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PAID PLACEMENT: ADVERTISING AND SEARCH ON THE INTERNET*

Yongmin Chen and Chuan He

Paid placement, where advertisers bid payments to a search engine to have their products displayed prominently among the results of a keyword search, has emerged as a predominant form of advertising on the Internet. This article studies a model of product differentiation in which the auction of advertisement positions is embedded in a market game of consumer search. In equilibrium, more relevant sellers for a given keyword bid more and their paid placement by the search engine reveals information about the relevance of their products. This results in efficient sequential search by consumers and increases total output. We also find that the search engine's revenue may have an inverted U-shape with respect to the match probability of the most relevant seller.

Paid placement, online advertising, in which links to advertisers' products appear prominently among the results of a keyword search, has emerged as a predominant form of Internet advertising. Under paid-placement auction (also called position auction), sellers (advertisers) bid payments to a search engine to be placed on its 'recommended' list for a keyword search. A group of advertisers who bid more than the rest are selected for placement.[1] The rapid growth of paid-placement advertising has made it one of the most important Internet institutions and has led to enormous commercial success for search engines. For example, Google, which derives most of its revenue from paid-placement advertising, received $28.2 billion advertisement revenues in 2010 (Google financial statement) and it has a larger market capitalisation than the big three US auto manufacturers combined.

The popularity and importance of paid-placement advertising raises several interesting questions. How do sellers form their bidding strategies? How does paid-placement advertising affect consumer search and welfare? And what determines the revenue of a search engine in equilibrium? We develop a market equilibrium model that addresses these questions in this article. In this model, consumers search for their desired product varieties, and a search engine serves as a useful intermediary that provides information about the relevance of different sellers' products.

We consider a game in which differentiated sellers first bid payments to a search engine to be placed on its list of search outcomes associated with a particular keyword (product). Only a small number of sellers are listed due to the limited number of positions available on the list. Sellers differ in their 'relevance', which we model as the probability that any consumer will find a seller's product to be her desired variety. Each consumer is *ex ante*

* Corresponding author: Yongmin Chen, Department of Economics, University of Colorado, Boulder, CO 80309, USA. Email: yongmin.chen@colorado.edu.

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[1] An advertisement is often placed in a coloured box on the top right of the first search results page, and it is referred to as a 'paid-placement advertisement' in the popular press (Coy, 2006). Sometimes, the advertisements also appear as coloured results at the top of the first search results page.

uncertain about which seller's product will match their preference and how much they are willing to pay for the product. By searching (inspecting) a seller's website, the consumer will learn about the seller's product and price. But there are search costs to inspect a seller's website; hence, a consumer needs to form a search strategy and, if a search yields a match, a purchase strategy. On the other hand, sellers take into account consumers' search and purchase behaviour when choosing pricing and bidding strategies.

At a separating equilibrium of this model, a seller bids more for placement when his product is more relevant for a given keyword, and the placement of sellers by the search engine reveals information about the relevance of different sellers. At a partially separating equilibrium, more relevant sellers bid the same amount to be placed on the search engine's list in random order. Both types of equilibria result in more efficient (sequential) search by consumers and increase total output, compared with the situation where paid-placement advertising is absent. Depending on the extent to which sellers differ in relevance, either the separating equilibrium or the partially separating equilibrium may lead to higher profit for the search engine.

The auction of advertisement positions by a search engine has been studied in a recent paper by Edelman *et al.* (2007), which demonstrates that the auction mechanism for paid-placement advertising is one of the generalised second price auction.[2] We also model the position auction as a second price auction, where a winning bidder for an advertisement position pays the next highest bid; but a major difference in our analysis is that we embed the bidding process in a market game, where consumers' search and purchase decisions, as well as sellers' pricing decisions, are all determined endogenously. Consequently, the values of sellers in being placed on the advertisement list and being placed at different positions are endogenous.[3] Our model and main results first appeared in an earlier version that was circulated as a NET Institute working paper (Chen and He, 2006). There have been many related recent contributions, where the 'prominence' of some sellers, whether acquired through paid-placement auctions or through other means, plays important roles in market equilibrium. For example, Athey and Ellison (forthcoming) also study position auctions in a framework of consumer search, with a particular focus on incomplete information and on auction design issues. Eliaz and Spiegler (2011) depart from auction considerations and instead study how a monopoly search engine may optimally control the quality of the search pool through properly setting price-per-click. In another direction, Armstrong *et al.* (2009) study the effects of making one firm prominent in a general model of consumer search and find that the prominent firm has higher profit but lower price than other firms. Rhodes (2011) shows that a prominent firm earns significantly more profit than other sellers even when consumers' cost of searching and comparing products is essentially zero. Armstrong and Zhou (2011) further investigate alternative ways that a firm can become prominent and their implications for market performance.[4]

Our model is also related to the literature on advertising. Advertising in our model conveys product information, as, for instance, in Nelson (1974), Grossman and Shapiro

[2] See also Varian (2007) for a related contribution.

[3] For studies of auctions with endogenous valuations, see, for instance, Lewis (1983), Krishna (1993) and Chen (2000).

[4] See also Wilson (2010) for a model, where a firm may establish its 'prominence' by choosing and advertising a low firm-specific consumer search cost.

(1984), Meurer and Stahl (1994) and Anderson and Renault (2006). The information conveyed by the advertisements through paid placement, however, is about the relevance of a seller's product relative to a particular keyword search and is thus unique to the Internet institution. Advertising by the sellers acts as a device to coordinate consumer search and the more consumers a seller can attract in turn enables the seller to bid more payment to be placed by the search engine. As it is the more relevant sellers who can benefit more from attracting more consumers to visit their websites, in equilibrium the more relevant sellers indeed bid more and are placed on the search engine's list; it would indeed be rational for consumers to search based on paid-placement advertising to find their desired product. This is related to the result in Bagwell and Ramey (1994), where advertising coordinates consumers to search stores that have lower marginal costs and hence lower prices; expecting more consumers, these stores indeed have the incentive to invest in reducing marginal costs.

We set up our model in Section 1. Section 2 studies market equilibrium, where we characterise consumers' equilibrium search and purchase decisions and firms' equilibrium pricing and bidding strategies. Section 3 analyses the implications of paid-placement advertising for the search engine's profit, consumer welfare and efficiency. Section 4 considers an extended model where sellers have different costs, which generates price dispersion under pure strategies. Section 5 concludes.

1. The Model

There are $m \geq 4$ differentiated sellers, selling to a unit mass of consumers at a constant marginal cost c. Given a particular keyword, the m sellers' products have different 'relevance' for consumers. With probability β_i, seller i's product matches the preference of any randomly chosen consumer, in which case the consumer's valuation for the seller's product is v, which is the realisation of a random variable with cdf $F(v)$ and pdf $f(v)$ on $[\underline{v}, \bar{v}]$, where $0 \leq \underline{v} < \bar{v}$; with probability $1 - \beta_i$, seller i's product does not match the preference of the consumer, in which case the consumer's valuation for the seller's product is zero. Consumers learn about their v only when they find the desired product. We call β_i the match (or relevance) probability of seller i and make the simplifying assumption that β_i is independent of $F(v)$ and is independent and identical for every consumer.

Without loss of generality, let

$$\beta_1 \geq \beta_2 \geq \cdots \geq \beta_m,$$

and refer seller i as type i. Each seller is privately informed about his type, while the distribution of seller types and possible values of β_i are common knowledge. For analytical tractability, we shall assume

$$\beta_i = \begin{cases} \gamma^{i-1}\beta & \text{for} \quad i = 1, 2, \ldots, I \\ \gamma^I \beta & \text{for} \quad i = I+1, \ldots, m \end{cases},$$

where $\beta, \gamma \in (0,1)$ and $2 \leq I \leq m$. Thus, the match probability decreases among the sellers at a constant rate γ for I sellers, then it becomes constant for the rest of the sellers.

We denote seller i by S_i. Each consumer is *ex ante* uncertain about which seller's product is desirable for their preference. They also do not know the match probability of any particular seller. But they can find out whether the seller's product is desirable by visiting the seller's website. They can also decide which sellers' websites to visit by first searching through a search engine with a keyword for the product, and the search engine then shows a list of paid-placement advertising sellers. Sellers are differentiated by their different relevance (matching probabilities) with respect to a particular keyword.[5] A seller can choose to pay the search engine to be included in the list (E). There are $n \leq m$ positions on E, E_1, E_2,…,E_n, that the search engine can auction to the sellers in a second price auction, where the seller who bids the most gets listed the highest (at E_1) and pays the second highest bid, the seller who bids the second highest gets listed the second highest (at E_2) and pays the third highest bid and so on. In other words, let the bids of the sellers in descending order be b_j, $j = 1,\ldots,m$. Then, sellers S_j will be included on E with the order $j = 1,2,\ldots,n$. We assume $n = 3 = I$, although it is straightforward to extend our analysis to any arbitrary n and I. Thus, by assumption, there are three positions on E, E_1, E_2 and E_3; and $\beta_i = \gamma^{i-1}\beta$ for $i = 1,2,3$ but $\beta_i = \gamma^3\beta$ for $i \geq 4$.

