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G. W. F. Hegel: *Philosophical Dissertation on the Orbits of the Planets* (1801)

Preceded by the 12 Theses Defended on August 27, 1801

Translated, with Foreword and Notes, by Pierre Adler

Foreword

> "People are pleased with the force of gravity's being known; and, thoroughly instructed, above all by the rather distressing story of the apple that fell before Newton, that heavenly bodies revolve in their orbits less because of the force common to the world, which Kepler and other philosophers established as one and the same, than in virtue of the everyday force which pulls stones toward the earth, people derive assurance against the sky, forgetting of course that an apple was present at the beginning of the universal misfortune of human kind and at the inception of the misfortune of Troy, in turn a bad omen for the philosophical sciences." Hegel, *On the Orbits of the Planets*

1. *Why Publish a Translation of "De Orbitis Planetarum"?*

Why publish this cryptic, seldom read, discussed or studied text of Hegel's, this scandalous piece of writing, which may be cause for embarrassment to Hegelians who know of its existence and which was deleted from inclusion in the German paperback edition (Theorie Werkausgabe, Suhrkamp Verlag) of Hegel's works?

One may adduce several answers. Firstly, with it Hegel began his itinerary as a university professor: it is his *Habilitationsschrift*. Secondly, in it Hegel formulates for the first time some elements of a *Naturphilosophie*: the writing shows that *naturphilosophische* considerations formed a vital and intrinsic constituent of Hegel's thought at its very inception. Thirdly, Hegel's understanding of Newton and his ideas about matter and motion in his later writings do not undergo any significant change from what they are in the

dissertation. Fourthly, the text might serve as a reminder that Hegel's philosophy of nature may not simply be severed from the rest of his organically ordered system, that Hegel's thoughts about nature and the conceptual organization of the natural sciences should not be evaded by students of his thought. Fifthly, it is one of the few attempts made by a German Idealist to investigate a concrete scientific question. Lastly, there is a sense in which Hegel's philosophy of nature constitutes—regardless of the question of its validity—one of the last attempts made by a philosopher to articulate the unity of physics and philosophy, and the dissertation was the first step taken by Hegel in that attempt.

The theses' connection to the dissertation will be mentioned below.

2. *The Latin Text*

Published in 1801, *On the Orbits of the Planets* was written in Latin. The impurity of Hegel's Latin does not make for easy translating. If to that, one adds the obscurity of much of the content, the difficulty of the task is compounded. My rendition keeps as close as possible to the text, although I did not hesitate to break up Hegel's very involved and long periods into shorter sentences.

In my endeavor, I benefited from the work of three previous translators: Georg Lasson, François De Gandt and Wolfgang Neuser.[1] I also derived much valuable help in understanding the text from De Gandt's and Neuser's respective exegetical and historical work on it. Although these two authors do not concur on all matters, their books should be read by anyone interested in *De Orbitis*.[2] More specific debts to their work are acknowledged in my notes to the translation.

The Latin text of *De Orbitis Planetarum* is available in the following two editions of Hegel's works:

> *Sämtliche Werke*, edited by Georg Lasson (Leipzig: Felix Meiner Verlag, 1928), volume 1, *Erste Druckschriften*, pp. 347–401. Lasson's rendition faces the original.
>
> *Sämtliche Werke*, edited by Hermann Glockner (Stuttgart: Frommanns Verlag Günther Holzboog, 1958), volume 1, *Aufsätze aus dem kritischen Journal der Philosophie und andere Schriften aus der Jenenser Zeit*, pp. 1–29.

Moreover, the original text is now available alongside Neuser's

translation.[3] Neuser had the Latin text typeset from a copy of the first edition of the dissertation. It faithfully reproduces both the number of words per line and the number of lines per page of that edition.

As for the theses, they may be found on page 404 of the first volume of the Lasson edition as well as in Neuser's book. They are not included in the Glockner edition.

The first draft of my translation was based on the text of the Glockner edition. The final draft was checked against the original published by Neuser. The rendition of the theses is based on the text of the Lasson edition. Here too, I checked the translation against the Latin version published by Neuser. I should add that I adopted Neuser's rendition of '*omnibus numeris*' in my translation of the twelfth thesis.

3. *The Defense*

Our text inaugurates Hegel's career as a university professor. In order to obtain the license to teach at the University of Jena, where he arrived in January of 1801, Hegel had, among other things, to defend and publish a dissertation. To this effect, he wrote *Dissertatio Philosophica de Orbitis Planetarum*. The defense took place on August 27, 1801, which happens to be Hegel's birthday. Present at the examination were Schelling and his brother Karl, then a student at Jena. However, according to the records of the University of Jena, Hegel did not defend the text devoted to the question of the orbits of the planets on that day, but the twelve theses published along with the dissertation. The theses deal with such topics as logic, philosophy of nature, theory of knowledge and ethics. Although the text of the dissertation was not ready on August 27, its title was known. The university received a printed copy of it on October 18. Clearly, then, Hegel must have written the dissertation during the month of September. In so doing he most certainly used preparatory notes and reflections gathered earlier in the year and over the course of the summer—if not prior to 1801.[4]

4. *Hegel's Scandalous Hypothetical Statement and the Asteroid Ceres*

In addition to two introductory paragraphs, the dissertation comprises three increasingly shorter parts, which Hegel briefly describes

in the second paragraph. The original does not identify these parts with the help of Roman numerals, as I have done in the translation.

The first part proposes a philosophical critique of basic notions of Newtonian mechanics and of the astronomical conceptions that rest on it. The second one unfolds a "philosophical construction" of the concept of solar system, in which Hegel relies on Kepler's and Schelling's thought. It contains some of the most cryptic passages of the whole work.

The relation of the third part to the preceding two is hardly articulated by Hegel save for his saying that knowledge of the laws of nature rests on the belief that there obtains an isomorphism between reason and nature. In it Hegel puts forth a (modified) Pythagorean series of numbers borrowed from Plato's *Timaeus*. The numbers making up this series are supposed to designate the distances of the planets from the sun. Hegel offers his series as a replacement for the Titius-Bode series, which, in view of its arithmetical character, he deems unphilosophical.[5] Now, the Titius-Bode series anticipated there being a celestial body between the orbits of Mars and Jupiter. Hegel, however, claims that on the assumption that the series proposed by him be more consonant with the rational structure of nature than the Titius-Bode one, i.e., on the assumption that the Demiurge conformed nature to it, the large interval between Mars (fourth planet) and Jupiter (fifth planet) need no longer puzzle anyone, for the Pythagorean series accounts for it. We quote Hegel's infamous statement: "If this series is an order of nature truer than the arithmetical progression, then it is manifest that there is a large space between the fourth and fifth positions and that no planet is lacking there." For this assertion—and probably for a few more to be found in *De Orbitis*—Hegel was accused of impeding the progress of science, of ruling out the existence of a celestial body on the basis of *a priori* ruminations, of not even being worthy of untying Newton's shoelaces, etc.[6] Indeed, Hegel's series, if accepted, leaves no room for a heavenly body between Mars and Jupiter. But to his detriment, on January 1, 1801, Giuseppe Piazzi (1746–1826) discovered an object, which he first construed to be a comet and subsequently the planet anticipated by the Titius-Bode series between the orbits of Mars and Jupiter.[7] Piazzi made his observation at Palermo, Sicily. He called the object of his discovery Ceres, after the tutelary divinity of that island. It turns out that Ceres is indeed an asteroid located between Mars and Jupiter. The news of this discovery was announced in a Jena journal on May 6, 1801, as well as in two other German publica-

tions,[8] but Hegel knew nothing of it. In June of the same year, Franz von Zach (1754–1832) published a more extensive treatment of the discovery in *Monatliche Correspondenz zur Beförderung der Erd- und Himmelskunde.*[9] According to Neuser, Hegel "later" became acquainted with this last report.[10] In the first edition of the *Encyclopedia* (1817), Hegel alluded to his dissertation in these terms: "What I attempted to do with this topic [that of the series of planetary distances] in an earlier dissertation, I can no longer regard as satisfactory."[11]

NOTES

1. G.W.F. Hegel, *Über die Planetenbahnen*, translated by Georg Lasson, in G. W. F. Hegel, *Sämtliche Werke*, Band I, *Erste Druckschriften*, ed. Georg Lasson (Liepzig: Felix Meiner Verlag, 1928), pp. 347–401. G.W.F. Hegel, *Les Orbites des Planètes*, translated, with an introduction, notes and appendices, by François De Gandt (Paris: Librairie Philosophique J. Vrin, 1979). G.W.F. Hegel, *Dissertatio Philosophica de Orbitis Planetarum. Philosophische Erörterung über die Planetenbahnen*, translated, with introduction and commentary, by Wolfgang Neuser (Weinheim: Acta humaniora d. VHC, 1986). There also exists an Italian translation: G.W.F. Hegel, *Le orbite di planeti*, translated, with introduction and commentary, by A. Negri (Roma, 1984).
2. For further bibliographical information, see the bibliographies in De Gandt's and Neuser's books.
3. See note 1.
4. See T.G. Bucher, "Wissenschaftstheoretische Überlegungen über Hegels Planetenschrift," *Hegel Studien* 18 (1983), pp. 65–137, p. 72 (in particular note 35) and Neuser, *op. cit.*, pp. 2–3. As Bucher, Neuser mentions that in composing his thesis Hegel drew on a sizeable manuscript of his devoted to Newton and Kepler, as well as on a study of the calculus. These writings were in German. Neuser estimates that they were destroyed by Hegel's son Immanuel in 1855 (see *ibid.*, p. 63 note 7).
5. For the Titius-Bode series and the history leading to it, see Michael Martin Nieto, *The Titius-Bode Law of Planetary Distances: its History and Theory* (New York: Pergam Press, 1972). The history in question involves Kepler, Christian Freihen von Wolf, Lambert and Kant.
6. See note 37 to my translation.
7. Neuser, *op. cit.*, p. 53.
8. Bucher, *op. cit.*, p. 68 note 15, and p. 91.
9. *Ibid.*, pp. 68–69, and 91.
10. Neuser, *op. cit.*, p. 69 note 143.
11. Bucher, *op. cit.*, p. 92. Bucher points out that the reference to the dissertation was deleted from the 1827 edition (p. 92 note 90). When quoting this passage, Neuser refers to the 1817 edition of the *Encyclopedia of the Philosophical Sciences*: see Neuser, *op. cit.*, p. 69 note 123, and p. 70 note 158. This text is also cited on page 1 of Neuser's book.

