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ON THE STANDARD LENGTHS OF ANGLE BISECTORS AND THE ANGLE BISECTOR THEOREM

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ABSTRACT. In this paper the author unveils several alternative proofs for the standard lengths of Angle Bisectors and Angle Bisector Theorem in any triangle, along with some new useful derivatives of them.

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1. INTRODUCTION

In this paper the author introduces alternative proofs for the standard length of Angle Bisectors and the Angle Bisector Theorem in classical Euclidean Plane Geometry, on a concise elementary format while promoting the significance of them by acquainting some prominent generalized side length ratios within any two distinct triangles existed with some certain correlations of their corresponding angles, as new lemmas. Within this paper 8 new alternative proofs are exposed by the author on the angle bisection, 3 new proofs each for the lengths of the Angle Bisectors by various perspectives with also 5 new proofs for the Angle Bisector Theorem.

1.1. The Standard Length of the Angle Bisector

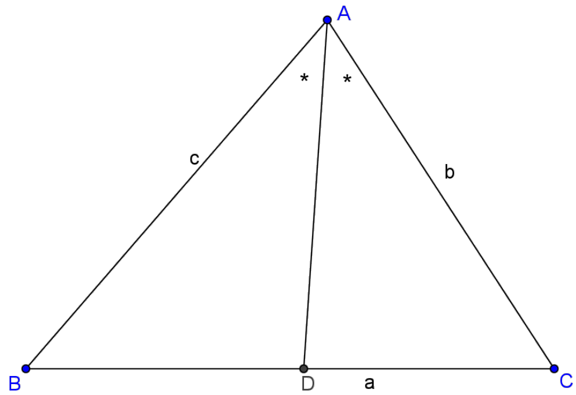

Figure 1.1: A natural triangle with an angle bisector

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The length of the angle bisector of a standard triangle such as AD in figure 1.1 is $AD^2 = AB \cdot AC - BD \cdot DC$, or $AD^2 = bc\left[1 - (a^2/(b+c)^2)\right]$ according to the standard notation of a triangle as it was initially proved by an extension of the angle bisector up to the circumcircle of the triangle. Nevertheless within this analysis the author adduces 3 new alternative methods in order to obtain the standard length of the angle bisector using some elementary Euclidean Geometry techniques without even being used trigonometry or vector Algebra at least just a little bit as follows.

Main Results

2. First Alternative Method: Proof of Figure 2

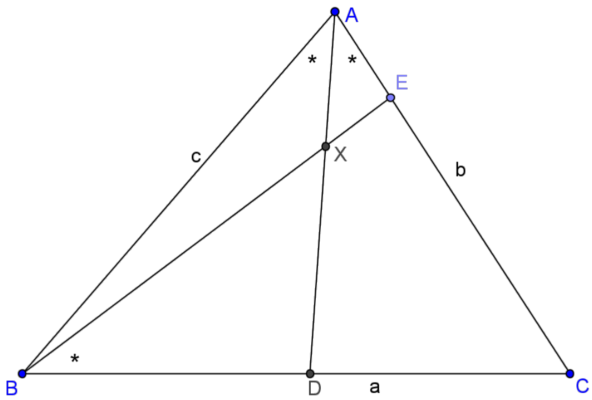


Figure 2: After being constructed the BE line

$$\widehat{ADB} = \widehat{CAD} + \widehat{ACB} = \widehat{BAD} + \widehat{ACB}$$
$$\widehat{AEX} = \widehat{ACB} + \widehat{CBE} = \widehat{ACB} + \widehat{CAD} = \widehat{ACB} + \widehat{BAD}$$

Likewise $\widehat{ADB} = \widehat{AEX}$ and since $\widehat{BAD} = \widehat{CAD}$, the triangles ABD and AXE are similar. Hence $\frac{AD}{AE} = \frac{AB}{AX}$. So, by replacing $AX = AD - DX$, $AD^2 = AD \cdot DX + AB \cdot AE$. Substituting $AE = AC - CE$,

$$AD^2 = AB \cdot AC - AB \cdot EC + AD \cdot DX \tag{2.1}$$

The angles $\widehat{CAD}$ and $\widehat{CBE}$ are equal and the angle ACB is common for both the triangles ADC and BEC. Hence the triangles ADC and BEC are similar. Likewise $\frac{EC}{DC} = \frac{BC}{AC}$ means $\frac{EC}{BC} = \frac{DC}{AC}$ and since AD is an angle bisector, $\frac{AB}{AC} = \frac{BD}{DC}$ (standard ratio). So, $\frac{DC}{AC} = \frac{BD}{AB}$. Hence $\frac{EC}{BC} = \frac{BD}{AB}$, means that

