The New Temple Geometry Problems in Hirotaka's Ebisui Files

Miroslaw Majewski (馬裕祺)
mirek.majewski@yahoo.com

Jen-Chung Chuan (全任重)
jcchuan@math.nthu.edu.tw

Nishizawa Hitoshi (西澤 一)
nisizawa@toyota-ct.ac.jp

Abstract: *For quite a while we are witnessing development of Hirotaka files. These are PDF documents with hundreds of geometry problems developed by a Japanese mathematician Hirotaka Ebisui. At the same time there are very few mathematicians, or even geometers, aware of existence of these files, and even less people who recognize their value. The objective of this paper is to make a brief analysis of these files, examine roots of the geometry problems in the Hirotaka files, and point out their value.*

1. Introduction

Let us start with a question to the reader, or even two questions. Can you imagine a mathematician who proved a nice theorem, then wrote it down or painted it on a wooden tablet, went with it to a church, or a temple, and hung it there? If you are a teacher of mathematics, can you imagine that you asked some of your students to take their mathematics homework, and carry it to a local shrine? Certainly most of you cannot imagine such practices and, I believe, never heard about such things. However, there is such a country where mathematicians and enthusiasts of mathematics used to carry their best theorems to a shrine, and hung them somewhere on the wall, or under a roof. This practice happened long time ago, and sporadically it happens also today. Such theorems were usually painted on a smooth piece of wooden plank, and sometimes they were even framed. If there was a geometric construction, then quite often the picture was very colorful. Sometimes such a picture contained decorations with flowers, plants, mountains, etc. Some of them were real pieces of art. It was probably a way to thank gods for the moment of enlightenment while solving the problem.

It is worth to notice that such mathematical tablet contained typically a theorem, its description and usually name of the person who proved it. The proof was usually left with the author. Therefore, such mathematical tablet was a kind of challenge for other people who attended the shrine – "*look I proved this, I am a clever person, can you also prove it*?"

The country we are talking about is Japan from the Edo period. The tablets that Japanese enthusiasts of mathematics used to hang in shines were called *sangaku*, which simply means a mathematical tablet. After the Edo period this custom vanished almost completely, but even now in some temples we can find sangaku from our times. They are, of course, different than those from the Edo period. Also the mathematics on modern sangaku is a bit different from the one developed during the Edo time. The Japanese mathematics from this period was called *wasan,* and in many aspects it was different from the western mathematics called there *yosan*.

Fig. 1 *An old Shinto shrine with sangaku*

In order to understand this phenomenon we shall briefly examine history of Japan, and especially beginning of the Edo period. This will give us an idea how mathematics was divided into wasan and yosan. We will also look on the origins of sangaku.

Nowadays, in the West, geometry problems displayed on sangaku are often called Japanese temple geometry problems. This explains part of the title of this paper. However, there is still more to explain.

2. The origins of wasan

Let us go back to the year 1600. This is the time where we should start looking for origins of wasan. At this time Japan was under control of *daimyo*, or in western terms warlords, who were still fighting about dominancy. Some of them were very powerful and the country was in almost continuous turmoil.

In 1600, during the famous Sekigahara battle daimyo were defeated by Tokugawa Ieyasu. This was the starting point to a new period in the history of Japan – a period of almost 250 years without wars in Japan. After the battle Tokugawa Ieyasu moved to a small, at this time, provincial town Edo, today's Tokyo. The country was united and many changes started taking place.

We have to notice that it was the time when Spanish, Portuguese and Dutch tried to settle down in Japan, and strengthen their trade. At the same time missionaries from these countries wanted to convert as many souls as possible. As we probably guess, the trade with foreigners was not a problem for anybody. However, converting people to Christianity was not very much welcomed by the two main religions in Japan – Shinto and Buddhists. This was in fact the main source of tensions in the country. In order to keep people calm Tokugawa Ieyasu issued an edict ordering Portuguese and Spanish to leave Japan, removing missionaries, to destroy all Christian churches, and forbidding Christianity in Japan. Tokugawa Ieyasu died a few years later, but his grandson Tokugawa Iemitsu finished the task of removing foreigners. In 1641 there were practically no foreigners in Japan. Only a small group of Dutch merchants, members of the East India Company, was left in a small, artificially made 200x70m island in Nagasaki waterfront. All these changes started a new period in Japan, sometimes called *sakoku*, or a closed country. All exchange of information between Japan and rest of the world was forbidden. Foreigners had no right to enter Japan, and Japanese had no right to travel outside of the country.

In this moment we have to notice that for centuries Japan was under the strong influence of China. There were many things that came to Japan from China: science, scientific and technical innovations, art, and Buddhism. This way came to Japan also many mathematical discoveries. This includes the most famous Chinese mathematics textbook *Jiuzhang Suanshu*, or *The Nine Chapters of Mathematical Art*, and of course *soroban* – a very specific form of abacus. As we may suspect closing the country also stopped any type of exchange with China.

Closing the country had not only negative effects. The most important – it stopped fights both internal as well as those with foreigners. It also forced, and in fact helped, Japanese to develop their own forms of art, and science. The local art, science and culture started developing rapidly. This concerns also mathematics. For the above mentioned reasons this period also has another name *genroku*, that means renaissance.

> **Genroku** (元禄) was a Japanese era name after *Jōkyō* and before *Hōei.* This period spanned the years from 1688 through 1704. The reigning emperor was Higashiyama-*tennō* (東山天皇). The years of *Genroku* are generally considered to be the Golden Age of the Edo Period. The previous almost hundred years of peace and seclusion in Japan had created relative economic stability. The arts and architecture flourished. A sense of optimism is suggested in the era name choice of *Genroku* meaning "Original happiness". (Source Wikipedia)

If we look closer into Japanese history we will see that during this time they developed famous Kabuki opera, Noh dance, tea ceremonies, the famous garden architecture, flower arranging, several famous painting schools including the famous *Katsushika Hokusai* an *ukiyo-e* painter, poetry including the most wonderful type of Japanese poetry *haiku*, and literature. These are things in the Japanese culture that we in the West appreciate the most.