Dissertationi Philosophicae

de

Orbitis Planetarum

Praemissae Theses

Quas

Rectore Academiae Magnificentissimo
Serenissimo Principe ac Domino

Domino
Carolo Augusto
Duce Saxoniae Juliaci Cliviae Montium
Angariae et Guestphaliae Rel.

Consentiente
Amplissimo Philosophorum Ordine

Pro Licentia Docendi
Rite Obtinenda

Publice Defendet
Die XXVII. Aug. a. MDCCCI

Ge. Wilh. Frid. Hegel

Philosophiae Doctor

Socio Assumto
Carolo Schelling
Wirtemb.

Jenae
Typis Prageri et Soc.

Theses

Appended to the

Philosophical Dissertation

On the Orbits of the Planets

Which,

The Rector of the Academy Being
The Very Magnificent and Most Noble Prince and Master

Charles Augustus
Duke of Saxony, of Juelich, of Cleves,
of the Mountains of Hungary and Westphalia, etc.,

With the Consent of the Great Order of Philosophers,
With a View to Obtain,
in Keeping with Custom,

The License to Teach,

Georg Wilhelm Friedrich Hegel,

Doctor of Philosophy,

Will Defend Publicly
On the 27th of August, 1801,

Karl Schelling
Being the Partner.

Jena
Prager Press and Co.

THESES

I.

Contradiction is the rule of the true, non-contradiction is the rule of the false.

II.

The syllogism is the principle of Idealism.

III.

The square is the law of nature, the triangle is the law of mind (*mens*).

IV.

In true Arithmetic there is no room for addition other than unity's being added to the dyad, and no room for subtraction other than the dyad's being removed from the triad, and no room for the triad that is a sum, nor for the unity that is difference.

V.

Just as the magnet is the natural lever, so too the gravitation of the planets toward the sun is the pendulum of nature.

VI.

The idea is synthesis of the infinite and the finite, and the whole of philosophy consists in ideas (*est in ideis*).

VII.

Critical philosophy lacks Ideas; it is an imperfect form of Scepticism.

VIII.

The matter of the postulate of reason, which critical philosophy exhibits, destroys this very philosophy, and is the principle of Spinozism.

IX.

The state of nature is not unjust; on that ground one must leave it.

X.

The principle of moral science resides in our having to revere fate (*reverentia fato*).

XI.

Virtue excludes innocence of action (*agendi*) and of passion (*patiendi*).*

XII.

Absolute morality is in every respect incompatible with virtue.

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* Translator's remark: 'Action' and 'passion' are opposed as 'agent' and 'patient'.

Dissertatio Philosophica

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Orbitis Planetarum

Quam

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Pro Licentia Docendi
Rite Obtinenda
Publico Examini

submittit

Ge. Wilh. Frid. Hegel

Philosophiae Doctor.

Jenae
Typis Prageri et Soc.
MDCCCI

Philosophical Dissertation

on

The Orbits of the Planets

Submitted,

The Rector of the Academy Being
The Very Magnificent and Most Noble Prince and Master

Charles Augustus
Duke of Saxony, of Juelich, of Cleves,
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With the Consent
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1801

Although, with the exception of the celestial bodies, all the other [348–349]
bodies generated by nature, being perfect in their genus, express the shape (*species*) of the Universe,[1] they are not self-sufficient in regards to the first force of nature, which is gravitation, and perish suppressed by the force of the whole. The celestial bodies, however, not counting among the glebe laden bodies and bearing within them their center of gravity more perfectly, proceed through the light air in the manner of the Gods: no other expression of reason (*ratio*)[2] is more sublime and purer than this animal which we call the system of the sun, nor is any expression of reason more worthy of philosophical contemplation. And the praise bestowed upon Socrates by Cicero for having brought philosophy from the sky and introduced it into the life and dwellings of men, either is to be considered of little value, or must be interpreted in such a way that we say that philosophy cannot be of any merit to the life and dwellings of men unless it descends from the sky, and that efforts must be geared toward elevating it to the sky.

The restricted space of a dissertation is not sufficient to the treatment of such a vast subject. It allows only the transmission of its elements. Under these conditions, I shall concern myself with an elucidation of the first concepts[3] on which the physical part of the science of astronomy customarily depends. Thereafter, I shall expose what the true philosophy establishes concerning the make-up of the solar system, especially as regards the orbits of the planets. Lastly, by means of an illustrious example taken from ancient philosophy, I shall show what value philosophy has in the determination of the mathematical relations (*rationes*) among quantities.

I

Anyone who accedes to this part of Physics easily sees that it is a [350–351]
mechanics of the sky rather than a physics, and that the laws exhibited by this astronomical science draw their origin from another science, the mathematical one, rather than being truly drawn out of nature itself, or constructed by reason (*ratio*). After the felicitous talent of our great Kepler had discovered the laws by which the planets are made to turn in their orbits, Newton was put forward as having demonstrated the same laws by geometrical and

not physical reasons (*rationes*) and nevertheless as having given Astronomy to Physics. He did not in any way introduce the force of gravitation, which he wanted to be the same as the centripetal or attractive force, into this part of physics—for all physicists before him established that the relation of the planets to the sun is a true relation (*ratio*), i.e., a real and physical force—but he compared the *quantity* of the force of gravity, which experience reveals in the bodies making up a part of our earth, with the *quantity* of the celestial movements, and he carried all this out by means of mathematical proportions (*rationes*) through geometry and computation.[4] With respect to such a conjunction of physics and mathematics, one must above all be warned to be careful not to confuse the purely mathematical relations with the physical ones (*rationes*), thinking rashly that the lines used by geometry to construct demonstrations of its theorems are forces or directions of forces. To be sure, the whole of mathematics must not be considered as purely ideal or formal, but as at the same time real and physical. For the relations (*rationes*) among quantities exhibited by mathematics, precisely because they are reasons (*rationes*), are inherent in nature, and if they are understood, are laws of nature. But the analysis and the explication of the whole, which are removed from the perfection of nature, must be distinguished from the very reason (*ratio*) of the whole, for—in view of the fact that the geometrical part of mathematics abstracts from time, and the arithmetical part abstracts from space, the former [352–353]
constituting the geometrical whole by the principle of space only, the latter constituting the arithmetical whole by the principle of time alone—the relations (*rationes*) characteristic of the knowledge of formal wholes are separated from the true relations of nature, in which time and space are conjoined. As for higher geometry,[5] which brings together analytical calculation and geometry, and which is born from the very necessity of measuring the relations (*rationes*) of space and time taken together, it suppresses[6] this separation only negatively through the concept of the infinite, does not offer a true synthesis of the two, and, in its negotiation, does not at all steer away from the formal method of geometry and arithmetic. For that reason let us not confuse what belongs to the proper and formal relations (*rationes*) of mathematical knowledge with physical relations (*rationes*) by attributing a physical reality to those relations the reality of which is only mathematical.

To be sure, Newton not only gave the title "mathematical principles of natural philosophy"[7] to a most illustrious work in which he described the laws of motion and gave an example of these in the system of the world, but he warned repeatedly that "I . . . use the words Attraction, Impulse or Propension of any sort towards a center promiscuously, and indifferently, one for another; considering those forces not physically, but mathematically: wherefore the reader is not to imagine that by those words I anywhere take upon me to define the kind, or the manner of any action, the causes or the physical reason thereof, or that I attribute forces, in a true and physical sense, to certain centers (which are only mathematical points); when at any time I happen to speak of centers as attracting, or as endued with attractive powers."[8] Now, what concept Newton had of Physics is obvious from this alone that "perhaps these attractions, physically speaking, may more truly be called impulses."[9] As for us, we esteem that impulses pertain to mechanics and not to true physics—about the difference of these two sciences more will be said later. For now we warn that, if he wished to develop mathematical relations (*rationes*), it is to be seen with astonishment that he used the word 'force' at all; for the [354–355] quantities of the phenomenon pertain to mathematics, but the knowledge of force belongs to physics. In actuality, believing that he everywhere defined proportions (*proportiones*) of forces, he erected a composite edifice out of mathematics and physics, in which it is difficult to separate what belongs to physical science and what truly has accrued to that science.

As for Kepler, he knew that gravitation is a common quality of bodies, that the attraction of the moon is the cause of the ebb and flow of the ocean, and that irregularities of the lunar motion originate in the conjoined forces of the sun and the earth; and if he, who was endowed with a rather pure love and sensibility for philosophy and the sciences, had been able to bear the confusion which, as we shall see, arises from the positing (*positio*) of the gravitational, centripetal and centrifugal forces, it would have been very easy for him to provide the pure and mathematical expression of the immortal laws discovered by him with a physical shape.[10] Indeed, the law which he gave (i.e., that the areas measured by the vector radii of the bodies in circular motion are proportional to the times) he would have been able to transmute into the form (*species*) of a physical law (i.e., that gravity is in proportion (*in ratione*) to the

arcs belonging to equal sectors); and since the total surfaces of the circles A and a are as the squares of the radii, $1/A : 1/a$ will be as $r^2 : R^2$. Since $1/A$ and $1/a$ express the quantity of motion, and, if you wish, the quantity of the centripetal force, he could have said that the force of gravitation or centripetal force stands in inverse ratio to the radii, or distances. However, he who wants to have instead of a true demonstration, the demonstration given by Newton of the proposition that "the areas, which revolving bodies describe by radii drawn to an immovable centre of force . . . are proportional to the times . . .,"[11] is not to be envied that complacency. For that demonstration results in the arcs', as much as the areas', being proportional to the times; but it had to be shown that the areas [356–357] only, and not at all the arcs, are proportional to the times.

