ROBERT BROWN AND THE POLLEN STUFF

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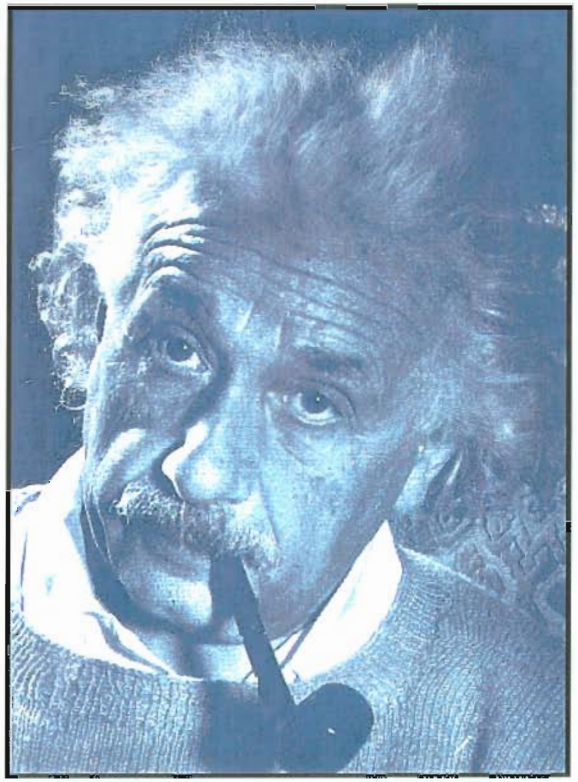
Robert Brown (1773-1858)


Brownian Motion

Robert Brown



Albert Einstein

Marian Smoluchowski

This is the view Brown obtained in 1828, when he first recognised the cell nucleus. It shows about twenty epidermal cells, and the nucleus can clearly be seen within each cell. Three stomata can also be clearly seen - these are the breathing pores through which a plant exchanges gases with the atmosphere.

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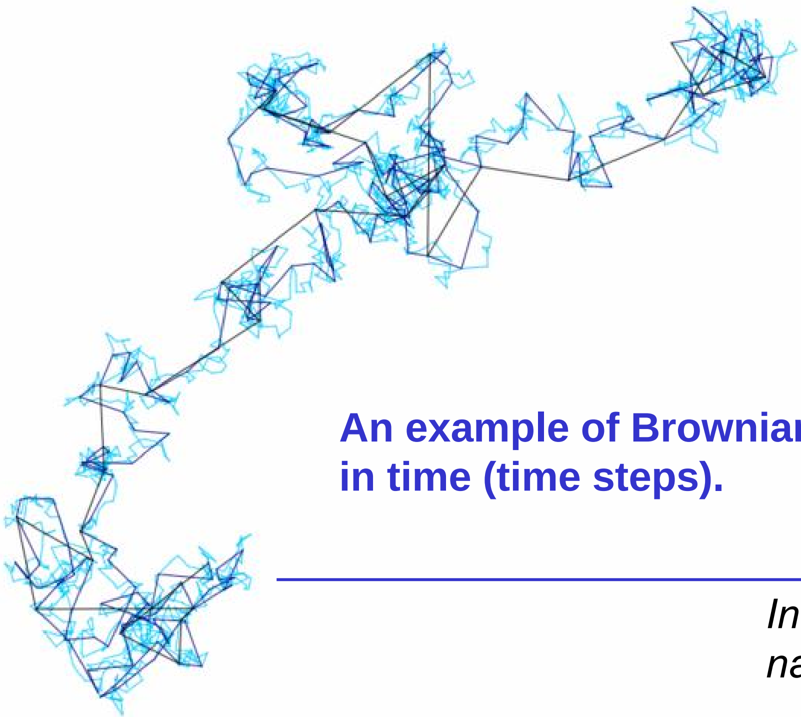
Brownian motion

Robert Brown (1773-1858)

In 1827, the botanist Robert Brown published a study *”A brief account of microscopical observations on the particles contained in the pollen of plants...”,* where we reported his observations of irregular, jittery motion of small (clay) particles in pollen grains.