Here comes a very interesting problem. This is a long period of peace. What to do with the huge crowd of samurai who lost their lords, jobs, and know nothing but how to kill opponents with their long and short swords? This is rather hard question. Just imagine what was expected that a superb man should know? At this time a man should know medicine, art, poetry, tea ceremony, dance and music, arithmetic and of course how to read and write. Therefore it was a huge challenge for a samurai to find a new goal in his life. However, many of them succeeded in this new reality and they became painters, some of them even famous painters, actors, poets, writers, physicians and teachers. The Western readers of this paper can observe this transformation of samurai in a very famous movie series *Musashi* (NHK, 2003).

> **Miyamoto Musashi** (宮本 武蔵) also known as Shinmen Takezō, Miyamoto Bennosuke, or by his Buddhist name Niten Dōraku, was a Japanese swordsman and samurai famed for his duels and distinctive style. Musashi, as he was often simply known, became renowned through stories of his excellent swordsmanship in numerous duels, even from a very young age. He was the founder of the Hyōhō Niten Ichi-ryū or Niten-ryū style of swordsmanship and the author of *The Book of Five Rings* (五輪書 Go Rin No Sho), a book on strategy, tactics, and philosophy that is still studied today (source Wikipedia).

As a result of all these changes many of the samurai started creating schools, known as *juku*. Most of them were small country schools with few students. Major topics taught there were reading, writing and arithmetic. Who attended these schools? Usually, the students were adults, starting from samurai, craftsmen, and merchants to countrymen. This was the place where there are roots of sangaku and the origins of specific mathematics that we can find on sangaku tablets.

3. How wasan is related to the western mathematics

Looking at the history of Japanese mathematics in this period we can find two kinds of mathematics – the one, very picturesque, on sangaku tablets developed by juku students as well as a more serious mathematics similar, in many aspects, to the mathematics developed at this time in Europe. Surprisingly, mathematicians in Europe and in Japan were working on similar problems. History of Japanese mathematics is not the main topic of this paper; therefore we will mention here only a few major facts. Readers interested in the history of Japanese mathematics can find a good source of information in the book by Smith & Makami (see [1]).

Now, let us briefly look what Japanese mathematicians did during this time. They developed a theory of determinants even more advanced than the one created by *Leibnitz* later in Europe. They discovered many geometry theorems that were discovered later in Europe by *Casey*, *Malfatti*, or *Soddy*. They perfected soroban calculations including multiplication and division of numbers. In fact, they were very good in handling arithmetical operations on big numbers, as well as equations of very high orders. They developed a theory that is similar to our integral calculus. They published books on calculating areas and volumes of complicated objects obtained from intersection of solids. Some of the most famous Japanese mathematicians of this period are *Seki Kowa* or *Seki Takakazu*, *Yoshida Mitsuyoshi*, *Immamura Tomoaki*, *Muromatsu Shigekyo* and many others. In 1627 *Yoshida Mitsuyoshi* published the book *Jink-ki*, which means *Big and Small Numbers*. Jink-ki was probably the most popular mathematical book in Japan and it had more than 300 editions.

> **Seki Takakazu** (関孝和, 1642 – December 5, 1708), also known as **Seki Kōwa** (関孝和), was a Japanese mathematician in the Edo period. Seki laid foundations for the subsequent development of Japanese mathematics known as *wasan*; and he has been described as Japan's "Newton." He created a new algebraic notation system, and also, motivated by astronomical computations, did work on infinitesimal calculus and Diophantine equations. A contemporary of Gottfried Leibniz and Isaac Newton, Seki's work was independent. His successors later developed a school dominant in Japanese mathematics until the end of the Edo era. (Source Wikipedia)

4. A mini gallery of sangaku

Before we will examine some examples of sangaku it would be interesting to find response to a few questions. Who created sangaku, why were they posted in shrines, and how did they look?

As we mentioned already at this time in Japan there were two main religions – Shinto, a traditional religion in Japan, and Buddhism – the religion that came from China with hundreds of other commodities. In Shinto, as well as in Buddhism, there is no one god. There are many gods or spirits named Kami. As the legends say Kami like horses. Therefore, some believers who couldn't afford a live horse were bringing to a Shinto shrine a picture of a horse painted on a wooden tablet. Some of these tablets were real pieces of art. During the Edo period, instead of horses, people started bringing sangaku to the Shinto and Buddhist temples. This shows us that mathematics and mathematical theorems were valued as something extremely beautiful for those people.

Who were those people? Inscriptions on existing sangaku show that there was quite a large diversity of sangaku authors. The authors were children and adults, people well educated and people with very basic knowledge of mathematics, merchants and craftsmen, men and women. Mathematical problems posted there were from very simple facts to very advanced mathematical theorems.

Some of the sangaku were just simple pieces of wood with some basic text and usually a picture if the content was related to geometry. Some other sangaku were prepared on large wooden boards, with many theorems, floral decorations and pictures. There were very small sangaku and huge ones a few meters long with massive amount of information on them.

Let us examine a few selected examples. A good source of sangaku pictures is the Japanese web site [I7]. The information posted there is in Japanese. However, we can get some sense of it if we open the web pages in Google Chrome and allow the program to translate them for us. On this web site we can also find information about *sangi* – the system of calculations with wooden rods that was used during the Edo period. Some of the presented here sangaku are quite new, some other are copies or reconstructions of old sangaku from the Edo period and later times. It is sad that after the opening of Japan to the West wasan was abandoned almost entirely in favor of western mathematics and this wonderful custom of developing and bringing sangaku to the temples almost completely disappeared. However, we still can find sangaku from our times.

Example 1 Sangaku from Isaniwa Jinjya shrine

Fig. 2 *Sangaku from the Isaniwa Jinjya shrine*, *year 1937*, *size 107x77 cm*, http://isaniwa.ddo.jp

Here we have a simple problem from elementary geometry. The text is written in *kambun* a language that is similar to traditional Chinese, and it can be translated as follows.

> **Right:** Given four (equal) middle circles contained in the big circle, and a small circle inside the four. Find the radius of the big circle in terms of the radius of the small circle.
>
> **Middle:** Solution
>
> **Left:** December 1937, place Yanamachi, Matsuyama city, Ehime prefecture, Isaniwa shrine.

We will come back to this sangaku in the next section of this paper.