I think that the famous decomposition of forces, bereft as it is of physical meaning in most cases, must be counted among the things that are important in mathematical proofs. For since the mechanical direction of motion can really arise from opposed directions of several forces, not only does it not follow from that that the direction of the living force (*vis viva*) arises from opposed forces, but such a mechanical relation (*ratio*), in accordance with which a body would be pushed by forces alien to it, must be considered entirely alien to the living force (*vis viva*). But when Newton, who dissects into parts the light which nature wanted (*voluit*) to be simple, similarly decomposes other simple forces and calls forces the lines used in the construction of the theorems bearing on the quantities of these simple forces, the physicists justifiably wonder how through the mathematical manipulation of the phenomenon there arises such a multiplicity of forces, which nature ignores. When nearly the entire science of mechanics and astronomy rests upon this decomposition and on the parallelogram of forces constructed from it, the very scope of the science, perfected in itself and in agreement with the phenomena of nature, seems to prove that hypothesis in such a way that its principle gains the greatest credit, since its use appears to be manifold—although, when considered in itself, it lacks in plausible reason (*ratio*). Subsequently, we shall see the true reason (*ratio*) why what a certain force brings about must be brought to view through a square, and why all quantities referring to the force must be brought to view through relations (*rationes*) that follow from the construction of the square. At this point, let it suffice to note that the decomposition into other

lines, of a simple phenomenon exhibited by a straight or curved line is a mathematical postulate, which commends itself abundantly to mathematics by its manifold advantages, but the principle of which depends on another science; that one must not make a judgment about the principle on the basis of its use and consequences and that no physical meaning, on account of mathematical advantages [358–359] alone, must be attributed to the lines into which the direction of a force exhibited by a line is decomposed in virtue of this postulate.

But no other origin of centripetal force—insofar as it is distinguished from gravity—and of centrifugal force is manifest than the one drawn out of the decomposition of the direction of motion into mathematical lines. Indeed, an infinitely small part of a circle is inscribed in a parallelogram in such a way that it forms the diagonal of the parallelogram, and that the lines of the sides of the parallelogram are on the one hand the tangent and the chord or sine, the chord being equal to the tangent in the last ratio (*ultima ratio*), and on the other hand, the versed sine[12] and the secant, which is equal to the versed sine in the last ratio. To these lines a physical reality is attributed so that the efficacy of centrifugal force is posited in place of the tangent and the efficacy of centripetal force instead of the versed sine. We shall look firstly at the reality of the centrifugal force.

Assuredly, it is manifest that the geometrical necessity of a tangent line in no way produces the necessity of a physical tangential force. Pure geometry does not modify the true form of the circle; it does not compare the circumference with the radius; nor does it know the circumference via the radius; rather it compares and knows the lines determined by the relation of the circumference to the radius. On the other hand, the geometry that attempts to subject the circle to computation, and to express numerically the relation of the circumference to the radius, seeks refuge in the hypothesis of a regular polygon of infinite sides, in such a way, however, that it suppresses at the same time this very polygon and the straight lines by means of the concepts of infinite and of last ratio. What is there to say if the geometry from which the decomposition of the circle into a multiplicity of straight lines originates, simply treats this concept as a hypothesis, the straight lines vanishing as the parallelogram is reduced to infinite smallness, and how could the physical reality of such lines proceed from that geometry?

If now, no reason (*ratio*) of geometry having been educed, we inquire into the physical reality of centrifugal force, let us not strive after a philosophical construction of centrifugal force by that experimental philosophy that Newton or rather the whole of England always considered to be the best—better yet, which they always considered to be the one and only philosophy. They can and [360–361] want to confirm the hypothesis of such a force by experience alone. However, nothing is more distressing than the examples by which they undertake to accomplish this. Newton and his followers give especially the example of the stone that "whirled about in a sling, endeavors to recede from the hand that turns it; and by that endeavor, distends the sling . . . and as soon as it is let go, flies away."[13] Then they illustrate centrifugal force with another example, that of a leaden ball which "projected from the top of a mountain by the force of gunpowder, with a given velocity, and in a direction parallel to the horizon, is carried in a curved line to the distance of two miles before it falls to the ground . . .; and by increasing the velocity, we may at pleasure increase the distance to which it might be projected, and diminish the curvature of the line which it might describe, till at last it should fall at the distance of 10, 30 or 90 degrees . . . or lastly, so that it might never fall to the earth, but go forwards into the celestial spaces, and proceed in its motion *in infinitum*."[14] The last example presents the concept of rectilineal motion, which everyone can imagine without example. Both examples draw this concept from the action of projecting from which one accedes to this concept by the shortest path by defining centrifugal force as the force that projects a body in a straight line; but neither example shows even a trace of such a force in nature.

It may be, however, that philosophy *a priori* deduces what the experimental method, which calls itself philosophy, undertakes to know falsely and with unfelicitous success from experiments, seeking as it does with blind zeal and by means of the senses the simulacrum of the true concepts of philosophy. It must be assumed that the opposition between the attractive and repulsive forces offered itself to the gaze of this ignorant philosophy and was applied to its theory of motion. But in reality philosophy attributes this difference of forces to matter in such a way that it makes gravity, or identity itself, the condition of those forces. How far the construction of the movement of the planets is from that reason (*ratio*) is obvious from the fact that centrifugal force, which is [362–363]

directed in a rectilineal motion, is attributed to the other body without the central body's being at all taken into account. Consequently, there cannot be any principle of the union of these two forces, nor—since they have the character of contradictory opposites—can it be explained why they are not opposed along a straight line, but along an angle which splits the straight line of their opposition into two. But it is undeniable that these forces, because they lack a common principle, are purely ideal and in no way physical. Therefore, when this experimental philosophy attempts to construct the phenomenon out of forces that have exactly nothing in common and are alien to each other, it may not appeal to the opposed forces of true philosophy. Indeed, the relation (*ratio*) of the latter forces is completely different. True philosophy repudiates the principle of experimental philosophy, this principle being sought in a mechanics imitating nature on the level of dead matter and effecting a synthesis of absolutely different forces in any body. Now what pertains to the imitation of nature must be entirely cast aside in the cognition of nature itself, and in physics room must be given to neither chance nor whim. The sun, the planets and the comets will have to be said to have come together without any necessity, but by pure chance, if their motion is explained by the relation (*ratio*) of centripetal and centrifugal forces.

Although physics has drawn the concepts of force tending toward a center and of tangential force, from a geometrical relation (*ratio*), this method of constructing the phenomenon out of absolute opposites is in no way to be considered the geometrical method. Geometry does not attempt to construct the circle or another curve with lines joining under a right angle or some other angle. Rather it considers the circle or curve in question as given, and from those givens it makes visible the determinate relations (*rationes*) of the other lines. Physical science ought to imitate perfectly this true method of positing the whole and of deducing from it the relations (*rationes*) among the parts, but not at all the method of composing [364–365
the whole out of opposed forces, i.e., out of parts. Moreover, how could it happen that this physical astronomy, which arrives at its laws by means of Mathematics, does not truly follow Mathematics? In reality, when it believes that it speaks about centrifugal force, centripetal force or gravitation, it always speaks about the whole phenomenon. Not only geometry, which states that a certain line is equal to the root of the sum of two squares, speaks not of any

singular line but of a hypothenuse, i.e., of a part determined by the Whole, which is a rectangular triangle, thus distinguishing this part from the Whole as well as from the other parts; but also one and the same phenomenon of whole motion is determined by the quantity of centripetal, centrifugal and gravitational force, in such a way that it makes no difference whether one solves some problem by taking one's bearings by the quantity of gravitation, of centripetal force, or of centrifugal force—and that these distinct forces are names with which it would be more appropriate to dispense. From the emptiness of such a distinction arises all the confusion and perplexity encountered in the explanation of phenomena. In this, one will grasp a manifest contradiction: in the fact that the effect of centripetal force is brought to view by the versed sine, and the effect of the centrifugal force by the tangent, while at the same time each of the two forces is said to be equal to the other. To suppress this contradiction, one cannot have recourse to the first ratio (*prima ratio*) of nascent quantities and the last ratio (*ultima ratio*) of evanescent quantities, in which the relation (*ratio*) among the arc, the versed sine and the tangent would be a relation of equality such that those lines could each be used instead of the other. For the first and last ratio is only a relation of equality if it is null, if there is a place for neither the arc, nor the versed sine, nor the tangent, nor the difference of the forces under discussion: centrifugal force is precisely only equal to centripetal force when the quantity of the total motion is in reality expressed by the quantity of the one or of the other, and when the relation (*ratio*) between those forces, their difference and their names have become empty.

Therefore concerning the vacuity of this difference, it is first of all [366–367]
acknowledged that centripetal force is the same as gravitation—and Newton was the only one to put so much effort in obtaining their identity. Hence, the physical construction of the phenomenon of the motion of celestial bodies, which attributes the entire phenomenon to gravitation and posits two factors in gravitation, the centripetal and centrifugal forces, amounts to nothing; for one of the two factors is posited to be equal to the total force. Secondly, given that the law of centripetal force, according to which it is in inverse ratio (*ratio inversa*) to the distances, demands that the total quantity of motion stand in the same ratio, it includes and at the same time contains the tangential direction imputed to centrifugal force; for it is established that circular motion is not the effect of

the sole propension toward a center, but is composed of the center oriented and tangential directions. Now since the total quantity of motion is attributed to centripetal force and determined by its quantity, it becomes obvious that this force is not opposed to centrifugal force but that the entire phenomenon is expressed by it. Because of this, in the geometrical construction the effect of centripetal force is exhibited by the surface of the whole triangle, one of the factors of which is the tangential line, or by the sector. To what extent, however, it is necessary that in a mathematical relation (*ratio*) one force be posited as equal to the other, or as truly the whole, is obvious from the fact that the total quantity of opposing forces must not only be measured by what one of the forces actually accomplishes, but also by the effect it would have produced had it not been impeded by the opposing force; and that in reckoning one must add to each force the effect brought about by the other. As such, the true quantity of centripetal force must be exhibited not only by the versed sine, but also by the tangent—or by the product of the two, the diagonal line—just as the true quantity of the centrifugal force must be exhibited not by the tangent line alone, but also by the versed sine—or by the product of the two. From that one may then establish that centrifugal force stands in inverse ratio to the distances, and whether one attributes the phenomenon to centripetal or centrifugal force, the solution to any problem will always be the same. [368–369

From the law according to which each of the two forces is in inverse ratio to the distances, it is manifest that the forces do not stand in such an opposition to each other as mechanical physics required for its construction of the phenomenon of motion. For of the two opposed forces, one diminishes when the other increases. However, since the versed sine and the tangent augment or decrease simultaneously, we understand firstly that the total phenomenon is described and determined by one force alone or by the other; and secondly that these forces depend on a third one, which is their true principle and their identity; or rather that neither centripetal nor centrifugal force is defined, and that the phenomenon is not constructed out of those factors, but that the quantity of the entire phenomenon of motion is posited.