He repeated the same experiment with particles of dust, showing that the motion could not be due to the pollen particles being alive.



Although several people worked on this phenomenon over the years, a proper physical explanation of it had to wait for almost 80 years.

An example of Brownian motion of a particle, recorded for three different resolutions in time (time steps).

Incidentally, Robert Brown was also the first to note the ubiquitous nature of a part of eukaryotic cells which he named the ”cell nucleus”.

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Jan Ingen-Housz (1730-1799)

William Sutherland (1859-1911)

Johann Ingen-Housz

K. K. Hofraths und Leibarztes, der Königl. Gesellschaft der Wissenschaften zu London, der Batavischen Gesellschaft der Experimentalphilosophie zu Rotterdam 2c. 2c. Mitglieds

Vermischte Schriften

physisch-medicinischen Inhalts.

Uibersetzt und herausgegeben von Nicolaus Carl Molitor.

Zweyte, verbesserte und mit ganz neuen Abhandlungen vermehrte Auflage.

Mit Kupfertafeln.

Zweyter Band.

Wien, gedruckt und verlegt bey Christian Friderich Wappler.

1784.

To see clearly how one can deceive one's mind on this point if one is not careful, one only has to place a drop of alcohol in the focal point of a microscope and introduce a little finely ground charcoal therein, and one will see these corpuscules in a confused, continuous and violent motion, as if they were animalcules which move rapidly around.

Stokes-Einstein-Sutherland equation?

$$D = \frac{k_B T}{6\pi\eta R}$$

William Sutherland (1859-1911)

As a historical sidenote, Einstein did not, in fact, have the precedence on the result above. Earlier in 1905 (March), a Scotsman/Australian named William Sutherland published a very similar derivation of the Stokes-Einstein equation (which he had publicly presented in a conference already in 1904).

It is not known, why the Stokes-Einstein equation is not known today as the Stokes-Einstein-Sutherland equation instead (although some authors have recently suggested it).

Sutherland's article was published in the *Philosophical Magazine*, a prestigious and well known journal. In addition, in 1905 he was already quite famous. For example, he was one of the two people outside Europe (the other one was J. Willard Gibbs) who were invited to a conference held in honor of Ludwig Boltzmann in 1906. (Einstein was not).

You can get Sutherland's paper, in addition to other Brownian-motion-related articles, from Peter Hänggi's web page:
http://www.physik.uni-augsburg.de/theo1/hanggi/History/BM-History.html

Einstein relation

In 1905, Albert Einstein published his PhD thesis on osmotic pressure. Developing the ideas therein further, later that year he published one of his ground-breaking papers of that year: the theory of Brownian motion.

Deriving a result, which nowadays is called a fluctuation-dissipation theorem, Einstein showed that the diffusion coefficient of a particle undergoing Brownian motion is

$$D = \frac{k_B T}{\xi}$$

← Friction factor of the particle; the frictional force is given by $\boldsymbol{F}_{drag} = - \xi \boldsymbol{v}$

Specifically,

$$D = \frac{k_B T}{6\pi\eta R}$$

For a spherical particle much larger than the sovent molecules (Stokes-Einstein equation)

The relation between the diffusion coefficient *D* and the displacement of a particle undergoing Brownian motion is

$$D = \lim_{t\to\infty} \frac{1}{2dt}\langle r^2(t)\rangle$$

whence, for long enough times *t*

$$\langle r^2(t)\rangle = 2dtD$$

Langevin and Einstein
in 1911

In the same year Albert Einstein correctly identified Brownian motion (such motion, visible only under a microscope, is the incessant, random movement of micrometre-sized particles immersed in a liquid) as due to imbalances in the forces on a particle resulting from molecular impacts from the liquid. Shortly thereafter, Langevin formulated a theory in which the minute fluctuations in the position of the particle were due explicitly to a random force. Langevin's approach proved to have great utility in describing molecular fluctuations in other systems, including nonequilibrium thermodynamics.