Example 2 Sangaku from the Takemizuke shrine

Fig. 3 *Reconstructed sangaku from the Takemizuke shrine, Nagano prefecture,* 450x200*cm*

Below we enclose an attempt of translation of selected parts:

Three Mutually Tangent Circles of the Same Size (first on the left)

Given: [Not sure] some algebraic relation between the radius of the red circle.
Find: the radius of the big circle.

Six Circles Inside a Right Triangle (second on the left)

Given:
(1) Six circles with two yellow circles having the same size.
(2) [Not sure] The sum of radii of three smaller circles multiplied with the radius of the blue circle equals the sum of the sides plus 64216 歩.
(3) All tangencies are as indicated.
Find: the sides of the triangle.
Answer: Two sides are 38寸 5分 6厘 6毫 and 82寸 5分 2厘 7毫.

Four Circles Inside a Triangle (third problem on the left)

Given:
(1) Four circles with two red circles having the same size.
(2) Blue circle has radius 8, each of the two red circles has radius 4寸 5分.
(3) All tangencies are as indicated.
Find: the sides of the triangle.
Answer: 41寸 4分 2厘 7毫, 29寸 5分 3厘 3毫, 15寸 5分 3厘 4毫

Example 3 Sangaku from Io shrine

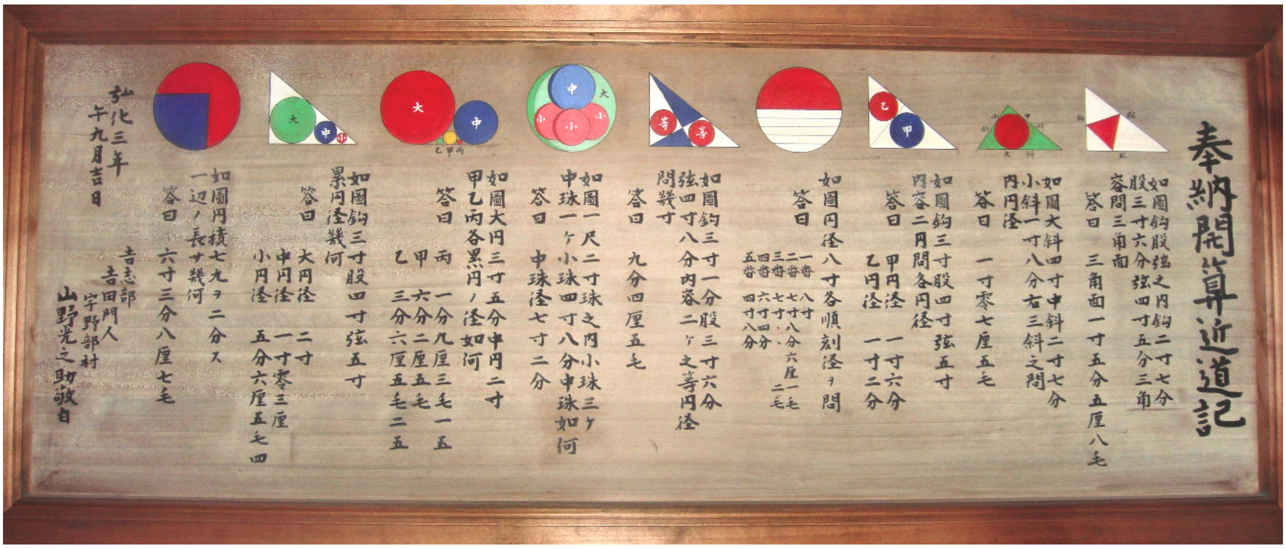


Fig. 4 *Sangaku from Io shrine, Osaka prefecture, year 1846, size* 182x60cm

Two triangles (first from the right)

Given: (1) the length of hypotenuse (4寸 5分 3角) , (2) the sides(2寸 7分 and 3寸 6分).
Find: the area of the [regular] triangle.
Solution: (1寸 5分 5厘 8毛)
[**Comment:** Although each character stating the length remains the same now as in 1846, it is unclear if they are decimal or not, not to say what is the equivalent length given in metric system.]

In-Circle (Second from the right)

Given: sides of a triangle (4寸，2寸 7分，1寸 8分),
Find: the radius of the inscribed circle (1寸 7厘 5毛).
[**Comment:** (1) It is unknown if 寸，分, 厘, and 毛 are decimal units or not. (2) There is no claim to the exactness of the figures.]

Two Circles in a 3-4-5 Triangle (third from the right)

Given: a 3-4-5 triangle and two mutually tangent circles 甲，乙 as indicated.
Find: the radius of circles 甲, 乙.
Solution: radius of circle 甲=1.6, radius of circle 乙=1.2.

Example 4 Collection of sangaku problems from Istukushima shrine

Fig. 5 *Four sangaku problems from Istukushima shrine, year 1885, size of each of them* is 40x40cm

Seven Circles and a Semi-Circle (top left)

Given: (1) three circles marked 水 of the same size, (2) two circles marked 土 have the same radius 1, (3) circles 金 and 火 and the semicircle are tangent as displayed.
Find: the radius of the circle marked 火.
Answer: 3 (with undecipherable reasons.)

Tangential Chords (bottom left)

Given: Semi-circle and [the maximum] inscribed circle 甲.
Find: the length of the tangential chord marked 斜.
Solution: adding the radii of the circles 乙 and 丙 [and some arithmetic operations].

Five Circles, Two Semi-Circles in a Rectangle (top right)

Given: (1) circles marked 乙 have the same radius, (2) circles marked 丙 have the same radius, (3) two semi-circles have the same diameter as the length of the rectangle (circle marked 甲 has diameter the width of the rectangle.). All tangencies are as shown.
Find: the area of the black region (in terms of the area of the circle marked 丙).

Rhombus and Five Circles (bottom right)

Given: (1) Big circle 甲 is tangent to the two tangential circles 乙 of radius 4 and to two circles 丙 of the same radius. (2) The rhombus (with two diagonal vertices lying on circle 甲) is tangent to circles 乙, 丙 as shown.
Find: the radius of circle 丙.,
Answer: 1.