How bereft of true sense the opposition between centripetal force and centrifugal force and its exhibition by the versed sine and the tangent are, can be seen especially in the explanation of the change

of velocity of one and the same body revolving in an ellipse. Since in an ellipse the relation (*ratio*) between the arrow exhibiting centripetal force and the tangent making visible centrifugal force is not the same everywhere, the change of velocity is usually explained by a disturbance in the equilibrium of the forces. At both middle points the same relation (*ratio*) between arrow and tangent obtains, as well as the same velocity. At the Aphelion and the Perihelion, on the contrary, the relation between arrow and tangent is the same, whereas the velocities differ the most.[15] In this respect, you might especially wonder at the fact that although all rests upon mathematical proofs, some contend, as we saw above, that centrifugal force stands in inverse ratio to the square of the distances, and others even claim that it stands in inverse ratio to the cube.

In this method for the explanation of the diverse velocities of both one and the same planet and all bodies turning in an orbit, there becomes known the reason for the empirical, which reason is always in agreement with itself (*illa empiriae sibi semper constans ratio*) and itself turns in a circle: the different velocities of the [370–371]
planets are indeed known on the basis of the difference in intensity of the forces, whereas the various intensities of the forces are known on the ground of the different velocities.

Let us also note another quite illustrious use to which centrifugal force is put. Indeed, by means of it one usually explains the phenomenon of the greater slowness of the pendulum in the lower geographical latitudes—and this philosophy[16] knows that gravity is lesser there. The explanation of this phenomenon by means of a gravity lesser at the equator and increasing as the square of the sine of the latitude, is given in such a way that centripetal force at the equator is said to be not equal to gravity, but smaller by 1/289, which part is imputed to centrifugal force. Now that part is found as follows: the arc described in one second by a body revolving uniformly in a circle in a day of 23 hours, 56′ and 4″, and at a distance of 16, 695, 539 feet from the center, measures 1436.2 feet, and its versed sine amounts to 0.0523 feet or 7.54 lines. But as on our earth, at the latitude of Paris the fall of a body in one second covers about 15.5 feet or 2174 lines,[17] and as the centripetal force is known by the distance covered by a falling body in a given time and expressed by the versed sine, the difference between the former versed sine and the latter one will be such that the former is

the 1/289 part of the latter: the former is attributed to centrifugal force, which elsewhere we saw expressed by the tangent line.[18] But as we realized earlier that one force may be substituted for the other at our convenience, and that the forces may be used interchangeably without modifying the laws, nothing prevents us from taking the smaller versed sine as the efficacy of the centripetal force, from adding to it gravity, and from saying that gravity augmented and not diminished by that amount is the cause of the slowing of the pendulum at the equator and that the weights of bodies increase and do not decrease in the lower latitudes. Measuring and explaining the phenomenon are carried out just as well in this manner. As experience teaches that the oscillation of the pendulum clock moves more slowly in the lower latitudes, and as the oscillations derive from the gravity that causes the fall of bodies, [372–373
they [19] want gravity to be smaller (*minorem esse gravitatem . . . volunt*) on account of the motion of the pendulum of same length and weight's being slowed down. However, the motion of a pendular body is not a simple fall: lest the fall immediately produce a straight line, the weight is impeded; it is suspended and thrown not from the point of suspension but from the side. As such, the vertical direction of the line changes into a curved line, arising if you wish, from the centripetal and centrifugal forces, by which we say the horizontal or tangential direction is produced. Why, then, do we not explain the slowed oscillations at the equator by the fact that impeding the difference produced by the vertical line of fall, or impeding the horizontal motion, and if you wish, impeding the centrifugal force, there stands a bigger obstacle, which must be posited in nothing else but in the stronger propensity for the vertical line, i.e., in a greater centripetal force at lower latitudes, which binds itself to the vertical line by a stronger effort, restores the suppressed vertical line, and overcomes more swiftly the direction contrary to it? Lastly, let us say that those things agree remarkably well with the figure of an earth wider elsewhere than at the equator, the diameter of which is shorter than the axis.[20] Consequently, as the pendulum suspended at a lower latitude is closer to a larger mass, it undergoes a stronger attraction, and tends with a greater weight toward the earth and toward the vertical line; and it cannot diverge from the vertical line as easily as a body attracted by a smaller mass at higher parallels assumes a lateral motion.

It would take too long to discuss the distinction made by Newton between motive and accelerative force, with which he seems to conceal, among other things, the fact that in the most celebrated application of the law of centrifugal force to the motion of the moon, the planets and their satellites he does not take into account their masses. As such, it is manifest that this law of gravitation is a mere [374–375] law of the phenomenon of motion, and not a law of force, for it is necessary that the effect of a force depend not only on the law of force, but also on mass; and the phenomena cannot agree with the law of force alone. Others, it is true, when they have to explain the comparison of the law with lunar motion, use the relation (*ratio*) of the lunar mass to the terrestrial mass. Then, they conjecture that the different masses of the planets do not change the law, which they want to pertain to force only, because when compared with the mass of the sun, the masses of the planets are very small. They think that the same relation (*ratio*) obtains among the satellites when compared with the planets around which they gyrate. However, they measure the density of a planet on the basis of the velocity of the satellites and on the basis of its relation (*ratio*) to the distance, just as they measure the density of the sun on the basis of the same relation to the planets.

In the same way as we have shown that centripetal force and centrifugal force can be substituted for each other in the explanation of phenomena, that a diminution in gravity can be replaced by an increase in gravity, and that the phenomena which are explained by a decreasing force of gravity can be explained by an augmenting force of gravity, similarly the law according to which the force of gravity is said to be in inverse ratio to the square (*in reciproca duplici ratione*) of the distances can be transposed in such a way that we say the force of gravity stands in direct ratio to the square (*in directa duplici ratione*) of the distances. Indeed, if gravity is said to diminish at a greater distance, only one factor is considered in the evaluation of gravity, i.e., velocity. As the velocity is lower at a greater distance, gravity is said to be lesser. But we must at the same time evaluate the magnitude of the force on the basis of the magnitude of the distance from which it acts; and of the force acting at a double distance, we must say that it is four times larger. Consequently, given the usual expression of the law of the force of gravity, if it is said that this force diminishes or augments on the basis of the considered magnitude of velocity

alone, the distance having not at all been applied to determining the relation (*ratio*) between augmentation and diminution, nor to ascertaining the concept itself of augmentation and diminution; then we could with equal right leave out of account velocity in [376–377]
ascertaining magnitude, saying that a force acting at a greater distance is greater and is in direct ratio (*in directa ratione*) to the distances. As is the case with the lever, the two factors of which are distance and weight in inverse ratio (*in inversa ratione*), gravity may be said arbitrarily to augment or to diminish when the distance increases. For at a greater distance, in order that there be equilibrium, the weight is less—that which Newton calls the motive force—and gravity is, therefore, less; or at a greater distance, gravity is greater, for the same weight at a greater distance belongs to a greater force.

From all these considerations, we first of all obtain the result that the distinction between centrifugal and centripetal forces is empty, but that the laws put forth as laws of centripetal and centrifugal forces, are in reality mathematical laws of motion, distorted by the physical appearance (*species*) of forces and by being called forces. It then follows that an increase or decrease is ill-attributed to the force of gravity, and that neither quantity, nor some quantitative relation (*ratio*) to any other thing—not even space and time—is suited to gravity itself. Of gravity we must say that it is one and the same thing which exists in the form of two factors, space and time, or even so to speak, in the form of quiescent space and space engendered by motion in time. Moreover, all quantitative difference and relation (*ratio*) belong to those factors, one of which augments while the other diminishes; and no relation (*ratio*) or proportion can obtain between the factors unless it obtains between factors posited within one and the same thing. And their absolute identity cannot vary, either augment or diminish. This shows how much purer the talent and natural inclination of Kepler were: he posited nothing but the ratio (*ratio*) of those factors that can truly increase and decrease and did not spoil the pure and truly celestial expression of these relations (*rationes*) by means of the determination of the quantities of gravity, of which there is no quantity. Nonetheless, the large accumulation of mathematics and [378–379]
the extended and fruitful use of Mathematics—in Astronomy especially—have made appealing to scholars the confusion initiated by Newton between mathematics and physics. People are pleased

with the force of gravity's being known; and, thoroughly instructed, above all by the rather distressing story of the apple that fell before Newton, that heavenly bodies revolve in their orbits less because of the force common to the world, which Kepler and other philosophers established as one and the same, than in virtue of the everyday force which pulls stones toward the earth, people derive assurance against the sky, forgetting of course that an apple was present at the beginning of the universal misfortune of human kind and at the inception of the misfortune of Troy, in turn a bad omen for the philosophical sciences.