The timing of the game is as follows. Sellers, having learned their private β_i, first bid to be listed on E. The chosen sellers are listed on E. Sellers then simultaneously and independently choose their prices, which are not observed by any consumer until the consumer searches the sellers' websites. Consumers then decide whether and how to search the websites; they may possibly use information from E. There are costs for consumers to search the websites of sellers. The cost for each consumer to conduct their jth search is t_j, $j = 1,\ldots,m$. A consumer makes a unit purchase if and when they find their desired product, the price does not exceed their realised v, and searching further does not yield a higher expected surplus for them. All players are risk neutral. We make the following technical assumptions:

A1. *There is a unique p^o such that*

$$p^o = \arg\max_{p\in[c,\bar{v}]}(p-c)[1-F(p)]. \tag{1}$$

A2.

$$t_j = \begin{cases} t & \text{for } j = 1,2,3,4 \\ t^h & \text{for } j > 4 \end{cases}, \tag{2}$$

where

$$t < \gamma^3\beta\int_{p^o}^{\bar{v}}(v-p^o)f(v)\,\mathrm{d}v < t^h. \tag{3}$$

A sufficient, but not necessary, condition for **A1** is that the hazard rate $[f(p)]/[1-F(p)]$ is monotonically increasing, which is satisfied for many familiar distributions such as uniform, exponential, and normal distributions. **A2** captures the idea that a consumer's

[5] A seller could be more relevant because the particular brands he carries, or simply because he carries a larger number of product varieties, for a given keyword.

marginal search cost becomes higher after some searches, perhaps due to 'capacity constraint' in her time that can be used for search. We define

$$\pi^o \equiv (p^o - c)[1 - F(p^o)]. \tag{4}$$

2. Equilibrium Analysis

A profile of strategies in our model consists of a search and purchase strategy by each consumer, a bidding strategy by seller S_i and a pricing strategy by S_i, for all i. After observing the placement of sellers, buyers have beliefs about the relevance (type) of different sellers. An equilibrium (perfect Bayesian equilibrium or PBE) is a profile of strategies, together with a system of beliefs by buyers, such that each player is optimising, and buyers' beliefs are consistent with the strategies and placement of sellers.

We start our analysis with consumers' search strategies. Suppose that the sellers placed on E are in the order of their relevance, namely that S_i takes the positions of E_i for $i = 1,2,3$. Suppose further that all sellers set their prices equal to p^o. Then, a consumer's expected return from searching E_i is

$$\gamma^{i-1}\beta \int_{p^o}^{\bar{v}} (v - p^o) f(v)\,\mathrm{d}v, \text{ for } i = 1, 2, 3,$$

and their expected return from searching any randomly selected seller not listed on E is

$$\gamma^3\beta \int_{p^o}^{\bar{v}} (v - p^o) f(v)\,\mathrm{d}v.$$

Since

$$t < \gamma^3\beta \int_{p^o}^{\bar{v}} (v - p^o) f(v)\,\mathrm{d}v < t^h,$$

from **A2**, it is optimal for each consumer to search sequentially, in the order of E_1, E_2, E_3, and then one randomly selected seller not listed on E. They stop searching either when they find their desired product or if they have conducted these four searches without finding their desired product. When the consumer finds that a seller's product matches their needs, they purchase the product if $v \geq p^o$; but does not purchase if $v < p^o$. Since their v is the same for the desired product from any seller, they will not conduct additional searches once their search has yielded a match. We therefore have the following.

Lemma 1. *Suppose that S_1, S_2, S_3 are placed on E in descending order and other sellers are not placed on the list. Suppose further that each seller's price is p^o. Then, it is optimal for each consumer to sequentially search E_1, E_2, E_3 and one randomly selected seller not listed on E. They stop searching either when they find their desired product, in which case they purchase if and only if $v \geq p^o$, or when they have conducted these four searches without finding their desired product.*

We next consider sellers' pricing strategies, given consumers' search and purchase behaviour described in Lemma 1. If a seller's product matches a consumer's needs, then the seller's price that maximises his expected profit from this consumer, without knowing the consumer's realised v, is p^o. As a consumer will purchase the seller's product if $v \geq p^o$, p^o must be the optimal price for the seller,[6] independent of whether the seller is listed on E or what his position on E is.

Therefore, given consumers' search and purchase behaviour described in Lemma 1, if S_1, S_2 and S_3 are placed at E_1, E_2 and E_3, the expected profits of S_i, excluding their payments to the search engine, are

$$\begin{aligned}
\pi_1 &= \beta\pi^o, \\
\pi_2 &= (1-\beta)\gamma\beta\pi^o = (1-\beta)\gamma\pi_1, \\
\pi_3 &= (1-\gamma\beta)(1-\beta)\gamma^2\beta\pi^o = (1-\gamma\beta)\gamma\pi_2, \\
\pi_k &= \frac{1}{m-3}(1-\gamma^2\beta)(1-\gamma\beta)(1-\beta)\gamma^3\beta\pi^o = \frac{1-\gamma^2\beta}{m-3}\gamma\pi_3, \quad \text{for } k = 4,\ldots,m.
\end{aligned} \tag{5}$$

Notice that in our model we can interpret the sellers not being placed on E as appearing possibly in the 'free' search results of the search engine. In equilibrium, when consumers search as in Lemma 1, a seller is visited by a consumer through free search only after the consumer has visited the sellers on E, and only with probability $1/(m-3)$, which approaches 0 as the number of sellers (m) becomes large. Therefore, adding the 'outside' option of being displayed as a 'free' search result by the search engine does not change our results in the context of our model. In reality, however, the issue of 'free' search result is more complicated. It is possible that the more relevant sellers are also displayed more prominently among the free search results, which makes them more likely to be visited by consumers even without paid-placement advertising. On the other hand, 'free' search for a keyword may display a lot of information that is not related to specific sellers, which reduces the chance that a seller is visited by consumers. Because of these opposing effects, in reality, the presence of a 'free' search option can potentially either increase or reduce a seller's expected payoff when not placed on E. We expect that the qualitative nature of our results remain valid with such considerations.[7]

We also notice that the analysis of bidding strategies here differs from the usual second price auction, since there are multiple positions to be auctioned and the values of E_1, E_2, E_3 and not winning the bid are endogenous for the bidders, depending on who will be placed at alternative positions. To determine how each seller will bid to be placed on E, we look for an equilibrium where $b_1 > b_2 > b_3 > b_k$ for $k = 4,\ldots,m$ and S_i ($i = 1, 2, 3$) bids the value of being placed at E_i. In such a possible equilibrium, given the placement rule and consumers' search behaviour, S_4's expected profit from not being placed on E's is π_4. If S_4 is placed at E_3 to replace S_3's position, his expected profit would be

$$(1-\beta)(1-\gamma\beta)\gamma^3\beta\pi_o = \gamma\pi_3.$$

[6] This is a familiar result in the search literature, following the seminal work of Diamond (1971). Our model captures the situation, where consumers' search for relevance dominates search for price. Our analysis does not depend crucially on each firm charging p^o. The qualitative nature of our results will be the same as long as firms' optimal price is constant or, as we show in Section 5, the distribution of firms' prices is on a sufficiently small interval relative to consumers' search costs.

[7] It is possible that a seller may initially benefit more from paid placement on E but over time, as it becomes well known by consumers, it has less need for paid-placement advertising. Our model abstracts away from such dynamic considerations.