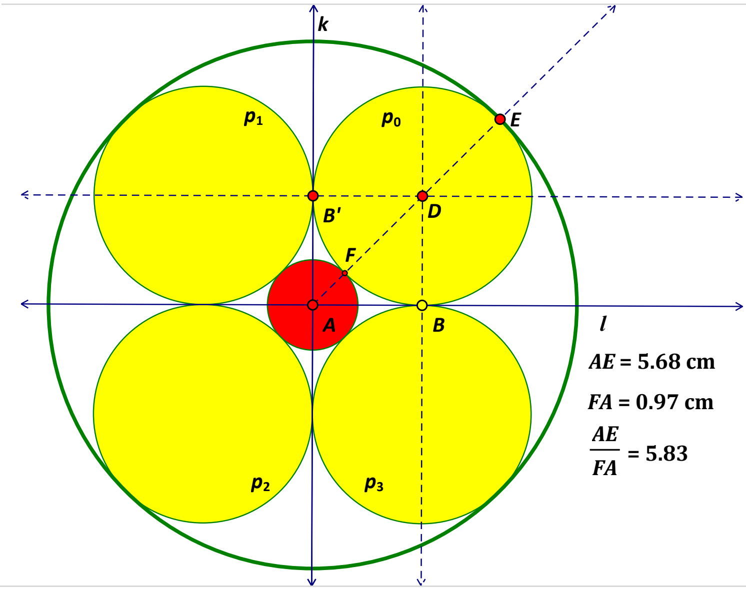
5. Constructing sangaku with Dynamic Geometry software

Nowadays most of the problems that appear on the old sangaku were solved already. We can find a number of publications with detailed solutions of these problems; see for example books [2] and [3], as well as web sites [I1, I2, and I3]. Many of them are wonderful examples for our students. Solving them quite often requires only a basic knowledge of *Euclidean* geometry including *Thales* and *Pythagoras* theorems. For the harder problems we may need more advanced tools, or methods, from elementary geometry, for example knowledge of an inversion, sometimes calculus can be quite useful.

We can also look on sangaku problems through dynamic geometry software, try to construct some of them and verify with a computer program. Many of the sangaku problems are not so simple to construct like we might think looking on the pictures. Especially those where we have circles inscribed into something more complicated than a triangle can lead to very elaborate constructions. There is definitely one thing that we notice looking on the pictures enclosed on sangaku. They are very much different than this what we used to see in geometry textbooks in the West.

Each Dynamic Geometry program gives us different construction tools and different approach. For example, in Cabri or GSP we have all classical construction tools and verification of a mathematical property is only possible on the level of real numbers. At the same time Geometry Expressions[1], in short GX, allows us to use constraints that are simply relations that we can apply to our objects, e.g. make them perpendicular, tangent, etc. In Geometry Expressions it is possible to verify a mathematical property in the same way like in Cabri and GSP. But we can also obtain a formula or formulae describing our construction. In many cases this is just this what the authors of sangaku problems asked for. Let us look on a simple example and see how both methods work.

Example 6a Construction of sangaku problem from Isaniwa Jinjya shrine using GSP



Construction in GSP

Construct perpendicular lines l and k

Construct circle p_0 tangent to l and k

Create reflections of the circle about the lines l and k in order to obtain circles p_1, p_2, and p_3

Construct a bisector of the angle B'AB, this way we will obtain two new points F and E

Use point F to draw a circle with radius AF and center A

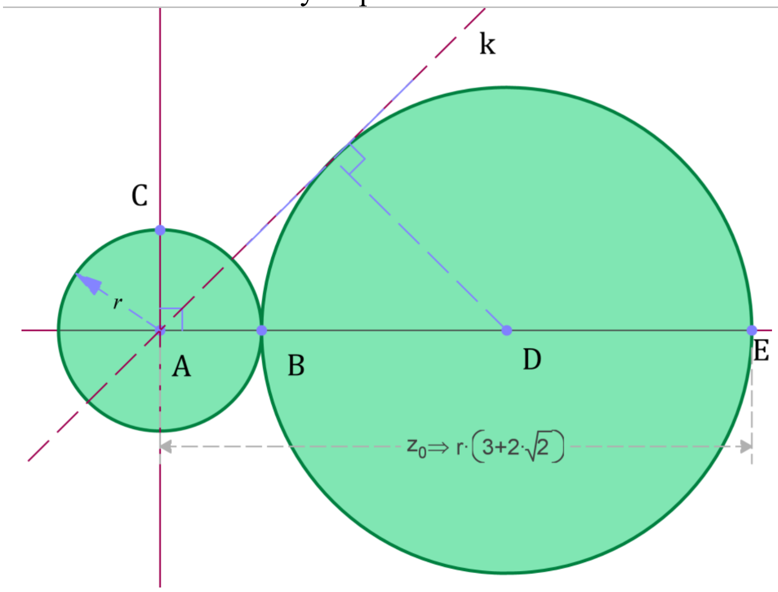
Use point E to draw large circle with radius AE and center A.

[1] Geometry Expressions is the most recent software for Dynamic Geometry. It uses constraints, which are in fact declarations of relations between objects. For example we can draw two intersecting lines and use perpendicularity constraint to make them perpendicular, we can draw a circle and a line and apply tangency constraint to make them tangent, etc. Another important feature of GE is the ability to calculate results in symbolic form. For example we can construct a circle inscribed in a triangle and obtain a formula expressing radius, or area, of the circle in terms of the sides of the triangle.

Now we can try to find out how these two radii are related. We can, for example, ask GSP to calculate the length of each radius, AE and AF, and then calculate value of the fraction AE/FA. In the above construction we obtained AE=5.68 cm, AF=0.97 cm, and AE/AF=5.83. By moving one of the construction points, e.g. point B, we can verify that the value of fraction AE/AF will not change. This is all. But if we wish to get the formula describing relation between these two radii we have to use the most classical tools in mathematics – paper, pencil and our brains.

Example 6b Construction of sangaku problem from Isaniwa Jinjya shrine using Geometry Expressions

In Geometry Expressions, we can still use the same construction methods like in GSP. However, we can also use tools specific for symbolic geometry. Here is one of the possible constructions that we can do in Geometry Expressions.



Construction in GX

Draw two perpendicular lines and a small circle with the center in the lines intersection point

Create a perpendicular bisector of the segment CB, we will obtain the line k

Select any point D on one of the lines (here the horizontal one) and draw a circle with the center in D and radius BD

Select the new circle and line k, and apply the tangency constraint

Declare radius of the small circle as r

Ask Geometry Expressions to calculate the formula for the length of the segment AE, this is the radius of the large circle that we didn't draw.