Consequently, although one must deem that the science of astronomy, insofar as it pertains to Mathematics, owes much to Newton, the physical appearance (*species*), with which he clad the mathematical relations, must be separated from those relations, and what part of truth resides therein must be examined by philosophy. Now, of the experimental philosophy, which the English temperament, as well as Newton, Locke and others who expressed that temperament in their writings, alone understand, I shall adduce an example, which concerns our topic. In order to refute a theorem of Descartes, Aristotle and others, claiming that the weights of bodies depend on the forms of matter, and to prove that weights are not a function (*non in ratione*) of the form but of the quantity of matter, Newton performed the following experiments. Putting pairs of equal weights of gold, silver, sand and wheat, etc., into two identical containers so as to avoid the inequality of the air's resistance, he constructed pendula entirely alike in length, weight, shape and resistance to air. What is known by means of pendula with like shape, length and resistance to air? The equality of or the difference between weights. Therefore, as he had made the weights of the pendular bodies equal, he happily [380–381]
discovered that the weights of the bodies were equal, thinking that such experimenting and philosophizing refuted the philosophers who established that only different forms belong to one and the same matter. On the basis of this example, we understand that experimental philosophy altogether ignores what true philosophy wants for itself. With the help of the same principle the true source of centripetal and centrifugal force is also explained. In the science of mechanics, which is alien to the life of nature, there can be no other primitive concept of matter than death, which they call inertial force, i.e., indifference towards rest and motion. This matter

is nothing but the most abstract concept of the object or of the absolutely opposed. Hence, they externally add to matter all the diversity which they apprehend in it, even the one known through motion. Moreover, on the basis of experiments and induction they know that gravity is a quality of universal matter: according to the second rule of philosophizing stated by Newton, "the same causes characterize natural effects of the same genus," e.g., "the descent of stones in Europe and America"; and according to the third rule, "the qualities . . . proper to . . . bodies, upon which one may perform experiments, are to be esteemed the qualities of bodies in general."[21] Therefore, while experience teaches that matter has weight, and since it is patent that the proportion (*ratio*) of gravity in the stone falling to earth differs from the proportion of gravity in the heavenly bodies—first of all in the bodies belonging to the system of our sun and not falling to earth—they state that there is another cause of this phenomenon, i.e., centrifugal force. This experimental philosophy, which ignores nature as well as the origin of gravity and of the impulse toward an infinite horizontal line, which it affirms is centrifugal force—one must indeed permit this philosophy to attribute all things to God, but one must require that it philosophize correctly about God and the rationality of his actions (*ejusque agendi ratione*), and that, ignoring nature, it know God truly. Now God's action is neither external nor mechanical nor arbitrary nor fortuitous. One must therefore firmly hold that the forces that, according to experimental philosophy, God gave to matter, truly dwell in matter, that they constitute the [382–383
nature of matter, this nature being an immanent and internal principle of opposed forces. But in reality mechanics flees before that concept, understanding neither God nor true force nor what the internal and the necessary are, but repeating that inert matter is moved always by an external impulse, or what amounts to the same thing, by forces alien to matter itself. As mechanics deals with external causes, and does not conceive of nature by way of reason (*ratio*), it cannot reach the principle of identity, which posits difference within itself. Having finally returned to philosophy, this principle restored philosophy itself, separated mechanics from physics, and gave physics back to philosophy, a physics not distinguished from mechanics merely by the name of dynamics. Let us understand the elements of the system of the planets through this principle. We present this understanding here briefly.

II

Gravity constitutes matter in such a way that matter is objective gravity. One and the same matter sundering itself into two poles forms a line of cohesion and assumes diverse shapes (*species*) over a series of unfoldings because of the different relation (*ratio*)[22] of the factors. From that real difference in gravity, we shall distinguish a second one, the ideal difference, namely that of the potencies of time and space; for when a twofoldness has been posited, a double twofoldness, one of the poles, the other of the potencies, or four regions, must be posited.

Firstly, we shall speak of the line of cohesion. In constituting this line, gravity posits itself in all points, which are simultaneously diversified among themselves[23] in accordance with the reciprocal relation (*ratio*) of factors, and brings forth a series of nodes[24] and centers out of itself. Of course, each of these does not lack the other multiplicity of relations (*rationes*), but contains them subjected to the power of its proper principle, under its law and individual organization. The solar system, which expresses such a line, is larger than the other lines, for, the line of cohesion being here [384–385] fragmented, each body carries its center of gravity within itself—assuredly not with absolute power but with a power greater than that of other bodies. For there exists no body which, albeit a whole in itself, does not depend on others and is not a part and an organ of a larger system. That is why not perfect but maximal freedom and independence from gravity belong to the celestial bodies. Therefore it is not due to some chance occurrence that the planets, having wandered through infinite space along a rectilinear path and fortuitously passing in the vicinity of the sun, were forced under its law and thereby into circular orbits. Nor does the hypothesis of a centrifugal force keep them remote from the sun, but forming one originary system with the sun, they are both contained and kept separate by a true force of cohesion.

The center of forces is distinct from the point of indifference. When the point of indifference is expressed—as in the magnet and thus in the lever, which imitates the natural line of magnetism within dead matter—it turns out to be the median point. For as indifference is neutral, it exerts no force, to which the condition of difference belongs. That is why the centers of forces are posited

within the line, but not in its middle, and these are the bodies. For a body is nothing else but the phenomenon of a physical force, or of a true idea (*vera idea*). Assuredly, Newton thought that the center of gravity, i.e., of indifference, should for that reason not be posited[25] in the sun, since it is moved slightly from its location by the attractions of the planets. Indeed, since to explain the motion of the celestial bodies he supposes nothing but the mutual attractions of those bodies, by which hypothesis a center is not immediately posited, he cannot succeed in demonstrating the propositions concerning curvilinear motion without positing a center of orbits. In section XI of book I, in which he broaches "the motions of bodies tending to each other with centripetal forces,"[26] he supposes that [386–387
the actions of the attracted and attracting bodies are reciprocal, so that none may stay at rest, but he also supposes that "both bodies through that reciprocal attraction revolve *as if* about a common center of gravity";[27] and he appeals to the fourth corollary of the Laws, where one discovers only that the common center of gravity of two or several bodies does not change its state of motion or rest as a result of the reciprocal actions of the bodies upon one another, and where a necessity for a true and real center or for a central body is in no way to be found. This common center of gravity is thus a purely mathematical point, and that the sun is the center of forces or is very close to that center must be attributed not to necessity, but to the chance occurrence which endowed it with the largest mass. The immensity of the solar mass, a concept to which that of density belongs, in turn rests upon the hypothesis that every force depends upon mass. However, physical philosophy teaches that the true center of forces is necessarily the source of light, and that the true force and virtue of the sun must be posited in that source. We have said that the center of forces is not posited[28] in the middle. For just as two external poles are constituted through a line of cohesion, so also two internal centers of forces are constituted. We know this internal duality in the culminating points of magnets and in the foci of the ellipse, the main axis of which is the true line of magnetism.[29] Those culminating points are disposed in such a way that each of them lies closer to the opposite pole than to the pole upon which it exerts its force, i.e., that therefore the internal pole $+M$ lies between the point of indifference and the external pole $-M$, and, similarly, that the internal pole $-M$ lies between the middle point and the external pole $+M$. However, since the system of the planets is a broken line of cohesion and does not form one

continuous body, and since as we shall see, one and the same body effects both poles, there is only one real culminating point of forces, namely the sun at one of the foci of the ellipse, the other focus of which is bereft of light and purely mathematical. Thus the line of [388–389]
natural magnetism turns into[30] the form of the natural pendulum, as the mechanical pendulum effects an incomplete lever—having lost its other pole, which a suspended body succumbing to gravity cannot engender. This rectilinear and virtual (*virtualis*), but not rigid, series of bodies having been posited as the basis of the total system, we notice that the bodies are referred to each other, and that although they form a system and not one body, they do not satisfy nature, which wants the force existing here in the form of a line to take the form of a body.

The real difference between the poles and the line of cohesion once understood, we now turn to the other difference, the ideal one or difference between potencies, between subject and object. If matter is so conceived that space is full, it lacks form, and space and matter are nothing but the abstract concept of objectivity (*objectivum*); in order that the physical and real concept of matter be intelligible, it must also be posited in the form of subjectivity, and the point must be posited in space, a point which is indeed an abstraction from space, but in such a way that at the same time it refers to space. In the concept of matter as filled and, so to speak, dense and therefore quiescent space, is admittedly contained the concept of a resistance to other matter intruding into the same place, but that concept is purely negative and empty. For once space is filled, any principle of change and of resistance is suppressed and must therefore be sought elsewhere. In order that real matter may be intelligible to us, there must be added to the abstract concept of space, the contrary form, that of subjectivity, which we shall designate by the more Latin word '*mens*', and by the word 'point' if subjectivity is referred to space. In this way the point—or, under the form proper to self-differentiation, time—constitutes along with space the elements of matter, which does not result from the combination of those elements, but is their principle. Through the internal and primitive identity and difference of the opposed potencies of coming into being and withering away[31]—for the poles are at rest—the necessity of change and motion becomes intelligi- [390–391]
ble. Change is indeed nothing other than the eternal restitution of identity out of difference, and the renewed production of differ-

ence—contraction and expansion. Now the other potency, *mens*, which produces itself in abstraction from space, is time, and insofar as it refers this production of itself to space it constitutes the line. This line is the *mens* producing itself—but in a subjective form and shut within itself. *Mens* assumes a form perfect and natural for it in that it turns into[32] its contrary, space, and constitutes the plane, which, because we posited no difference other than that between *mens* and extension, lacks all other difference and is square.

Reflection seems to be alien to this transition of time into space, abstracting as it does from things themselves in mathematics, and deeming as it does to compare the numbers and measures of things but not the incommensurable things themselves; time and space appear to be such incommensurables to reflection. However, although geometry and computation are forgetful of the things themselves, and manipulate only lines and numbers, which are discovered by operations of computation or by geometrical demonstrations, lines and numbers are given a meaning pertaining to the things themselves, so that it is patent that not only quantities but the things themselves have been compared. Moreover, mathematics makes use of this reciprocal transition of incommensurables into each other under another form: it extends the line into the plane and the plane into the body. Generally, it conceals this identity of incommensurables by the word 'infinite', stating that the plane consists of innumerable lines, etc. Furthermore, expressing the relations (*rationes*) of many numbers by infinite series, it acknowledges that it has exceeded the absolute diversity of reflection and compared incommensurables. Notably, however, the geometry called higher geometry reduces the plane to the line, and both to the infinitely small, i.e., to the point; while analysis constructs the line—even the infinite line—out of points. But how the line arises out of the point, and the plane out of the line, and so forth, is not conceived otherwise than by calling upon the concept of motion, i.e., after space and time have first been identically posited. We have seen that the line is *mens* producing [392–393]
itself in the subjective form appropriate to it and that its transition into its truly objective shape (*species*) is the square. The product, on the other hand, which pertains to *natura naturata*,[33] is the cube. If we abstract completely from *mens*, there are indeed three dimensions of space producing itself. The body which is becoming

is square, the body which is, is cube. As the relation (*ratio*) of bodies separated from each other is the line, i.e., the subjective relation lacking an objective form, when they suppress this difference and so constitute themselves into a single body by falling upon each other, they change the line into a square. That is why the law of fall is the relation (*ratio*) of the square of the distance, or of the line changed into a square.