Therefore, S_4 is willing to bid

$$\Delta_4 \equiv \gamma\pi_3 - \pi_4 = \gamma\pi_3 - (1-\gamma^2\beta)\frac{\gamma}{m-3}\pi_3 = \left(1 - \frac{1-\gamma^2\beta}{m-3}\right)\gamma\pi_3, \tag{6}$$

to be placed at E_3. On the other hand, to keep his current position, S_3 is willing to bid

$$\Delta_3 = \pi_3 - (1-\beta)(1-\gamma\beta)(1-\gamma^3\beta)\frac{\gamma^2\beta}{m-3}\pi^o = \left(1 - \frac{1-\gamma^3\beta}{m-3}\right)\pi_3. \tag{7}$$

We have

$$\Delta_3 - \Delta_4 = \left(1 - \frac{1-\gamma^3\beta}{m-3}\right)\pi_3 - \left(1 - \frac{1-\gamma^2\beta}{m-3}\right)\gamma\pi_3 = \frac{(1-\gamma)(m-4)}{(m-3)}\pi_3 \geq 0,$$

where the inequality holds strictly if $m > 4$. Thus, if S_3 bids Δ_3, the increase of his profit from not being on E to being at E_3, or the value of E_3 to him, is Δ_3. Taking the proposed equilibrium placement as given, S_3 outbids S_4 for E_3. The expected payoff for S_3 at this proposed equilibrium would be $\pi_3 - \Delta_4$.

For S_2, his expected payoff to be placed at E_3 would be $(1-\beta)(1-\gamma^2\beta)\gamma\beta\pi^o - \Delta_4$. To keep his position at E_2, S_2 is thus willing to bid

$$\begin{aligned}\Delta_2 &= \pi_2 - [(1-\beta)(1-\gamma^2\beta)\gamma\beta\pi^o - \Delta_4] \\ &= (1-\beta)\gamma\beta\pi^o - (1-\beta)(1-\gamma^2\beta)\gamma\beta\pi^o + \Delta_4 \\ &= (1-\beta)\gamma^3\beta^2\pi^o + \left(1 - \frac{1-\gamma^2\beta}{m-3}\right)\gamma\pi_3.\end{aligned} \tag{8}$$

For S_1, his expected payoff to be placed at E_2 would be $(1-\gamma\beta)\beta\pi^o - \Delta_3$. To keep his position at E_1,S_1 is willing to bid

$$\Delta_1 = \pi_1 - [(1-\gamma\beta)\beta\pi^o - \Delta_3] = \gamma\beta^2\pi^o + \left(1 - \frac{1-\gamma^3\beta}{m-3}\right)\pi_3. \tag{9}$$

Theorem 1 establishes that bidding Δ_i is indeed an equilibrium strategy for S_i, $i = 1, 2, 3, 4$.

Theorem 1. *Assume* $\beta \geq \max\{2-(1/\gamma), (1-\gamma)/(2-\gamma)\} \equiv \underline{\beta}(\gamma)$. *Then, there is an equilibrium in which seller* S_i *bids to pay the search engine*

$$\begin{aligned} b_1 &= \gamma\beta^2\pi^o + \left(1 - \frac{1-\gamma^3\beta}{m-3}\right)\pi_3, \\ b_2 &= (1-\beta)\gamma^3\beta^2\pi^o + \left(1 - \frac{1-\gamma^2\beta}{m-3}\right)\gamma\pi_3, \\ b_3 &= \left(1 - \frac{1-\gamma^3\beta}{m-3}\right)\pi_3, \\ b_k &= \left(1 - \frac{1-\gamma^2\beta}{m-3}\right)\gamma\pi_3, \quad k = 4, \ldots, m. \end{aligned} \tag{10}$$

S_1, S_2, S_3 are placed at E_1, E_2, E_3 and pay b_2, b_3 and b_4, respectively. Each seller's price is p^o, and each consumer searches and purchases as described in Lemma 1.

The proof for Theorem 1 is contained in the Appendix. Basically, one needs to show that, given the bids of other sellers, no seller can benefit by bidding differently from his equilibrium bid. This involves showing that S_k, $k = 4,\ldots,m$ would not want to bid sufficiently more to be placed at E_1, E_2 or E_3; that S_3 neither would want to bid sufficiently more to be placed at E_2 or E_1, nor would he want to lower his bid to be not placed on E; and similarly for S_2 and S_1. The additional parameter restriction, which is satisfied if $\beta \geq \max\left\{\frac{1}{2}, \gamma\right\}$, provides a sufficient but not necessary condition (when $m > 4$) for the existence of such an equilibrium. The intuition for this restriction is that if β is too small relative to γ, the sellers will become too similar in their relative relevance, which makes the condition that no seller will mimic the other seller's bidding strategy difficult to satisfy. While many retailers now offer a large assortment of products, it is common that they specialise in different product lines and/or different brands for a given product line so as to differentiate themselves from each other. For instance, even category killers, such as Home Depot and Lowes, offer many non-overlapping products. This suggests that given a particular keyword, the relevance of sellers can still be very different (i.e. the value of γ is smaller than β) to consumers despite the fact that these sellers all have a large variety of products.

It turns out that there are several possible perfect Bayesian equilibria depending on consumers' beliefs.

PROPOSITION 1. (*i*) *If consumers' belief is that sellers on E are in descending order of relevance and are more relevant than those not on E, then the separating equilibrium characterised in Theorem* 1 *is also the unique equilibrium of the model.* (*ii*) *If consumers' belief is that sellers on E are in random order of relevance but are more relevant than those not on E, then there is a partially separating equilibrium, where S_1, S_2, S_3 bid the same amount and are placed on E in random order.* (*iii*) *If consumers' belief is that sellers on E are in random order of relevance and are not more relevant than those not on E, then there is a pooling equilibrium where all sellers bid zero.*

The belief systems stated in Proposition 1 may not be exhaustive but they seem to be most natural. The pooling equilibrium, although theoretically possible, is apparently counterfactual and uninteresting. Search engines, such as Google, derive substantial revenues from paid-placement advertising, which contradicts the pooling equilibrium outcome. We henceforth focus on the separating and the partially separating equilibria, both of which appear plausible. While the separating equilibrium implies that consumers search sellers on E in descending order, at the partially separating equilibrium consumers search sellers on E randomly.[8] Indeed, anecdotal evidence suggests that some consumers search paid-placement advertisements in descending order while others search randomly. In the next Section, we investigate the profit implications of the two alternative equilibria for the search engine and their implications for consumer welfare and efficiency.

[8] An alternative explanation for the partially separating equilibrium is that consumers use some other features not present in the model, such as the summary description included with the URL, to make their selection.

3. Search Engine's Profit, Consumer Welfare and Efficiency

In the separating equilibrium, the search engine's profit is

$$\begin{aligned}\pi_E &= b_2 + b_3 + b_4\\ &= (1-\beta)\gamma^3\beta^2\pi^o + \left[\left(1-\frac{1-\gamma^2\beta}{m-3}\right)\gamma + \left(1-\frac{1-\gamma^3\beta}{m-3}\right) + \left(1-\frac{1-\gamma^2\beta}{m-3}\right)\gamma\right]\pi_3\\ &= (1-\beta)\gamma^3\beta^2\pi^o + \frac{(m-4)(1+2\gamma)+3\beta\gamma^3}{m-3}(1-\beta)(1-\gamma\beta)\gamma^2\beta\pi^o. \end{aligned} \tag{11}$$

Therefore, treating m as a continuous variable, we have:

$$\frac{\partial \pi_E}{\partial m} = (1+2\gamma-3\gamma^3\beta)(1-\beta)(1-\gamma\beta)\gamma^2\beta\frac{\pi^o}{(m-3)^2} > 0, \tag{12}$$

$$\begin{aligned}\lim_{m\to\infty}\frac{\partial \pi_E}{\partial \beta} &= (1-\beta)[\gamma\beta + (1+2\gamma)(1-\gamma\beta)]\gamma^2\beta\pi^o\\ &= [6\beta^2\gamma^2 - 2(2\gamma+2\gamma^2+1)\beta + 2\gamma + 1]\gamma^2\pi^o.\end{aligned}$$

Solving β from

$$6\beta^2\gamma^2 - 2(2\gamma+2\gamma^2+1)\beta + 2\gamma + 1 = 0,$$

we obtain

$$\hat{\beta}(\gamma) \equiv \frac{1}{6\gamma^2}\left(1+2\gamma+2\gamma^2 - \sqrt{4\gamma+2\gamma^2-4\gamma^3+4\gamma^4+1}\right), \tag{13}$$

which decreases in γ, with $\lim_{\gamma\to 0}\hat{\beta}(\gamma) = \frac{1}{2}$ and $\lim_{\gamma\to 1}\hat{\beta}(\gamma) = (5-\sqrt{7})/6$.

Thus, when m is large, π_E has an inverted U-shaped relationship with respect to β: it increases in β for $\beta < \hat{\beta}(\gamma)$ but decreases in β for $\beta > \hat{\beta}(\gamma)$.

