

2020 年度日本政府(文部科学省)奨学金留学生選考試験

QUALIFYING EXAMINATION FOR APPLICANTS FOR THE JAPANESE
GOVERNMENT (MEXT) SCHOLARSHIP 2020

学科試験 問題

EXAMINATION QUESTIONS

(学部留学生)

UNDERGRADUATE STUDENTS

数 学 (A)

MATHEMATICS(A)

注意 ☆試験時間は **60 分**。

PLEASE NOTE: THE TEST PERIOD IS **60 MINUTES**.

Nationality		No.		Marks
Name	(Please print full name, underlining family name)			

1. Answer the following questions and fill in your responses in the corresponding boxes on the answer sheet.

- (1) For sets A , B , and C , we assume $C \subset A$. The number of elements in set A is 66, that in set A but not in set C is 47, that in set B but not in set C is 42, that in set C but not in set B is 8, and that in set A but not in either set B or set C is 31. Then, the number of elements included in set A , B , or C is [1-1].

- (2) Consider a graph of the function $y = x^2$ in xy -plane. The minimum distance between point $(0, 4)$ on the y -axis and points on the graph is [1-2]. You should rationalize the denominator in the answer.

- (3) Consider a graph of the function $y = 2x^2 - 6x + 2$ in xy -plane. Next, consider another graph that is symmetrical to the previous graph with respect to the line $x = 2$. The graph that is symmetrical to the latter graph with respect to the line $y = 3$ is described as

$$y = a_2x^2 + a_1x + a_0.$$

Then, we have $a_2 = \text{[1-3]}$, $a_1 = \text{[1-4]}$, and $a_0 = \text{[1-5]}$.

- (4) Let x and y be integers. We assume that $x + y$ is a multiple of 2, $x + 4y \leq 17$, and $3x + 2y \leq 21$. Then, $x + 2y$ is maximum when $x = \text{[1-6]}$ and $y = \text{[1-7]}$. The maximum value is [1-8].

- (5) Consider two graphs of the functions $y = \frac{1}{8}x^2 - 2$ and $y = \frac{1}{2}x^2 - 8$ in xy -plane. We describe a common tangent line of the two graphs in xy -plane as

$$y = a_1x + a_0.$$

We assume that the x -coordinates of both tangential points are positive. Then, we have $a_1 = \boxed{[1-9]}$ and $a_0 = \boxed{[1-10]}$.

- (6) When we set $t = \cos x$ for a function $f(x) = \cos 2x + \cos 3x$, $f(x)$ has an expression in t as follows:

$$\boxed{[1-11]} t^3 + \boxed{[1-12]} t^2 + \boxed{[1-13]} t + \boxed{[1-14]}.$$

- (7) For a set $A = \{2, 3, 5\}$, we randomly choose one element in A three times. Let B_k be a number of the k -th trial in the three trials and let $C = B_1 \times B_2 \times B_3$ be the product of $\{B_i\}$. The probability that C is an odd number is $\frac{\boxed{[1-15]}}{27}$ and the probability that C is a multiple of 5 is $\frac{\boxed{[1-16]}}{27}$.

- (8) A function $f(x) = x(x-6)^2$ has the extreme values at $\boxed{[1-17]}$ and $\boxed{[1-18]}$, where $\boxed{[1-17]} < \boxed{[1-18]}$. If we define $g(x) = |f(x)|$ and we consider the numbers of different real solutions of the equation $g(x) = a$ of x according to a constant a , then the maximum number of real solutions is $\boxed{[1-19]}$.

- (9) For eight data 1, 1, 3, 5, 6, 8, 9, 15, the sample mean is $\boxed{[1-20]}$. If we define a deviation as the difference of each data from the sample mean, the sum of squares of the deviations is $\boxed{[1-21]}$ and the mean is $\boxed{[1-22]}$.

2. Take two points B and C on the circumference of a circle, whose center is denoted by O. We assume that the three points O, B, and C are not collinear. We consider a straight line that is tangential with the circumference on point B. We define point A on the tangential line, such that $\angle ABC > \frac{\pi}{2}$. Furthermore, line CA is a bisector of both angles $\angle BAO$ and $\angle BCO$. We assume that the lengths of edges AB and CB are 2 and 1, respectively. We denote the intersection of lines OB and CA by D. And we denote the lengths of edges BD and OD by x and y , respectively. Answer the following questions and fill in your responses in the corresponding boxes on the answer sheet. They should be simplified as much as possible.

- (1) From the assumption that edge AD is a bisector of angle $\angle BAO$, the length OA is described as $\boxed{[2-1]}$ by using x, y .
- (2) From the assumption that edge CD is a bisector of angle $\angle BCO$, y is described as $\boxed{[2-2]}$ by using x .
- (3) Therefore, we have

$$x = 4 - \boxed{[2-3]},$$

$$y = \frac{\boxed{[2-4]}}{3} - 4.$$

3. We divide the sequence of natural numbers $1, 2, 3, \dots$ as follows:

$$\underbrace{1}_{\text{first group}} \quad | \quad \underbrace{2, 3, 4}_{\text{second group}} \quad | \quad \underbrace{5, 6, 7, 8, 9}_{\text{third group}} \quad | \quad \dots$$

Here the n -th group ($n = 1, 2, 3, \dots$) has $(2n - 1)$ elements. Let a_n be the first number in the n -th group and let S_n be the total sum of the numbers in the n -th group. Answer the following questions for the sequences $\{a_n\}, \{S_n\}$.

(1) For the sequence $\{a_n\}$, the n -th term is

$$a_n = \boxed{[3-1]} n^3 + \boxed{[3-2]} n^2 + \boxed{[3-3]} n + \boxed{[3-4]}.$$

(2) 2678 is in the $\boxed{[3-5]}$ -th group, and in the group it is the $\boxed{[3-6]}$ -th term.

(3) For the sequence $\{S_n\}$, the n -th term is

$$S_n = \boxed{[3-7]} n^3 + \boxed{[3-8]} n^2 + \boxed{[3-9]} n + \boxed{[3-10]}.$$