$$AB \cdot EC = BC \cdot BD \tag{2.2}$$

The angles $\widehat{CBE}$ and $\widehat{BAD}$ are equal and the angle ADB is common for both the triangles ABD and BDX. Hence, the triangles ABD and BDX are similar. Hence $\frac{BD}{AD} = \frac{DX}{BD}$. So,

$$AD \cdot DX = BD^2 \tag{2.3}$$

Hence by substituting above values from (2.2) and (2.3) to (2.1) AD^2 becomes $AD^2 = AB \cdot AC - BC \cdot BD + BD^2 = AB \cdot AC - BD \cdot (BC - BD) = AB \cdot AC - BD \cdot DC$. Likewise $AD^2 = AB \cdot AC - BD \cdot DC$, and since $\frac{c}{b} = \frac{BD}{DC}$ (as AD is the bisector) $BD = \frac{ac}{b+c}$ and $DC =$

$\frac{ab}{b+c}$, thus by replacing those values $AD^2 = bc - (\frac{a^2bc}{(b+c)^2})$, hence $AD^2 = bc\left(1 - (\frac{a^2}{(b+c)^2})\right)$. Likewise the proof is completed.
When angle $\widehat{CBE} > \widehat{ABC}$, means $\frac{\widehat{A}}{2} > \widehat{B}$, the E point will lie on extended CA. Thus above correlation $AD^2 = bc\left(1 - (\frac{a^2}{(b+c)^2})\right)$ can easily be proved exactly as it has been proved earlier.
Hence it can be easily adduced the lengths of the **Bisectors of** angle **ABC** and the angle **ACB** such that $\left[ac\left(1 - (\frac{b^2}{(a+c)^2})\right)\right]^{\frac{1}{2}}$ and $\left[ab\left(1 - (\frac{c^2}{(a+b)^2})\right)\right]^{\frac{1}{2}}$ respectively, comparing with AD.

3. Second Alternative Method: Proof of Figure 3

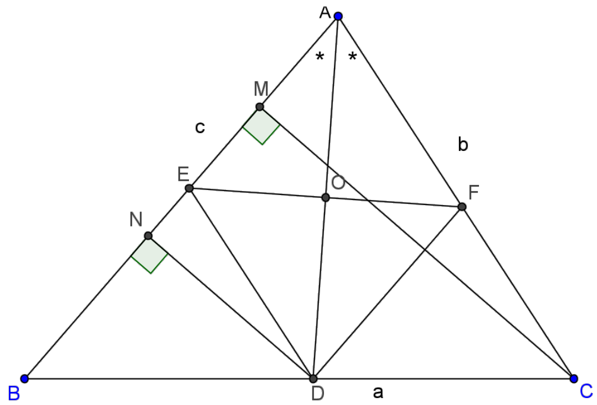


Figure 3: Creating parallel lines

AEDF is a parallelogram(as DE and DF are parallel to AC and AB respectively). So, $\widehat{ADE} = \widehat{CAD}$ and $\widehat{ADF} = \widehat{BAD}$, since $\widehat{BAD} = \widehat{CAD}$ and $\widehat{ADE} = \widehat{ADF}$. Thus angle $\widehat{BAD} = \widehat{CAD} = \widehat{ADE} = \widehat{ADF}$. So, the AEDF parallelogram becomes a Rhombus and because of that $AE = ED = DF = AF$ while AD and EF diagonals are perpendicular for each of them at O as angle $\widehat{AOE} = 90°$. Moreover $AO = OD = \frac{AD}{2}$.
$\frac{BD}{DC} = \frac{c}{b}$ (As AD is the Angle Bisector). Hence

$$BD = \frac{ac}{b+c} \tag{3.1}$$

Angle $\widehat{ABC}$ is common for both the triangles BDE and ABC. Angle $\widehat{EDB} = \widehat{ACB}$ (DE and AC are parallel). Likewise the triangles BDE and ABC are similar. Therefore $\frac{DE}{b} = \frac{BD}{a}$, and replacing in (3.1), we get $DE = \frac{bc}{b+c}$. One obtains:

$$DE = DF = AE = AF = \frac{bc}{b+c}. \tag{3.2}$$

Angle $\widehat{BAD}$ is common for both the triangles ADN and AOE and angle $\widehat{AOE} = \widehat{AND} = 90°$. Hence the triangles ADN and AOE are similar, so that $\frac{AE}{AD} = \frac{AO}{AN} = \frac{AD}{2 \cdot AN}$. Thus

$$AD^2 = 2AN \cdot AE \tag{3.3}$$

Angle $\widehat{AND} = \widehat{AMC} = 90^\circ$ and $\widehat{BAC} = \widehat{BED}$ (since DE and AC are parallel). Hence the triangles DEN and AMC are similar, so that $\frac{EN}{AM} = \frac{DE}{b}$, replacing by (3.2),

$$EN = \frac{AM \cdot c}{b + c} \tag{3.4}$$

Considering the triangle BMC, $a^2 = CM^2 + BM^2 = b^2 - AM^2 + (c - AM)^2 = b^2 + c^2 - 2c \cdot AM$, thus

$$AM = \frac{b^2 + c^2 - a^2}{2c} \tag{3.5}$$

Substituting from (3.5) to (3.4), one obtains:

$$EN = \frac{b^2 + c^2 - a^2}{2(b + c)} \tag{3.6}$$

$AN = AE + EN$, thus by replacing from (3.2) and (3.6),

$$AN = \frac{(b + c)^2 - a^2}{2(b + c)} \tag{3.7}$$

Substituting from (3.2) and (3.7) to (3.3), we get: $AD^2 = \frac{2bc}{b+c} \cdot \left[\frac{(b+c)^2 - a^2}{2(b+c)}\right]$.
Simplifying this further$AD^2 = bc\left(1 - \left(\frac{a^2}{(b+c)^2}\right)\right)$. Thus the proof is successfully completed.

4. Third Alternative Method: Proof of Figure 4

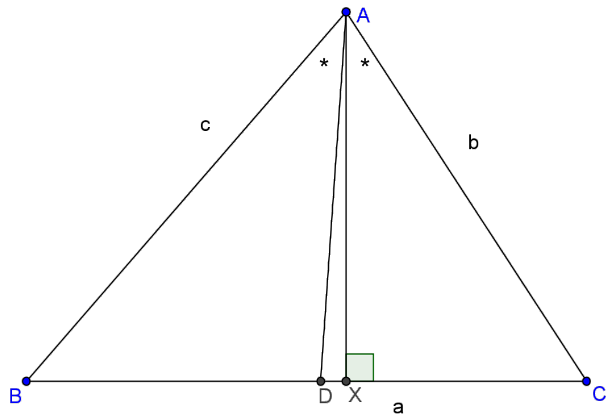


Figure 4: Natural triangle with a perpendicular

$\frac{BD}{DC} = \frac{c}{b}$ (as AD is the angle bisector), so that $BD = \frac{ac}{b+c}$ and

$$DC = \frac{ab}{b + c} \tag{4.1}$$

Considering the triangle ABX,

$$c^2 = AX^2 + BX^2 = AD^2 - DX^2 + (BD + DX)^2 = AD^2 + BD^2 + 2BD \cdot DX,$$

thus

$$2BD \cdot DX = c^2 - AD^2 - BD^2 \tag{4.2}$$

Considering the triangle AXC,

$$b^2 = AX^2 + CX^2 = AD^2 - DX^2 + (DC - DX)^2 = AD^2 + DC^2 - 2DC \cdot DX,$$

thus

$$2DC \cdot DX = AD^2 + DC^2 - b^2 \tag{4.3}$$

From (4.2) and (4.3), one obtains:

$$\frac{BD}{DC} = \frac{c^2 - AD^2 - BD^2}{AD^2 + DC^2 - b^2}. \tag{4.4}$$

Replacing from (4.1), we get:

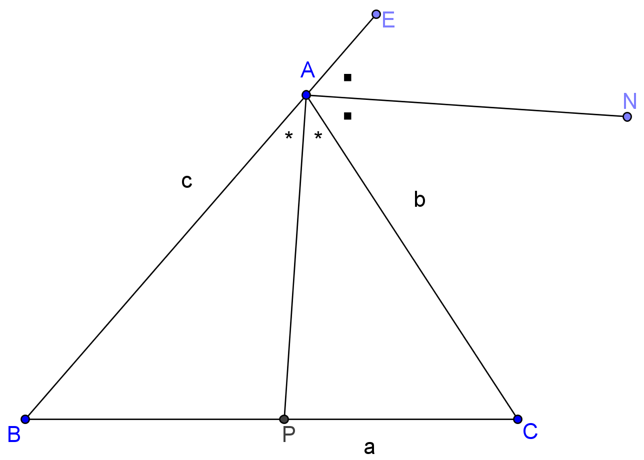
$$\frac{c}{b} = \frac{c^2 - AD^2 - \left(\frac{ac}{b+c}\right)^2}{AD^2 + \left(\frac{ab}{b+c}\right)^2 - b^2}.$$