Next, we consider the comparative static of π_E with respect to γ. Since under a higher γ sellers are more similar in terms of relevance, γ is a proxy for the extent of competition. We have:

$$\lim_{m\to\infty}\frac{\partial \pi_E}{\partial \gamma} = 2\pi^o\beta\gamma(1-\beta)(1+3\gamma-4\beta\gamma^2), \tag{14}$$

which is positive for $\beta, \gamma \in (0,1)$. That is, when m is large, the search engine's profits unambiguously increase with γ. Intuitively, as more sellers with similar product offerings compete for the limited slots of paid-placement advertisements, they have to bid more, and the search engine benefits from such competition.

In the partially separating equilibrium, the search engine's profit is

$$\begin{aligned}\pi_E^p &= 3\Delta\\ &= \left[\frac{\beta^2\gamma - 3\beta\gamma - 3\beta\gamma^2 - 3\beta + \beta^2\gamma^2 + \beta^2\gamma^3 + 9}{3} - \frac{3(1-\beta)(1-\gamma\beta)(1-\gamma^2\beta)}{m-3}\right]\gamma^3\beta\pi^o\end{aligned}$$

where Δ is the equilibrium bid of S_1, S_2 and S_3. Δ is defined in (A.1) and is derived in the proof of Proposition 1 in the Appendix. Treating m as a continuous variable, we have:

$$\frac{\partial \pi_E^p}{\partial m} = 3(1-\beta)(1-\beta\gamma^2)\frac{1-\beta\gamma}{(m-3)^2}\beta\gamma^3\pi^o > 0. \tag{15}$$

In contrast to π_E, when m is large, π_E^p is monotonically increasing in β, as can be seen from the following:

$$\lim_{m\to\infty}\frac{\partial \pi_E^p}{\partial \beta} = [3-\beta(2-\beta\gamma)(\gamma^2+\gamma+1)]\gamma^3\pi^o > 0. \tag{16}$$

Similar to π_E, when m is large, π_E^p is monotonically increasing in γ:

$$\lim_{m\to\infty}\frac{\partial \pi_E^p}{\partial \gamma} = \frac{1}{3}\pi^o\beta\gamma^2[\beta\gamma(4\beta-15\gamma+5\beta\gamma+6\beta\gamma^2-12)+27-9\beta], \tag{17}$$

which can be shown to be positive for $\beta, \gamma \in (0,1)$.

The search engine can be better off under either the separating equilibrium or the partially separating equilibrium. When the number of firms (m) is large enough, we can compare the profits of the search engine under the two equilibria as follows:

$$\lim_{m\to\infty}(\pi_E-\pi_E^p) = \frac{1}{3}[3(1-\beta-\gamma)-\beta^2\gamma^2(\gamma^2+\gamma-5)-3\beta\gamma(1+\gamma-\gamma^2)]\beta\gamma^2\pi^o,$$

which is positive for any given $\beta < 1$ if γ is sufficiently small but negative if γ is sufficiently large.

In other words, when m is large, the search engine's profit is higher under the separating equilibrium when γ is sufficiently small, or when firms differ substantially in relevance. Otherwise, the search engine can make more profit under the partially separating equilibrium. We summarise the above discussions with the following.

PROPOSITION 2. (*i*) *Under either the separating equilibrium or the partially separating equilibrium, the search engine's profit, π_E or π_E^p, is strictly increasing in the number of firms, m.* (*ii*) *Under the separating equilibrium, when m is large, π_E is increasing in the match probability β for $\beta \in (\underline{\beta}(\gamma), \hat{\beta}(\gamma)]$ but is decreasing in β for $\beta \in (\hat{\beta}(\gamma), 1)$.* (*iii*) *Under the partially separating equilibrium, π_E^p is monotonically increasing in β when m is large.* (*iv*) *$\pi_E \geq \pi_E^p$ when γ is small and m is large.*

The intuitions for Proposition 2 are as follows. For (*i*), as more sellers are present in the market, a seller is less likely to be selected randomly by a buyer and thus placement on the search engine's recommended list is more valuable. This motivates the sellers to bid more for placement, increasing the search engine's revenue. For (*ii*), notice that an increase in β has a positive effect on the value of being placed at E_1 but has two opposite effects on the value of being placed at E_2 and E_3: while it increases the probability of match when a consumer visits the seller's website, it also reduces the probability that the consumer will visit E_2 or E_3, since the consumer is more likely to purchase at E_1. The balance of these effects results in the search engine's revenue being first increasing and then decreasing in β. For (*iii*), under the partially separating equilibrium, S_1, S_2, S_3 all bid the same amount and the higher β is, the more each of them is willing to bid to be placed on E. Thus the search engine's profit is monotonically increasing in relevance under the partially separating equilibrium. Finally, for (*iv*), when γ is small, the value of S_i being placed on E decreases dramatically as β increases and, since under the partially separating equilibrium S_1, S_2 and S_3 pay the value of S_4 , not the next highest seller's value, the search engine's revenue is higher under the separating equilibrium.

We can also investigate consumer welfare and efficiency properties of the two equilibria. One way to evaluate the efficiency property of paid-placement advertising is to see how it impacts consumer search costs to achieve a given probability of finding the desired product. Under the separating equilibrium, for this probability to be β, $1-(1-\beta)(1-\gamma\beta)$ and $1-(1-\beta)(1-\gamma\beta)(1-\gamma^2\beta)$, a consumer needs to incur search costs in the amount of, respectively, t, $2t$ and $3t$. Under the partially separating equilibrium, the consumer also attains a probability of

$$1-(1-\beta)(1-\gamma\beta)(1-\gamma^2\beta),$$

by searching all the sellers listed on the search engine but attains lower probabilities of finding a match in the first two searches than under the separating equilibrium.

Without paid-placement advertising and for large m, the probability of a match from each search is approximately

$$\frac{1}{m}[1+\gamma+\gamma^2+(m-3)\gamma^3]\beta \approx \gamma^3\beta.$$

The probability of achieving a match from τ searches is approximately $1-(1-\gamma^3\beta)^\tau$.

Thus, to achieve any particular probability of match, the expected search time, or the expected search cost, is lower under paid-placement advertising.

Another way to evaluate the efficiency property of paid-placement advertising is to see how it impacts expected output. The expected output under paid-placement advertising (either the separating equilibrium or the partially separating equilibrium) is

$$q_h = [1-(1-\beta)(1-\gamma\beta)(1-\gamma^2\beta)(1-\gamma^3\beta)][1-F(p^o)], \tag{18}$$

whereas the expected output without paid-placement advertising is approximately

$$q_l = [1-(1-\gamma^3\beta)^4][1-F(p^o)] < q_h. \tag{19}$$

We therefore have the following.

Remark 1. *Paid-placement advertising leads to more efficient consumer search and to higher total output. The separating equilibrium generally yields higher welfare gain than the partially separating equilibrium.*

Our analysis sheds light on the search engine's strategies. Specifically, the search engine should recruit as many firms as possible to participate in the paid-placement mechanism. In addition, if the market supports the fully separating equilibrium, then it may be optimal for the search engine to restrict the precision of keywords such that the resulting match probability for the most relevant seller is not too high.

Our stylised model abstracts away from other important considerations in the paid-placement mechanism. For example, Google determines advertisement placement using a quality measure of the sellers (click-through rate) in addition to bids. The fact that the search engine may also consider other factors, such as the sellers' quality of service and/or prices in determining the sellers' placement, is likely to reinforce our main conclusion that paid-placement advertising provides useful information to consumers that facilitates consumer search and improves efficiency. Consideration of these other factors may also support our finding that the search engine's revenue may not always be monotonically increasing in sellers' relevance.

4. Extension: Heterogeneous Costs

As is well known in the literature (Diamond, 1971), with costly consumer search a market with many firms may nevertheless sustain a single monopoly price. Consistent with this tradition, our main model has the property that in equilibrium all sellers charge the same price, p^o. However, we do observe price dispersion in practice. As such, it is useful to demonstrate the robustness of our results in the presence of price dispersion.

We now consider a simple modification to our model. Instead of assuming the same cost for all sellers, assume that sellers may have different costs. More specifically, we assume that each seller's constant marginal cost c_i is the realisation of a random variable distributed on $[\underline{c}, \bar{c}]$, with cdf and pdf $G(\cdot)$ and $g(\cdot)$, respectively; and each seller learns its cost realisation after bidding for E.[9]

For any $c_i \in [\underline{c}, \bar{c}]$, let

$$p^o(c_i) = \arg\max_{p \in [c_i, \bar{v}]} (p - c_i)[1 - F(p)], \tag{20}$$

$$\tilde{\pi}^o = \int_{\underline{c}}^{\bar{c}} [p^o(c) - c]\{1 - F[p^o(c)]\} \, \mathrm{d}G(c), \tag{21}$$

where with a slight abuse of notation, we have used c to also denote the random unit cost of any seller. Then, if seller i is listed as E_1, its expected profit is $\beta_i \tilde{\pi}^o$, provided that consumers first visit E_1.