If we did the construction exactly the way as it was described above, Geometry Expressions will produce the formula $R = r(3 + 2\sqrt{2})$. We can easily check that $3 + 2\sqrt{2} \cong 5.828$, and this is exactly the same value that we got in GSP. We have to notice that in GX, like in any other Computer Algebra System, the obtained result may be different depending on the order of calculations. Therefore, if we do the same construction using slightly different order of steps, we may get a different result. For the above example we can get, for example, a result like the one below.

$$R = r + \frac{2r\sqrt{2+\sqrt{2}}}{\sqrt{2-\sqrt{2}}} \cong 5.828r$$

As we can easily see the obtained formula is equivalent to the one that we got originally but not necessary the simplest one.

The example analyzed in this section is very simple. However, our goal was to emphasize differences of constructing and verifying sangaku problems using different software. In many cases a very simple looking sangaku problem may turn into a very complex, and sometimes, extremely hard to obtain geometric construction.

6. The chapter where we talk about Hirotaka Ebisui

Hirotaka Ebisui is a Japanese mathematician, known as a person who is fascinated with the plane geometry. In the last few years he produced a number of geometric constructions that hold a mathematical property. Many of Hirotaka's constructions remind us the pictures that we have seen on sangaku tablets from the Edo period and later. The major difference is that his works are always constructions, while the ones on sangaku tablets were always drawings. It is very difficult to evaluate how many such problems Hirotaka Ebisui created. For the last year one of the authors (M.M) have been collecting all Hirotaka's PDF files and combining them in one large document about 220 pages, and each page containing 3 to 4 theorems. He has also been recreating many of Hirotaka's constructions in GSP and sometimes in Geometry Expressions. Each of them turns out to be a true mathematical property.

Fig. 6 *Hirotaka Ebisui with Professor Gunter Weiss from TU in Dresden*

Original Hirotaka's Ebisui documents are usually very hard to understand and as a result only very few people are able to get a sense from them and appreciate his works.

One day Prof. Gunter Weiss from Technical University in Dresden, in Germany, wrote to me[2]: "*I personally am convinced that Hirotaka is a genius with an incredible instinct for extending and generalizing elementary geometric problems in an interesting and surprising way. He stands in the tradition of the so-called "Japanese Temple Geometry" both, concerning the elementary geometric topics as well as in his behavior not to publish his findings others than showing them to friends with the words "I found a new theorem, please enjoy".*

He has never had a "scientific pupil" and his work is not "mathematical mainstream" at all. But it is so fascinating!

[2] Personal communication with M. Majewski, published with permission of Prof. Gunter Weiss

All this together makes me a true lover of his work, which expresses Japanese philosophy and tradition AND the beauty of pure simple geometry, which has no other aim then Japanese cherry blossoms.

He cannot be forced to work on a certain problem one proposes to him, but he is like a never stopping fountain, when one listens to him and gets stimulated to work on his findings. He very seldom proves his "theorems" and seems to be quite content, if his graphics-software constantly shows the incidences if zoomed. But he also knows the "geometer's toolbox" for elementary geometry very well (Desargues' and Pappus' and Brianchon' theorems, angle at circumference, power of a point with respect to a circle ...) and sometimes adds a proof, if asked for that.

I believe that his findings really are new and that nobody else dealt with this material before him. So the findings (- I call them findings and not theorems -) are due to him."

7. Ten Hirotaka's geometry problems

In this chapter we will show a selection of Hirotaka's Ebisui constructions with a simple explanation what is there, and what should be proved. In order to make these problems a bit more similar to the sangaku problems from Edo period some fills were added. We selected only those problems that are simple enough to be understood by an average high school student. We are not giving here any proofs leaving them to the readers of this paper. However, each of them was confirmed in some way using GSP tools.

Before we will proceed with the examples, it would be important to explain the convention used while developing the enclosed pictures. We use here specific color scheme that cannot be seen in the black-and-white printing. Therefore, we also vary the thickness of lines and points.

Medium thick lines, green, are the starting point of the construction. Thin lines, usually blue, sometimes dashed are the construction lines. In many cases, if these lines are not needed any more, we hide them. However, sometimes we leave them in order to show better how the construction was done. The thick, red lines, are the final ones that usually carry some property to be proved or are part of the final object, e.g. a triangle, rectangle, etc., with specific properties to be proved.

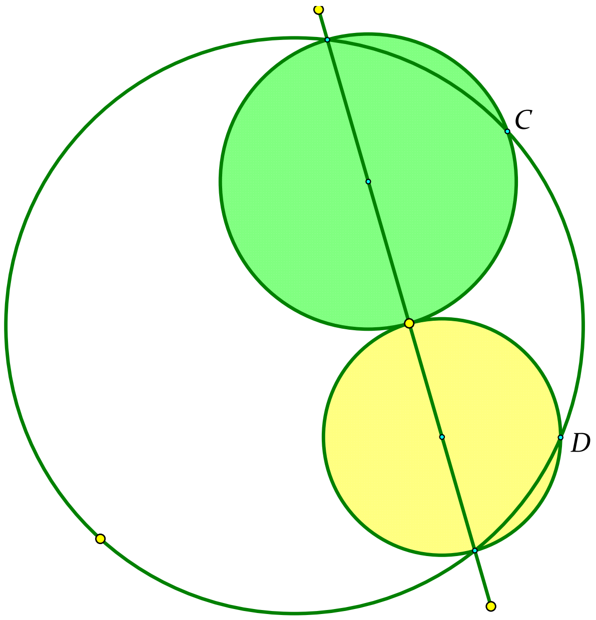
Usually we hide all points that do not matter for the whole construction. We leave only selected points to emphasize some important features, e.g. point on a segment, on a circle, etc. These points are always small and with light blue filling. Finally the medium size points with yellow filling are those construction points that in GSP file, or in GSP online applet, can be moved in order to check what will happen with the property to be proved.