Langevin equation (1)

Let us consider the dynamics of a single colloidal particle under continuous bombardment by the solvent molecules

$$\frac{d\vec{r}}{dt} = \vec{v}(t)$$

$$\frac{d\vec{p}}{dt} = -\xi\vec{v}(t) + \vec{f}(t)$$

Instantaneous random force

$$\frac{d^2\vec{r}}{dt^2} = -\left(\frac{\xi}{m}\right)\frac{d\vec{r}}{dt} + \frac{\vec{f}(t)}{m}$$

Langevin equation

The random force *f(t)* satisfies the following conditions:

$$\langle f \rangle = \langle fx \rangle = \langle fv \rangle = 0$$

$$\langle f(t) f(t') \rangle = \Gamma \delta(t - t')$$

Obtained through a fluctuation-dissipation theorem for the problem at hand.

Paul Langevin and Albert Einstein, two friends who illuminated the physics of the same phenomenon in two quite different ways.

Langevin equation (2)

For the displacement (*x,y,z*) of a Brownian particle $\langle x \rangle = \langle y \rangle = \langle z \rangle = 0$

However, for the mean-square displacements we have $\langle x^2 \rangle = \langle y^2 \rangle = \langle z^2 \rangle$

And for the 3D-displacement we have the relation $\langle r^2 \rangle = 3\langle x^2 \rangle$

Since the total displacement of a Brownian particle is obtained with any of the one-dimensional displacements, let us write the Langevin equation in the x-direction as

$$m\left(\frac{d^2x}{dt^2}\right) = -\xi\left(\frac{dx}{dt}\right) + f \qquad \text{(L1)}$$

Using the relations

$$x\left(\frac{dx}{dt}\right) = \frac{1}{2}\left(\frac{dx^2}{dt}\right) \quad \text{and} \quad x\left(\frac{d^2x}{dt^2}\right) = \frac{1}{2}\left(\frac{d^2x^2}{dt^2}\right) - \left(\frac{dx}{dt}\right)^2$$

We can multiply eq. (L1) with x and rewrite it as

$$\frac{m}{2}\left(\frac{d^2x^2}{dt^2}\right) - m\left(\frac{dx}{dt}\right)^2 = -\frac{\xi}{2}\left(\frac{dx^2}{dt}\right) + fx \qquad \text{(L2)}$$

Langevin equation (3)

We then take the average of eq. (L2), employing the well-known resulst from the equipartition theorem

$$\left\langle \frac{1}{2}m\left(\frac{dx}{dt}\right)^2 \right\rangle = \frac{1}{2}k_B T$$

and further using the notation

$$\alpha = \frac{\langle dx^2 \rangle}{dt}$$

we obtain a first-order differential equation

$$\left(\frac{d\alpha}{dt}\right) + \frac{\xi}{m}\alpha = \frac{2k_B T}{m} \qquad \text{(L3)}$$

for which the general solution is

$$\alpha = \frac{2k_B T}{\xi} + C\exp\left(-\frac{\xi}{m}t\right) \qquad \text{(L4)}$$

Finally, for times $t \gg \frac{\xi}{m}$, integrating eq. (L4) over time we obtain the result

$$\langle x^2 \rangle = \frac{2k_B T}{\xi}t \qquad \text{or} \qquad \langle x^2 \rangle = 2Dt$$

Ito-SDE:

$$d\underline{x} = \underline{a}(\underline{x}(t);t)\,dt + \underline{\underline{b}}(\underline{x}(t);t) \bullet d\underline{w}(t)$$