We modify assumption **A2** to assume

$$\mathbf{A2'}.\ \gamma\beta \int_{\underline{c}}^{\bar{c}} [p^o(\bar{c}) - p^o(c)] g(c) \, \mathrm{d}c < t < \gamma^3 \beta \int_{\underline{c}}^{\bar{c}} \left\{ \int_{p^o(c)}^{\bar{v}} [v - p^o(c)] f(v) \, \mathrm{d}v \right\} g(c) \, \mathrm{d}c < t^h.$$

A2′ requires that the cost dispersion is not too large, so that in equilibrium a consumer stops searching once they find their desired product and they do not search more than four times. Notice that **A2′** becomes **A2** when $[\underline{c}, \bar{c}]$ converges to a constant c.

[9] This way, bidding by sellers does not signal sellers' costs (prices), allowing us to focus on the role of paid-placement advertising in revealing product relevance. We are not aware of evidence suggesting that sellers with paid-placement advertising have systematically higher or lower costs (prices) and, under our formulation, all sellers have the same expected price in equilibrium.

First, suppose that S_1, S_2, S_3 are placed at E_1, E_2, E_3, respectively, and S_k are not placed on E's list, $k = 4,\ldots,m$. Suppose further that S_i prices at $p^o(c_i)$. Then, it is optimal for consumers to search sequentially, in the order of E_1, E_2, E_3, and then randomly choose non-listed sellers. If a consumer finds their desired product at a particular seller, their expected return from having another search cannot exceed

$$\gamma\beta\int_{\underline{c}}^{\bar{c}}[p^o(\bar{c}) - p^o(c)]g(c)\,\mathrm{d}c,$$

which is less than t by assumption. On the other hand, conditional on having not found a match, a consumer's expected return from searching a non-listed seller is

$$\gamma^3\beta\int_{\underline{c}}^{\bar{c}}\left\{\int_{p^o(c)}^{\bar{v}}[v - p^o(c)]f(v)\,\mathrm{d}v\right\}g(c)\,\mathrm{d}c,$$

which is larger than t but less than t^h by assumption. Therefore, given the suggested placement of sellers and their prices, it is optimal for each consumer to search sequentially at most four sellers, in the order of E_1, E_2, E_3, and a randomly chosen non-listed seller; they stop searching either when they find a match or when they have searched four times; and they make a purchase if they find a match and their v is at or above the seller's price.

Next, given the search and purchase behaviour of consumers, it is optimal for S_i to set $p^o(c_i)$. Hence, at the time of bidding for placement, the expected profit of S_i from any consumer who visits S_i is simply $\tilde{\pi}^o$.

Finally, to establish the equilibrium, we need to show that each seller bids optimally and the bidding by the sellers indeed results in the proposed order of placement under the second price auction. At the proposed equilibrium, the expected profits of sellers, excluding their bidding payments, are

$$\begin{aligned}
\tilde{\pi}_1 &= \beta\tilde{\pi}^o,\\
\tilde{\pi}_2 &= (1-\beta)\gamma\beta\tilde{\pi}^o = (1-\beta)\gamma\tilde{\pi}_1,\\
\tilde{\pi}_3 &= (1-\gamma\beta)(1-\beta)\gamma^2\beta\tilde{\pi}^o = (1-\gamma\beta)\gamma\tilde{\pi}_2,\\
\tilde{\pi}_k &= \frac{1}{m-3}(1-\gamma^2\beta)(1-\gamma\beta)(1-\beta)\gamma^3\beta\tilde{\pi}^o = \frac{1-\gamma^2\beta}{m-3}\gamma\tilde{\pi}_3, \quad \text{for } k = 4,\ldots,m.
\end{aligned} \tag{22}$$

If S_4 is placed at E_3, his expected profit would be $(1-\beta)(1-\gamma\beta)\gamma^3\beta\tilde{\pi}^o$. Thus, S_4 is willing to bid

$$\tilde{\Delta}_4 = (1-\beta)(1-\gamma\beta)\gamma^3\beta\tilde{\pi}^o - \frac{(1-\beta)(1-\gamma\beta)(1-\gamma^2\beta)}{m-3}\gamma^3\beta\tilde{\pi}^o = \left(1 - \frac{1-\beta\gamma^2}{m-3}\right)\gamma\tilde{\pi}_3,$$

to be placed at E_3. On the other hand, to keep his position at E_3, S_3 is willing to bid

$$\tilde{\Delta}_3 = \tilde{\pi}_3 - (1-\beta)(1-\gamma\beta)(1-\gamma^3\beta)\frac{\gamma^2\beta}{m-3}\tilde{\pi}^o = \left(1 - \frac{1-\gamma^3\beta}{m-3}\right)\tilde{\pi}_3.$$

Similarly, as in our earlier analysis where all sellers have the same constant marginal cost, to keep their positions at E_2 and E_1, S_2 and S_1 are willing to bid, respectively,

$$\tilde{\Delta}_2 = (1-\beta)\gamma^3\beta^2\tilde{\pi}^o + \left(1 - \frac{1-\gamma^2\beta}{m-3}\right)\gamma\tilde{\pi}_3,$$

$$\tilde{\Delta}_1 = \gamma\beta^2\tilde{\pi}^o + \left(1 - \frac{1-\gamma^3\beta}{m-3}\right)\tilde{\pi}_3.$$

Therefore, analogous to Theorem 1, we have the following.

PROPOSITION 3. *Assume that* $\beta \geq \max\{2-(1/\gamma),\ (1-\gamma)/(2-\gamma)\}$. *Then, the game with heterogeneous seller costs has an equilibrium, where* S_i *bids*

$$\begin{aligned} b_1 &= \gamma\beta^2\tilde{\pi}^o + \left(1 - \frac{1-\gamma^3\beta}{m-3}\right)\tilde{\pi}_3, \quad b_2 = (1-\beta)\gamma^3\beta^2\tilde{\pi}^o + \left(1 - \frac{1-\gamma^2\beta}{m-3}\right)\gamma\tilde{\pi}_3, \\ b_3 &= \left(1 - \frac{1-\gamma^3\beta}{m-3}\right)\tilde{\pi}_3, \quad b_k = \left(1 - \frac{1-\gamma^2\beta}{m-3}\right)\gamma\tilde{\pi}_3, \quad k = 4, \ldots, m. \end{aligned} \tag{23}$$

S_1, S_2, S_3 are placed at E_1, E_2, E_3 and pay b_2, b_3 and b_4, respectively. S_i charges price $p^o(c_i)$. Each consumer searches sequentially, in the order of E_1, E_2, E_3 and then a randomly selected non-listed seller; stops searching when they find a match or have searched four sellers; and purchases if the price of the product that matches their needs does not exceed their valuation for the product.

The proof of Proposition 3 is the same as the proof of Theorem 1 in the Appendix, except replacing π^o and π_3 there by $\tilde{\pi}^o$ and $\tilde{\pi}_3$. Notice that in equilibrium sellers tend to have different prices, depending on the realisation of their costs, and the expected price of each seller is

$$\tilde{p} = \int_{\underline{c}}^{\bar{c}} p^o(c)\,\mathrm{d}G(c).$$

In the literature, price dispersion is often generated in models with mixed strategies, where some consumers purchase only from particular sellers (due to loyalty or imperfect information), whereas other consumers purchase only from the seller with the lowest prices (Rosenthal, 1980; Varian, 1980; Stahl, 1989; Baye and Morgan, 2001; Chen and Zhang, forthcoming). An important exception is Reinganum (1979), where a price distribution is generated by a set of firms with different marginal costs choosing pure strategies.[10] Our modified model here has followed the approach of Reinganum in considering possibly different marginal costs for different firms. Unlike her model, where in equilibrium each consumer only searches once, consumers engage in sequential search here because firms sell differentiated products and each consumer searches for the variety matching their preference.