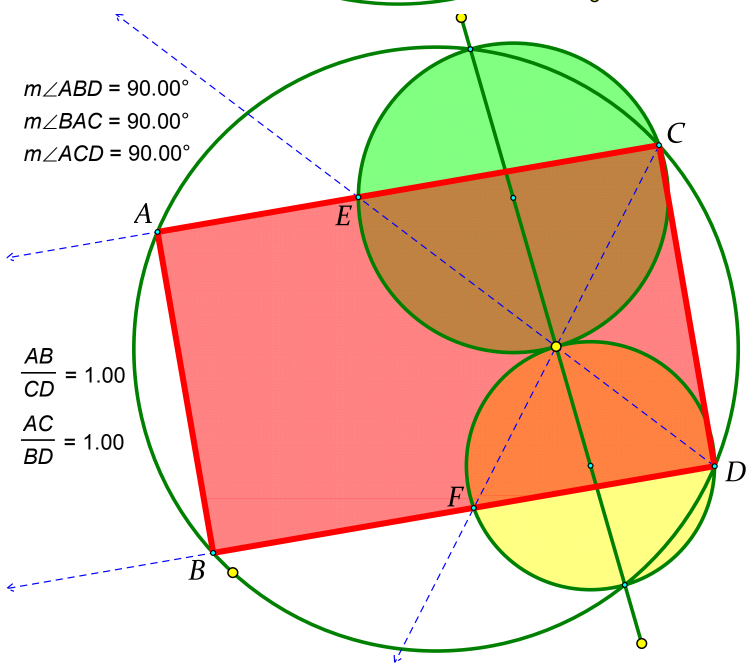
Finally, fills do not follow the above convention. We added them in order to make these constructions similar to the Edo period sangaku pictures.

The numbers for each problem are the original numbers used by Hirotaka Ebisui. In order to avoid some confusion there is one thing that we should mention here. It happens quite often that a few of his constructions may carry the same number.

Some of the presented here problems are a bit more complicated than the remaining. Therefore, we illustrate them using two pictures. The first picture shows the initial stage of the construction, which is with what we start and the second picture, shows the final construction.

Problem HI 001





HI 001

Create a large circle, a segment crossing it, select a point on the segment and construct two small circles passing through the selected point and points of intersection of the segment with the large circle. Centers of the small circles should be located on the segment.

Construct rays from C and D passing through the point of tangency of both small circles. We will obtain two new points E and F.

Construct rays CE and DF, and intersection points A and B on the large circle.

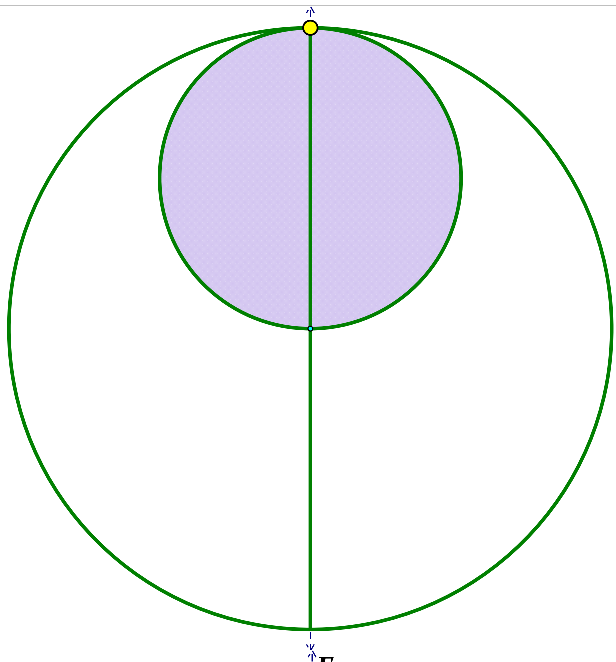
Connect points ABCD and fill the area inside.

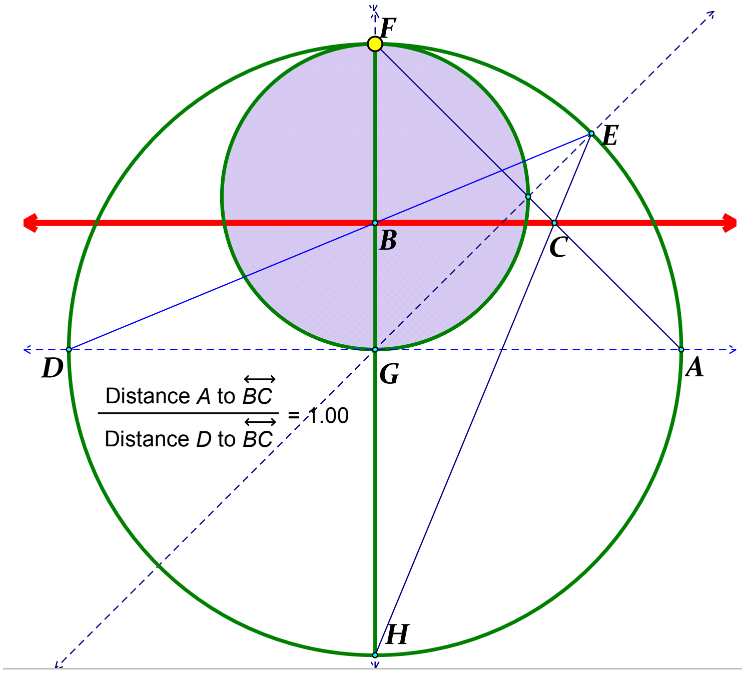
The objective of this problem is to prove that ABCD is a rectangle. In GSP we can measure angles ABD, BAC and ACD. GSP shows that each of them is 90 degrees.

We can also check the proportions AB/CD and AC/BD and GSP shows that each of them is equal to 1.

These properties hold if we move the segment, change size of any circle as long as the segment intersects with the large circle and radius of any circle is strictly larger than 0.

Problem HI 003





HI 003

Start with two tangent circles, smaller inside the large one and passing through the center of the large circle. The vertical segment connects the point of tangency with the other side of the circle and passes through centers of both circles.

Draw a line perpendicular to the segment and passing through the center of the large circle (point G). This way we obtain two new points A and D.

Construct the bisector of the angle AGF, you will obtain a new point E on the edge of the large circle.

