationships between the chords and the locally well formed voice-leading that connects the structurally, but indirectly related, adjacent chords. In this case, the chromatic baseline serves to maintain tonal coherence between the chords. Capturing this structural relationship will require to formulate structural constraints which govern these cases of locally well-formed adjacency which may underpin structural coherence - since both structural relationships and local voice-leading equally govern tonal coherence [8].

In order to cover additional features of harmonic rhythm and to be applicable for the variety of surface phenomena (involving local dissonance treatment like suspensions) which originate from properties of voice-leading, a number of transformational rules, and structural

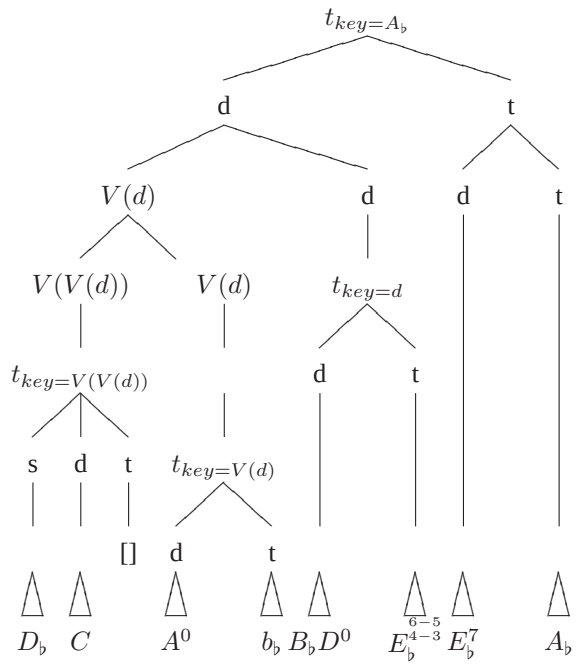

Fig. 4. Analysis of Chopin's Waltz op.69, No.1 from Fig.6

constraints governing harmonic metrical alignment could be added.

V. Discussion

While some of the rules proposed may be arguable, this paper mainly aims to make a contribution in showing the possibility and advantages of a hierarchical, generative and functional approach to describe and formalise harmonic progressions and relationships. The fact that the simple and compact set of rules outlining a general cadential context suffices for the explanation of many examples accords with the statement in [8], [5] that tonal harmony is fundamentally grounded in elaborations of cadential harmony. Moreover, whereas Piston's linear progression table cannot account for many phenomena (like Fig.3 and Fig.4), most of the chord progressions in Piston's table can be accounted for by the rules proposed here. Unlike a linear understanding, the approach can also account for phenomena of tonicisation and modulation. Music theory does not draw a clear line between these two concepts; however, one might conceptualise modulations as key changes that happen on higher levels in the tree (and, therefore, do have impact on a larger number of subordinated chords), whereas tonicisation might be treated as more local, low level passing key changes.

The presented approach is closely related to the GTTM in various aspects. It is governed by recursive, hierarchical principles and captures progressive and prolongational dependency relationships between harmonic elements. However, whereas the GTTM is not a generative grammar since there are no generative rules given, the structure proposed here is a generative grammar in its full sense; the parsing trees also indicate the constructional steps involved in the deduction. Moreover, it is possible to account for some harmonic and key properties of some large scale formal structures such as binary forms (or Fig.2).

It is important to stress that this approach specifies a constructional/compositional grammar rather than a cognitive system. Compositional practice in the common practice period shows organisation in harmonic and large-scale key dependencies which may be formalised in the proposed way. However, from a music psychological perspective it is not clear whether these constructional principles correspond with music cognition. Whereas Cook's strongly debated [10] article [11] finds musicians not able to perceive large-scale key dependencies, other evidence [12] may suggest some evidence for large-scale key dependency perception in musician and nonmusician subjects. It remains a matter of experimental investigation to assess the degree to which the rules given above may be part of (non)musicians implicit knowledge of musical structure.

References

[1] F. Lerdahl and R. Jackendoff, *A Generative Theory of Tonal Music*. Cambridge, MA: MIT Press, 1983.
[2] W. Piston, *Harmony*. New York: W.W.Norton & Company, 1948.
[3] J. Bharucha and C. Krumhansl, "The representation of harmonic structure in music: Hierarchies of stability as a function of context," *Cognition*, vol. 13, pp. 63–102., 1983.
[4] M. Rohrmeier, "Towards modelling movement in music: Analysing properties and dynamic aspects of pc set sequences in bach's chorales," Master's thesis, University of Cambridge, 2005.
[5] R. Gauldin, *Harmonic Practice in Tonal Music*. New York: Norton, 1997.
[6] M. Steedman, "The blues and the abstract truth: Music and mental models," in *Mental Models In Cognitive Science*, A. Garnham and J. Oakhill, Eds. Mahwah, NJ: Erlbaum, 1996, pp. 305–318.
[7] A. Forte, *Tonal Harmony in Concept and Practice*. Holt, Rinehart and Winston, 1962.
[8] E. Aldwell and C. Schachter, *Harmony and Voice Leading*, 2nd ed. San Diego: Harcourt Brace Jovanovich, 1989.
[9] D. Temperley, "Bayesian models of musical structure and cognition," *Musicae Scientiae*, vol. 8, pp. 175–205, 2004.
[10] R. Gjerdingen, "An experimental music theory?" in *Rethinking Music*, N. Cook and M. Everist, Eds. Oxford: Oxford University Press, 1999, pp. 161–170.
[11] N. Cook, "The perception of large-scale tonal closure," *Music Perception*, vol. 5, pp. 197–206, 1987.
[12] M. Woolhouse, I. Cross, and T. Horton, "The perception of non-adjacent harmonic relations," in *International Conference an Music Perception and Cognition 9*, 2006.

Appendix

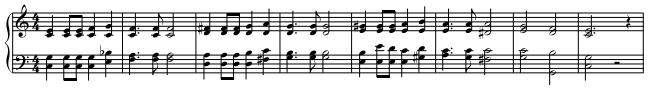

Fig. 5. from Bortnianski, Tibje Pajom, mm.9-16

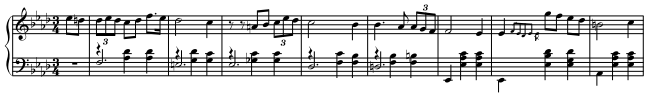

Fig. 6. from Chopin's Valse, op.69 No.1 mm.1-8