Thus simplifying this further

$$AD^2(b+c) = bc^2 - b \cdot \left(\frac{ac}{b+c}\right)^2 + b^2c - c \cdot \left(\frac{ab}{b+c}\right)^2$$

so, one obtains:
$AD^2(b+c) = bc(b+c) - \frac{a^2bc}{b+c}$. Thus $AD^2 = bc - \frac{a^2bc}{(b+c)^2}$.
Likewise
$AD^2 = bc\left(1 - \left(\frac{a^2}{(b+c)^2}\right)\right)$. Hence the proof is again successively completed.

Remark 1. *The length of the* ***External Angle Bisector*** *can be assumed with the use of (4.4) as follows.*



AN is a part of the external angle bisector of angle $\widehat{BAC}$. Let's we imagine that the extended angle bisector AN will meet the extended BC line at **D**, then the length of the external angle bisector becomes AD.

Proof. $\frac{BD}{DC} = \frac{c}{b}$ (As AD is the external bisector), hence $\frac{BC+DC}{DC} = \frac{c}{b}$, so that $\frac{BC}{DC} = \frac{c-b}{b}$. Whence $CD = \frac{ab}{c-b}$.
Using (4.4) to the ***ABD***, $\frac{BC}{DC} = \frac{c^2-AC^2-BC^2}{AC^2+CD^2-AD^2}$.

Since $AC = b$, $BC = a$ and $CD = \frac{ab}{c-b}$,
$\frac{c-b}{b} = \frac{c^2-b^2-a^2}{b^2-(ab/(c-b))^2-AD^2}$, simplifying this further from several steps, $AD^2 \cdot (c-b)^2 + bc \cdot (c-b)^2 = a^2bc$, whence the distance of the ***External Angle Bisector*** AD can be adduced such that, $AD^2 = bc\left[\left(\frac{a}{c-b}\right)^2 - 1\right]$. ∎

5. First Alternative Proof for the Angle Bisector Theorem

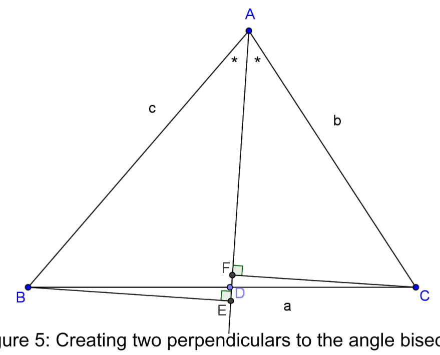


Figure 5: Creating two perpendiculars to the angle bisector

Angles $\widehat{BDE}$ and $\widehat{CDF}$ are equals (vertically opposite angle) and $\widehat{BED} = \widehat{CFD} = 90°$ (BE and CF are perpendiculars), thus the triangles BDE and CDF are similar.
So that,

$$\frac{BD}{DC} = \frac{BE}{CF} \tag{5.1}$$

Angles $\widehat{BAD}$ and $\widehat{CAD}$ are equals (as AD is the angle bisector) and $\widehat{AEB} = \widehat{AFC} = 90°$ (BE and CF are perpendiculars), thus the triangles ABE and AFC are similar.
So that