5. Conclusion

One of the great promises of the Internet is its efficiency in disseminating information. More information, however, can be a mixed blessing for consumers, as evidenced by, for

[10] Also see Arbatskaya (2007) for a model where consumers have different search costs and search firms sequentially in a predetermined order, with price dispersion in equilibrium.

instance, the intrusion of junk e-mails to our lives. For the Internet to be a beneficial medium, therefore, the information it delivers should go to consumers who exhibit such information needs. More specifically, efficiency requires that consumers who search for information receive information from the most relevant sources. Indeed, the ability to deliver relevant information to consumers who search for information is unique to the Internet. Such characteristics may also exist in other media but are far more costly.

Paid-placement advertising, where a search engine acts as an intermediary between firms and consumers, facilitates the transmission of information from firms to consumers and has enjoyed phenomenal commercial success. This article has developed a market equilibrium model that uncovers the economic forces behind the success of this important Internet institution. When consumers must engage in costly search to find their desired product variety, they face the issue of how to search various sellers who carry different product varieties. Depending on consumers' beliefs, advertising through paid placement enables sellers to reveal either full or partial information about their product relevance to consumers. A seller with a more relevant product expects a higher probability of a sale from the visit of a consumer to the seller's website and hence a higher expected profit from attracting such a consumer; this motivates the seller to bid more and to receive a higher advertisement placement position. Moreover, since consumers do not learn a seller's price until visiting the seller's website, in equilibrium the expected price from each seller is the same. Therefore, it is optimal for consumers to search sellers sequentially, according to their placement on the search engine's list. In the case of partial information revelation, the most relevant sellers bid the same amount, just enough to gain a slot on the search engine's list, in which case it is also optimal for consumers to search the listed sellers randomly. In equilibrium, paid-placement advertising leads to more efficient search by consumers and to higher total output. Our analysis also sheds light on the search engine's strategies. We show that it is more profitable for the search engine to list the most relevant sellers in descending order when sellers differ substantially in relevance, otherwise the search engine can be better off to place the most relevant sellers randomly. In addition, we demonstrate that there can be an inverted U-shaped relationship between the search engine's profit and relevance, implying that the search engine's profit may be maximised when the keyword relevance is set at some intermediate level.

We have also considered an extension to the model. When sellers *ex post* have different marginal costs, there is price dispersion in the market, even though each seller still sets a monopoly price based on his realised marginal cost. The fact that the next seller may not offer the same product match that a consumer desires diminishes their expected return from searching further for a lower price. Consequently, under the assumption that the cost (price) dispersion is relatively small, a consumer will not search further once they have found their desired product. A basic result of our analysis, that paid-placement advertising enables sellers to reveal their product relevance through bidding and leads to efficient consumer search, continues to hold.

There are several directions for future research. One possibility is to allow competition among search engines, which could affect the bidding incentives of sellers and could address issues such as whether competition will lead to the adoption of efficient information dissemination mechanisms. For tractability, we have used a specific functional form of the distribution of relevance among sellers and assumed that products either match or have no value; it would be desirable to extend our model to allow more general forms of relevance and more general distributions of consumer values for different products. Furthermore, it would be interesting to empirically evaluate the assumptions and implications of our analysis. For instance, our analysis predicts that sellers placed higher by a search engine for a keyword search will have higher expected sales for the product, which is empirically testable.

Appendix

Proof of Theorem 1

Given the placement of S_1, S_2, S_3 at E_1, E_2, E_3 and each seller's price p^o, each consumer's search and purchase behaviour is optimal. Given consumer behaviour, each seller's price is optimal. Thus, our proof will be complete if it is shown that no seller can benefit from bidding differently. Since it is a second price auction, we need only be concerned with deviations by S_i that would change the placement of S_i. Let π_i^j be seller i's payoff at position E_j, including i's bidding payment.

First consider S_4 (or any S_i for $i \geq 4$). To be placed at E_3, S_4 needs to bid at least b_3; his expected payoff at E_3, after paying b_3, is

$$\pi_4^3 = (1-\beta)(1-\gamma\beta)\gamma^3\beta\pi^o - \left(1 - \frac{1-\gamma^3\beta}{m-3}\right)\pi_3 = \gamma\pi_3 - \left(1 - \frac{1-\gamma^3\beta}{m-3}\right)\pi_3.$$

On the other hand, the expected profit of S_4 from not being on E is

$$\pi_4 = \frac{1-\gamma^2\beta}{m-3}\gamma\pi_3.$$

We have

$$\pi_4^3 - \pi_4 = \gamma\pi_3 - \left(1 - \frac{1-\gamma^3\beta}{m-3}\right)\pi_3 - \frac{1-\gamma^2\beta}{m-3}\gamma\pi_3 = -\frac{(1-\gamma)(m-4)}{(m-3)}\pi_3 \leq 0,$$

where the inequality holds strictly if $m > 4$. Hence, S_4 has no incentive to switch positions with S_3. Similarly, we have

$$\begin{aligned}
\pi_4^2 - \pi_4 &= (1-\beta)\gamma^3\beta\pi^o - \left[(1-\beta)\gamma^3\beta^2\pi^o + \left(1 - \frac{1-\gamma^2\beta}{m-3}\right)\gamma\pi_3\right] - \frac{1-\gamma^2\beta}{m-3}\gamma\pi_3 \\
&= (1-\beta)^2\gamma^3\beta\pi^o - \gamma\pi_3 = (1-\beta)^2\gamma^3\beta\pi^o - \gamma(1-\gamma\beta)(1-\beta)\gamma^2\beta\pi^o \\
&= -(1-\beta)(1-\gamma)\gamma^3\beta^2\pi^o < 0,
\end{aligned}$$

and

$$\begin{aligned}
\pi_4^1 - \pi_4 &= \gamma^3\beta\pi^o - \left[\gamma\beta^2\pi^o + \left(1 - \frac{1-\gamma^3\beta}{m-3}\right)\pi_3\right] - \frac{1-\gamma^2\beta}{m-3}\gamma\pi_3 \\
&= \gamma\beta\pi^o\left[(\gamma^2-\beta) - (1-\gamma\beta)(1-\beta) + \frac{1-\gamma}{m-3}(1-\gamma\beta)(1-\beta)\gamma\right] \\
&= \gamma\beta\pi^o\left\{-\gamma(1-\gamma)\left[1 - \frac{(1-\gamma\beta)(1-\beta)}{m-3}\right] + \beta[\gamma + \gamma^2(1-\beta) - 1]\right\} < 0,
\end{aligned}$$

if $1 \geq \gamma[1 + \gamma(1-\beta)]$, which holds if $\beta \geq \max\{2 - (1/\gamma), (1-\gamma)/(2-\gamma)\}$. It follows that S_4 has no incentive to switch positions with S_2 or S_1. Thus, S_4 cannot benefit from any deviation.

Next, consider S_3. If S_3 switches positions with S_4, its payoff would be π_3^4; while its payoff at E_3, after paying E, is $\pi_3 - b_4$. We have:

$$\pi_3^4 - (\pi_3 - b_4) = \frac{1}{m-3}(1-\gamma^3\beta)(1-\gamma\beta)(1-\beta)\gamma^2\beta\pi^o - \left[\pi_3 - \left(1 - \frac{1-\gamma^2\beta}{m-3}\right)\gamma\pi_3\right]$$
$$= -\frac{1}{m-3}(m-4)(1-\beta\gamma)(1-\gamma)(1-\beta)(\pi^o)\beta\gamma^2 < 0.$$

Hence, S_3 has no incentive to switch position with S_4. Similarly, we have

$$\pi_3^2 - (\pi_3 - b_4) = \left[(1-\beta)\gamma^2\beta\pi^o - (1-\beta)\gamma^3\beta^2\pi^o - \left(1 - \frac{1-\gamma^2\beta}{m-3}\right)\gamma\pi_3\right]$$
$$- \left[\pi_3 - \left(1 - \frac{1-\gamma^2\beta}{m-3}\right)\gamma\pi_3\right]$$
$$= (1-\beta)\gamma^2\beta\pi^o - (1-\beta)\gamma^3\beta^2\pi^o - (1-\gamma\beta)(1-\beta)\gamma^2\beta\pi^o = 0,$$

and

$$\pi_3^1 - (\pi_3 - b_4) = \gamma^2\beta\pi^o - \gamma\beta^2\pi^o - \left(1 - \frac{1-\gamma^3\beta}{m-3}\right)\pi_3 - \left[\pi_3 - \left(1 - \frac{1-\gamma^2\beta}{m-3}\right)\gamma\pi_3\right]$$
$$= \gamma^2\beta\pi^o - \gamma\beta^2\pi^o - (2-\gamma)(1-\gamma\beta)(1-\beta)\gamma^2\beta\pi^o + \frac{1-\gamma}{m-3}(1-\gamma\beta)(1-\beta)\gamma^2\beta\pi^o$$
$$\leq \gamma^2\beta\pi^o - \gamma\beta^2\pi^o - (2-\gamma)(1-\gamma\beta)(1-\beta)\gamma^2\beta\pi^o + (1-\gamma)(1-\gamma\beta)(1-\beta)\gamma^2\beta\pi^o$$
$$= -\gamma\beta^2\pi^o(\beta\gamma^2 - \gamma^2 - \gamma + 1) \leq 0,$$

if $1 \geq \gamma[1 + \gamma(1-\beta)]$. It follows that S_3 has no incentive to switch positions with S_2 or S_1. Thus, S_3 cannot benefit from any deviation.