But in all this there is room for another difference: either the difference of the two bodies is really suppressed or it subsists; in other words, out of the two bodies there arises one body (*unum corpus*) that is either real or ideal. The real body is effected by free fall, the ideal one by circular motion. In the case of fall, the element of the square is simply exhibited by the sum of the units of time, or by a line divided in accordance with an arbitrary measure and expressed by numbers. In the case of circular motion, however, which produces an ideal body, the difference between the bodies, and thus in one respect between time and space, remains. Time brings about periodic time, while space brings about the distance between bodies. Now, periodic time must be brought together with the space that the body covers and that forms an angle with the space of the distance. This synthesis, which brings about the quantity of motion, is the square itself. There are therefore two elements in what is called the matter of motion, which expresses the whole relation (*ratio*) between two bodies moving about each other: namely, the line of the distance and the square of the motion. Hence the quantity of the whole, which is formed by the union of these two elements, will be the cube or body. Since gravity is always one and the same, the cube of all the planets will be the same whenever we speak about them. From that one easily produces Kepler's very famous law.

From what we have taught the philosophical lemmas of mathematics must be drawn, and from there too must be derived the demonstrations of the theorems, which found nearly all of [394–395] applied mathematics and which up to our time have been lacking true demonstrations, which cannot be provided mathematically. We wanted to make an attempt in this direction through the concepts we have unfolded. The common decomposition of forces, the mathematical truth and necessity of which are postulated and bereft of physical truth, rests on this exposition of the synthesis of space and time and on the transition of the mind or the line into the

square. From there an easy path is open to the laws of mechanics, which transfers the physical laws to dead matter. But the laws must be sought in nature—not in mechanics, which imitates nature. We now return to our topic.

The relation (*ratio*) among the distances of the celestial bodies,[34] of which we shall soon speak, is therefore determined by the line of cohesion. Separated from each other, their masses form centers of density opposed to the rarity of the ether, points of extreme contraction opposed to extreme expansion. Hence physicists ascribe absolute elasticity and repulsive force to the ether, while attributing attractive force to bodies, to which alone they refer the force of gravity, and not to the ether. The primitive identity of nature strives to suppress this opposition between extreme density and extreme rarity, and the phenomenon of this opposition, that is, the separation among bodies. Now, the virtual (*virtualis*) line strives to turn into[35] the square, and to assume a form and a body. This striving is the phenomenon of motion. Since nature wanted the system of celestial bodies not to coagulate into a single mass, nor to fall into the sad state of a *natura naturata* and share in the lot of its bodies, but rather wanted that system to be a living expression of reason (*ratio*) and an image of reason (*ejusque imaginem*), curvilinear motion does not produce a real body, but an ideal one, i.e., a square. As such, the bodily shape (*corpus*) assumed by the line of the celestial bodies is nothing other than the space embraced by the bodies gyrating in their orbits. Consequently, if we wish to define circular motion by its opposite, we shall say that it is a suppressed body, or that it is the reduction of the body or the cube by the square, and that this concept expresses [396–397] Kepler's sublime law.

In the formal circle, the concept of equal distance from a point brings about the circumference. The primitive character of the circle consists in no diameter and no point of the circumference's surpassing the infinitely many other diameters and points. Hence, if only the difference of the bodies and not the effort of nature to bind them into a single body is posited, then it is impossible to arrive at motion by proceeding from the primitive line of cohesion. But if the circle could be constructed mechanically from the attractive force of the central body and the centrifugal force of the body revolving along its orbit, how could one arrive at the

preeminence of any diameter, to the line of cohesion and its culminating points, and to the ellipse?

Although in the solar system the bodies are separated and the suppressed rigid line of cohesion turns into motion,[36] the force of the line in no way loses itself in the indifference of the formal circle's diameters. But the line manifests its force in constituting itself in the axis of the orbits, and it manifests its polarity in the change of motion which is slowed down by one of poles and accelerated by the other. It slows down at the Aphelion, where the force of the culminating point, the sun, is greatest; it accelerates at the Perihelion, where the force is the least, but where the indwelling force (*insita vis*) of the body is greatest. The perturbations of planetary motion must be referred to this: they are formations the cohesion of which is weaker, transitory[37] and easily subjected to a first cohesion.

Lastly, as we opposed the ideal difference between the potencies to the real difference in magnetism, we must observe briefly that the real difference itself also exists as a double difference, that a real line is formed from the Occident to the Orient, and similarly the line of those bodies called Comets, which revolve in orbits with immense apsides because the Orient and the Occident stand under the law of the difference between potencies.

III

There remains for me to add certain remarks concerning the [398–399]
relation (*ratio*) among the distances of the planets. Those distances seem to be a matter of experience alone. But in truth the measure and number of nature cannot be alien to reason (*ratio*): the study and knowledge of the laws of nature rest on nothing other than our believing that nature has been formed by reason (*ratio*) and our being convinced of the identity of all laws of nature. When those who seek the laws on the basis of experience and induction chance upon the form (*species*) of a law, they acknowledge this identity between reason (*ratio*) and nature by the joy they experience in the face of the discovered law; and if other phenomena do not sufficiently agree with the law, they acknowledge it by the way in which they doubt the experiments and aspire to bring the two into harmony. Our subject, the relation (*ratio*) among the distances of

the planets, illustrates this point: since the distances among the planets present a certain relation (*ratio*) typical of an arithmetical progression, and since no planet in nature corresponds to the fifth member of the progression, a planet is thought to exist between Mars and Jupiter, and unknown to us, to wander through the celestial spaces; and it is sought after with zeal.

As this progression is arithmetical and does not follow the generation of numbers out of themselves—i.e., it does not follow the powers—it in no way pertains to philosophy. We know that the Pythagoreans extensively elaborated the philosophical relations (*rationes*) among numbers. This gives us license to adduce a series of numbers handed down to us and preserved in each of the two books called *Timaeus*. Granted, Timaeus does not relate these to the planets, but he estimates that the Demiurge conformed the Universe to their relation (*ratio*). The series of numbers is as follows: 1, 2, 3, 4, 9, 16, 27. I may posit 16 instead of the 8 that we find in the text. If this series is an order of nature truer than the arithmetical progression, then it is manifest that there is a large space between the fourth and the fifth positions and that no planet is lacking there.[38] [400–401]

Now, to convey briefly the rest of the matter, you will discover that the cube roots of the fourth powers of these numbers (not to omit the unit, let it be posited as $\sqrt[3]{3}$) are the relations (*rationes*) among the distances of the planets:[39]

1.4 2.56 4.37 6.34 18.75 40.34 81.

You also see that the satellites of Jupiter are distant from one another in the same relation (*ratio*) as the progression of the first four planets, except that the fourth satellite somewhat exceeds its own number.

Moreover, a different relation (*ratio*), which is quite worthy of notice, obtains among the satellites of Saturn: the periods of the first four stand in the relation (*ratio*) of the square roots of 1, 2, 4, 8 and their distances stand in the relation of the cube roots of the same numbers. And if you want the numbers themselves of the periodic times, you obtain $\sqrt{2^9}$, $\sqrt{2^{10}}$, $\sqrt{2^{11}}$, $\sqrt{2^{12}}$, $\sqrt{22}$, $\sqrt{32}$, $\sqrt{45}$,$\sqrt{64}$. The fifth satellite, as does the fifth planet, modifies the formal progression; and while the distances of the first four are as the cubic roots of 1, 2, 4, 8, i.e., 1, 1.26, 1.63, 2; $\sqrt[3]{8}$ pertains to the fourth, $\sqrt{8}$ pertains to the fifth or $\sqrt[3]{16.32}$, and the series of

cubes, the roots of which expresses the relation (*ratio*) among the distances, are:

$$1, 2, 2^2, 2^3, (2^4{:}2^5), 2^8, (2^{12}{:}2^{13})$$

namely: $\ldots\ldots\ldots\ldots\ldots\ldots 2^{(\frac{9}{2})} \ldots\ldots\ldots 2^{(\frac{25}{2})}$.

TRANSLATOR'S NOTES

Abbreviations of works referred to in the notes:

OPDG: G.W.F. Hegel, *Les Orbites des Planètes*, translated, with an introduction, notes and appendices, by François De Gandt (Paris: Librairie Philosophique J. Vrin, 1979).

OPL: G.W.F. Hegel, *Über die Planetenbahnen*, translated by Georg Lasson, in G.W.F. Hegel, *Sämtliche Werke*, Band I, *Erste Druckschriften*, ed. Georg Lasson (Leipzig: Felix Meiner Verlag, 1928), pp. 347–401. Lasson's rendition faces the Latin text.

OPN: G.W.F. Hegel, *Dissertatio Philosophica de Orbitis Planetarum. Philosophische Erörterung über die Planetenbahnen*, translated, with introduction and commentary, by Wolfgang Neuser (Weinheim: Acta humaniora d. VHC, 1986). As its title makes plain, this book contains the Latin text. Worthy of note is that the Latin text was typeset from a copy of the first edition in the possession of the *Staatsbibliothek Preussischer Kulturbesitz*, located in Berlin.