$$\frac{c}{b} = \frac{BE}{CF} \tag{5.2}$$

From (5.1) and (5.2) we get: $\frac{BD}{DC} = \frac{c}{b}$

6. Second Alternative Proof for the Angle Bisector Theorem

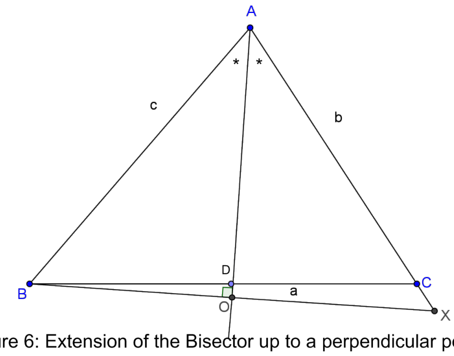


Figure 6: Extension of the Bisector up to a perpendicular point

$\widehat{BAO} = \widehat{XAO}$ (as AD is the angle bisector) and $\widehat{AOB} = \widehat{AOX} = 90°$ (AO is perpendicular to BX). Therefore the triangles AOB and AOX are congruent. Hence $BO = OX$ and $AB = c = AX$.
Moreover, from $BO = OX$ and $\widehat{DOB} = \widehat{DOX} = 90°$ we get the congruence of the triangles BOD and XOD. So that, $BD = DX$. Thereafter since $AB = AX$, $BD = DX$ and AD is common for both the triangles ABD and ADX, then the triangles ABD and ADX are also congruent. Hence the areas of the triangles ABD and ADX are the same which means $\Delta ABD \equiv \Delta ADX$.
The ratios of the respective areas of triangles are as follows:

$$\frac{Area_{\Delta ADX}}{Area_{\Delta ADC}} = \frac{AX}{AC} = \frac{c}{b}$$

Since $\Delta ABD \equiv \Delta ADX$, $\frac{Area_{\Delta ABD}}{Area_{\Delta ADC}} = \frac{c}{b}$. We observe that $\frac{Area_{\Delta ABD}}{Area_{\Delta ADC}} = \frac{BD}{DC}$ easily. Likewise $\frac{BD}{DC} = \frac{c}{b}$.

7. Third Alternative Proof for the Angle Bisector Theorem Using Figure 6

Using Menelaus theorem in the triangle BCX, one obtains:

$$\frac{AX}{AC} \cdot \frac{DC}{BD} \cdot \frac{BO}{OX} = 1$$

Replacing relevant values and since $BO = OX$ we get $\frac{c}{b} \cdot \frac{DC}{BD} = 1$.
Thus

$$\frac{BD}{DC} = \frac{c}{b}.$$

8. Fourth Alternative Proof for the Angle Bisector Theorem

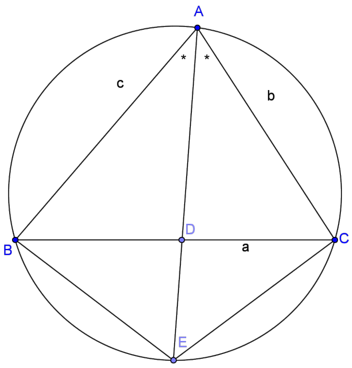


Figure 7: Extension of the angle bisector up to the circumcircle

$\widehat{BAE} = \widehat{CAE}$ (as AD is the Angle Bisector). Moreover, $\widehat{BAE} = \widehat{CAE} = \widehat{BCE}$ and $\widehat{CAE} = \widehat{BAE} = \widehat{CBE}$. (as ABCD is a cyclic quadrilateral).
So that $\widehat{CBE} = \widehat{BCE}$, hence

$$BE = EC \tag{8.1}$$

Since $\widehat{CBE} = \widehat{BAE}$ and $\widehat{AEB}$ is common for both the triangles ABE and BDE, likewise the triangles ABE and BDE are similar. So that

$$\frac{BD}{c} = \frac{BE}{AE} \tag{8.2}$$

Since $\widehat{BCE} = \widehat{CAE}$ and $\widehat{AEC}$ is common for both the triangles AEC and EDC, likewise the triangles AEC and EDC are similar. So that

$$\frac{DC}{b} = \frac{EC}{AE} \tag{8.3}$$

Since $BE = EC$ from (8.1) then (8.2) and (8.3) coincide. Thus $\frac{BD}{c} = \frac{DC}{b}$ means $\frac{BD}{DC} = \frac{c}{b}$.

9. Fifth Alternative Proof for the Angle Bisector Theorem

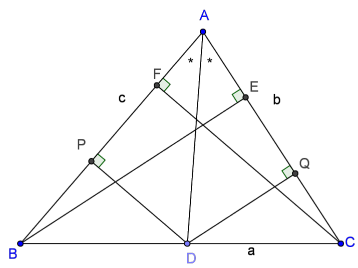


Figure 8: Creating several perpendiculars to AB and AC

$\widehat{BAD} = \widehat{CAD}$ (AD is the angle bisector) and $\widehat{APD} = \widehat{AQD} = 90°$, AD is common for both the triangles APD and ADQ, likewise APD and ADQ are congruent. So that $DP = DQ$. Angle $\widehat{ABC}$ is common for both the triangles BDP and BCF and $\widehat{BPD} = \widehat{BFC} = 90°$, likewise the triangles BDP and BCF are similar. Hence $\frac{BD}{a} = \frac{DP}{CF}$, so that