Next, consider S_2. If S_2 switches positions with S_4, its payoff would be π_2^4; while its payoff at E_2, after paying for E, is $\pi_2 - b_3$. We have:

$$\pi_2^4 - (\pi_2 - b_3) = \frac{1}{m-3}(1-\gamma^3\beta)(1-\gamma^2\beta)(1-\beta)\gamma\beta\pi^o$$
$$- \left[(1-\beta)\gamma\beta\pi^o - \left(1 - \frac{1-\gamma^3\beta}{m-3}\right)(1-\gamma\beta)(1-\beta)\gamma^2\beta\pi^o\right]$$
$$= \frac{-1}{m-3}[(m-4)(1-\gamma) + (m-3)\beta\gamma^2 + \beta\gamma^3(1-\gamma)](1-\beta)\beta\gamma\pi^o < 0.$$

Hence, S_2 has no incentive to switch positions with S_4. Similarly, we have

$$\pi_2^3 - (\pi_2 - b_3) = (1-\gamma^2\beta)(1-\beta)\gamma\beta\pi^o - (1-\beta)\gamma\beta\pi^o + \frac{m-4}{m-3}(1-\gamma)(1-\gamma\beta)(1-\beta)\gamma^2\beta\pi^o$$
$$< (1-\gamma^2\beta)(1-\beta)\gamma\beta\pi^o - (1-\beta)\gamma\beta\pi^o + (1-\gamma)(1-\gamma\beta)(1-\beta)\gamma^2\beta\pi^o$$
$$= \gamma^2\beta(1-\beta)(\beta\gamma^2 - 2\beta\gamma - \gamma + 1)\pi^o \leq 0,$$

if $\gamma[1 + \beta(2-\gamma)] \geq 1$, which holds if $\beta \geq \max\{2 - (1/\gamma), (1-\gamma)/(2-\gamma)\}$. Furthermore,

$$\pi_2^1 - (\pi_2 - b_3) = \gamma\beta\pi^o - \gamma\beta^2\pi^o - \left(1 - \frac{1-\gamma^3\beta}{m-3}\right)\pi_3 - \left[(1-\beta)\gamma\beta\pi^o - \left(1 - \frac{1-\gamma^3\beta}{m-3}\right)\pi_3\right]$$
$$= \gamma\beta\pi^o - \gamma\beta^2\pi^o - (1-\beta)\gamma\beta\pi^o = 0.$$

It follows that S_2 has no incentive to switch positions with S_3 or S_1. Thus, S_2 cannot benefit from any deviation.

Finally, consider S_1. If S_1 switches positions with S_4, its payoff would be π_1^4; while its payoff at E_1, after paying for E, is $\pi_1 - b_2$. We have:

$$\begin{aligned}
\pi_1^4 - (\pi_1 - b_2) &= \beta(\pi^o)(\gamma^3 - \beta\gamma^4 - \beta^2\gamma^3 + \beta^2\gamma^4 - 1) + \frac{(1-\beta\gamma^2)(1-\beta\gamma)(\gamma+\gamma^2+1)(1-\gamma)\pi^o\beta}{m-3} \\
&\le \beta(\pi^o)(\gamma^3 - \beta\gamma^4 - \beta^2\gamma^3 + \beta^2\gamma^4 - 1) + (\beta\gamma^2 - 1)(\beta\gamma - 1)(\gamma+\gamma^2+1)(1-\gamma)\pi^o\beta \\
&= -\gamma\beta^2\pi^o(\gamma - \gamma^4 + 1 - \beta\gamma^3 + \beta\gamma^5) < 0.
\end{aligned}$$

Hence, S_1 has no incentive to switch position with S_4. Similarly, we have

$$\begin{aligned}
\pi_1^3 - (\pi_1 - b_2) &= (1-\gamma^2\beta)(1-\gamma\beta)\beta\pi^o - b_4 - (\pi_1 - b_2) \\
&= (1-\gamma^2\beta)(1-\gamma\beta)\beta\pi^o - \left(1 - \frac{1-\gamma^2\beta}{m-3}\right)\gamma\pi_3 - \beta\pi^o \\
&\quad + (1-\beta)\gamma^3\beta^2\pi^o + \left(1 - \frac{1-\gamma^2\beta}{m-3}\right)\gamma\pi_3 \\
&= -\gamma\beta^2(\pi^o)(1+\gamma-\gamma^2) < 0,
\end{aligned}$$

$$\begin{aligned}
\pi_1^2 - (\pi_1 - b_2) &= (1-\gamma\beta)\beta\pi^o - \beta\pi^o + (1-\beta)\gamma^3\beta^2\pi^o - \left(1 - \frac{1-\gamma^3\beta}{m-3}\right)\pi_3 + \left(1 - \frac{1-\gamma^2\beta}{m-3}\right)\gamma\pi_3 \\
&= -\gamma\beta^2(\pi^o)(\beta\gamma^2 - \gamma^2 + 1) - \frac{m-4}{m-3}(1-\gamma)(1-\gamma\beta)(1-\beta)\gamma^2\beta\pi^o < 0.
\end{aligned}$$

It follows that S_1 has no incentive to switch positions with S_3 or S_2. Thus, S_1 cannot benefit from any deviation.

In sum, none of the sellers can benefit from any deviation when $\beta \ge \max\{2 - (1/\gamma), (1-\gamma)/(2-\gamma)\}$.

Proof of Proposition 1

We show that the three forms of consumer beliefs are consistent with the specified strategies and placement of sellers, thereby constituting the corresponding PBEs.

(1) Consumers believe that sellers are placed on E in descending order of relevance and search E sequentially in that order. There is a unique PBE, which is the separating equilibrium.
From the proof of Theorem 1, the equilibrium bids of the sellers as stated in Theorem 1, the sellers' placements and the consumers' belief and search behaviour, constitute a PBE. We next show that the equilibrium is unique under the stated consumer belief. First, there can be no other separating equilibrium in which S_1, S_2, S_3 are placed on E but not in the order of E_1, E_2, E_3. Suppose to the contrary that there is such an equilibrium. Then, if a less relevant seller, say S_3, bids more and is placed at a higher position on E, consumers would optimally search the lower placed but more relevant seller(s) before S_3 at such an equilibrium (because consumers have correct beliefs in equilibrium), which means that S_3 could benefit by lowering its bid and its placement position on E, contradicting the equilibrium assumption. If, on the other hand, all three sellers bid the same amount and are placed on E in random order, consumers would have the same expected payoff from any order of search on E. But if in this case consumers will search in the order of E_1, E_2, E_3, any of the S_i, $i = 1, 2, 3$, will have the incentive to deviate by bidding a little more in order to be placed at the top, again contradicting the equilibrium assumption. Next, there can be no equilibrium in which some S_k with $k > 3$, say S_4, is placed on E and S_4 bids differently from the other two sellers placed on E. Suppose to the contrary that there is such an equilibrium. Then, S_4 must bid at least as high as the highest bidder not listed on E. But at such an equilibrium buyers would search randomly from the sellers not on E , before searching S_4, as the expected match probability from

sellers not listed on E would be higher than that of S_4. This implies that S_4 would benefit from a deviation that lowers his bid (or would refrain from bidding) so that he will be placed on E, contradicting the equilibrium assumption. Finally, it is straightforward to show that there also can be no equilibrium in which some S_k with $k > 3$, say S_4, is placed on E and S_4 bids the same amount as at least one other seller placed on E.

(2) Consumers believe that sellers placed on E are in random order of relevance but are more relevant than those who are not placed on E. Consumers first search E randomly and there is a partially separating equilibrium.
We need to show that S_1, S_2, S_3 outbid other sellers and are placed on E in random order. Such bidding strategies and placement of sellers are consistent with consumers' belief and their corresponding search behaviour.