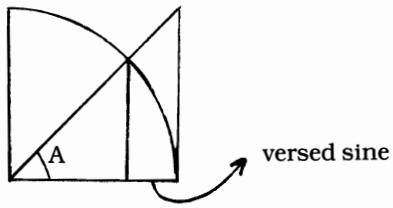
Both De Gandt's and Neuser's translations indicate the pagination of the Lasson edition. So does mine: the even numbers refer to the Latin, the odd ones to the German.

1. Firstly, a word on the rendition of '*species*'. I initially thought that I would translate this term by 'shape' throughout, for I thought that Hegel was using it as the Latin equivalent of '*Gestalt*'. In other words, I assumed that Hegel was consistently using '*species*' as a technical term meant to convey his interpretation of nature as a hierarchical system of interrelated shapes or figures that form the order of nature or *kosmos*—the pendant, so to speak, in the philosophy of nature, of the shapes or forms of consciousness or of *Geist*. This assumption, however, cannot be made: e.g., at *OPL*, pp. 376 and 378, there are two occurrences of the word and in both it is used pejoratively, namely as designating an appearance, a semblance. Although I reserved the English 'form' for the Latin '*forma*', I had to use 'form' to translate some occurrences of '*species*', i.e., at *OPL*, pp. 354, 392 and 398. As a result of this, I have always indicated the word '*species*' after its translations. On the understanding of nature as a system of shapes, see *OPDG*, pp. 61–70.

   Secondly, all words beginning with a capital in the translation are also capitalized in the original. This applies to the theses as well.
2. The term '*ratio*' occurs with great frequency in this text. It is, however, said in several ways. Depending on the context, it may mean either 'reason', or 'relation', or 'ratio', or 'proportion'. In view of this equivocity, I have indicated the Latin term in parentheses after its rendition, except in those places where it is

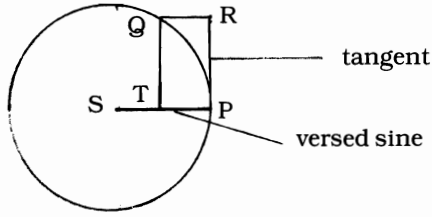
contextually obvious that it is being used. Unless I include a specific construction in which it occurs, the word is indicated in its nominative forms only.

3. Throughout the translation, 'concept' renders '*notio*'.
4. Here 'computation' renders '*calculus*'. It also renders the two occurrences of the word at *OPL*, p. 390. In those three cases, could Hegel have meant what is designated by the term 'calculus' in English, namely differential or integral calculus? Neuser thought so, for he translates by '*Differentialrechnung*' in these three cases. See also *OPL*, p. 352: '*analyticus calculus*', which I rendered by 'analytical computation'; and p. 358: '*circulum calculo subjicere*', which I translated by 'to subject the circle to computation'.
5. 'Higher geometry' renders '*sublimiore geometria*'. Later in the text, Hegel uses the expression '*altior geometria*': see *OPL*, p. 390.
6. The infinitive form of the verb rendered by 'suppresses' is '*tollere*', its principal parts being *tollo, sustuli, sublatum*. The English verb 'to sublate' stems from this verb. Both Lasson and Neuser use '*aufheben*' to render it. I must add that 'suppress' always renders '*tollere*', except in the very first sentence.
7. Hegel uses no quotation marks at all in his text. All quotation marks are mine. Nor does he capitalize the title of Newton's epoch-making work.
8. (Not only does Hegel not make use of quotation marks, he often does not identify texts by other authors. He never indicates any page numbers.) Isaac Newton, *Mathematical Principles of Natural Philosophy*, translated by Andrew Motte; translation revised by Florian Cajori (Berkeley: University of California Press, 1962), volume I, pp. 5–6 (Definition VIII). Hereinafter referred to as *Principles*. Isaac Newton, *Philosophiae Naturalis Principia Mathematica*, the third edition, with variant readings, assembled and edited by A. Koyré and I.B. Cohen, 2 volumes (Harvard: Harvard University Press, 1972), volume I, p. 46. Hereinafter referred to as *Principia*.
9. *Principles*, volume I, p. 164 (translation modified); *Principia*, volume I, p. 266.
10. Syntactically, it might have been more correct to render the last words of this sentence as follows: "to disguise the physical shape (*species*) of the immortal laws discovered by him with a pure and mathematical expression." That is the option taken by Wolfgang Neuser, OPN, pp. 87–88, line 32–line 2. It seems to me, however, that that rendition conflicts with the next sentence's words 'he would have been able to transmute into the form (*species*) of a physical law'.
11. *Principles*, volume I, p. 40 (Section II, Proposition I); *Principia*, volume I, p. 88.
12. The contemporary equivalent of versed sine is: $1 - \cos A$.



Newton's decomposition of the uniform circular trajectory of a body under the action of a central force is as follows. PQ is covered in a very small interval of time. The versed sine measures the deflection undergone by the inertial trajectory of the body. Had the body not had any inertial motion, the line

denoted by the versed sine would represent the trajectory of the body under the sole action of the central or centripetal force S. The tangent line PR is the geometrical representation of the path that would have been travelled by the body in the absence of centripetal force, i.e., under the sole action of the inertial velocity.



This is not to say that the tangent represents a path covered in the absence of force: the uniform rectilinear path is covered under the action of an internal (or indwelling or inherent) force (*vis insita*). More precisely, one must say that Newton uses the Latin term for force (*vis*) in two senses: in the sense that it has in the statement of Law II of *Principia* and in the sense which the word has in '*vis insita*' or '*vis inertiae*', these last two phrases having the same referent, as is made plain by Definition VIII of *Principia*. The force of Law II causes a change in a body's state (of motion or rest), produces a change in momentum, whereas the *vis insita* "is the power by which it [the body] endeavors to persevere in its state of being at rest or moving uniformly in a straight line, and is proportional to the quantity of the body," (my translation of a text from *De motu corporum*, a preparatory manuscript for *Principia*, quoted on page 67 of I.B. Cohen, *Introduction to Newton's 'Principia'* (Cambridge: Harvard University Press, 1978), in a section entitled "Newton's Use of 'Vis Insita'," in which the author discusses the translation of '*vis insita*' and presents the meanings of its participle). In other words, the internal force or force of inertia does not operate any change in the state of a body: on this twofold notion of force in Newton, see I.B. Cohen, *The Newtonian Revolution* (Cambridge: Cambridge University Press, 1980), pp. 190–193.

Hegel, however, shows himself incapable of understanding Newtonian inertia, since he consistently imputes to Newton the claim that the tangent represents centrifugal force. In this he is thoroughly mistaken, for Newton never says anything of the sort. To the extent that Hegel fails to appreciate the role, the nature and the theoretical import of the principle of inertia in Newtonian physics, he simply fails to understand the conception of natural motion instituted by the seventeenth century and which displaced the Aristotelian interpretation of motion. Hegel's urge to make motion intelligible seems to be greater than his ability to let Newton's texts speak. These texts have replaced the Aristotelian quest for making motion intelligible, with a mathematical treatment of change of state or inertial velocity, rest being a limiting case of inertial velocity. It follows that those portions of *De Orbitis Planetarum* which are based on the confusion between centrifugal force and inertia are vitiated, which is to say that the critique directed against Newton is ill-taken. The situation is worsened when one notes that the analysis of non-circular

trajectories—e.g., the ellipitical paths described by the planets—exhibits greater complexity and that Hegel does not grasp the differences between the analysis of a circular trajectory and that of any orbit. For example, in an ellipse, the line PS is not perpendicular to the tangent—except at the perihelion and the aphelion. In these cases, Newton does not use the expression '*sinus versus*' but the term '*sagitta*', 'arrow'. On this decisive question, see *OPDG*, pp. 71–86, 136 note 19, 145 note 39, 185–188 and the relevant section of André Doz's commentary on Hegel's treatment of the category of measure at the end of the Logic of Being in *The Science of Logic*, G. W. F. Hegel, *La théorie de la mesure*, translated and commented by André Doz (Paris: Presses Universitaires de France, 1970), pp. 178–190. For a very clear and stimulating presentation of Newton's three-tiered analysis of central forces in *Principia*, consult François De Gandt's article "The Geometrical Treatment of Central Forces in Newton's *Principia*" in this issue of *Graduate Faculty Philosophy Journal*.

In fairness to Hegel, one must say that his bungles concerning centrifugal force are not restricted to him, that they are tied to the confusing history of the notion of force from Huygens to Kant: see e.g., I.B. Cohen, *The Newtonian Revolution*, *op. cit.*, p. 82: "Apart from the fact that Huygens had not really got the message of the *Principia*, and still thought of an interplay or counterbalancing of centripetal and centrifugal forces rather than the action of a centripetal force on a body with inertial motion. . . ." One gets a first sense of this history in *OPDG*, pp. 185–188. To my knowledge, the discussion of this topic of Hegel's dissertation that is most generous to Hegel and most sensitive to his historical situation is Wolfgang Neuser's. Although Neuser's approach to all three parts of the work is most willing to understand it, it nearly never falls into apologetics, is always very knowledgeable and is forthright about Hegel's mistakes (down to Hegel's—interpolation?—mistakes in the computation of the series of numbers that are to express the distances of the planets from the sun, *OPN*, p. 51). At any rate, Neuser reads the first part of the dissertation in this way. He thinks and attempts to corroborate that the target that lies behind Hegel's criticisms of Newton is the eighteenth century version of Newtonian physics, i.e., the conceptual presentation of Newtonian physics at the hands of Martin, Maclaurin, De La Caille and D'Alembert: see *OPN*, pp. 5 and 7–23. The following statement of Neuser's sums up his interpretation fairly well, *ibid.*, p. 17: "Der Sinn der Hegelschen Kritik läge dann darin, dass er eine Inkonsistenz zwischen den Nachfolgern Newtons und Newton aufzeigt, die Unvereinbarkeit von dynamischem und statischem Konzept anspricht und eine willkürliche Aufsplitterung von Kräften kritisiert, deren physikalische Bedeutung im Hypothetischen bleibt, deren mathematische Nützlichkeit aber unbestritten ist." This way of reading Hegel's critique of Newton in *De Orbitis* has been indicated, albeit not pursued, by André Doz, *op. cit.*, p. 179: "et s'il [Hegel] vise Newton, il est difficile de savoir jusqu'à quel point il a su discerner ce qui revenait à Newton lui-même et ce qui revenait à certains de ses vulgarisateurs: toujours est-il qu'il ne s'est pas soucié de les dissocier."