$$BD \cdot CF = a \cdot DP \tag{9.1}$$

Angle $\widehat{ACB}$ is common for both the triangles DCQ and BEC and $\widehat{DQC} = AngleBEC = 90°$, likewise the triangles DCQ and BEC are similar. Hence $\frac{DC}{a} = \frac{DQ}{BE}$, so that $BE \cdot DC = a \cdot DQ$ and since $DP = DQ$ as it proved earlier, we get:

$$BE \cdot DC = a \cdot DP \tag{9.2}$$

From (9.1) and (9.2), one obtains $BE \cdot DC = BD \cdot CF$, so that

$$\frac{BD}{DC} = \frac{BE}{CF} \tag{9.3}$$

Angle $\widehat{BAC}$ is common for both the triangles ABE and AFC and $\widehat{AFC} = \widehat{AEB} = 90°$, likewise the triangles ABE and AFC are similar. Hence

$$\frac{BE}{CF} = \frac{c}{b} \tag{9.4}$$

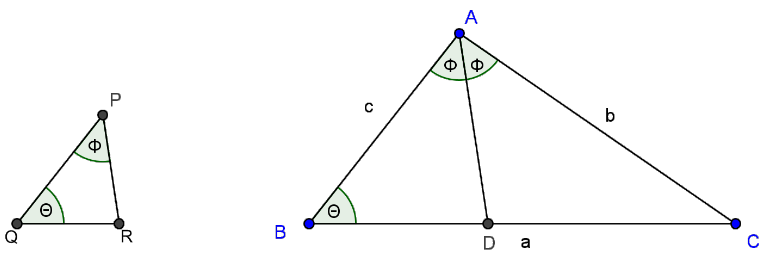
From (9.3) and (9.4), one obtains $\frac{BD}{DC} = \frac{c}{b}$.

Remark – When AD becomes the **External Angle Bisector**, it can be easily speculated and proved that $\frac{BD}{DC} = \frac{c}{b}$ as before within each of those 5 new proofs presented for the Internal Angle Bisector by a slight difference of AD line in each figure.

10. Several Derivatives of the Lengths of Angle Bisectors

Let us imagine that ΔPQR and ΔABC are any two distinct Euclidean triangles such that angle $\widehat{BAC} = 2\widehat{QPR} = 2\phi$ and $\widehat{ABC} = \widehat{PQR} = \theta$. $BC = a, AC = b, AB = c$. Draw the AD angle bisector up to D on BC. Hereafter we divulge the following **6** significant **Lemmas** emerged using the Angle Bisection.

$$BD = \frac{ac}{b+c} \left(since \frac{BD}{DC} = \frac{c}{b} \right)$$



$$AD^2 = bc\left(1 - \frac{a^2}{(b+c)^2}\right)$$

and using the similar triangles PQR and ABD, we get $\frac{PR}{AD} = \frac{PQ}{c}$
Hence, **Lemma 1** can be denoted as

$$\frac{PR}{PQ} = \frac{AD}{c} = \frac{\left[bc\left(1 - \frac{a^2}{(b+c)^2}\right)\right]^{\frac{1}{2}}}{c} = \left[\frac{b(b+c-a)(a+b+c)}{c(b+c)^2}\right]^{\frac{1}{2}}$$

Moreover $\frac{PR}{AD} = \frac{QR}{BD}$.

So that **Lemma 2** can be denoted as,

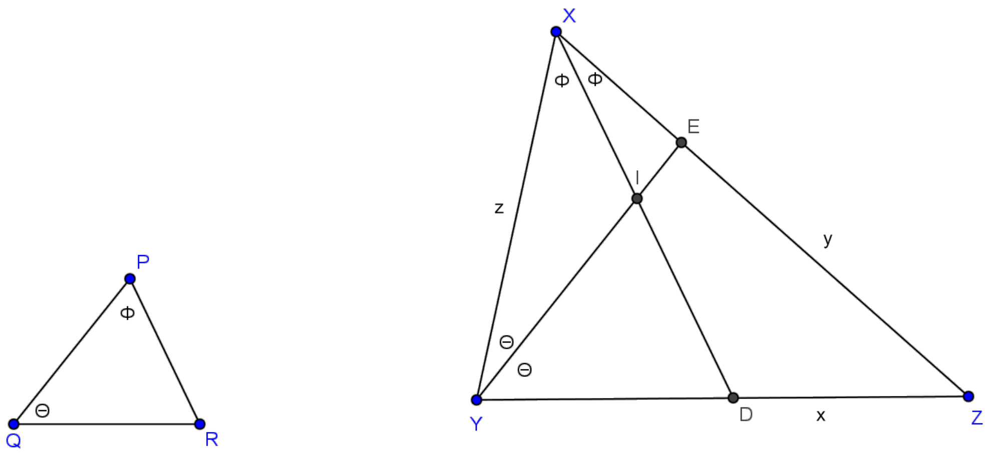
$$\frac{QR}{PR} = \frac{BD}{AD} = \frac{ac}{b+c}\frac{1}{\left[bc\left(1 - \frac{a^2}{(b+c)^2}\right)\right]^{\frac{1}{2}}} = a\left[\frac{c}{b(b+c-a)(a+b+c)}\right]^{\frac{1}{2}}$$