If S_I, $I = 4,\ldots,m$, is placed on E to randomly replace an S_i's position, his expected profit would be

$$b_I = \frac{1}{3}\left\{\begin{matrix} 1+\frac{1}{3}[(1-\beta)+(1-\gamma\beta)+(1-\gamma^2\beta)]+ \\ \frac{1}{3}[(1-\beta)(1-\gamma\beta)+(1-\beta)(1-\gamma^2\beta)+(1-\gamma\beta)(1-\gamma^2\beta)] \end{matrix}\right\}\gamma^3\beta\pi^o.$$

Therefore, S_I is willing to bid

$$\begin{aligned}\Delta &\equiv b_I - \pi_4 \\ &= b_I - \frac{1}{m-3}(1-\beta)(1-\gamma\beta)(1-\gamma^2\beta)\gamma^3\beta\pi^o = \gamma^3\beta\pi^o\cdot \\ &\left(\begin{matrix} \frac{1}{3}\left\{\begin{matrix} 1+\frac{1}{3}[(1-\beta)+(1-\gamma\beta)+(1-\gamma^2\beta)]+ \\ \frac{1}{3}[(1-\beta)(1-\gamma\beta)+(1-\beta)(1-\gamma^2\beta)+(1-\gamma\beta)(1-\gamma^2\beta)] \end{matrix}\right\} \\ -\frac{1}{m-3}(1-\beta)(1-\gamma\beta)(1-\gamma^2\beta) \end{matrix}\right)\end{aligned} \tag{A.1}$$

to be placed on E. On the other hand, to keep his current position, S_3 is willing to bid

$$\begin{aligned}\bar{\Delta} \equiv& \frac{1}{3}\left\{1+\frac{1}{2}[(1-\beta)+(1-\gamma\beta)]+(1-\beta)(1-\gamma\beta)\right\}\gamma^2\beta\pi^o \\ &-(1-\beta)(1-\gamma\beta)(1-\gamma^3\beta)\frac{\gamma^2\beta}{m-3}\pi^o \\ =&\left(\begin{matrix}\frac{1}{3}\left\{1+\frac{1}{2}[(1-\beta)+(1-\gamma\beta)]+(1-\beta)(1-\gamma\beta)\right\} \\ -\frac{1}{m-3}(1-\beta)(1-\gamma\beta)(1-\gamma^3\beta)\end{matrix}\right)\gamma^2\beta\pi^o.\end{aligned}$$

We have

$$\begin{aligned}\bar{\Delta}-\Delta &= \left[\begin{matrix}\frac{1}{6}(2\beta^2\gamma^3+4\beta^2\gamma^2+6\beta^2\gamma-6\beta\gamma^2-12\beta\gamma-9\beta+18) \\ -\frac{1}{m-3}(1-\beta)(1-\gamma\beta)\end{matrix}\right](1-\gamma)\gamma^2\beta\pi^o \\ &\equiv \Psi(1-\gamma)\gamma^2\beta\pi^o > 0,\end{aligned}$$

where we notice that $\bar{\Delta}-\Delta$ is minimised when $m = 4$, in which case

$$\begin{aligned}\Psi &= \frac{1}{6}(2\beta^2\gamma^3+4\beta^2\gamma^2+6\beta^2\gamma-6\beta\gamma^2-12\beta\gamma-9\beta+18)-(1-\beta)(1-\gamma\beta) \\ &= \frac{1}{3}\beta^2\gamma^3+\frac{2}{3}\beta^2\gamma^2-\beta\gamma^2-\beta\gamma-\frac{1}{2}\beta+2>0.\end{aligned}$$

Thus, if S_3 bids $\bar{\Delta}$, the increase of his profit from not being on E to being on E, or the value of being on E to him, is $\bar{\Delta}$. Taking the proposed equilibrium placement as given, S_3 outbids S_I for being placed on E. The expected payoff for S_3 at this proposed equilibrium will be

$$\frac{1}{3}\left\{1+\frac{1}{2}[(1-\beta)+(1-\gamma\beta)]+(1-\beta)(1-\gamma\beta)\right\}\gamma^2\beta\pi^o-\Delta.$$

Similarly, S_1 and S_2 also outbid S_I.

(3) Consumers believe that sellers placed on E are in random order of relevance and are not more relevant than those who are not placed on E. Consumers search randomly, and there is a pooling PBE.

In this case, all sellers bid zero to be placed on E and are randomly chosen to be placed on E. Consumers' belief is thus consistent with the placement of sellers and their search behaviour is optimal. Given consumers' search strategy, the bidding strategy of sellers is also optimal. The proposed is thus a pooling equilibrium.

University of Colorado at Boulder
Leeds School of Business, University of Colorado at Boulder

References

Anderson, S. and Renault, R. (2006). 'Advertising content', *American Economic Review*, vol. 96(1), pp. 96–113.
Arbatskaya, M. (2007). 'Ordered search', *RAND Journal of Economics*, vol. 38(1), pp. 119–26.
Armstrong, M., Vickers, J. and Zhou, J. (2009). 'Prominence and consumer search', *RAND Journal of Economics*, vol. 40(2), pp. 209–33.
Armstrong, M. and Zhou, J. (2011). 'Paying for prominence', Economic Journal, vol. 121(556), pp. F368–95.
Athey, S. and Ellison, G. (forthcoming). 'Position auctions with consumer search', *Quarterly Journal of Economics*.
Bagwell, K. and Ramey, G. (1994). 'Coordination economies, advertising, and search behavior in retail markets', *American Economic Review*, vol. 84(3), pp. 498–517.
Baye, M. and Morgan, J. (2001). 'Information gatekeepers on the Internet and the competitiveness of homogeneous product markers', *American Economic Review*, vol. 91(3), pp. 454–74.
Chen, Y. (2000). 'Strategic bidding by potential competitors: will monopoly persist?' *Journal of Industrial Economics*, vol. 48(2), pp. 161–75.
Chen, Y. and He, C. (2006). 'Paid placement advertising and search on the Internet', Net Institute Working Paper No. 06-02.
Chen, Y. and Zhang, T. (forthcoming) 'Equilibrium price dispersion with heterogeneous searchers', *International Journal of Industrial Organization*.
Coy, P. (2006). 'The secret to Google's success: its innovative auction system has ad revenues soaring', *Business Week*, March 6.
Diamond, P.A. (1971). 'A model of price adjustment', *Journal of Economic Theory*, vol. 3(2), pp. 156–68.
Edelman, B., Ostrovsky, M. and Schwarz, M. (2007). 'Internet advertising and the generalized second price auction: selling billions of dollars worth of keywords', *American Economic Review*, vol. 97(1), pp. 242–59.
Eliaz, K. and Spiegler, R. (2011) 'A simple model of search engine pricing', Economic Journal, vol. 121(556), pp. F329–39.
Grossman, G.M. and Shapiro, C. (1984). 'Informative advertising with differentiated products', *Review of Economic Studies*, vol. 51(1), pp. 63–81.
Krishna, K. (1993). 'Auctions with endogenous valuations: the persistence of monopoly revisited', *American Economic Review*, vol. 83(1), pp. 147–60.
Lewis, T. (1983). 'Preemption, divestiture, and forward contracting in a market dominated by a single firm', *American Economic Review*, vol. 73(5), pp. 1092–101.
Meurer, M. and Stahl, D.O. (1994). 'Informative advertising and product match', *International Journal of Industrial Organization* vol. 12(1), pp. 1–19.
Nelson, P. (1974). 'Advertising as information', *Journal of Political Economy*, vol. 82(4), pp. 729–54.
Reinganum, J.F. (1979). 'A simple model of equilibrium price dispersion', *Journal of Political Economy*, vol. 87(4), pp. 851–8.
Rhodes, A. (2011). 'Can prominence matter even in an almost frictionless market?', Economic Journal, vol. 121(556), pp. F297–F308.
Rosenthal, R.W. (1980). 'A model in which an increase in the number of sellers leads to a higher price', *Econometrica*, vol. 48(6), pp. 1575–9.
Stahl, D.O. (1989). 'Oligopolistic pricing with sequential consumer search', *American Economic Review*, vol. 79, pp. 700–12.
Varian, H. (1980). 'A model of sales', *American Economic Review*, vol. 70(4), pp. 651–9.
Varian, H. (2007). 'Position suctions', *International Journal of Industrial Organization*, vol. 25(6), pp. 1163–78.
Wilson, C. (2010). 'Ordered search and equilibrium obfuscation', *International Journal of Industrial Organization*, vol. 28(5), pp. 496–506.