13. *Principles*, volume I, pp. 2–3 (Definition V); *Principia*, volume I, p. 42.
14. *Principles*, volume I, p. 3 (Definition V); *Principia*, volume I, pp. 42–43.
15. In Latin, this sentence reads as follows: ". . . in Aphelio contra et in Perihelio eadem quidem est sagittae et tangentis, sed diversissima velocitas ratio." Wolfgang Neuser renders it in this way: "aber das Verhältnis von Brennstrahl

und Tangente ist im Aphel entgegensetzt und im Perihel gleich, aber das Verhältnis der Geschwindigkeiten ist völlig anders." He thus seems to be construing '*contra*' as an adjective. I know of only two uses of this word: an adverbial one and a prepositional one. I took it to be an adverb. As for '*velocitas ratio*', it is odd, because the text speaks of two velocities and of their one relation.

16. I.e., the experimental philosophy.
17. In this passage, Hegel employs French units of measurements no longer in use today, but prevalent in the physical treatises of his time (see *OPN*, pp. 52 and 154):
    1 toise = 6 (French) feet = 2.13 English yards = 1.95 meters
    1 foot = 144 lines
    1 foot = 0.3248394 meter.
18. The Latin here reads as follows: ". . . et vis centripeta per spatium, quod corpus in dato tempore cadendo percurrit, cognoscatur, et per sinum versum exponatur, inter priorem igitur et hunc sinum versum tanta sit differentia ut ille sit hujus pars 1/289: ille vi centrifugae tribuitur, quam alias per lineam tangentem exponi videmus."
19. Again, the experimental or natural philosophers.
20. The thesis of an earth that would be flatter at the equator was empirically confuted during the eighteenth century: by Maupertuis and Clairaut's surveying expedition to Lapland, and by La Condamine and Bouguer's to South America. The first one returned to France in 1738, the second one in 1742. The measurements brought back by Maupertuis confirmed Newton's prediction of the flattening of the earth at the poles. The earth is an oblate spheroid: it bulges at the equator and is flatter at the poles. The centripetal force exerted on an object at the poles is thus somewhat greater at the poles than at the equator. For the story of the testing of Newton's prediction in the eighteenth century, see T.B. Jones, *The Figure of the Earth* (Lawrence, Kansas: Coronado Press, 1967).
21. *Principles*, volume II, p. 398, translation modified; *Principia*, volume II, p. 550; "Regula II. Ideoque effectuum naturalium ejusdem generis eaedem assignandae sunt causae, quatenus fieri potest. . . . descensus lapidum in *Europa* et in *America* . . ." and p. 552: "Regula III. Qualitates corporum quae intendi et remitti nequeunt, quaeque corporibus omnibus competunt in quibus experimenta instituere licet, pro qualitatibus corporum universorum habendae sunt."
22. What does Hegel mean by '*ratio*' here? I grant that rendering it by 'relation' is somewhat vague. Should we understand it as 'proportion'?
23. 'Among themselves' renders '*in se invicem*'.
24. 'Node' is a technical term belonging to astronomy: it denotes the point at which the orbit of a planet or a comet intersects the plane of the ecliptic (the sun's apparent annual path). The expression 'nodal line' designates the line connecting the ascending and descending nodes. However, Neuser, *OPN*, p. 157, points out that the concept of nodal line has a more general sense in Hegel: it denotes any determinate series arising out of a continuum through qualified discreting.
25. The infinitive form of the verb rendered by 'posited' is '*ponere*'. I kept to this rendition throughout the text. In this particular sentence, one may feel that 'located' would have been a more fitting translation. It seems to me that the specific use of '*ponere*' in the next seven sentences justifies my decision.

26. The words in quotation marks render the title of section XI of book I of *Principles*, volume I, p. 164; *Principia*, volume I, p. 266.
27. *Principles*, volume I, p. 164, translation modified; *Principia*, volume I, p. 266. I had to change the Motte-Cajori translation so as to accommodate the modification that Hegel introduced into Newton's text. Here is how Newton's text differs from Hegel's version of it:
    Newton: "ambo . . . quasi attractione mutua, circum gravitatis centrum commune revolvantur . . ."
    Hegel's reworked version: "ambo autem attractione ista mutua quasi circum gravitatis commune centrum revolvi . . .".
    If we disregard the negligible additions of '*autem*' and '*ista*', and the change of the verb's mood, the important alteration consists in Hegel's having displaced '*quasi*'. Whereas Newton speaks of a quasi-attraction, Hegel makes him speak of a quasi-center.
28. See note 24.
29. See *OPN*, pp. 158–159, for a very helpful illustration of and commentary on these considerations on magnetism.
30. 'Turns into' renders '*transit*'. '*Transire*' means 'to go beyond', 'to go over'. The German rendition of '*transit*' would be '*geht über*', '*übergehen*' being precisely a verb often used by Hegel.
31. The initial words of this sentence read as follows: "Ex hac interna et primitiva oppositarum potentiarum orientis et occidentis . . . identitate et differentia. . . ." Neuser, *OPN*, p. 129, translates them in this way: "Aus der inneren und ursprünglichen Identität und Differenz der entgegengesetzten Potenzen des Entstehens und Vergehens. . . ." In English: "Through the internal and primitive identity and difference of the opposed potencies of coming into being and withering away. . . ." Neuser thus construes the two participles as having their ordinary verbal senses, and not as nominalized participles designating the cardinal points, as do Lasson (*OPL*, p. 389) and De Gandt (*OPDG*, p. 157). I have adopted his rendition. Later in the text Hegel does use these two words to designate cardinal points, see *OPL*, p. 396.
32. Same remark as in note 29.
33. I have refrained from translating this expression, for I can think of no elegant and brief translation that would do it justice. If there existed a verb such as 'to nature' in English, we might render this phrase by 'nature natured'. Spinoza uses it in *Ethica*: for example, at I, Prop. XXIX, Scholium.
34. In other words, the relation meant is the relation among the planets' distances *from the sun*. This phrasing occurs again in the third part.
35. 'To turn into' renders '*abire*', 'to go away', 'to leave', and figuratively 'to disappear', 'to leave a state for another one', 'to turn into'. When the verb is used in these last two senses, it occurs in conjunction with the preposition '*in*'. That is precisely the case here. Neuser translates it by '*übergehen*'.
36. The verb occurring here is the same as the one commented on in note 35. 'Turns into motion' translates '*in motum abeat*'. In German this would be rendered by '*in die Bewegung übergeht*', the literal translation of which is 'passes over into motion' or 'goes over into motion'.
37. 'Transitory' renders '*cito praetereuntis*', the literal sense of which is 'swiftly going beyond'. Neuser renders this phrase with '*schnell vorübergehenden*'.
38. This is in all likelihood the one sentence of Hegel's dissertation that brought

upon him most of the wrath, ridicule and charges of incompetence, sophistry, intellectual bankruptcy and of deriving empirical facts from concepts, prompted by this work (for some examples, see *OPN*, p. 1). Neuser reminds us of a grammatical fact that should have been obvious to all. The sentence in question is a hypothetical one: its consequent holds only on the assumption that its antecedent obtains (*ibid.*, p. 51). Neuser's thorough and provocative discussion of the third part of the dissertation is very much worth reading.

In appealing to such numerical considerations in his effort to display reason in natural things, Hegel had predecessors. His prior spiritual kins include Rheticus, Kepler, Francesco Sizi and Christiaan Huygens himself—in other words, they comprise some of the weightier executors of the modern mathematical projection of nature. Rheticus, Copernicus's disciple, accounted for his master's system by pointing out that its having six planets is due to six's being the first perfect number, i.e., to six's being the sum of its divisors (6 = 1 + 2 + 3). Kepler proposed another justification for the number of planets: in his view, God created the solar system in accordance with the order of the five nested perfect solids. Sizi was convinced that there were exactly seven planets. This convinction was founded on the presence of the number seven in certain natural wholes: for example, the seven openings in the head (ears, eyes, nostrils and mouth), the seven primary components of the alchemical genus of metals, etc. After Huygens discovered that Saturn had a satellite, he deemed that the solar system was complete and symmetrical, for the respective number of the primary and secondary planets was now six. In his creation, God had abided by the principle of perfect numbers. In this I am endebted to I.B. Cohen, *The Newtonian Revolution*, *op. cit.*, pp. 20–21. In the last note to this catalog of theologico-numerico-physical thoughts Cohen mentions the Titius-Bode law. That the Titius-Bode regularity was also explicitly embedded in a theological context for both Titius and Bode is shown in *OPN*, pp. 56–57.

39. In reference to the numbers making up this series, one must mention Neuser's suggestion, *ibid.*, p. 52: "Nun hat Hegel zwar einerseits eine mathematische Umrechnung zugrunde gelegt, andererseits aber gesagt, dass die Planetenabstände nur aus der Erfahrung zu nehmen sind. Dann wäre eine vernünftige Interpretation, dass die Umrechnung der Timaiosschen Zahlen nur ein Angleichen an ein anderes Masssystem bedeute und Hegels Zahlen eine Reihe absoluter Zahlen seien, deren Masseinheit Hegel verschwiegen habe und bei der jeder einzelne Wert einen empirischen Fehler enthalte. Diese Interpretation ist stimmig, wenn man als Masseinheit (10^8) 'Pariser Fuss' (1 Pariser Fuss = 0,32485m) unterstellt, eine Masseinheit, die Hegel in der Habilitationsschrift im übrigen in Anlehnung an Newton schon benutzt hat. Diese Masseinheit ist auch in den damaligen physikalischen Schriften durchhaus üblich. Vergleicht man darüber hinaus Hegels Werte mit den damals bekannten Abständen, so sind die Hegelschen Werte mit einer maximalen Abweichung von rund 8,3 % recht gut."