$\frac{PQ}{c} = \frac{QR}{BD}$, thus **Lemma 3** can be denoted as,

$$\frac{QR}{PQ} = \frac{BD}{c} = \frac{\frac{ac}{b+c}}{c} = \frac{a}{b+c}$$

Let us imagine again that ΔPQR and ΔXYZ are another two distinct Euclidean triangles such that $\widehat{YXZ} = 2\widehat{QPR}$ and $\widehat{XYZ} = 2\widehat{PQR}$. $YZ = x, XZ = y, XY = z$. Draw the XD and YE angle bisectors up to D on YZ and E on XZ respectively while they intersect at I.

$$DY = \frac{xz}{y+z}, DZ = \frac{xy}{y+z}.$$



$\frac{DI}{XI} = \frac{DY}{YX} = \frac{\frac{xz}{y+z}}{z} = \frac{x}{y+z}$, hence $XI = XD \cdot \frac{y+z}{x+y+z}$ and the value of $XD = \left[zy\left(1 - \frac{x^2}{(y+z)^2}\right)\right]^{\frac{1}{2}}$,
thus $XI = \frac{\left(yz\left((y+z)^2 - x^2\right)\right)^{\frac{1}{2}}}{y+z} \cdot \frac{y+z}{x+y+z} = \frac{\left(yz\left((y+z)^2 - x^2\right)\right)^{\frac{1}{2}}}{x+y+z}$
From the fact that ΔPQR and ΔXYI are similar triangles, one obtains: $\frac{PQ}{z} = \frac{PR}{XI}$ and $\frac{PR}{PQ} = \frac{XI}{z}$
therefore, **Lemma 4** can be adduced as

$$\frac{PR}{PQ} = \frac{(yz((y+z)^2 - x^2))^{\frac{1}{2}}}{z(x+y+z)} = \sqrt{\frac{y(y+z-x)}{z(x+y+z)}}$$

$\frac{YI}{IE} = \frac{z}{XE} = \frac{z}{\frac{zy}{x+z}} = \frac{x+z}{y}$, hence $YI = \frac{x+z}{x+y+z} \cdot YE$, moreover the value of

$$YE = \left[xz\left(1 - \frac{y^2}{(x+z)^2}\right)\right]^{\frac{1}{2}},$$

hence $YI = \frac{\left[xz((x+z)^2-y^2)\right]^{\frac{1}{2}}}{x+y+z}$.
$\frac{PR}{XI} = \frac{QR}{YI}$,

$$\frac{QR}{PR} = \frac{YI}{XI} = \frac{\left[xz((x+z)^2 - y^2)\right]^{\frac{1}{2}}}{x+y+z} \cdot \frac{x+y+z}{\left[yz((y+z)^2 - x^2)\right]^{\frac{1}{2}}},$$

thus **Lemma 5** can be adduced as,

$$\frac{QR}{PR} = \sqrt{\frac{x(x+z-y)}{y(y+z-x)}}$$

$$\frac{QR}{YI} = \frac{QP}{z}, \frac{QR}{PQ} = \frac{IY}{z} = \frac{\left[xz((x+z)^2 - y^2)\right]^{\frac{1}{2}}}{z(x+y+z)}.$$

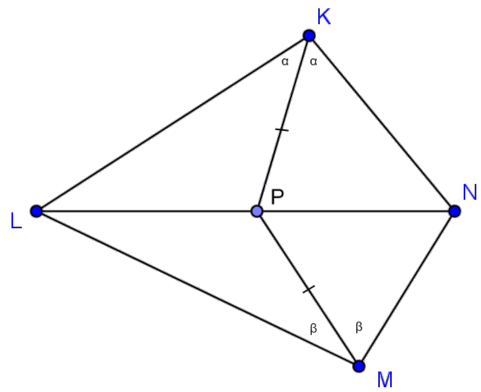
Therefore **Lemma6** can be adduced as

$$\frac{QR}{PQ} = \sqrt{\frac{x(x+z-y)}{z(x+y+z)}}$$

Considering these above ratios it is very unambiguous that the ratios of one particular triangle can be adduced from the lengths of other corresponding triangle, consequently these propositions can be diffused and used towards many significant improvements of Advanced Classical Euclidean Geometry.