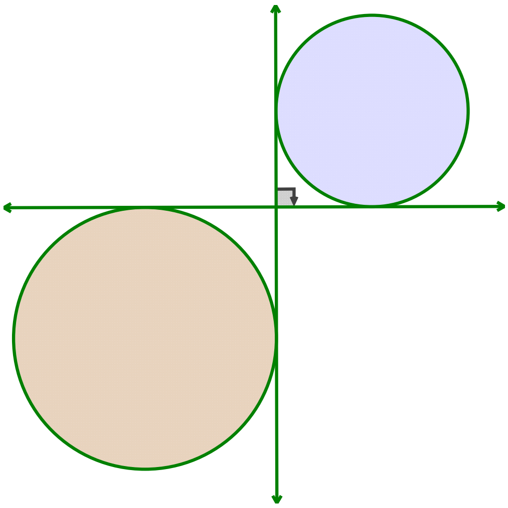
Create segments ED, EH and AF.

Finally, create a line passing through points B (intersection of ED with FH) and C (intersection of bisector and segment AF).

The objective of this problem is to prove that the line BC is parallel to the line AD or perpendicular to FG.

Calculations in GSP show that distances of points A and D from the line BC are exactly the same and independent of the radius of the circle.

Problem HI 004

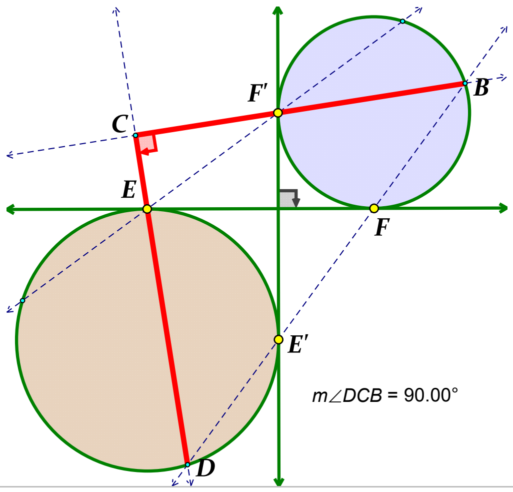


HI 004

This is one of the nicest problems in Hirotaka files. It is repeated in Hirotaka files in a few different versions.

The starting point of this problem are two perpendicular lines, and two circles tangent to both lines and located in the opposite quarters of the plane.

Construct lines connecting points of tangency, each on different circle. You will obtain lines EF' and FE' as well as two new points B and D.

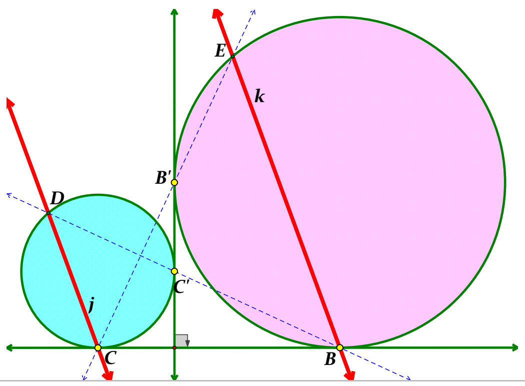


Now construct lines BF' and DE obtain the intersection point C, hide the lines, and draw segments CB and CD.

The objective of this problem is to prove that segments CB and CD are perpendicular.

GSP evaluated this angle as 90 degrees and its value does not change if we change the size of any of these two circles.

Constructions of the following examples are slightly simpler. Therefore, we present them in a short form.



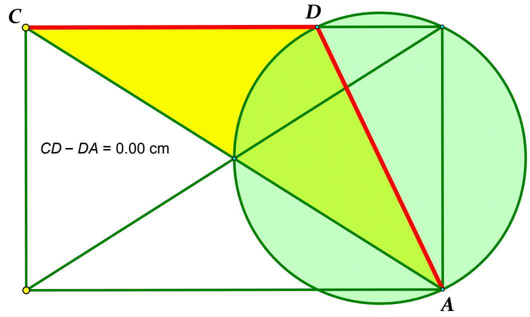
HI 028

This problem is just another version of the problem HI 004. Here both tangent circles are on the same side of one of the two perpendicular lines.

Connect the tangency points to obtain the two dashed lines.

Finally draw lines passing through points of intersection of the dashed lines with circles (lines k and j).

The objective is to prove that lines k and j are parallel.



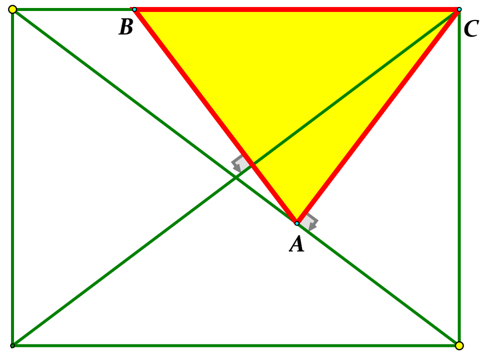
HI 122

This is another simple and nice problem.

Start with a rectangle and a circle. Two adjacent points of the rectangle are on the circle.

One of the intersection points of the rectangle with the circle label as D, construct triangle ADC.

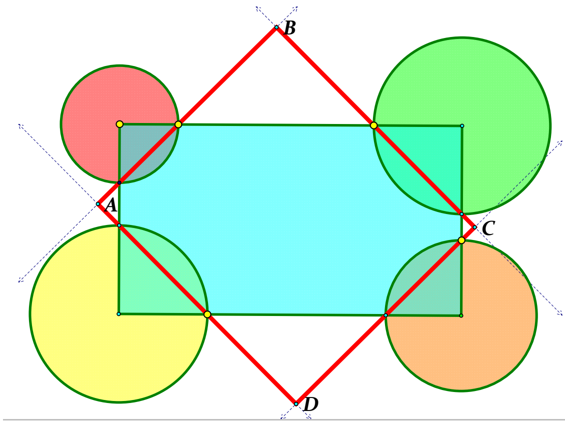
The objective is to prove that the ADC is an isosceles triangle.



HI 290

Start with a rectangle and its diagonals. From one of the vertices construct a segment perpendicular, here CA, to one of the diagonals, and from its other end, here point A, draw another segment perpendicular to the other diagonal. Extend the new segment until it will contact the side of the rectangle, here it is AB.

The objective is to prove that CAB is an isosceles triangle.