$\Updownarrow$ equivalent

GSI-SDE

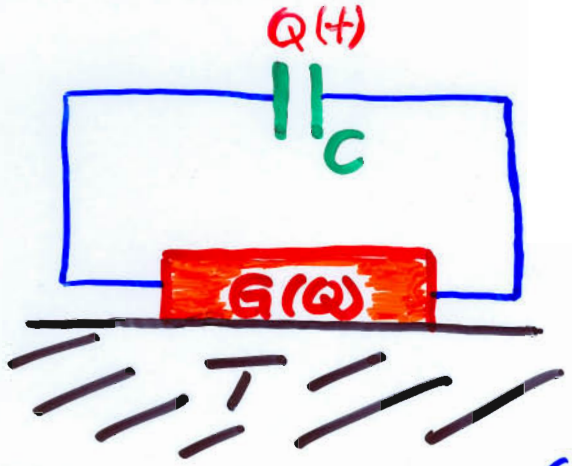
$$d\underline{x}(t) = \underline{a}(\underline{x}(t);t)\,dt - (1-\lambda)\,\underline{F}(\underline{x}(t);t) + \underline{\underline{b}}(x(t);t) \overset{GSI}{\otimes} d\underline{w}(t)$$

λ: finite $(-\infty, +\infty)$

$\lambda = 1$: Ito

$\lambda = 1/2$: Stratonovich

$\lambda = 0$: HANGGI



nonlinear conductance

VOLTAGE $V = \frac{Q}{C}$

$$\dot{Q} = -G(Q)\frac{Q}{C}$$

CHARGE FLUCTUATIONS $Q(t)$

EQUILIBRIUM $\bar{p}(Q) = Z^{-1}\exp\left(-\frac{Q^2}{2C}/kT\right)$

?¿ DYNAMICS!?

PHYSICS: ⟷ $\frac{\partial p_t(Q)}{\partial t} + \frac{\partial}{\partial Q} J(Q,t) = 0$

$$\dot{p}_t(Q) = -\frac{\partial}{\partial Q}\left[-G(Q)\frac{Q}{C}p_t(Q) - kTG(Q)\frac{\partial}{\partial Q}p_t(Q)\right]$$

continuity - eq.

$$\frac{dQ}{dt}^{Ito} = -G(Q)\frac{Q}{C}dt + kT\frac{dG(Q)}{dQ} + \sqrt{2kTG(Q)} \bullet dW(t)/dt$$