11. An Elementary Problem On the Angle Bisection

Suppose that KLMN is a quadrilateral at which the point P is located on its LN diagonal such that $KP = PM$, being both KP and PM are the angle bisectors of the opposite angles $\widehat{LKN}$ and $\widehat{LMN}$ respectively. Then it is proved that $KL = LM$ and $KN = NM$ and $\widehat{LKN} = \widehat{LMN}$.



Let angle $\widehat{LKP} = \alpha$ and angle $\widehat{LMP} = \beta$

Proof of the correlation of KLMN Quadrilateral
KP and PM are the bisectors of angle $\widehat{LKN}$ and $\widehat{LMN}$ respectively, whence

$$KP^2 = KL \cdot KN - LP \cdot PN$$

$$PM^2 = ML \cdot MN - LP \cdot PN$$

Since $KP = PM$, $KL \cdot KN - LP \cdot PN = ML \cdot MN - LP \cdot PN$,

$$KL \cdot KN = ML \cdot MN \tag{11.1}$$

As KP is the angle bisector, $\frac{KL}{KN} = \frac{LP}{PN}$, and as PM is the angle bisector, $\frac{ML}{MN} = \frac{LP}{PN}$, whence

$$\frac{KL}{KN} = \frac{ML}{MN} \tag{11.2}$$

By the use (11.1) and (11.2), $KL = LM$ and $KN = MN$ and whence by the congruence of triangles it is proved that $\alpha = \beta$, hence $\widehat{LKN} = \widehat{LMN}$.

Conclusion of Remarks- *The readers are kindly encouraged to have a precise look at those interlocutory derivatives mentioned in 10 and 11 as well as particularly in [1], [5] and [7] of the references to grasp a better comprehension about the significance of the Angle Bisector Theorems on which some felicitous correlations are often emerged in Advanced Euclidean Geometry.*

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References

[1] G.W.I.S Amarasinghe, A Prominent Correlation On the Extended Angle Bisector, *Journal of the World Federation of National Mathematics Competitions*, **24(1)**, 2011, pp.33 – 36.
[2] G.W.I.S Amarasinghe, Solutions for the problem 43.3, *Mathematical Spectrum*, **43(3)**, 2011, pp.138 – 139.
[3] G.W.I.S. Amarasinghe, Advanced Plane Geometry Research 1, *Proceedings of the 66th Annual Sessions of Sri Lanka Association for the Advancement of science(SLAAS)*, **66**, 2010, pp.77.

[4] G.W.I.S. Amarasinghe, Advanced Plane Geometry Research 2, *Proceedings of the 66th Annual Sessions of Sri Lanka Association for the Advancement of science(SLAAS)*, **66**, 2010, pp.82.
[5] G.W.I.S. Amarasinghe, The Jungle Gym, Problem 260, A Parametric Equation, *Math Horizons*, **18(4)**, 2011, pp.30.
[6] G.W.I.S. Amarasinghe, Advanced Plane Geometry Research 3: Alternative Proofs for the Standard Theorems in Plane Geometry, *Proceedings of the 66th Annual Sessions of Sri Lanka Association for the Advancement of Science(SLAAS)*, **66**, 2010, pp.78.
[7] G.W.I.S Amarasinghe, A New Theorem On Any Right Angled Cevian Triangle, *Journal of the World Federation of National Mathematics Competitions,* **24(2)**, 2011, pp.29-37.
[8] http://www.cut-the-knot.org/triangle/ABisector.shtml
[9] R.H Buchholz, On Triangles with rational altitudes, angle bisectors or medians, *Bulletin of the Australian Mathematical Society,* **45(3**), 1992, pp.525 – 526.
[10] S.H Wu, Z.H Zhang, A Class of Inequalities Related to the Angle Bisectors and the Sides of a Triangle, *Journal of Inequalities in Pure and Applied Mathematics*, **7(3)**, *Article 108*, 2006.

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