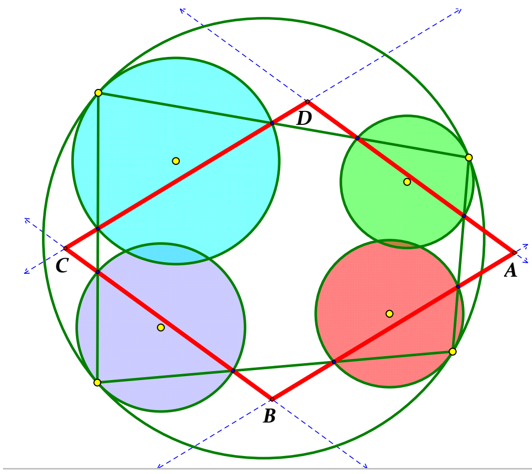
HI 222

A rectangle and four circles are given. Each vertex of the rectangle is a center of one circle.

For each of the circles draw a line passing through points of intersection of the circle with sides of the rectangle.

Construct a polygon ABCD using the four new lines and their intersection points.

The objective is to prove that the polygon ABCD is a rectangle.

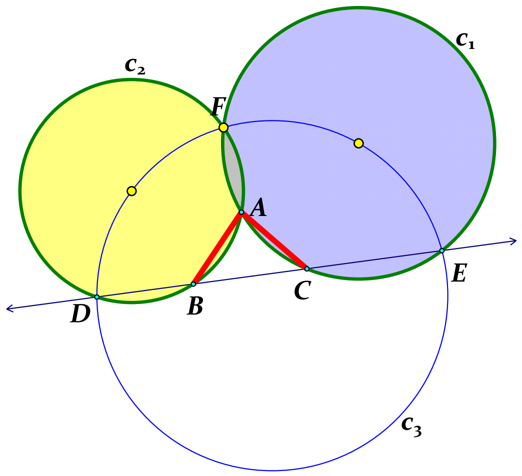


HI 287

This problem is in some sense similar to the problem HI 222. Start with a large circle and four smaller circles inside the large circle and tangent to it. Create a polygon connecting the four points of tangency.

For each small circle draw a line passing through points of intersection of the circle with the sides of the polygon. Use these lines to create a new polygon, on the picture it is ABCD.

The objective is to prove that ABCD is a parallelogram.



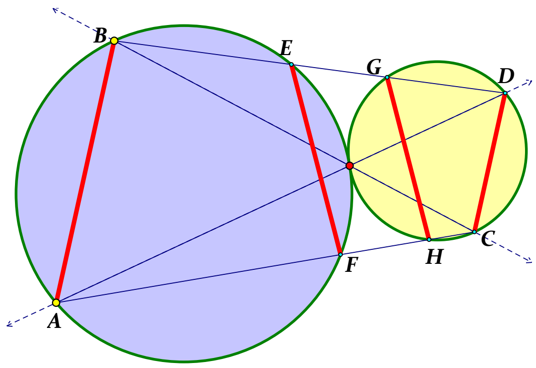
HI 069

This is Hirotaka's favorite problem. He published it on one the problem solving portals in Japan and he got quite enthusiastic response.

Two intersecting circles c_1 and c_2 are given. Through one of the points of intersection, here point F, and centers of the circles construct a new circle c_3. Points of intersection c_3 with two existing circles label as D and E.

Draw line passing through points D and E. The line intersects the two circles c_1 and c_2 in two new points B and C. Connect points A with B and A with C.

The objective is to prove that $|AB|=|AC|$



HI 386

There are given two tangent circles. On one of them choose arbitrarily two points A and B.

Construct two lines each one passing through the tangency point of circles and points A and B respectively. Label points of intersection of the line with the other circle as D and C. Draw segments BD and AC.

Draw segments connecting opposite points on the edge of each circle, i.e. AB, EF, GH, and DC.

The objective is to prove that $AB||DC$ and $EF||GH$.

8. Conclusions

We could continue for many hours, days, and perhaps months, explorations of the Hirotaka files. However, we do not have so much time and space in the conference proceedings. We, the authors, believe that the provided here examples prove the value of Hirotaka works. Some of Hirotaka's problems can be used by teachers as exercises in problem solving for mathematically gifted high school, as well as university, students. Some other of his problems can be a starting point for advanced research in elementary geometry. Many of them should be considered as new valuable theorems in elementary geometry.

9. References

[1] *A history of Japanese Mathematics*, D. E. Smith & Y. Mikami, Cosimo, Cosimo Classics series, ed. 2007.

[2] *Sacred Mathematics – Japanese Temple Geometry*, H. Fukagawa & T. Rothman, Princeton University Press, ed. 2008.

[3] *Japanese Temple Geometry Problems*, H. Fukugawa and D. Pedoe, Charles Babbage Research Foundation, Winnipeg, Canada, 1989.

[4] *Japanese Temple Geometry*, T. Rothman, Scientific American, 30, 2003.

Internet resources

[I1] *Sangakus*, R. Nolla & R. Masip, Recursos de Geometria, Department of Mathematics, Taragona, Spain, 2009, `http://www.xtec.cat/~rnolla/Sangaku/SangWEB/PDF/Sangak_4.pdf`

[I2] *SANGAKUS*, Contemplació i raó, R. Nolla, `http://www.xtec.es/~rnolla/Sangaku/Sangakus3b.pdf`

[I3] *Mark's Math and Excel pages*, `http://www.mfdabbs.pwp.blueyonder.co.uk/`

[I4] *Sangaku*, Wikipedia, `http://en.wikipedia.org/wiki/Sangaku`

[I5] *Sangaku*, `http://www.sangaku.info/`

[I6] *Sangaku page* at `http://www.cut-the-knot.org/pythagoras/Sangaku.shtml`

[I7] *Japanese Wasan web site at* `http://www.wasan.jp/`

[I8] *Hirotaka files at* `http://mirek.majewscy.net/hirotaka_files/`

The Authors

Miroslaw Majewski (馬裕祺)
mirek.majewski@yahoo.com
School of Arts & Sciences,
New York Institute of Technology, Abu Dhabi campus, UAE

Jen-Chung Chuan ()
jcchuan@math.nthu.edu.tw
Department of Mathematics,
National Tsing Hua University, Hsinchu, Taiwan

Nishizawa Hitoshi (西澤 一)
nisizawa@toyota-ct.ac.jp
Department of Electrical and Electronic Engineering
Toyota National College of Technology, Toyota, Japan