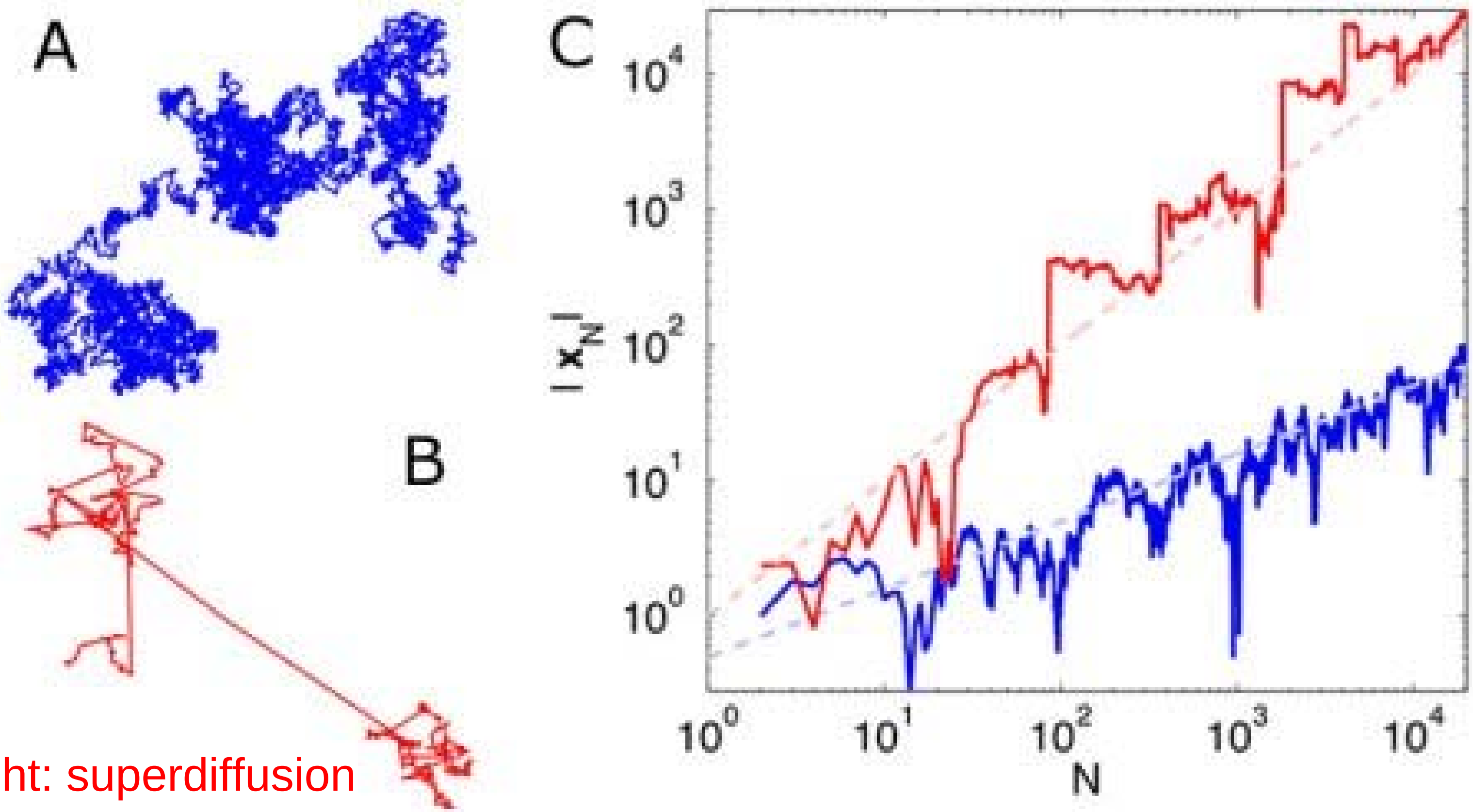
$$\frac{dQ}{dt}^{S} = -G(Q)\frac{Q}{C}dt + \frac{1}{2}kT\frac{dG(Q)}{dQ} + \sqrt{2kTG(Q)} \circ dW(t)/dt$$

$$\frac{dQ}{dt}^{HANGGI} = -G(Q)\frac{Q}{C}dt + 0 + \sqrt{2kTG(Q)} \overset{\lambda=0}{\otimes} dW(t)/dt$$

P. H., HELV. PHYS. ACTA 51: 183 (1978)

P. H., H. THOMAS, PHYS. REP. 88: 207-319 (82) · SECT. VI

normal Brownian motion



Levy flight: superdiffusion

Measurement of the Translational and Rotational Brownian Motion of Individual Particles in a Rarefied Gas

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We measured the free Brownian motion of individual spherical and the Brownian rotation of individual nonspherical micrometer-sized particles in rarefied gas. Measurements were done with high spatial and temporal resolution under microgravity conditions in the Bremen drop tower so that the transition from diffusive to ballistic motion could be resolved. We find that the translational and rotational diffusion can be described by the relation given by Uhlenbeck and Ornstein [Phys. Rev. **36**, 823 (1930)]. Measurements of rotational Brownian motion can be used for the determination of the moments of inertia of small particles.

$$\langle \Delta x^2 \rangle = 2D\Delta t\left(1 - \frac{\tau_f}{\Delta t} + \frac{\tau_f}{\Delta t}\exp\left[-\frac{\Delta t}{\tau_f}\right]\right)$$